Computer algebra independent integration tests

7-Inverse-hyperbolic-functions/7.3-Inverse-hyperbolic-tangent/7.3.2-d-x-^m-a+b-arctanh-c-x^n-^p

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May 24, 2020 Compiled on May 24, 2020 at 6:20pm

Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [243]. This is test number [192].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 88.89 (216)	% 11.11 (27)
Mathematica	% 95.06 (231)	% 4.94 (12)
Maple	% 80.66 (196)	% 19.34 (47)
Maxima	% 43.62 (106)	% 56.38 (137)
Fricas	% 60.08 (146)	% 39.92 (97)
Sympy	% 30.45 (74)	% 69.55 (169)
Giac	% 51.03 (124)	% 48.97 (119)

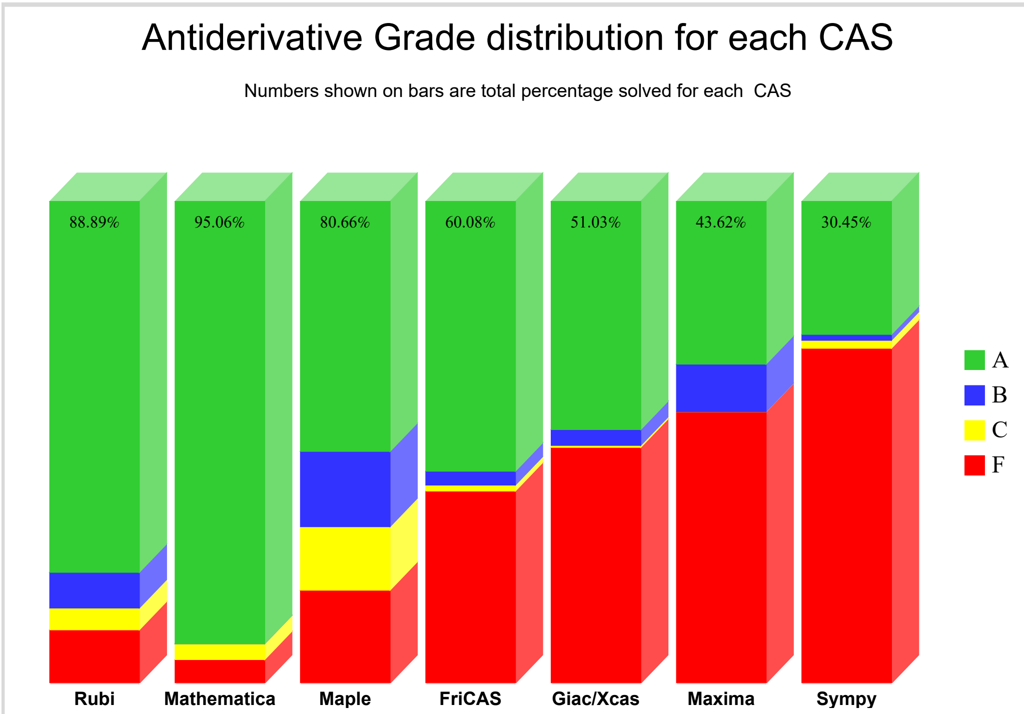
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

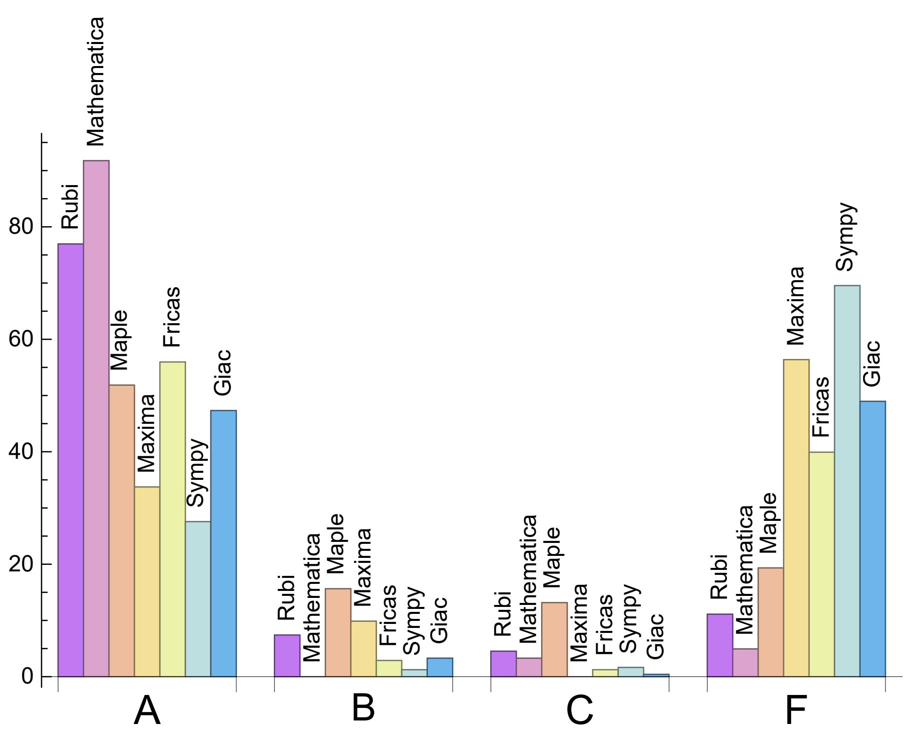
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	76.95	7.41	4.53	11.11
Mathematica	91.77	0.	3.29	4.94
Maple	51.85	15.64	13.17	19.34
Maxima	33.74	9.88	0.	56.38
Fricas	55.97	2.88	1.23	39.92
Sympy	27.57	1.23	1.65	69.55
Giac	47.33	3.29	0.41	48.97

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.43	199.6	1.33	74.	1.
Mathematica	0.47	121.66	1.07	99.	1.11
Maple	0.56	3909.02	39.16	81.	1.22
Maxima	1.04	177.82	1.77	76.5	1.51
Fricas	1.79	332.4	3.56	141.	2.92
Sympy	17.01	188.93	3.09	80.	1.47
Giac	1.27	119.34	1.41	90.	1.6

1.4 list of integrals that has no closed form antiderivative

{43, 44, 46, 47, 48, 49, 94, 95, 97, 98, 130, 131, 133, 134, 182, 183, 185, 186, 231, 232, 234, 235, 239, 240, 242, 243}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {64, 65, 66, 67, 69, 70, 77, 78, 116, 117, 118, 119, 121, 122, 123, 124, 125, 126, 143, 144, 145, 146, 148, 149, 155, 171, 172, 174, 175}

Mathematica {14, 16, 18, 20, 22, 24, 25, 26, 27, 28, 29, 31, 32, 33, 34, 65, 67, 69, 73, 74, 77, 78, 80, 81, 117, 119, 121, 123, 124, 125, 126, 128, 129, 144, 146, 147, 148, 150, 151, 152, 153, 154, 155, 156, 172, 173, 174, 178, 179, 202, 203, 204, 205, 207, 208}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in-

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

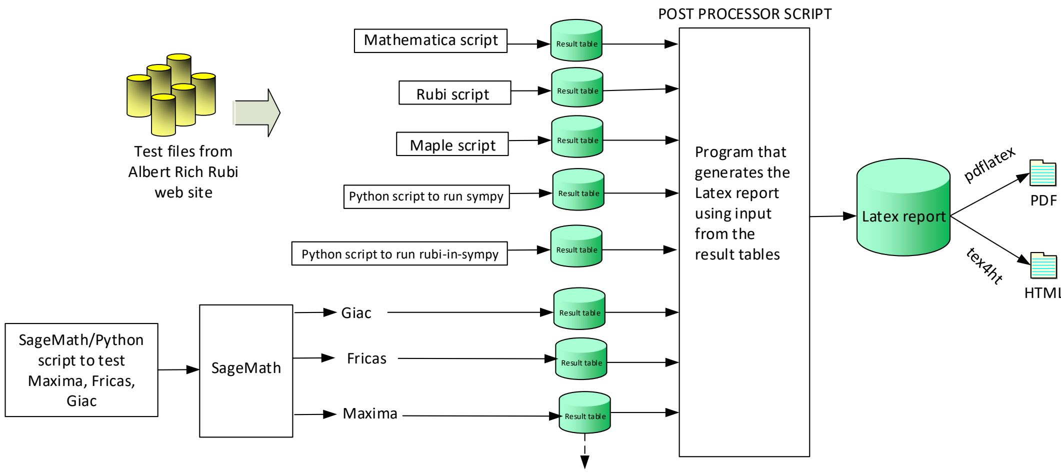
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

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June 22, 2018

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 68, 71, 72, 73, 74, 75, 76, 79, 82, 83, 84, 85, 86, 87, 88, 89, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 120, 127, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 147, 154, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 173, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 199, 206, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 222, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243 }

B grade: { 65, 67, 69, 77, 78, 117, 119, 121, 123, 124, 125, 126, 144, 146, 148, 155, 172, 174 }

C grade: { 64, 66, 70, 116, 118, 122, 143, 145, 149, 171, 175 }

F grade: { 80, 81, 90, 91, 92, 93, 128, 129, 150, 151, 152, 153, 156, 195, 196, 197, 198, 200, 201, 202, 203, 204, 205, 207, 208, 221, 223}

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 73, 74, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 88, 89, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 152, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 178, 179, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 228, 229, 230, 231, 232, 233, 234, 235, 237, 238, 239, 240, 241, 242, 243 }

B grade: { }

C grade: { 31, 33, 80, 128, 151, 153, 227, 236 }

F grade: { 71, 72, 75, 76, 90, 91, 92, 93, 176, 177, 180, 181}

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 18, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 67, 82, 83, 84, 85, 86, 87, 88, 89, 94, 95, 97, 98, 99, 100, 101, 102, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 119, 130, 131, 133, 134, 135, 136, 137, 138, 140, 141, 142, 148, 157, 158, 159, 160, 162, 163, 164, 165, 166, 167, 168, 169, 170, 174, 182, 183, 185, 186, 187, 188, 189, 190, 192, 193, 194, 209, 210, 211, 212, 213, 214, 215, 216, 218, 219, 220, 231, 232, 234, 235, 238, 239, 240, 242, 243 }

B grade: { 7, 13, 14, 15, 16, 17, 20, 21, 22, 23, 29, 54, 66, 70, 78, 118, 122, 126, 139, 143, 144, 145, 146, 149, 155, 161, 175, 191, 195, 196, 197, 198, 200, 201, 217, 221, 227, 236 }

C grade: { 19, 24, 25, 26, 27, 28, 30, 31, 32, 33, 34, 77, 103, 125, 147, 150, 151, 152, 153, 154, 156, 199, 202, 203, 204, 205, 206, 207, 208, 222, 233, 237 }

F grade: { 45, 64, 65, 68, 69, 71, 72, 73, 74, 75, 76, 79, 80, 81, 90, 91, 92, 93, 96, 116, 117, 120, 121, 123, 124, 127, 128, 129, 132, 171, 172, 173, 176, 177, 178, 179, 180, 181, 184, 223, 224, 225, 226, 228, 229, 230, 241 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 15, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 64, 97, 98, 99, 100, 101, 102, 104, 105, 106, 108, 110, 112, 114, 116, 133, 134, 135, 136, 137, 138, 140, 141, 142, 143, 145, 157, 158, 159, 160, 162, 163, 164, 171, 185, 186, 187, 188, 189, 190, 192, 193, 194, 195, 196, 209, 210, 211, 212, 214, 220, 231, 232, 234, 235, 238, 242, 243

B grade: { 17, 21, 23, 66, 70, 118, 122, 149, 175, 191, 197, 198, 200, 201, 202, 203, 204, 207, 208, 217, 221, 223, 236, 237 }

C grade: { }

F grade: { 7, 14, 16, 18, 19, 20, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 54, 58, 59, 60, 61, 62, 63, 65, 67, 68, 69, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 103, 107, 109, 111, 113, 115, 117, 119, 120, 121, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 139, 144, 146, 147, 148, 150, 151, 152, 153, 154, 155, 156, 161, 165, 166, 167, 168, 169, 170, 172, 173, 174, 176, 177, 178, 179, 180, 181, 182, 183, 184, 199, 205, 206, 213, 215, 216, 218, 219, 222, 224, 225, 226, 227, 228, 229, 230, 233, 239, 240, 241 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 15, 17, 21, 23, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 46, 47, 48, 50, 51, 52, 53, 55, 56, 57, 58, 59, 61, 62, 63, 64, 66, 70, 82, 83, 84, 85, 86, 87, 88, 89, 94, 95, 97, 98, 99, 100, 101, 102, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 118, 122, 130, 131, 133, 134, 135, 136, 137, 138, 140, 141, 142, 143, 145, 149, 157, 158, 159, 160, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 175, 182, 183, 185, 186, 187, 188, 189, 190, 192, 193, 194, 195, 196, 197, 201, 209, 210, 211, 212, 214, 218, 219, 220, 231, 232, 234, 235, 238, 239, 240, 242, 243 }

B grade: { 60, 198, 200, 221, 223, 227, 236 }

C grade: { 213, 215, 216 }

F grade: { 7, 14, 16, 18, 19, 20, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 45, 49, 54, 65, 67, 68, 69, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 90, 91, 92, 93, 96, 103, 117, 119, 120, 121, 123, 124, 125, 126, 127, 128, 129, 132, 139, 144, 146, 147, 148, 150, 151, 152, 153, 154, 155, 156, 161, 172, 173, 174, 176, 177, 178, 179, 180, 181, 184, 191, 199, 202, 203, 204, 205, 206, 207, 208, 217, 222, 224, 225, 226, 228, 229, 230, 233, 237, 241 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 15, 17, 21, 23, 43, 44, 46, 47, 48, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 70, 135, 136, 137, 138, 140, 141, 142, 143, 145, 149, 157, 159, 162, 163, 164, 165, 166, 167, 168, 169, 170, 175, 192, 193, 200, 201, 210, 232, 234, 238 }

B grade: { 209, 211, 212 }

C grade: { 37, 158, 160, 171 }

F grade: { 7, 14, 16, 18, 19, 20, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 38, 39, 40, 41, 42, 45, 49, 54, 65, 67, 68, 69, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 139, 144, 146, 147, 148, 150, 151, 152, 153, 154, 155, 156, 161, 172, 173, 174, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 194, 195, 196, 197, 198, 199, 202, 203, 204, 205, 206, 207, 208, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 233, 235, 236, 237, 239, 240, 241, 242, 243 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 15, 17, 38, 39, 40, 41, 42, 43, 44, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 63, 64, 66, 94, 95, 97, 98, 99, 100, 101, 102, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 118, 130, 131, 133, 134, 135, 136, 137, 138, 140, 141, 142, 143, 145, 157, 158, 159, 160, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 182, 183, 185, 186, 187, 188, 189, 190, 192, 193, 194, 209, 210, 211, 212, 214, 216, 218, 219, 220, 231, 232, 234, 235, 238, 239, 240, 242, 243 }

B grade: { 60, 61, 62, 85, 86, 87, 88, 89 }

C grade: { 213 }

F grade: { 7, 14, 16, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 45, 54, 65, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 90, 91, 92, 93, 96, 103, 117, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 132, 139, 144, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 161, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 184, 191, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 215, 217, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 233, 236, 237, 241 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	81	67	95	153	63	104
normalized size	1	1.	1.37	1.14	1.61	2.59	1.07	1.76
time (sec)	N/A	0.033	0.01	0.007	0.969	1.954	2.132	1.173

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	62	60	74	153	68	84
normalized size	1	1.	1.09	1.05	1.3	2.68	1.19	1.47
time (sec)	N/A	0.043	0.009	0.007	0.966	2.056	1.616	1.156

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	70	58	82	127	53	92
normalized size	1	1.	1.46	1.21	1.71	2.65	1.1	1.92
time (sec)	N/A	0.029	0.009	0.007	0.96	2.025	1.167	1.235

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	51	51	59	127	58	72
normalized size	1	1.	1.11	1.11	1.28	2.76	1.26	1.57
time (sec)	N/A	0.035	0.008	0.007	0.968	1.969	0.945	1.232

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	59	49	68	104	42	80
normalized size	1	1.	1.59	1.32	1.84	2.81	1.14	2.16
time (sec)	N/A	0.017	0.007	0.007	0.985	1.908	0.592	1.209

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	29	41	97	27	54
normalized size	1	1.	1.	0.97	1.37	3.23	0.9	1.8
time (sec)	N/A	0.013	0.003	0.002	0.976	1.947	0.387	1.172

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	24	47	0	0	0	0
normalized size	1	1.	0.92	1.81	0.	0.	0.	0.
time (sec)	N/A	0.015	0.009	0.013	0.	0.	0.	0.

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	39	45	53	116	41	63
normalized size	1	1.	1.08	1.25	1.47	3.22	1.14	1.75
time (sec)	N/A	0.026	0.008	0.01	0.976	1.87	0.946	1.152

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	59	49	61	95	36	77
normalized size	1	1.	1.59	1.32	1.65	2.57	0.97	2.08
time (sec)	N/A	0.022	0.008	0.01	0.979	1.933	0.74	1.202

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	59	59	66	140	70	80
normalized size	1	1.	1.09	1.09	1.22	2.59	1.3	1.48
time (sec)	N/A	0.036	0.008	0.01	0.967	1.928	1.561	1.206

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	70	58	81	117	46	92
normalized size	1	1.	1.46	1.21	1.69	2.44	0.96	1.92
time (sec)	N/A	0.027	0.008	0.01	0.987	1.926	1.166	1.214

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	70	68	82	166	80	92
normalized size	1	1.	1.08	1.05	1.26	2.55	1.23	1.42
time (sec)	N/A	0.041	0.009	0.01	0.973	1.979	2.782	1.222

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	164	314	290	437	211	259
normalized size	1	1.	1.13	2.17	2.	3.01	1.46	1.79
time (sec)	N/A	0.327	0.068	0.023	1.013	2.188	4.115	1.302

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	162	162	161	306	0	0	0	0
normalized size	1	1.	0.99	1.89	0.	0.	0.	0.
time (sec)	N/A	0.302	0.452	0.016	0.	0.	0.	0.

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	132	278	255	354	168	215
normalized size	1	1.	1.17	2.46	2.26	3.13	1.49	1.9
time (sec)	N/A	0.223	0.059	0.015	0.977	2.097	2.244	1.257

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	130	130	122	270	0	0	0	0
normalized size	1	1.	0.94	2.08	0.	0.	0.	0.
time (sec)	N/A	0.201	0.257	0.013	0.	0.	0.	0.

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	90	239	213	269	114	163
normalized size	1	1.	1.2	3.19	2.84	3.59	1.52	2.17
time (sec)	N/A	0.114	0.062	0.016	0.994	2.233	1.089	1.213

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	74	74	82	123	0	0	0	0
normalized size	1	1.	1.11	1.66	0.	0.	0.	0.
time (sec)	N/A	0.101	0.143	0.049	0.	0.	0.	0.

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	120	701	0	0	0	0
normalized size	1	1.	1.03	5.99	0.	0.	0.	0.
time (sec)	N/A	0.264	0.072	0.214	0.	0.	0.	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	71	71	94	248	0	0	0	0
normalized size	1	1.	1.32	3.49	0.	0.	0.	0.
time (sec)	N/A	0.145	0.144	0.021	0.	0.	0.	0.

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	101	253	204	300	126	0
normalized size	1	1.	1.26	3.16	2.55	3.75	1.58	0.
time (sec)	N/A	0.133	0.068	0.019	1.004	2.367	1.455	0.

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	130	130	145	339	0	0	0	0
normalized size	1	1.	1.12	2.61	0.	0.	0.	0.
time (sec)	N/A	0.231	0.344	0.02	0.	0.	0.	0.

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	164	290	302	386	184	0
normalized size	1	1.	1.4	2.48	2.58	3.3	1.57	0.
time (sec)	N/A	0.228	0.069	0.026	0.998	2.38	2.601	0.

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	247	247	305	1330	0	0	0	0
normalized size	1	1.	1.23	5.38	0.	0.	0.	0.
time (sec)	N/A	0.964	0.704	0.754	0.	0.	0.	0.

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	262	262	383	1275	0	0	0	0
normalized size	1	1.	1.46	4.87	0.	0.	0.	0.
time (sec)	N/A	0.774	0.735	0.823	0.	0.	0.	0.

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	185	185	245	1245	0	0	0	0
normalized size	1	1.	1.32	6.73	0.	0.	0.	0.
time (sec)	N/A	0.569	0.492	0.602	0.	0.	0.	0.

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	197	197	250	1177	0	0	0	0
normalized size	1	1.	1.27	5.97	0.	0.	0.	0.
time (sec)	N/A	0.449	0.52	0.477	0.	0.	0.	0.

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	123	123	161	6097	0	0	0	0
normalized size	1	1.	1.31	49.57	0.	0.	0.	0.
time (sec)	N/A	0.25	0.284	0.342	0.	0.	0.	0.

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	108	108	161	261	0	0	0	0
normalized size	1	1.	1.49	2.42	0.	0.	0.	0.
time (sec)	N/A	0.217	0.26	0.082	0.	0.	0.	0.

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	178	1470	0	0	0	0
normalized size	1	1.	0.97	7.99	0.	0.	0.	0.
time (sec)	N/A	0.449	0.128	0.113	0.	0.	0.	0.

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	102	102	196	1583	0	0	0	0
normalized size	1	1.	1.92	15.52	0.	0.	0.	0.
time (sec)	N/A	0.268	0.321	0.183	0.	0.	0.	0.

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	123	123	192	5098	0	0	0	0
normalized size	1	1.	1.56	41.45	0.	0.	0.	0.
time (sec)	N/A	0.294	0.272	0.316	0.	0.	0.	0.

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	200	200	323	1838	0	0	0	0
normalized size	1	1.	1.62	9.19	0.	0.	0.	0.
time (sec)	N/A	0.498	0.861	0.849	0.	0.	0.	0.

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	187	187	295	1281	0	0	0	0
normalized size	1	1.	1.58	6.85	0.	0.	0.	0.
time (sec)	N/A	0.626	0.639	0.873	0.	0.	0.	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	128	107	0	660	0	0
normalized size	1	1.	1.03	0.86	0.	5.32	0.	0.
time (sec)	N/A	0.086	0.073	0.036	0.	2.339	0.	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	115	93	0	586	0	0
normalized size	1	1.	1.08	0.88	0.	5.53	0.	0.
time (sec)	N/A	0.067	0.055	0.011	0.	2.253	0.	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	114	89	0	509	586	0
normalized size	1	1.	1.08	0.84	0.	4.8	5.53	0.
time (sec)	N/A	0.061	0.05	0.012	0.	2.229	20.006	0.

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	98	70	0	487	0	119
normalized size	1	1.	1.15	0.82	0.	5.73	0.	1.4
time (sec)	N/A	0.052	0.032	0.013	0.	2.324	0.	1.162

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	99	78	0	494	0	127
normalized size	1	1.	1.16	0.92	0.	5.81	0.	1.49
time (sec)	N/A	0.055	0.043	0.013	0.	2.16	0.	1.27

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	107	94	0	558	0	163
normalized size	1	1.	1.	0.88	0.	5.21	0.	1.52
time (sec)	N/A	0.063	0.06	0.014	0.	2.212	0.	1.343

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	108	94	0	583	0	159
normalized size	1	1.	1.01	0.88	0.	5.45	0.	1.49
time (sec)	N/A	0.066	0.048	0.014	0.	2.28	0.	1.265

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	122	108	0	628	0	188
normalized size	1	1.	0.98	0.86	0.	5.02	0.	1.5
time (sec)	N/A	0.078	0.066	0.015	0.	2.452	0.	1.423

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.024	3.519	1.024	0.	0.	0.	0.

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.023	2.321	0.914	0.	0.	0.	0.

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	59	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.032	0.067	0.925	0.	0.	0.	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.027	0.253	0.654	0.	0.	0.	0.

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	0.5	0.658	0.	0.	0.	0.

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.005	0.441	0.201	0.	0.	0.	0.

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.024	0.326	0.378	0.	0.	0.	0.

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	78	66	93	135	58	105
normalized size	1	1.	1.44	1.22	1.72	2.5	1.07	1.94
time (sec)	N/A	0.039	0.016	0.022	0.953	1.999	29.879	1.206

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	53	45	62	134	85	77
normalized size	1	1.	1.1	0.94	1.29	2.79	1.77	1.6
time (sec)	N/A	0.035	0.014	0.009	0.982	2.059	21.035	1.17

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	67	57	78	112	48	93
normalized size	1	1.	1.56	1.33	1.81	2.6	1.12	2.16
time (sec)	N/A	0.03	0.013	0.012	0.955	2.02	14.08	1.184

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	42	37	50	108	71	66
normalized size	1	1.	1.14	1.	1.35	2.92	1.92	1.78
time (sec)	N/A	0.015	0.007	0.003	0.956	2.065	10.068	1.188

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	28	124	0	0	0	0
normalized size	1	1.	0.93	4.13	0.	0.	0.	0.
time (sec)	N/A	0.033	0.013	0.029	0.	0.	0.	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	45	49	55	130	80	69
normalized size	1	1.	1.12	1.22	1.38	3.25	2.	1.72
time (sec)	N/A	0.026	0.011	0.013	0.981	2.002	17.27	1.172

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	65	55	69	103	41	90
normalized size	1	1.	1.59	1.34	1.68	2.51	1.	2.2
time (sec)	N/A	0.027	0.011	0.013	0.959	1.939	12.464	1.195

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	61	63	69	150	97	88
normalized size	1	1.	1.09	1.12	1.23	2.68	1.73	1.57
time (sec)	N/A	0.034	0.011	0.015	0.975	2.11	34.431	1.333

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	93	53	0	473	185	103
normalized size	1	1.	1.43	0.82	0.	7.28	2.85	1.58
time (sec)	N/A	0.035	0.024	0.011	0.	2.009	19.363	1.274

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	91	51	0	448	581	101
normalized size	1	1.	1.44	0.81	0.	7.11	9.22	1.6
time (sec)	N/A	0.033	0.019	0.01	0.	2.229	12.964	1.277

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	57	37	0	394	139	112
normalized size	1	1.	1.3	0.84	0.	8.95	3.16	2.55
time (sec)	N/A	0.024	0.02	0.007	0.	2.074	8.394	1.195

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	75	42	0	385	699	107
normalized size	1	1.	1.63	0.91	0.	8.37	15.2	2.33
time (sec)	N/A	0.026	0.017	0.013	0.	2.125	15.145	1.225

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	91	51	0	440	813	132
normalized size	1	1.	1.44	0.81	0.	6.98	12.9	2.1
time (sec)	N/A	0.033	0.027	0.014	0.	2.134	21.291	1.352

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	91	51	0	458	833	123
normalized size	1	1.	1.44	0.81	0.	7.27	13.22	1.95
time (sec)	N/A	0.032	0.027	0.013	0.	2.141	30.884	1.533

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	A	F	A	A	A	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD
size	125	636	146	0	293	375	206	236
normalized size	1	5.09	1.17	0.	2.34	3.	1.65	1.89
time (sec)	N/A	1.547	0.073	180.	1.007	2.043	39.731	1.503

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	146	536	132	0	0	0	0	0
normalized size	1	3.67	0.9	0.	0.	0.	0.	0.
time (sec)	N/A	1.296	0.278	180.	0.	0.	0.	0.

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	A	B	B	A	A	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD
size	91	524	106	247	251	292	163	186
normalized size	1	5.76	1.16	2.71	2.76	3.21	1.79	2.04
time (sec)	N/A	0.965	0.06	0.163	0.976	2.113	19.12	1.382

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	A	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	94	207	99	144	0	0	0	0
normalized size	1	2.2	1.05	1.53	0.	0.	0.	0.
time (sec)	N/A	0.514	0.066	0.035	0.	0.	0.	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	141	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.336	0.101	0.133	0.	0.	0.	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	87	237	119	0	0	0	0	0
normalized size	1	2.72	1.37	0.	0.	0.	0.	0.
time (sec)	N/A	0.632	0.157	180.	0.	0.	0.	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	A	B	B	A	A	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD
size	88	360	111	257	236	324	175	0
normalized size	1	4.09	1.26	2.92	2.68	3.68	1.99	0.
time (sec)	N/A	1.057	0.083	0.188	1.003	2.164	26.066	0.

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1173	1173	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	2.334	9.308	0.216	0.	0.	0.	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1129	1129	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	2.061	9.058	0.184	0.	0.	0.	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	958	958	566	0	0	0	0	0
normalized size	1	1.	0.59	0.	0.	0.	0.	0.
time (sec)	N/A	1.468	2.562	0.125	0.	0.	0.	0.

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	942	942	566	0	0	0	0	0
normalized size	1	1.	0.6	0.	0.	0.	0.	0.
time (sec)	N/A	1.335	3.369	0.175	0.	0.	0.	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1102	1102	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.842	2.554	0.175	0.	0.	0.	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1176	1176	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.99	3.012	0.167	0.	0.	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	C	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	141	479	185	751	0	0	0	0
normalized size	1	3.4	1.31	5.33	0.	0.	0.	0.
time (sec)	N/A	4.216	0.464	0.269	0.	0.	0.	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	B	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	134	390	213	298	0	0	0	0
normalized size	1	2.91	1.59	2.22	0.	0.	0.	0.
time (sec)	N/A	2.323	0.222	0.007	0.	0.	0.	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	211	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.56	0.191	0.138	0.	0.	0.	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	C	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	125	0	222	0	0	0	0	0
normalized size	1	0.	1.78	0.	0.	0.	0.	0.
time (sec)	N/A	0.78	0.414	0.194	0.	0.	0.	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	139	0	218	0	0	0	0	0
normalized size	1	0.	1.57	0.	0.	0.	0.	0.
time (sec)	N/A	1.558	0.29	0.231	0.	0.	0.	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	317	317	241	302	0	124	0	0
normalized size	1	1.	0.76	0.95	0.	0.39	0.	0.
time (sec)	N/A	0.334	0.125	0.022	0.	2.293	0.	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	317	317	240	292	0	109	0	0
normalized size	1	1.	0.76	0.92	0.	0.34	0.	0.
time (sec)	N/A	0.3	0.093	0.016	0.	2.272	0.	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	301	227	280	0	80	0	0
normalized size	1	1.	0.75	0.93	0.	0.27	0.	0.
time (sec)	N/A	0.251	0.072	0.011	0.	2.16	0.	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	285	285	227	273	0	72	0	666
normalized size	1	1.	0.8	0.96	0.	0.25	0.	2.34
time (sec)	N/A	0.242	0.059	0.013	0.	2.229	0.	1.243

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	285	285	268	272	0	81	0	682
normalized size	1	1.	0.94	0.95	0.	0.28	0.	2.39
time (sec)	N/A	0.248	0.103	0.013	0.	2.266	0.	1.906

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	301	268	280	0	89	0	698
normalized size	1	1.	0.89	0.93	0.	0.3	0.	2.32
time (sec)	N/A	0.25	0.106	0.01	0.	2.268	0.	3.186

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	317	317	275	292	0	105	0	717
normalized size	1	1.	0.87	0.92	0.	0.33	0.	2.26
time (sec)	N/A	0.278	0.086	0.016	0.	2.237	0.	9.78

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	317	317	281	302	0	109	0	720
normalized size	1	1.	0.89	0.95	0.	0.34	0.	2.27
time (sec)	N/A	0.287	0.098	0.014	0.	2.236	0.	25.291

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	F	F	F(-2)	F	F(-1)	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	6327	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	61.946	0.313	0.	0.	0.	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	F	F	F(-2)	F	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	6177	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	59.351	0.213	0.	0.	0.	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	F	F	F(-2)	F	F(-1)	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	6334	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.029	94.015	0.272	0.	0.	0.	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	F	F	F(-2)	F	F(-1)	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	6520	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.03	50.738	0.275	0.	0.	0.	0.

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.024	1.751	0.122	0.	0.	0.	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.024	1.167	0.119	0.	0.	0.	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	64	0	0	0	0	0
normalized size	1	1.	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.04	0.066	0.117	0.	0.	0.	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.029	0.348	0.063	0.	0.	0.	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.027	0.414	0.068	0.	0.	0.	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	78	66	93	138	0	105
normalized size	1	1.	1.44	1.22	1.72	2.56	0.	1.94
time (sec)	N/A	0.038	0.02	0.009	0.953	2.042	0.	1.202

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	53	45	62	134	0	77
normalized size	1	1.	1.1	0.94	1.29	2.79	0.	1.6
time (sec)	N/A	0.036	0.016	0.008	1.021	1.916	0.	1.196

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	67	57	78	113	0	93
normalized size	1	1.	1.56	1.33	1.81	2.63	0.	2.16
time (sec)	N/A	0.03	0.016	0.012	0.98	1.799	0.	1.149

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	42	37	50	108	0	66
normalized size	1	1.	1.14	1.	1.35	2.92	0.	1.78
time (sec)	N/A	0.02	0.008	0.003	1.011	1.961	0.	1.169

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	28	92	0	0	0	0
normalized size	1	1.	0.93	3.07	0.	0.	0.	0.
time (sec)	N/A	0.034	0.014	0.029	0.	0.	0.	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	45	49	55	130	0	69
normalized size	1	1.	1.12	1.22	1.38	3.25	0.	1.72
time (sec)	N/A	0.027	0.012	0.013	1.021	1.98	0.	1.134

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	65	55	69	104	0	90
normalized size	1	1.	1.59	1.34	1.68	2.54	0.	2.2
time (sec)	N/A	0.027	0.018	0.013	1.013	1.936	0.	1.162

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	61	63	69	150	0	88
normalized size	1	1.	1.09	1.12	1.23	2.68	0.	1.57
time (sec)	N/A	0.036	0.012	0.014	0.965	2.157	0.	1.171

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	196	184	0	2611	0	279
normalized size	1	1.	1.13	1.06	0.	15.01	0.	1.6
time (sec)	N/A	0.22	0.042	0.015	0.	2.077	0.	1.317

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	136	99	143	674	0	147
normalized size	1	1.	1.35	0.98	1.42	6.67	0.	1.46
time (sec)	N/A	0.097	0.043	0.006	1.448	1.869	0.	1.184

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	187	159	0	598	0	223
normalized size	1	1.	1.13	0.96	0.	3.62	0.	1.35
time (sec)	N/A	0.197	0.052	0.01	0.	1.927	0.	1.298

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	196	172	144	367	0	169
normalized size	1	1.	1.7	1.5	1.25	3.19	0.	1.47
time (sec)	N/A	0.095	0.05	0.014	1.449	1.767	0.	1.218

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	198	186	0	659	0	281
normalized size	1	1.	1.12	1.06	0.	3.74	0.	1.6
time (sec)	N/A	0.284	0.03	0.013	0.	1.884	0.	1.493

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	198	114	161	414	0	170
normalized size	1	1.	1.69	0.97	1.38	3.54	0.	1.45
time (sec)	N/A	0.097	0.028	0.007	1.501	1.791	0.	1.186

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	187	177	0	622	0	269
normalized size	1	1.	1.13	1.07	0.	3.77	0.	1.63
time (sec)	N/A	0.249	0.026	0.01	0.	1.858	0.	1.31

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	183	105	149	312	0	143
normalized size	1	1.	1.76	1.01	1.43	3.	0.	1.38
time (sec)	N/A	0.083	0.029	0.005	1.528	1.829	0.	1.183

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	196	172	0	547	0	261
normalized size	1	1.	1.13	0.99	0.	3.14	0.	1.5
time (sec)	N/A	0.267	0.052	0.013	0.	1.673	0.	2.087

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	A	F	A	A	F(-1)	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD
size	125	636	146	0	293	381	0	236
normalized size	1	5.09	1.17	0.	2.34	3.05	0.	1.89
time (sec)	N/A	1.546	0.076	180.	0.986	1.855	0.	1.289

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	F	F	F	F(-2)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	146	536	132	0	0	0	0	0
normalized size	1	3.67	0.9	0.	0.	0.	0.	0.
time (sec)	N/A	1.309	0.272	180.	0.	0.	0.	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	A	B	B	A	F(-2)	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD
size	91	524	106	247	251	292	0	186
normalized size	1	5.76	1.16	2.71	2.76	3.21	0.	2.04
time (sec)	N/A	0.985	0.056	0.18	1.123	1.689	0.	1.198

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	A	F	F	F(-2)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	96	207	99	145	0	0	0	0
normalized size	1	2.16	1.03	1.51	0.	0.	0.	0.
time (sec)	N/A	0.592	0.152	0.004	0.	0.	0.	0.

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	143	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.335	0.111	0.144	0.	0.	0.	0.

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	F	F	F	F(-2)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	90	237	117	0	0	0	0	0
normalized size	1	2.63	1.3	0.	0.	0.	0.	0.
time (sec)	N/A	0.619	0.158	180.	0.	0.	0.	0.

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	A	B	B	A	F(-1)	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD
size	88	360	111	257	236	324	0	0
normalized size	1	4.09	1.26	2.92	2.68	3.68	0.	0.
time (sec)	N/A	1.06	0.083	0.201	0.997	1.771	0.	0.

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	F	F	F	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	144	420	159	0	0	0	0	0
normalized size	1	2.92	1.1	0.	0.	0.	0.	0.
time (sec)	N/A	1.311	0.363	0.181	0.	0.	0.	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	F	F	F	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	231	1421	334	0	0	0	0	0
normalized size	1	6.15	1.45	0.	0.	0.	0.	0.
time (sec)	N/A	6.35	0.47	0.217	0.	0.	0.	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	C	F	F	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	139	479	185	750	0	0	0	0
normalized size	1	3.45	1.33	5.4	0.	0.	0.	0.
time (sec)	N/A	4.204	0.299	0.316	0.	0.	0.	0.

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	B	F	F	F(-2)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	130	390	191	295	0	0	0	0
normalized size	1	3.	1.47	2.27	0.	0.	0.	0.
time (sec)	N/A	2.467	0.282	0.006	0.	0.	0.	0.

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	214	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.565	0.18	0.151	0.	0.	0.	0.

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	C	F	F	F	F(-1)	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	120	0	223	0	0	0	0	0
normalized size	1	0.	1.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.779	0.395	0.208	0.	0.	0.	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	F	F(-1)	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	136	0	218	0	0	0	0	0
normalized size	1	0.	1.6	0.	0.	0.	0.	0.
time (sec)	N/A	1.575	0.279	0.23	0.	0.	0.	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.025	1.81	0.125	0.	0.	0.	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.025	1.174	0.125	0.	0.	0.	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	64	0	0	0	0	0
normalized size	1	1.	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.043	0.062	0.119	0.	0.	0.	0.

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.028	0.349	0.07	0.	0.	0.	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.027	0.361	0.069	0.	0.	0.	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	67	62	77	113	46	84
normalized size	1	1.	1.34	1.24	1.54	2.26	0.92	1.68
time (sec)	N/A	0.032	0.01	0.013	0.979	1.668	1.759	1.168

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	50	67	57	117	49	66
normalized size	1	1.	1.11	1.49	1.27	2.6	1.09	1.47
time (sec)	N/A	0.033	0.008	0.013	0.961	1.769	1.109	1.168

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	56	53	59	90	36	72
normalized size	1	1.	1.44	1.36	1.51	2.31	0.92	1.85
time (sec)	N/A	0.02	0.008	0.01	0.97	1.687	0.487	1.141

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	48	39	85	24	57
normalized size	1	1.	1.	1.66	1.34	2.93	0.83	1.97
time (sec)	N/A	0.013	0.003	0.01	0.95	1.679	0.317	1.131

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	28	63	0	0	0	0
normalized size	1	1.	0.93	2.1	0.	0.	0.	0.
time (sec)	N/A	0.032	0.012	0.014	0.	0.	0.	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	38	37	50	115	39	66
normalized size	1	1.	1.09	1.06	1.43	3.29	1.11	1.89
time (sec)	N/A	0.021	0.008	0.004	0.957	1.694	1.852	1.133

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	60	57	70	103	44	77
normalized size	1	1.	1.4	1.33	1.63	2.4	1.02	1.79
time (sec)	N/A	0.027	0.009	0.007	0.955	1.761	2.114	1.164

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	62	57	74	144	68	85
normalized size	1	1.	1.09	1.	1.3	2.53	1.19	1.49
time (sec)	N/A	0.041	0.009	0.007	0.962	1.672	3.641	1.129

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	A	B	A	A	A	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD
size	123	812	131	328	255	350	158	209
normalized size	1	6.6	1.07	2.67	2.07	2.85	1.28	1.7
time (sec)	N/A	1.703	0.061	0.007	0.982	1.677	3.443	1.222

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	B	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	142	695	145	391	0	0	0	0
normalized size	1	4.89	1.02	2.75	0.	0.	0.	0.
time (sec)	N/A	1.389	0.303	0.032	0.	0.	0.	0.

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	A	B	A	A	A	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD
size	83	574	92	287	184	254	104	159
normalized size	1	6.92	1.11	3.46	2.22	3.06	1.25	1.92
time (sec)	N/A	1.041	0.047	0.025	0.976	1.762	1.05	1.211

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	B	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	74	370	97	282	0	0	0	0
normalized size	1	5.	1.31	3.81	0.	0.	0.	0.
time (sec)	N/A	0.402	0.122	0.012	0.	0.	0.	0.

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	133	133	114	780	0	0	0	0
normalized size	1	1.	0.86	5.86	0.	0.	0.	0.
time (sec)	N/A	0.314	0.092	0.162	0.	0.	0.	0.

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	A	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	87	205	101	144	0	0	0	0
normalized size	1	2.36	1.16	1.66	0.	0.	0.	0.
time (sec)	N/A	0.507	0.097	0.004	0.	0.	0.	0.

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	A	B	B	A	A	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD
size	87	707	119	284	223	288	124	0
normalized size	1	8.13	1.37	3.26	2.56	3.31	1.43	0.
time (sec)	N/A	1.247	0.067	0.016	0.989	1.867	1.962	0.

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	C	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	203	0	286	1410	0	0	0	0
normalized size	1	0.	1.41	6.95	0.	0.	0.	0.
time (sec)	N/A	4.39	0.603	0.332	0.	0.	0.	0.

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	C	C	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	217	0	316	2033	0	0	0	0
normalized size	1	0.	1.46	9.37	0.	0.	0.	0.
time (sec)	N/A	3.302	0.744	0.02	0.	0.	0.	0.

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	C	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	135	0	193	5536	0	0	0	0
normalized size	1	0.	1.43	41.01	0.	0.	0.	0.
time (sec)	N/A	2.245	0.318	0.278	0.	0.	0.	0.

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	C	C	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	108	0	198	1756	0	0	0	0
normalized size	1	0.	1.83	16.26	0.	0.	0.	0.
time (sec)	N/A	0.701	0.263	0.121	0.	0.	0.	0.

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	208	208	171	1631	0	0	0	0
normalized size	1	1.	0.82	7.84	0.	0.	0.	0.
time (sec)	N/A	0.509	0.191	0.07	0.	0.	0.	0.

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	B	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	126	387	205	298	0	0	0	0
normalized size	1	3.07	1.63	2.37	0.	0.	0.	0.
time (sec)	N/A	2.237	0.122	0.004	0.	0.	0.	0.

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	C	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	139	0	195	6645	0	0	0	0
normalized size	1	0.	1.4	47.81	0.	0.	0.	0.
time (sec)	N/A	2.102	0.334	0.266	0.	0.	0.	0.

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	73	64	84	122	51	96
normalized size	1	1.	1.35	1.19	1.56	2.26	0.94	1.78
time (sec)	N/A	0.042	0.014	0.016	0.99	1.75	18.497	1.332

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	50	65	57	124	75	70
normalized size	1	1.	1.11	1.44	1.27	2.76	1.67	1.56
time (sec)	N/A	0.033	0.012	0.019	0.968	1.711	13.194	1.245

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	62	55	66	97	41	84
normalized size	1	1.	1.44	1.28	1.53	2.26	0.95	1.95
time (sec)	N/A	0.03	0.011	0.006	0.96	1.585	15.366	1.254

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	39	52	46	99	61	63
normalized size	1	1.	1.15	1.53	1.35	2.91	1.79	1.85
time (sec)	N/A	0.018	0.007	0.016	0.982	1.647	10.123	1.386

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	28	154	0	0	0	0
normalized size	1	1.	0.93	5.13	0.	0.	0.	0.
time (sec)	N/A	0.033	0.014	0.03	0.	0.	0.	0.

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	42	37	50	126	76	70
normalized size	1	1.	1.14	1.	1.35	3.41	2.05	1.89
time (sec)	N/A	0.02	0.009	0.004	0.969	1.775	18.503	1.275

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	64	57	76	109	49	89
normalized size	1	1.	1.42	1.27	1.69	2.42	1.09	1.98
time (sec)	N/A	0.032	0.011	0.009	0.967	1.546	45.385	1.214

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	62	45	74	151	94	88
normalized size	1	1.	1.09	0.79	1.3	2.65	1.65	1.54
time (sec)	N/A	0.04	0.013	0.006	0.98	1.678	47.101	1.16

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	88	53	0	440	588	90
normalized size	1	1.	1.4	0.84	0.	6.98	9.33	1.43
time (sec)	N/A	0.035	0.021	0.014	0.	1.836	42.206	1.303

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	86	51	0	421	624	93
normalized size	1	1.	1.41	0.84	0.	6.9	10.23	1.52
time (sec)	N/A	0.033	0.018	0.008	0.	1.774	22.228	1.258

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	54	39	0	351	473	77
normalized size	1	1.	1.23	0.89	0.	7.98	10.75	1.75
time (sec)	N/A	0.024	0.016	0.01	0.	1.764	16.727	1.215

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	72	44	0	385	593	84
normalized size	1	1.	1.57	0.96	0.	8.37	12.89	1.83
time (sec)	N/A	0.031	0.018	0.01	0.	1.774	13.88	1.232

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	90	55	0	451	706	97
normalized size	1	1.	1.38	0.85	0.	6.94	10.86	1.49
time (sec)	N/A	0.037	0.031	0.012	0.	1.848	25.699	1.301

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	90	55	0	468	668	100
normalized size	1	1.	1.38	0.85	0.	7.2	10.28	1.54
time (sec)	N/A	0.038	0.024	0.011	0.	1.785	52.415	1.314

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	A	F	A	A	C	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD
size	94	599	104	0	212	278	151	180
normalized size	1	6.37	1.11	0.	2.26	2.96	1.61	1.91
time (sec)	N/A	1.296	0.06	180.	0.978	1.817	14.765	1.275

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	94	404	107	0	0	0	0	0
normalized size	1	4.3	1.14	0.	0.	0.	0.	0.
time (sec)	N/A	0.696	0.134	180.	0.	0.	0.	0.

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	144	144	177	0	0	0	0	0
normalized size	1	1.	1.23	0.	0.	0.	0.	0.
time (sec)	N/A	0.324	0.066	0.483	0.	0.	0.	0.

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	A	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	99	207	114	144	0	0	0	0
normalized size	1	2.09	1.15	1.45	0.	0.	0.	0.
time (sec)	N/A	0.529	0.09	0.004	0.	0.	0.	0.

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	A	B	B	A	A	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD
size	97	770	131	682002	247	308	172	0
normalized size	1	7.94	1.35	7030.95	2.55	3.18	1.77	0.
time (sec)	N/A	1.53	0.081	88.027	1.057	1.799	41.77	0.

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1214	1214	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	2.712	5.646	0.685	0.	0.	0.	0.

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1172	1172	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	2.208	2.978	0.641	0.	0.	0.	0.

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1549	1549	565	0	0	0	0	0
normalized size	1	1.	0.36	0.	0.	0.	0.	0.
time (sec)	N/A	2.249	3.406	0.549	0.	0.	0.	0.

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1117	1117	568	0	0	0	0	0
normalized size	1	1.	0.51	0.	0.	0.	0.	0.
time (sec)	N/A	2.15	3.037	0.656	0.	0.	0.	0.

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1263	1263	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	2.585	2.662	0.603	0.	0.	0.	0.

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1337	1337	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	2.79	2.61	0.665	0.	0.	0.	0.

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.025	2.437	0.664	0.	0.	0.	0.

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.025	1.625	0.38	0.	0.	0.	0.

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	68	0	0	0	0	0
normalized size	1	1.	0.91	0.	0.	0.	0.	0.
time (sec)	N/A	0.056	0.067	0.294	0.	0.	0.	0.

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.028	0.403	0.185	0.	0.	0.	0.

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	0.39	0.146	0.	0.	0.	0.

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	114	84	116	213	0	142
normalized size	1	1.	1.3	0.95	1.32	2.42	0.	1.61
time (sec)	N/A	0.042	0.031	0.026	1.023	1.787	0.	1.204

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	101	75	105	185	0	131
normalized size	1	1.	1.35	1.	1.4	2.47	0.	1.75
time (sec)	N/A	0.035	0.024	0.026	0.958	1.779	0.	1.17

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	88	66	93	159	0	119
normalized size	1	1.	1.42	1.06	1.5	2.56	0.	1.92
time (sec)	N/A	0.024	0.021	0.027	1.002	1.761	0.	1.194

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	42	50	72	131	0	90
normalized size	1	1.	1.08	1.28	1.85	3.36	0.	2.31
time (sec)	N/A	0.02	0.024	0.024	0.971	1.719	0.	1.22

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	63	82	0	0	0
normalized size	1	1.	1.	2.17	2.83	0.	0.	0.
time (sec)	N/A	0.032	0.011	0.036	1.493	0.	0.	0.

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	67	55	69	122	231	90
normalized size	1	1.	1.68	1.38	1.72	3.05	5.78	2.25
time (sec)	N/A	0.023	0.024	0.033	0.961	1.746	40.854	1.332

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	86	64	86	149	342	105
normalized size	1	1.	1.43	1.07	1.43	2.48	5.7	1.75
time (sec)	N/A	0.026	0.026	0.033	0.956	1.652	179.21	1.327

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	99	73	97	174	0	119
normalized size	1	1.	1.36	1.	1.33	2.38	0.	1.63
time (sec)	N/A	0.034	0.03	0.037	0.957	1.627	0.	1.399

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	B	A	A	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	211	0	224	396	358	662	0	0
normalized size	1	0.	1.06	1.88	1.7	3.14	0.	0.
time (sec)	N/A	0.025	0.112	0.056	0.988	2.126	0.	0.

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	B	A	A	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	173	0	194	358	325	567	0	0
normalized size	1	0.	1.12	2.07	1.88	3.28	0.	0.
time (sec)	N/A	0.024	0.104	0.049	1.005	2.187	0.	0.

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	B	B	A	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	129	0	160	317	290	479	0	0
normalized size	1	0.	1.24	2.46	2.25	3.71	0.	0.
time (sec)	N/A	0.014	0.086	0.051	1.027	2.204	0.	0.

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	B	B	B	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	85	0	115	272	236	382	0	0
normalized size	1	0.	1.35	3.2	2.78	4.49	0.	0.
time (sec)	N/A	0.006	0.061	0.05	1.014	1.991	0.	0.

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	164	742	0	0	0	0
normalized size	1	1.	1.13	5.12	0.	0.	0.	0.
time (sec)	N/A	0.317	0.083	0.316	0.	0.	0.	0.

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	B	B	B	A	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	85	0	129	292	235	375	680	0
normalized size	1	0.	1.52	3.44	2.76	4.41	8.	0.
time (sec)	N/A	0.024	0.11	0.054	1.02	1.834	42.93	0.

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	B	B	A	A	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	133	0	178	332	316	471	972	0
normalized size	1	0.	1.34	2.5	2.38	3.54	7.31	0.
time (sec)	N/A	0.024	0.128	0.062	0.985	1.924	179.052	0.

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	C	B	F	F(-1)	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	374	0	418	1518	2662	0	0	0
normalized size	1	0.	1.12	4.06	7.12	0.	0.	0.
time (sec)	N/A	0.023	1.194	1.179	4.152	0.	0.	0.

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	C	B	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	304	0	351	1423	2132	0	0	0
normalized size	1	0.	1.15	4.68	7.01	0.	0.	0.
time (sec)	N/A	0.024	0.789	0.326	3.732	0.	0.	0.

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	C	B	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	234	0	285	1339	1598	0	0	0
normalized size	1	0.	1.22	5.72	6.83	0.	0.	0.
time (sec)	N/A	0.014	0.527	0.286	3.391	0.	0.	0.

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	C	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	142	0	201	6235	0	0	0	0
normalized size	1	0.	1.42	43.91	0.	0.	0.	0.
time (sec)	N/A	0.006	0.277	0.388	0.	0.	0.	0.

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	248	1542	0	0	0	0
normalized size	1	1.	1.11	6.88	0.	0.	0.	0.
time (sec)	N/A	0.512	0.199	0.173	0.	0.	0.	0.

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	C	B	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	142	0	230	5199	713	0	0	0
normalized size	1	0.	1.62	36.61	5.02	0.	0.	0.
time (sec)	N/A	0.022	0.307	0.412	5.865	0.	0.	0.

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	C	B	F	F(-1)	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	234	0	333	1365	949	0	0	0
normalized size	1	0.	1.42	5.83	4.06	0.	0.	0.
time (sec)	N/A	0.023	0.692	0.31	6.649	0.	0.	0.

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	31	35	32	112	128	49
normalized size	1	1.	0.82	0.92	0.84	2.95	3.37	1.29
time (sec)	N/A	0.016	0.016	0.024	0.971	1.653	18.109	1.179

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	25	30	26	97	39	42
normalized size	1	1.	0.81	0.97	0.84	3.13	1.26	1.35
time (sec)	N/A	0.013	0.011	0.024	0.977	1.706	1.717	1.168

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	22	76	87	34
normalized size	1	1.	1.	0.85	1.1	3.8	4.35	1.7
time (sec)	N/A	0.009	0.008	0.025	0.954	1.708	0.892	1.182

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	29	24	100	126	41
normalized size	1	1.	1.	1.21	1.	4.17	5.25	1.71
time (sec)	N/A	0.01	0.018	0.03	0.958	1.808	2.417	1.164

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	222	194	0	4629	0	296
normalized size	1	1.	1.17	1.02	0.	24.36	0.	1.56
time (sec)	N/A	0.301	0.053	0.036	0.	10.99	0.	1.492

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	75	57	78	142	0	97
normalized size	1	1.	1.53	1.16	1.59	2.9	0.	1.98
time (sec)	N/A	0.034	0.026	0.029	0.995	1.856	0.	1.228

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	222	194	0	4730	0	0
normalized size	1	1.	1.17	1.02	0.	24.89	0.	0.
time (sec)	N/A	0.238	0.034	0.036	0.	10.529	0.	0.

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	141	179	0	4289	0	251
normalized size	1	1.	0.83	1.05	0.	25.23	0.	1.48
time (sec)	N/A	0.287	0.101	0.032	0.	10.769	0.	1.199

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	32	63	84	0	0	0
normalized size	1	1.	0.94	1.85	2.47	0.	0.	0.
time (sec)	N/A	0.033	0.02	0.036	1.594	0.	0.	0.

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	205	167	0	647	0	232
normalized size	1	1.	1.19	0.97	0.	3.76	0.	1.35
time (sec)	N/A	0.221	0.034	0.034	0.	1.933	0.	1.47

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	220	180	0	618	0	270
normalized size	1	1.	1.17	0.96	0.	3.29	0.	1.44
time (sec)	N/A	0.287	0.058	0.037	0.	1.996	0.	1.514

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	73	55	69	132	0	90
normalized size	1	1.	1.55	1.17	1.47	2.81	0.	1.91
time (sec)	N/A	0.032	0.029	0.036	0.985	1.878	0.	1.212

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	B	B	B	F(-1)	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	101	0	122	284	251	402	0	0
normalized size	1	0.	1.21	2.81	2.49	3.98	0.	0.
time (sec)	N/A	0.024	0.093	0.049	1.009	1.998	0.	0.

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	167	785	0	0	0	0
normalized size	1	1.	1.07	5.03	0.	0.	0.	0.
time (sec)	N/A	0.32	0.132	0.334	0.	0.	0.	0.

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	B	B	F(-1)	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	96	0	123	0	236	402	0	0
normalized size	1	0.	1.28	0.	2.46	4.19	0.	0.
time (sec)	N/A	0.024	0.139	0.28	0.998	1.891	0.	0.

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	73	0	0	0	0	0
normalized size	1	1.	1.11	0.	0.	0.	0.	0.
time (sec)	N/A	0.031	0.052	0.12	0.	0.	0.	0.

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	73	0	0	0	0	0
normalized size	1	1.	1.09	0.	0.	0.	0.	0.
time (sec)	N/A	0.024	0.045	0.21	0.	0.	0.	0.

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	58	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.028	0.032	0.191	0.	0.	0.	0.

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	39	76	0	456	0	0
normalized size	1	1.	1.08	2.11	0.	12.67	0.	0.
time (sec)	N/A	0.035	0.075	0.036	0.	1.819	0.	0.

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	66	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.032	0.061	0.102	0.	0.	0.	0.

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	73	0	0	0	0	0
normalized size	1	1.	1.04	0.	0.	0.	0.	0.
time (sec)	N/A	0.031	0.041	0.109	0.	0.	0.	0.

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	73	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.034	0.042	0.112	0.	0.	0.	0.

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	8.618	0.162	0.	0.	0.	0.

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.006	1.897	0.187	0.	0.	0.	0.

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	146	880	0	0	0	0
normalized size	1	1.	0.99	5.95	0.	0.	0.	0.
time (sec)	N/A	0.313	0.128	0.33	0.	0.	0.	0.

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.023	12.352	0.109	0.	0.	0.	0.

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.023	12.317	0.109	0.	0.	0.	0.

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	33	61	198	423	0	0
normalized size	1	1.	1.1	2.03	6.6	14.1	0.	0.
time (sec)	N/A	0.021	0.039	0.034	1.266	1.731	0.	0.

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	22	85	140	0	0	0
normalized size	1	1.	0.92	3.54	5.83	0.	0.	0.
time (sec)	N/A	0.021	0.011	0.095	0.989	0.	0.	0.

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	17	30	20	63	15	38
normalized size	1	1.	0.89	1.58	1.05	3.32	0.79	2.
time (sec)	N/A	0.006	0.002	0.043	0.955	1.607	0.258	1.18

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.024	7.971	0.232	0.	0.	0.	0.

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.024	5.791	0.214	0.	0.	0.	0.

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	88	77	0	0	0	0	0
normalized size	1	1.05	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.044	0.087	0.279	0.	0.	0.	0.

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.027	0.373	0.058	0.	0.	0.	0.

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	1.753	0.275	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [178] had the largest ratio of [2.417]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	3	1.	12	0.25
2	A	4	3	1.	12	0.25
3	A	4	3	1.	12	0.25
4	A	4	3	1.	12	0.25
5	A	3	3	1.	10	0.3
6	A	3	2	1.	8	0.25
7	A	1	1	1.	12	0.083
8	A	5	5	1.	12	0.417
9	A	3	3	1.	12	0.25
10	A	4	3	1.	12	0.25
11	A	4	3	1.	12	0.25
12	A	4	3	1.	12	0.25
13	A	16	7	1.	14	0.5

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#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
14	A	14	9	1.	14	0.643
15	A	11	7	1.	14	0.5
16	A	9	8	1.	14	0.571
17	A	6	5	1.	12	0.417
18	A	5	5	1.	10	0.5
19	A	6	5	1.	14	0.357
20	A	4	4	1.	14	0.286
21	A	8	7	1.	14	0.5
22	A	8	7	1.	14	0.5
23	A	13	8	1.	14	0.571
24	A	33	11	1.	14	0.786
25	A	24	11	1.	14	0.786
26	A	18	10	1.	14	0.714
27	A	12	9	1.	14	0.643
28	A	8	8	1.	12	0.667
29	A	5	6	1.	10	0.6
30	A	8	6	1.	14	0.429
31	A	5	6	1.	14	0.429
32	A	7	6	1.	14	0.429
33	A	14	11	1.	14	0.786
34	A	16	8	1.	14	0.571
35	A	7	6	1.	16	0.375
36	A	6	6	1.	16	0.375
37	A	6	6	1.	16	0.375
38	A	5	5	1.	16	0.312
39	A	5	5	1.	16	0.312
40	A	6	6	1.	16	0.375
41	A	6	6	1.	16	0.375
42	A	7	6	1.	16	0.375
43	A	0	0	0.	0	0.
44	A	0	0	0.	0	0.
45	A	2	2	1.	14	0.143
46	A	0	0	0.	0	0.
47	A	0	0	0.	0	0.
48	A	0	0	0.	0	0.
49	A	0	0	0.	0	0.
50	A	5	4	1.	14	0.286
51	A	4	3	1.	14	0.214
52	A	4	4	1.	14	0.286
53	A	2	2	1.	12	0.167
54	A	2	2	1.	14	0.143
55	A	5	5	1.	14	0.357
56	A	4	4	1.	14	0.286
57	A	4	3	1.	14	0.214

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
58	A	5	5	1.	14	0.357
59	A	5	5	1.	14	0.357
60	A	5	4	1.	10	0.4
61	A	4	4	1.	14	0.286
62	A	5	5	1.	14	0.357
63	A	5	5	1.	14	0.357
64	C	62	19	5.09	16	1.187
65	B	53	19	3.67	16	1.187
66	C	44	16	5.76	16	1.
67	B	28	12	2.2	14	0.857
68	A	7	6	1.	16	0.375
69	B	24	13	2.72	16	0.812
70	C	46	23	4.09	16	1.438
71	A	102	26	1.	16	1.625
72	A	86	26	1.	16	1.625
73	A	69	21	1.	12	1.75
74	A	47	21	1.	16	1.313
75	A	64	24	1.	16	1.5
76	A	77	24	1.	16	1.5
77	B	155	30	3.4	16	1.875
78	B	82	23	2.91	14	1.643
79	A	9	7	1.	16	0.438
80	F	0	0	N/A	0	N/A
81	F	0	0	N/A	0	N/A
82	A	17	14	1.	18	0.778
83	A	17	14	1.	18	0.778
84	A	16	13	1.	18	0.722
85	A	16	13	1.	18	0.722
86	A	16	13	1.	18	0.722
87	A	16	13	1.	18	0.722
88	A	17	14	1.	18	0.778
89	A	17	14	1.	18	0.778
90	F	0	0	N/A	0	N/A
91	F	0	0	N/A	0	N/A
92	F	0	0	N/A	0	N/A
93	F	0	0	N/A	0	N/A
94	A	0	0	0.	0	0.
95	A	0	0	0.	0	0.
96	A	3	3	1.	16	0.188
97	A	0	0	0.	0	0.
98	A	0	0	0.	0	0.
99	A	5	4	1.	14	0.286
100	A	4	3	1.	14	0.214
101	A	4	4	1.	14	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
102	A	2	2	1.	14	0.143
103	A	2	2	1.	14	0.143
104	A	5	5	1.	14	0.357
105	A	4	4	1.	14	0.286
106	A	4	3	1.	14	0.214
107	A	12	8	1.	14	0.571
108	A	9	8	1.	10	0.8
109	A	11	7	1.	14	0.5
110	A	9	9	1.	14	0.643
111	A	12	8	1.	14	0.571
112	A	9	9	1.	14	0.643
113	A	11	7	1.	12	0.583
114	A	8	8	1.	14	0.571
115	A	12	8	1.	14	0.571
116	C	62	19	5.09	16	1.187
117	B	53	19	3.67	16	1.187
118	C	44	16	5.76	16	1.
119	B	28	12	2.16	16	0.75
120	A	7	6	1.	16	0.375
121	B	24	13	2.63	16	0.812
122	C	46	23	4.09	16	1.438
123	B	59	24	2.92	16	1.5
124	B	239	32	6.15	16	2.
125	B	155	30	3.45	16	1.875
126	B	82	23	3.	16	1.438
127	A	9	7	1.	16	0.438
128	F	0	0	N/A	0	N/A
129	F	0	0	N/A	0	N/A
130	A	0	0	0.	0	0.
131	A	0	0	0.	0	0.
132	A	3	3	1.	16	0.188
133	A	0	0	0.	0	0.
134	A	0	0	0.	0	0.
135	A	5	4	1.	14	0.286
136	A	5	4	1.	14	0.286
137	A	4	4	1.	12	0.333
138	A	4	3	1.	10	0.3
139	A	2	2	1.	14	0.143
140	A	2	2	1.	14	0.143
141	A	4	4	1.	14	0.286
142	A	5	4	1.	14	0.286
143	C	88	34	6.6	16	2.125
144	B	73	34	4.89	16	2.125
145	C	58	32	6.92	14	2.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
146	B	31	14	5.	12	1.167
147	A	7	6	1.	16	0.375
148	B	28	12	2.36	16	0.75
149	C	66	23	8.13	16	1.438
150	F	0	0	N/A	0	N/A
151	F	0	0	N/A	0	N/A
152	F	0	0	N/A	0	N/A
153	F	0	0	N/A	0	N/A
154	A	9	7	1.	16	0.438
155	B	82	23	3.07	16	1.438
156	F	0	0	N/A	0	N/A
157	A	6	5	1.	14	0.357
158	A	5	4	1.	14	0.286
159	A	5	5	1.	14	0.357
160	A	3	3	1.	12	0.25
161	A	2	2	1.	14	0.143
162	A	2	2	1.	14	0.143
163	A	5	5	1.	14	0.357
164	A	5	4	1.	14	0.286
165	A	6	6	1.	14	0.429
166	A	6	6	1.	14	0.429
167	A	6	5	1.	10	0.5
168	A	5	5	1.	14	0.357
169	A	6	6	1.	14	0.429
170	A	6	6	1.	14	0.429
171	C	59	33	6.37	16	2.063
172	B	34	19	4.3	14	1.357
173	A	7	6	1.	16	0.375
174	B	28	12	2.09	16	0.75
175	C	67	23	7.94	16	1.438
176	A	97	33	1.	16	2.063
177	A	79	33	1.	16	2.063
178	A	99	29	1.	12	2.417
179	A	71	29	1.	16	1.812
180	A	104	30	1.	16	1.875
181	A	129	30	1.	16	1.875
182	A	0	0	0.	0	0.
183	A	0	0	0.	0	0.
184	A	4	4	1.	16	0.25
185	A	0	0	0.	0	0.
186	A	0	0	0.	0	0.
187	A	7	4	1.	16	0.25
188	A	6	4	1.	16	0.25
189	A	5	4	1.	14	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
190	A	5	4	1.	12	0.333
191	A	2	2	1.	16	0.125
192	A	4	4	1.	16	0.25
193	A	5	4	1.	16	0.25
194	A	6	4	1.	16	0.25
195	F	0	0	N/A	0	N/A
196	F	0	0	N/A	0	N/A
197	F	0	0	N/A	0	N/A
198	F	0	0	N/A	0	N/A
199	A	7	6	1.	18	0.333
200	F	0	0	N/A	0	N/A
201	F	0	0	N/A	0	N/A
202	F	0	0	N/A	0	N/A
203	F	0	0	N/A	0	N/A
204	F	0	0	N/A	0	N/A
205	F	0	0	N/A	0	N/A
206	A	9	7	1.	18	0.389
207	F	0	0	N/A	0	N/A
208	F	0	0	N/A	0	N/A
209	A	3	2	1.	12	0.167
210	A	3	2	1.	12	0.167
211	A	2	2	1.	12	0.167
212	A	4	4	1.	12	0.333
213	A	13	9	1.	16	0.562
214	A	5	5	1.	16	0.312
215	A	13	9	1.	14	0.643
216	A	13	8	1.	12	0.667
217	A	2	2	1.	16	0.125
218	A	12	8	1.	16	0.5
219	A	13	9	1.	16	0.562
220	A	5	5	1.	16	0.312
221	F	0	0	N/A	0	N/A
222	A	7	6	1.	18	0.333
223	F	0	0	N/A	0	N/A
224	A	2	2	1.	14	0.143
225	A	2	2	1.	12	0.167
226	A	3	2	1.	10	0.2
227	A	2	2	1.	14	0.143
228	A	2	2	1.	14	0.143
229	A	2	2	1.	14	0.143
230	A	2	2	1.	14	0.143
231	A	0	0	0.	0	0.
232	A	0	0	0.	0	0.
233	A	7	6	1.	16	0.375

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
234	A	0	0	0.	0	0.
235	A	0	0	0.	0	0.
236	A	2	2	1.	10	0.2
237	A	2	2	1.	10	0.2
238	A	3	3	1.	4	0.75
239	A	0	0	0.	0	0.
240	A	0	0	0.	0	0.
241	A	3	3	1.05	16	0.188
242	A	0	0	0.	0	0.
243	A	0	0	0.	0	0.

Chapter 3

Listing of integrals

3.1 $\int x^5 \left(a + b \tanh^{-1}(cx)\right) dx$

Optimal. Leaf size=59

$$\frac{1}{6}x^6 \left(a + b \tanh^{-1}(cx)\right) + \frac{bx^3}{18c^3} + \frac{bx}{6c^5} - \frac{b \tanh^{-1}(cx)}{6c^6} + \frac{bx^5}{30c}$$

```
[Out] (b*x)/(6*c^5) + (b*x^3)/(18*c^3) + (b*x^5)/(30*c) - (b*ArcTanh[c*x])/(6*c^6
) + (x^6*(a + b*ArcTanh[c*x]))/6
```

Rubi [A] time = 0.0329148, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5916, 302, 206}

$$\frac{1}{6}x^6 \left(a + b \tanh^{-1}(cx)\right) + \frac{bx^3}{18c^3} + \frac{bx}{6c^5} - \frac{b \tanh^{-1}(cx)}{6c^6} + \frac{bx^5}{30c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (b*x)/(6*c^5) + (b*x^3)/(18*c^3) + (b*x^5)/(30*c) - (b*ArcTanh[c*x])/(6*c^6
) + (x^6*(a + b*ArcTanh[c*x]))/6
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int x^5 \left(a+b\tanh^{-1}(cx)\right)\,dx &= \frac{1}{6}x^6\left(a+b\tanh^{-1}(cx)\right)-\frac{1}{6}(bc)\int \frac{x^6}{1-c^2x^2}\,dx\\ &= \frac{1}{6}x^6\left(a+b\tanh^{-1}(cx)\right)-\frac{1}{6}(bc)\int \left(-\frac{1}{c^6}-\frac{x^2}{c^4}-\frac{x^4}{c^2}+\frac{1}{c^6\left(1-c^2x^2\right)}\right)dx\\ &= \frac{bx}{6c^5}+\frac{bx^3}{18c^3}+\frac{bx^5}{30c}+\frac{1}{6}x^6\left(a+b\tanh^{-1}(cx)\right)-\frac{b\int \frac{1}{1-c^2x^2}\,dx}{6c^5}\\ &= \frac{bx}{6c^5}+\frac{bx^3}{18c^3}+\frac{bx^5}{30c}-\frac{b\tanh^{-1}(cx)}{6c^6}+\frac{1}{6}x^6\left(a+b\tanh^{-1}(cx)\right)\end{aligned}$$

Mathematica [A] time = 0.0095314, size = 81, normalized size = 1.37

$$\frac{ax^6}{6}+\frac{bx^3}{18c^3}+\frac{bx}{6c^5}+\frac{b\log(1-cx)}{12c^6}-\frac{b\log(cx+1)}{12c^6}+\frac{bx^5}{30c}+\frac{1}{6}bx^6\tanh^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (b*x)/(6*c^5) + (b*x^3)/(18*c^3) + (b*x^5)/(30*c) + (a*x^6)/6 + (b*x^6*ArcT
anh[c*x])/6 + (b*Log[1 - c*x])/(12*c^6) - (b*Log[1 + c*x])/(12*c^6)
```

Maple [A] time = 0.007, size = 67, normalized size = 1.1

$$\frac{x^6a}{6}+\frac{bx^6\mathrm{Artanh}\,(cx)}{6}+\frac{bx^5}{30\,c}+\frac{bx^3}{18\,c^3}+\frac{bx}{6\,c^5}+\frac{b\ln\,(cx-1)}{12\,c^6}-\frac{b\ln\,(cx+1)}{12\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arctanh(c*x)),x)
```

```
[Out] 1/6*x^6*a+1/6*b*x^6*arctanh(c*x)+1/30*b*x^5/c+1/18*b*x^3/c^3+1/6*b*x/c^5+1/
12/c^6*b*ln(c*x-1)-1/12/c^6*b*ln(c*x+1)
```

Maxima [A] time = 0.969474, size = 95, normalized size = 1.61

$$\frac{1}{6}\,ax^6+\frac{1}{180}\left(30\,x^6\,\mathrm{artanh}\,(cx)+c\left(\frac{2\left(3\,c^4x^5+5\,c^2x^3+15\,x\right)}{c^6}-\frac{15\,\log\,(cx+1)}{c^7}+\frac{15\,\log\,(cx-1)}{c^7}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/6*a*x^6 + 1/180*(30*x^6*arctanh(c*x) + c*(2*(3*c^4*x^5 + 5*c^2*x^3 + 15*x
)/c^6 - 15*log(c*x + 1)/c^7 + 15*log(c*x - 1)/c^7))*b
```

Fricas [A] time = 1.95422, size = 153, normalized size = 2.59

$$\frac{30\,ac^6x^6+6\,bc^5x^5+10\,bc^3x^3+30\,bcx+15\,\left(bc^6x^6-b\right)\log\left(-\frac{cx+1}{cx-1}\right)}{180\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/180*(30*a*c^6*x^6 + 6*b*c^5*x^5 + 10*b*c^3*x^3 + 30*b*c*x + 15*(b*c^6*x^6
 - b)*log(-(c*x + 1)/(c*x - 1)))/c^6
```

Sympy [A] time = 2.13222, size = 63, normalized size = 1.07

$$\begin{cases} \frac{ax^6}{6}+\frac{bx^6\,\mathrm{atanh}\,(cx)}{6}+\frac{bx^5}{30c}+\frac{bx^3}{18c^3}+\frac{bx}{6c^5}-\frac{b\,\mathrm{atanh}\,(cx)}{6c^6} & \text{for}\ c\neq 0 \\ \frac{ax^6}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*atanh(c*x)),x)
```

```
[Out] Piecewise((a*x**6/6 + b*x**6*atanh(c*x)/6 + b*x**5/(30*c) + b*x**3/(18*c**3
) + b*x/(6*c**5) - b*atanh(c*x)/(6*c**6), Ne(c, 0)), (a*x**6/6, True))
```

Giac [A] time = 1.173, size = 104, normalized size = 1.76

$$\frac{1}{12}\,bx^6\log\left(-\frac{cx+1}{cx-1}\right)+\frac{1}{6}\,ax^6+\frac{bx^5}{30\,c}+\frac{bx^3}{18\,c^3}+\frac{bx}{6\,c^5}-\frac{b\log\left(cx+1\right)}{12\,c^6}+\frac{b\log\left(cx-1\right)}{12\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] 1/12*b*x^6*log(-(c*x + 1)/(c*x - 1)) + 1/6*a*x^6 + 1/30*b*x^5/c + 1/18*b*x^
3/c^3 + 1/6*b*x/c^5 - 1/12*b*log(c*x + 1)/c^6 + 1/12*b*log(c*x - 1)/c^6
```

3.2 $\int x^4 \left(a+b\tanh^{-1}(cx)\right) dx$

Optimal. Leaf size=57

$$\frac{1}{5}x^5\left(a+b\tanh^{-1}(cx)\right)+\frac{bx^2}{10c^3}+\frac{b\log\left(1-c^2x^2\right)}{10c^5}+\frac{bx^4}{20c}$$

```
[Out] (b*x^2)/(10*c^3) + (b*x^4)/(20*c) + (x^5*(a + b*ArcTanh[c*x]))/5 + (b*Log[1
 - c^2*x^2])/(10*c^5)
```

Rubi [A] time = 0.0426034, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5916, 266, 43}

$$\frac{1}{5}x^5\left(a+b\tanh^{-1}(cx)\right)+\frac{bx^2}{10c^3}+\frac{b\log\left(1-c^2x^2\right)}{10c^5}+\frac{bx^4}{20c}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (b*x^2)/(10*c^3) + (b*x^4)/(20*c) + (x^5*(a + b*ArcTanh[c*x]))/5 + (b*Log[1
 - c^2*x^2])/(10*c^5)
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^4\left(a+b\tanh^{-1}(cx)\right)dx &= \frac{1}{5}x^5\left(a+b\tanh^{-1}(cx)\right)-\frac{1}{5}(bc)\int\frac{x^5}{1-c^2x^2}\,dx\\ &= \frac{1}{5}x^5\left(a+b\tanh^{-1}(cx)\right)-\frac{1}{10}(bc)\,\text{Subst}\left(\int\frac{x^2}{1-c^2x}\,dx,x,x^2\right)\\ &= \frac{1}{5}x^5\left(a+b\tanh^{-1}(cx)\right)-\frac{1}{10}(bc)\,\text{Subst}\left(\int\left(-\frac{1}{c^4}-\frac{x}{c^2}-\frac{1}{c^4\left(-1+c^2x\right)}\right)dx,x,x^2\right)\\ &= \frac{bx^2}{10c^3}+\frac{bx^4}{20c}+\frac{1}{5}x^5\left(a+b\tanh^{-1}(cx)\right)+\frac{b\log\left(1-c^2x^2\right)}{10c^5}\end{aligned}$$

Mathematica [A] time = 0.0085123, size = 62, normalized size = 1.09

$$\frac{ax^5}{5}+\frac{bx^2}{10c^3}+\frac{b\log\left(1-c^2x^2\right)}{10c^5}+\frac{bx^4}{20c}+\frac{1}{5}bx^5\tanh^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (b*x^2)/(10*c^3) + (b*x^4)/(20*c) + (a*x^5)/5 + (b*x^5*ArcTanh[c*x])/5 + (b
*Log[1 - c^2*x^2])/(10*c^5)
```

Maple [A] time = 0.007, size = 60, normalized size = 1.1

$$\frac{ax^5}{5}+\frac{bx^5\text{Artanh}\,(cx)}{5}+\frac{bx^4}{20\,c}+\frac{bx^2}{10\,c^3}+\frac{b\ln\,(cx-1)}{10\,c^5}+\frac{b\ln\,(cx+1)}{10\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arctanh(c*x)),x)
```

```
[Out] 1/5*a*x^5+1/5*b*x^5*arctanh(c*x)+1/20*b*x^4/c+1/10*b*x^2/c^3+1/10/c^5*b*ln(
c*x-1)+1/10/c^5*b*ln(c*x+1)
```

Maxima [A] time = 0.966233, size = 74, normalized size = 1.3

$$\frac{1}{5}\,ax^5+\frac{1}{20}\left(4\,x^5\,\text{artanh}\,(cx)+c\left(\frac{c^2x^4+2\,x^2}{c^4}+\frac{2\,\log\left(c^2x^2-1\right)}{c^6}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*a*x^5 + 1/20*(4*x^5*arctanh(c*x) + c*((c^2*x^4 + 2*x^2)/c^4 + 2*log(c^2
*x^2 - 1)/c^6))*b
```

Fricas [A] time = 2.05552, size = 153, normalized size = 2.68

$$\frac{2\,bc^5x^5\log\left(-\frac{cx+1}{cx-1}\right)+4\,ac^5x^5+bc^4x^4+2\,bc^2x^2+2\,b\log\left(c^2x^2-1\right)}{20\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/20*(2*b*c^5*x^5*log(-(c*x + 1)/(c*x - 1)) + 4*a*c^5*x^5 + b*c^4*x^4 + 2*b
*c^2*x^2 + 2*b*log(c^2*x^2 - 1))/c^5
```

Sympy [A] time = 1.61565, size = 68, normalized size = 1.19

$$\begin{cases} \frac{ax^5}{5} + \frac{bx^5 \operatorname{atanh}(cx)}{5} + \frac{bx^4}{20c} + \frac{bx^2}{10c^3} + \frac{b\log\left(x - \frac{1}{c}\right)}{5c^5} + \frac{b \operatorname{atanh}(cx)}{5c^5} & \text{for } c \neq 0 \\ \frac{ax^5}{5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*atanh(c*x)),x)
```

```
[Out] Piecewise((a*x**5/5 + b*x**5*atanh(c*x)/5 + b*x**4/(20*c) + b*x**2/(10*c**3
) + b*log(x - 1/c)/(5*c**5) + b*atanh(c*x)/(5*c**5), Ne(c, 0)), (a*x**5/5,
True))
```

Giac [A] time = 1.15635, size = 84, normalized size = 1.47

$$\frac{1}{10} bx^5 \log\left(-\frac{cx+1}{cx-1}\right) + \frac{1}{5} ax^5 + \frac{bx^4}{20\,c} + \frac{bx^2}{10\,c^3} + \frac{b\log\left(c^2x^2-1\right)}{10\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] 1/10*b*x^5*log(-(c*x + 1)/(c*x - 1)) + 1/5*a*x^5 + 1/20*b*x^4/c + 1/10*b*x^
2/c^3 + 1/10*b*log(c^2*x^2 - 1)/c^5
```

3.3 $\int x^3 \left(a+b\tanh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=48

$$\frac{1}{4}x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{bx}{4c^3}-\frac{b\tanh^{-1}(cx)}{4c^4}+\frac{bx^3}{12c}$$

```
[Out] (b*x)/(4*c^3) + (b*x^3)/(12*c) - (b*ArcTanh[c*x])/(4*c^4) + (x^4*(a + b*Arc
Tanh[c*x]))/4
```

Rubi [A] time = 0.0289794, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5916, 302, 206}

$$\frac{1}{4}x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{bx}{4c^3}-\frac{b\tanh^{-1}(cx)}{4c^4}+\frac{bx^3}{12c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (b*x)/(4*c^3) + (b*x^3)/(12*c) - (b*ArcTanh[c*x])/(4*c^4) + (x^4*(a + b*Arc
Tanh[c*x]))/4
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int x^3\left(a+b\tanh^{-1}(cx)\right)dx&=\frac{1}{4}x^4\left(a+b\tanh^{-1}(cx)\right)-\frac{1}{4}(bc)\int\frac{x^4}{1-c^2x^2}\,dx\\&=\frac{1}{4}x^4\left(a+b\tanh^{-1}(cx)\right)-\frac{1}{4}(bc)\int\left(-\frac{1}{c^4}-\frac{x^2}{c^2}+\frac{1}{c^4\left(1-c^2x^2\right)}\right)dx\\&=\frac{bx}{4c^3}+\frac{bx^3}{12c}+\frac{1}{4}x^4\left(a+b\tanh^{-1}(cx)\right)-\frac{b\int\frac{1}{1-c^2x^2}\,dx}{4c^3}\\&=\frac{bx}{4c^3}+\frac{bx^3}{12c}-\frac{b\tanh^{-1}(cx)}{4c^4}+\frac{1}{4}x^4\left(a+b\tanh^{-1}(cx)\right)\end{aligned}$$

Mathematica [A] time = 0.0086165, size = 70, normalized size = 1.46

$$\frac{ax^4}{4} + \frac{bx}{4c^3} + \frac{b\log(1-cx)}{8c^4} - \frac{b\log(cx+1)}{8c^4} + \frac{bx^3}{12c} + \frac{1}{4}bx^4\tanh^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (b*x)/(4*c^3) + (b*x^3)/(12*c) + (a*x^4)/4 + (b*x^4*ArcTanh[c*x])/4 + (b*Lo
g[1 - c*x])/(8*c^4) - (b*Log[1 + c*x])/(8*c^4)
```

Maple [A] time = 0.007, size = 58, normalized size = 1.2

$$\frac{x^4a}{4} + \frac{bx^4\text{Artanh}(cx)}{4} + \frac{bx^3}{12c} + \frac{bx}{4c^3} + \frac{b\ln(cx-1)}{8c^4} - \frac{b\ln(cx+1)}{8c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c*x)),x)
```

```
[Out] 1/4*x^4*a+1/4*b*x^4*arctanh(c*x)+1/12*b*x^3/c+1/4*b*x/c^3+1/8/c^4*b*ln(c*x-
1)-1/8/c^4*b*ln(c*x+1)
```

Maxima [A] time = 0.959693, size = 82, normalized size = 1.71

$$\frac{1}{4}ax^4 + \frac{1}{24}\left(6x^4\operatorname{artanh}(cx) + c\left(\frac{2\left(c^2x^3 + 3x\right)}{c^4} - \frac{3\log(cx+1)}{c^5} + \frac{3\log(cx-1)}{c^5}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/4*a*x^4 + 1/24*(6*x^4*arctanh(c*x) + c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x
 + 1)/c^5 + 3*log(c*x - 1)/c^5))*b
```

Fricas [A] time = 2.02546, size = 127, normalized size = 2.65

$$\frac{6ac^4x^4 + 2bc^3x^3 + 6bcx + 3\left(bc^4x^4 - b\right)\log\left(-\frac{cx+1}{cx-1}\right)}{24c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/24*(6*a*c^4*x^4 + 2*b*c^3*x^3 + 6*b*c*x + 3*(b*c^4*x^4 - b)*log(-(c*x + 1
)/(c*x - 1)))/c^4
```

Sympy [A] time = 1.16716, size = 53, normalized size = 1.1

$$\begin{cases} \frac{ax^4}{4} + \frac{bx^4 \operatorname{atanh}(cx)}{4} + \frac{bx^3}{12c} + \frac{bx}{4c^3} - \frac{b \operatorname{atanh}(cx)}{4c^4} & \text{for } c \neq 0 \\ \frac{ax^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c*x)),x)
```

```
[Out] Piecewise((a*x**4/4 + b*x**4*atanh(c*x)/4 + b*x**3/(12*c) + b*x/(4*c**3) -
b*atanh(c*x)/(4*c**4), Ne(c, 0)), (a*x**4/4, True))
```

Giac [A] time = 1.2352, size = 92, normalized size = 1.92

$$\frac{1}{8} bx^4 \log\left(-\frac{cx+1}{cx-1}\right) + \frac{1}{4} ax^4 + \frac{bx^3}{12\,c} + \frac{bx}{4\,c^3} - \frac{b \log(cx+1)}{8\,c^4} + \frac{b \log(cx-1)}{8\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] 1/8*b*x^4*log(-(c*x + 1)/(c*x - 1)) + 1/4*a*x^4 + 1/12*b*x^3/c + 1/4*b*x/c^
3 - 1/8*b*log(c*x + 1)/c^4 + 1/8*b*log(c*x - 1)/c^4
```

3.4 $\int x^2 \left(a+b\tanh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=46

$$\frac{1}{3}x^3\left(a+b\tanh^{-1}(cx)\right)+\frac{b\log\left(1-c^2x^2\right)}{6c^3}+\frac{bx^2}{6c}$$

```
[Out] (b*x^2)/(6*c) + (x^3*(a + b*ArcTanh[c*x]))/3 + (b*Log[1 - c^2*x^2])/(6*c^3)
```

Rubi [A] time = 0.03505, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5916, 266, 43}

$$\frac{1}{3}x^3\left(a+b\tanh^{-1}(cx)\right)+\frac{b\log\left(1-c^2x^2\right)}{6c^3}+\frac{bx^2}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (b*x^2)/(6*c) + (x^3*(a + b*ArcTanh[c*x]))/3 + (b*Log[1 - c^2*x^2])/(6*c^3)
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x^2\left(a+b\tanh^{-1}(cx)\right)\,dx &= \frac{1}{3}x^3\left(a+b\tanh^{-1}(cx)\right)-\frac{1}{3}(bc)\int\frac{x^3}{1-c^2x^2}\,dx\\
&= \frac{1}{3}x^3\left(a+b\tanh^{-1}(cx)\right)-\frac{1}{6}(bc)\,\text{Subst}\left(\int\frac{x}{1-c^2x}\,dx,x,x^2\right)\\
&= \frac{1}{3}x^3\left(a+b\tanh^{-1}(cx)\right)-\frac{1}{6}(bc)\,\text{Subst}\left(\int\left(-\frac{1}{c^2}-\frac{1}{c^2\left(-1+c^2x\right)}\right)dx,x,x^2\right)\\
&= \frac{bx^2}{6c}+\frac{1}{3}x^3\left(a+b\tanh^{-1}(cx)\right)+\frac{b\log\left(1-c^2x^2\right)}{6c^3}
\end{aligned}$$

Mathematica [A] time = 0.0079837, size = 51, normalized size = 1.11

$$\frac{ax^3}{3}+\frac{b\log\left(1-c^2x^2\right)}{6c^3}+\frac{bx^2}{6c}+\frac{1}{3}bx^3\tanh^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (b*x^2)/(6*c) + (a*x^3)/3 + (b*x^3*ArcTanh[c*x])/3 + (b*Log[1 - c^2*x^2])/(
6*c^3)
```

Maple [A] time = 0.007, size = 51, normalized size = 1.1

$$\frac{x^3a}{3}+\frac{bx^3\mathrm{Artanh}\,(cx)}{3}+\frac{bx^2}{6\,c}+\frac{b\ln(cx-1)}{6\,c^3}+\frac{b\ln(cx+1)}{6\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c*x)),x)
```

```
[Out] 1/3*x^3*a+1/3*b*x^3*arctanh(c*x)+1/6*b*x^2/c+1/6/c^3*b*ln(c*x-1)+1/6/c^3*b*
ln(c*x+1)
```

Maxima [A] time = 0.968135, size = 59, normalized size = 1.28

$$\frac{1}{3}\,ax^3+\frac{1}{6}\left(2\,x^3\,\mathrm{artanh}\,(cx)+c\left(\frac{x^2}{c^2}+\frac{\log\left(c^2x^2-1\right)}{c^4}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/3*a*x^3 + 1/6*(2*x^3*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x^2 - 1)/c^4))*b
```

Fricas [A] time = 1.96861, size = 127, normalized size = 2.76

$$\frac{bc^3x^3\log\left(-\frac{cx+1}{cx-1}\right)+2\,ac^3x^3+bc^2x^2+b\log\left(c^2x^2-1\right)}{6\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/6*(b*c^3*x^3*log(-(c*x + 1)/(c*x - 1)) + 2*a*c^3*x^3 + b*c^2*x^2 + b*log(
c^2*x^2 - 1))/c^3
```

Sympy [A] time = 0.944725, size = 58, normalized size = 1.26

$$\begin{cases} \frac{ax^3}{3} + \frac{bx^3 \operatorname{atanh}(cx)}{3} + \frac{bx^2}{6c} + \frac{b\log\left(x-\frac{1}{c}\right)}{3c^3} + \frac{b\operatorname{atanh}(cx)}{3c^3} & \text{for } c \neq 0 \\ \frac{ax^3}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c*x)),x)
```

```
[Out] Piecewise((a*x**3/3 + b*x**3*atanh(c*x)/3 + b*x**2/(6*c) + b*log(x - 1/c)/(
3*c**3) + b*atanh(c*x)/(3*c**3), Ne(c, 0)), (a*x**3/3, True))
```

Giac [A] time = 1.23154, size = 72, normalized size = 1.57

$$\frac{1}{6}\,bx^3\log\left(-\frac{cx+1}{cx-1}\right) + \frac{1}{3}\,ax^3 + \frac{bx^2}{6\,c} + \frac{b\log\left(c^2x^2-1\right)}{6\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] 1/6*b*x^3*log(-(c*x + 1)/(c*x - 1)) + 1/3*a*x^3 + 1/6*b*x^2/c + 1/6*b*log(c
^2*x^2 - 1)/c^3
```

3.5 $\int x \left(a+b \tanh^{-1}(cx)\right) dx$

Optimal. Leaf size=37

$$\frac{1}{2}x^2\left(a+b\tanh^{-1}(cx)\right)-\frac{b\tanh^{-1}(cx)}{2c^2}+\frac{bx}{2c}$$

```
[Out] (b*x)/(2*c) - (b*ArcTanh[c*x])/(2*c^2) + (x^2*(a + b*ArcTanh[c*x]))/2
```

Rubi [A] time = 0.0172118, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {5916, 321, 206}

$$\frac{1}{2}x^2\left(a+b\tanh^{-1}(cx)\right)-\frac{b\tanh^{-1}(cx)}{2c^2}+\frac{bx}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (b*x)/(2*c) - (b*ArcTanh[c*x])/(2*c^2) + (x^2*(a + b*ArcTanh[c*x]))/2
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int x\left(a+b\tanh^{-1}(cx)\right)dx &= \frac{1}{2}x^2\left(a+b\tanh^{-1}(cx)\right)-\frac{1}{2}(bc)\int\frac{x^2}{1-c^2x^2}\,dx\\ &= \frac{bx}{2c}+\frac{1}{2}x^2\left(a+b\tanh^{-1}(cx)\right)-\frac{b\int\frac{1}{1-c^2x^2}\,dx}{2c}\\ &= \frac{bx}{2c}-\frac{b\tanh^{-1}(cx)}{2c^2}+\frac{1}{2}x^2\left(a+b\tanh^{-1}(cx)\right)\end{aligned}$$

Mathematica [A] time = 0.0072913, size = 59, normalized size = 1.59

$$\frac{ax^2}{2}+\frac{b\log(1-cx)}{4c^2}-\frac{b\log(cx+1)}{4c^2}+\frac{1}{2}bx^2\tanh^{-1}(cx)+\frac{bx}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (b*x)/(2*c) + (a*x^2)/2 + (b*x^2*ArcTanh[c*x])/2 + (b*Log[1 - c*x])/(4*c^2)
 - (b*Log[1 + c*x])/(4*c^2)
```

Maple [A] time = 0.007, size = 49, normalized size = 1.3

$$\frac{ax^2}{2}+\frac{bx^2\text{Artanh}\,(cx)}{2}+\frac{bx}{2\,c}+\frac{b\ln\,(cx-1)}{4\,c^2}-\frac{b\ln\,(cx+1)}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c*x)),x)
```

```
[Out] 1/2*a*x^2+1/2*b*x^2*arctanh(c*x)+1/2*b*x/c+1/4/c^2*b*ln(c*x-1)-1/4/c^2*b*ln
(c*x+1)
```

Maxima [A] time = 0.98538, size = 68, normalized size = 1.84

$$\frac{1}{2}\,ax^2+\frac{1}{4}\left(2\,x^2\,\text{artanh}\,(cx)+c\left(\frac{2\,x}{c^2}-\frac{\log\,(cx+1)}{c^3}+\frac{\log\,(cx-1)}{c^3}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/2*a*x^2 + 1/4*(2*x^2*arctanh(c*x) + c*(2*x/c^2 - log(c*x + 1)/c^3 + log(c
*x - 1)/c^3))*b
```

Fricas [A] time = 1.90822, size = 104, normalized size = 2.81

$$\frac{2\,ac^2x^2+2\,bcx+\left(bc^2x^2-b\right)\log\left(-\frac{cx+1}{cx-1}\right)}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*c^2*x^2 + 2*b*c*x + (b*c^2*x^2 - b)*log(-(c*x + 1)/(c*x - 1)))/c^2
```

Sympy [A] time = 0.592059, size = 42, normalized size = 1.14

$$\begin{cases} \frac{ax^2}{2} + \frac{bx^2 \operatorname{atanh}(cx)}{2} + \frac{bx}{2c} - \frac{b \operatorname{atanh}(cx)}{2c^2} & \text{for } c \neq 0 \\ \frac{ax^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c*x)),x)
```

```
[Out] Piecewise((a*x**2/2 + b*x**2*atanh(c*x)/2 + b*x/(2*c) - b*atanh(c*x)/(2*c**
2), Ne(c, 0)), (a*x**2/2, True))
```

Giac [A] time = 1.20851, size = 80, normalized size = 2.16

$$\frac{1}{4} bx^2 \log\left(-\frac{cx+1}{cx-1}\right) + \frac{1}{2} ax^2 + \frac{bx}{2c} - \frac{b\log(cx+1)}{4c^2} + \frac{b\log(cx-1)}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] 1/4*b*x^2*log(-(c*x + 1)/(c*x - 1)) + 1/2*a*x^2 + 1/2*b*x/c - 1/4*b*log(c*x
 + 1)/c^2 + 1/4*b*log(c*x - 1)/c^2
```

3.6 $\int \left(a+b \tanh^{-1}(cx)\right) \, dx$

Optimal. Leaf size=30

$$ax+\frac{b \log \left(1-c^2 x^2\right)}{2 c}+b x \tanh^{-1}(c x)$$

```
[Out] a*x + b*x*ArcTanh[c*x] + (b*Log[1 - c^2*x^2])/(2*c)
```

Rubi [A] time = 0.0125908, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5910, 260}

$$ax+\frac{b \log \left(1-c^2 x^2\right)}{2 c}+b x \tanh^{-1}(c x)$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcTanh[c*x],x]
```

```
[Out] a*x + b*x*ArcTanh[c*x] + (b*Log[1 - c^2*x^2])/(2*c)
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned} \int \left(a+b \tanh^{-1}(cx)\right) \, dx &= ax+b \int \tanh^{-1}(cx) \, dx\\ &= ax+bx \tanh^{-1}(cx)-(bc) \int \frac{x}{1-c^2x^2} \, dx\\ &= ax+bx \tanh^{-1}(cx)+\frac{b \log \left(1-c^2x^2\right)}{2c} \end{aligned}$$

Mathematica [A] time = 0.0028612, size = 30, normalized size = 1.

$$ax+\frac{b \log \left(1-c^2 x^2\right)}{2 c}+b x \tanh^{-1}(c x)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcTanh[c*x],x]
```

```
[Out] a*x + b*x*ArcTanh[c*x] + (b*Log[1 - c^2*x^2])/(2*c)
```

Maple [A] time = 0.002, size = 29, normalized size = 1.

$$ax + bx\text{Artanh}\,(cx) + \frac{b\ln\left(-c^2x^2+1\right)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arctanh(c*x),x)
```

```
[Out] a*x+b*x*arctanh(c*x)+1/2*b*ln(-c^2*x^2+1)/c
```

Maxima [A] time = 0.975951, size = 41, normalized size = 1.37

$$ax + \frac{\left(2\,cx\,\text{artanh}\,(cx) + \log\left(-c^2x^2+1\right)\right)b}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c*x),x, algorithm="maxima")
```

```
[Out] a*x + 1/2*(2*c*x*arctanh(c*x) + log(-c^2*x^2 + 1))*b/c
```

Fricas [A] time = 1.94706, size = 97, normalized size = 3.23

$$\frac{bcx\log\left(-\frac{cx+1}{cx-1}\right) + 2\,acx + b\log\left(c^2x^2-1\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c*x),x, algorithm="fricas")
```

```
[Out] 1/2*(b*c*x*log(-(c*x + 1)/(c*x - 1)) + 2*a*c*x + b*log(c^2*x^2 - 1))/c
```

Sympy [A] time = 0.386652, size = 27, normalized size = 0.9

$$ax + b\left(\begin{cases} x\,\text{atanh}\,(cx) + \frac{\log(cx+1)}{c} - \frac{\text{atanh}\,(cx)}{c} & \text{for}\ c \neq 0 \\ 0 & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*atanh(c*x),x)
```

```
[Out] a*x + b*Piecewise((x*atanh(c*x) + log(c*x + 1)/c - atanh(c*x)/c, Ne(c, 0)),
 (0, True))
```

Giac [A] time = 1.17245, size = 54, normalized size = 1.8

$$\frac{1}{2}\left(x\log\left(-\frac{cx+1}{cx-1}\right)+\frac{\log\left(\left|c^2x^2-1\right|\right)}{c}\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c*x),x, algorithm="giac")
```

```
[Out] 1/2*(x*log(-(c*x + 1)/(c*x - 1)) + log(abs(c^2*x^2 - 1))/c)*b + a*x
```

3.7 $\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx$

Optimal. Leaf size=26

$$-\frac{1}{2}b\text{PolyLog}(2,-cx)+\frac{1}{2}b\text{PolyLog}(2,cx)+a\log(x)$$

```
[Out] a*Log[x] - (b*PolyLog[2, -(c*x)])/2 + (b*PolyLog[2, c*x])/2
```

Rubi [A] time = 0.0148648, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5912}

$$-\frac{1}{2}b\text{PolyLog}(2,-cx)+\frac{1}{2}b\text{PolyLog}(2,cx)+a\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/x,x]
```

```
[Out] a*Log[x] - (b*PolyLog[2, -(c*x)])/2 + (b*PolyLog[2, c*x])/2
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx=a\log(x)-\frac{1}{2}b\text{Li}_2(-cx)+\frac{1}{2}b\text{Li}_2(cx)$$

Mathematica [A] time = 0.0093242, size = 24, normalized size = 0.92

$$\frac{1}{2}b(\text{PolyLog}(2,cx)-\text{PolyLog}(2,-cx))+a\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])/x,x]
```

```
[Out] a*Log[x] + (b*(-PolyLog[2, -(c*x)] + PolyLog[2, c*x]))/2
```

Maple [B] time = 0.013, size = 47, normalized size = 1.8

$$a\ln(cx)+b\ln(cx)\,\text{Artanh}(cx)-\frac{b\text{dilog}(cx)}{2}-\frac{b\text{dilog}(cx+1)}{2}-\frac{b\ln(cx)\ln(cx+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/x,x)
```

```
[Out] a*ln(c*x)+b*ln(c*x)*arctanh(c*x)-1/2*b*dilog(c*x)-1/2*b*dilog(c*x+1)-1/2*b*
ln(c*x)*ln(c*x+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}b\int\frac{\log(cx+1)-\log(-cx+1)}{x}\,dx + a\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x,x, algorithm="maxima")
```

```
[Out] 1/2*b*integrate((log(c*x + 1) - log(-c*x + 1))/x, x) + a*log(x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\,\text{artanh}(cx)+a}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*x) + a)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{a+b\,\text{atanh}(cx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/x,x)
```

```
[Out] Integral((a + b*atanh(c*x))/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{b\,\text{artanh}(cx)+a}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)/x, x)
```

3.8 $\int \frac{a+b\tanh^{-1}(cx)}{x^2}\,dx$

Optimal. Leaf size=36

$$-\frac{a+b\tanh^{-1}(cx)}{x}-\frac{1}{2}bc\log\left(1-c^2x^2\right)+bc\log(x)$$

```
[Out] -((a + b*ArcTanh[c*x])/x) + b*c*Log[x] - (b*c*Log[1 - c^2*x^2])/2
```

Rubi [A] time = 0.0262703, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5916, 266, 36, 29, 31}

$$-\frac{a+b\tanh^{-1}(cx)}{x}-\frac{1}{2}bc\log\left(1-c^2x^2\right)+bc\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/x^2,x]

[Out] -((a + b*ArcTanh[c*x])/x) + b*c*Log[x] - (b*c*Log[1 - c^2*x^2])/2
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}(cx)}{x^2}\,dx = -\frac{a+b\tanh^{-1}(cx)}{x} + (bc)\int \frac{1}{x\left(1-c^2x^2\right)}\,dx$$
$$= -\frac{a+b\tanh^{-1}(cx)}{x} + \frac{1}{2}(bc)\,\text{Subst}\left(\int \frac{1}{x\left(1-c^2x\right)}\,dx, x, x^2\right)$$
$$= -\frac{a+b\tanh^{-1}(cx)}{x} + \frac{1}{2}(bc)\,\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^2\right) + \frac{1}{2}\left(bc^3\right)\text{Subst}\left(\int \frac{1}{1-c^2x}\,dx, x, x^2\right)$$
$$= -\frac{a+b\tanh^{-1}(cx)}{x} + bc\log(x) - \frac{1}{2}bc\log\left(1-c^2x^2\right)$$

Mathematica [A] time = 0.0079244, size = 39, normalized size = 1.08

$$-\frac{a}{x} - \frac{1}{2}bc\log\left(1-c^2x^2\right) + bc\log(x) - \frac{b\tanh^{-1}(cx)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])/x^2,x]
```

```
[Out] -(a/x) - (b*ArcTanh[c*x])/x + b*c*Log[x] - (b*c*Log[1 - c^2*x^2])/2
```

Maple [A] time = 0.01, size = 45, normalized size = 1.3

$$-\frac{a}{x} - \frac{b\mathrm{Artanh}\,(cx)}{x} - \frac{cb\ln\,(cx-1)}{2} + cb\ln\,(cx) - \frac{cb\ln\,(cx+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/x^2,x)
```

```
[Out] -a/x-b/x*arctanh(c*x)-1/2*c*b*ln(c*x-1)+c*b*ln(c*x)-1/2*c*b*ln(c*x+1)
```

Maxima [A] time = 0.97607, size = 53, normalized size = 1.47

$$-\frac{1}{2}\left(c\left(\log\left(c^2x^2-1\right) - \log\left(x^2\right)\right) + \frac{2\,\mathrm{artanh}\,(cx)}{x}\right)b - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*b - a/x
```

Fricas [A] time = 1.87, size = 116, normalized size = 3.22

$$-\frac{bcx\log\left(c^2x^2-1\right) - 2\,bcx\log\left(x\right) + b\log\left(-\frac{cx+1}{cx-1}\right) + 2\,a}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b*c*x*log(c^2*x^2 - 1) - 2*b*c*x*log(x) + b*log(-(c*x + 1)/(c*x - 1))
 + 2*a)/x
```

Sympy [A] time = 0.946261, size = 41, normalized size = 1.14

$$\begin{cases} -\frac{a}{x} + bc\log(x) - bc\log\left(x - \frac{1}{c}\right) - bc\operatorname{atanh}(cx) - \frac{b\operatorname{atanh}(cx)}{x} & \text{for}\ c \neq 0 \\ -\frac{a}{x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/x**2,x)
```

```
[Out] Piecewise((-a/x + b*c*log(x) - b*c*log(x - 1/c) - b*c*atanh(c*x) - b*atanh(
c*x)/x, Ne(c, 0)), (-a/x, True))
```

Giac [A] time = 1.15233, size = 63, normalized size = 1.75

$$-\frac{1}{2}bc\log\left(c^2x^2 - 1\right) + bc\log(x) - \frac{b\log\left(-\frac{cx+1}{cx-1}\right)}{2x} - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^2,x, algorithm="giac")
```

```
[Out] -1/2*b*c*log(c^2*x^2 - 1) + b*c*log(x) - 1/2*b*log(-(c*x + 1)/(c*x - 1))/x
- a/x
```

3.9 $\int \frac{a+b\tanh^{-1}(cx)}{x^3}\,dx$

Optimal. Leaf size=37

$$-\frac{a+b\tanh^{-1}(cx)}{2x^2}+\frac{1}{2}bc^2\tanh^{-1}(cx)-\frac{bc}{2x}$$

```
[Out] -(b*c)/(2*x) + (b*c^2*ArcTanh[c*x])/2 - (a + b*ArcTanh[c*x])/(2*x^2)
```

Rubi [A] time = 0.0216618, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5916, 325, 206}

$$-\frac{a+b\tanh^{-1}(cx)}{2x^2}+\frac{1}{2}bc^2\tanh^{-1}(cx)-\frac{bc}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/x^3,x]
```

```
[Out] -(b*c)/(2*x) + (b*c^2*ArcTanh[c*x])/2 - (a + b*ArcTanh[c*x])/(2*x^2)
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}(cx)}{x^3}\,dx &= -\frac{a+b\tanh^{-1}(cx)}{2x^2}+\frac{1}{2}(bc)\int \frac{1}{x^2\left(1-c^2x^2\right)}\,dx\\ &= -\frac{bc}{2x}-\frac{a+b\tanh^{-1}(cx)}{2x^2}+\frac{1}{2}\left(bc^3\right)\int \frac{1}{1-c^2x^2}\,dx\\ &= -\frac{bc}{2x}+\frac{1}{2}bc^2\tanh^{-1}(cx)-\frac{a+b\tanh^{-1}(cx)}{2x^2}\end{aligned}$$

Mathematica [A] time = 0.0083632, size = 59, normalized size = 1.59

$$-\frac{a}{2x^2}-\frac{1}{4}bc^2\log(1-cx)+\frac{1}{4}bc^2\log(cx+1)-\frac{b\tanh^{-1}(cx)}{2x^2}-\frac{bc}{2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])/x^3,x]
```

```
[Out] -a/(2*x^2) - (b*c)/(2*x) - (b*ArcTanh[c*x])/(2*x^2) - (b*c^2*Log[1 - c*x])/
4 + (b*c^2*Log[1 + c*x])/4
```

Maple [A] time = 0.01, size = 49, normalized size = 1.3

$$-\frac{a}{2\,x^2}-\frac{b\mathrm{Artanh}\,(cx)}{2\,x^2}-\frac{bc}{2\,x}-\frac{c^2b\ln\,(cx-1)}{4}+\frac{c^2b\ln\,(cx+1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/x^3,x)
```

```
[Out] -1/2*a/x^2-1/2*b/x^2*arctanh(c*x)-1/2*b*c/x-1/4*c^2*b*ln(c*x-1)+1/4*c^2*b*l
n(c*x+1)
```

Maxima [A] time = 0.978649, size = 61, normalized size = 1.65

$$\frac{1}{4}\left(\left(c\log\,(cx+1)-c\log\,(cx-1)-\frac{2}{x}\right)c-\frac{2\,\mathrm{artanh}\,(cx)}{x^2}\right)b-\frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*b - 1/
2*a/x^2
```

Fricas [A] time = 1.93332, size = 95, normalized size = 2.57

$$-\frac{2\,bcx-\left(bc^2x^2-b\right)\log\left(-\frac{cx+1}{cx-1}\right)+2\,a}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*b*c*x - (b*c^2*x^2 - b)*log(-(c*x + 1)/(c*x - 1)) + 2*a)/x^2
```

Sympy [A] time = 0.739748, size = 36, normalized size = 0.97

$$-\frac{a}{2x^2}+\frac{bc^2\,\mathrm{atanh}\,(cx)}{2}-\frac{bc}{2x}-\frac{b\,\mathrm{atanh}\,(cx)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/x**3,x)
```

```
[Out] -a/(2*x**2) + b*c**2*atanh(c*x)/2 - b*c/(2*x) - b*atanh(c*x)/(2*x**2)
```

<u>**Giac**</u> **[A]** time = 1.2016, size = 77, normalized size = 2.08

$$\frac{1}{4}\,bc^2\log\left(cx+1\right)-\frac{1}{4}\,bc^2\log\left(cx-1\right)-\frac{b\log\left(-\frac{cx+1}{cx-1}\right)}{4\,x^2}-\frac{bcx+a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^3,x, algorithm="giac")
```

```
[Out] 1/4*b*c^2*log(c*x + 1) - 1/4*b*c^2*log(c*x - 1) - 1/4*b*log(-(c*x + 1)/(c*x
 - 1))/x^2 - 1/2*(b*c*x + a)/x^2
```

3.10 $\int \frac{a+b\tanh^{-1}(cx)}{x^4}\,dx$

Optimal. Leaf size=54

$$-\frac{a+b\tanh^{-1}(cx)}{3x^3}-\frac{1}{6}bc^3\log\left(1-c^2x^2\right)+\frac{1}{3}bc^3\log(x)-\frac{bc}{6x^2}$$

```
[Out] -(b*c)/(6*x^2) - (a + b*ArcTanh[c*x])/(3*x^3) + (b*c^3*Log[x])/3 - (b*c^3*L
og[1 - c^2*x^2])/6
```

Rubi [A] time = 0.0360336, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5916, 266, 44}

$$-\frac{a+b\tanh^{-1}(cx)}{3x^3}-\frac{1}{6}bc^3\log\left(1-c^2x^2\right)+\frac{1}{3}bc^3\log(x)-\frac{bc}{6x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/x^4,x]
```

```
[Out] -(b*c)/(6*x^2) - (a + b*ArcTanh[c*x])/(3*x^3) + (b*c^3*Log[x])/3 - (b*c^3*L
og[1 - c^2*x^2])/6
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\tanh^{-1}(cx)}{x^4}\,dx &= -\frac{a+b\tanh^{-1}(cx)}{3x^3}+\frac{1}{3}(bc)\int \frac{1}{x^3\left(1-c^2x^2\right)}\,dx \\
&= -\frac{a+b\tanh^{-1}(cx)}{3x^3}+\frac{1}{6}(bc)\,\text{Subst}\left(\int \frac{1}{x^2\left(1-c^2x\right)}\,dx,x,x^2\right) \\
&= -\frac{a+b\tanh^{-1}(cx)}{3x^3}+\frac{1}{6}(bc)\,\text{Subst}\left(\int \left(\frac{1}{x^2}+\frac{c^2}{x}-\frac{c^4}{-1+c^2x}\right)dx,x,x^2\right) \\
&= -\frac{bc}{6x^2}-\frac{a+b\tanh^{-1}(cx)}{3x^3}+\frac{1}{3}bc^3\log(x)-\frac{1}{6}bc^3\log\left(1-c^2x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.008483, size = 59, normalized size = 1.09

$$
-\frac{a}{3x^3}-\frac{1}{6}bc^3\log\left(1-c^2x^2\right)+\frac{1}{3}bc^3\log(x)-\frac{bc}{6x^2}-\frac{b\tanh^{-1}(cx)}{3x^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])/x^4,x]
```

```
[Out] -a/(3*x^3) - (b*c)/(6*x^2) - (b*ArcTanh[c*x])/(3*x^3) + (b*c^3*Log[x])/3 -
(b*c^3*Log[1 - c^2*x^2])/6
```

Maple [A] time = 0.01, size = 59, normalized size = 1.1

$$
-\frac{a}{3x^3}-\frac{b\text{Artanh}(cx)}{3x^3}-\frac{c^3b\ln(cx-1)}{6}-\frac{bc}{6x^2}+\frac{c^3b\ln(cx)}{3}-\frac{c^3b\ln(cx+1)}{6}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/x^4,x)
```

```
[Out] -1/3*a/x^3-1/3*b/x^3*arctanh(c*x)-1/6*c^3*b*ln(c*x-1)-1/6*b*c/x^2+1/3*c^3*b
*ln(c*x)-1/6*c^3*b*ln(c*x+1)
```

Maxima [A] time = 0.967296, size = 66, normalized size = 1.22

$$
-\frac{1}{6}\left(\left(c^2\log\left(c^2x^2-1\right)-c^2\log\left(x^2\right)+\frac{1}{x^2}\right)c+\frac{2\,\text{artanh}(cx)}{x^3}\right)b-\frac{a}{3x^3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] -1/6*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/x^3)
*b - 1/3*a/x^3
```

Fricas [A] time = 1.92786, size = 140, normalized size = 2.59

$$
-\frac{bc^3x^3\log\left(c^2x^2-1\right)-2\,bc^3x^3\log(x)+bcx+b\log\left(-\frac{cx+1}{cx-1}\right)+2\,a}{6\,x^3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(b*c^3*x^3*log(c^2*x^2 - 1) - 2*b*c^3*x^3*log(x) + b*c*x + b*log(-(c*x
 + 1)/(c*x - 1)) + 2*a)/x^3
```

Sympy [A] time = 1.5614, size = 70, normalized size = 1.3

$$\begin{cases} -\frac{a}{3x^3} + \frac{bc^3\log(x)}{3} - \frac{bc^3\log\left(x-\frac{1}{c}\right)}{3} - \frac{bc^3\operatorname{atanh}(cx)}{3} - \frac{bc}{6x^2} - \frac{b\operatorname{atanh}(cx)}{3x^3} & \text{for}\ c \neq 0 \\ -\frac{a}{3x^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/x**4,x)
```

```
[Out] Piecewise((-a/(3*x**3) + b*c**3*log(x)/3 - b*c**3*log(x - 1/c)/3 - b*c**3*a
tanh(c*x)/3 - b*c/(6*x**2) - b*atanh(c*x)/(3*x**3), Ne(c, 0)), (-a/(3*x**3)
, True))
```

Giac [A] time = 1.20603, size = 80, normalized size = 1.48

$$-\frac{1}{6}\,bc^3\log\left(c^2x^2-1\right)+\frac{1}{3}\,bc^3\log(x)-\frac{b\log\left(-\frac{cx+1}{cx-1}\right)}{6\,x^3}-\frac{bcx+2\,a}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^4,x, algorithm="giac")
```

```
[Out] -1/6*b*c^3*log(c^2*x^2 - 1) + 1/3*b*c^3*log(x) - 1/6*b*log(-(c*x + 1)/(c*x
- 1))/x^3 - 1/6*(b*c*x + 2*a)/x^3
```

3.11 $\int \frac{a+b\tanh^{-1}(cx)}{x^5}\,dx$

Optimal. Leaf size=48

$$-\frac{a+b\tanh^{-1}(cx)}{4x^4}-\frac{bc^3}{4x}+\frac{1}{4}bc^4\tanh^{-1}(cx)-\frac{bc}{12x^3}$$

```
[Out] -(b*c)/(12*x^3) - (b*c^3)/(4*x) + (b*c^4*ArcTanh[c*x])/4 - (a + b*ArcTanh[c
*x])/(4*x^4)
```

Rubi [A] time = 0.0266312, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5916, 325, 206}

$$-\frac{a+b\tanh^{-1}(cx)}{4x^4}-\frac{bc^3}{4x}+\frac{1}{4}bc^4\tanh^{-1}(cx)-\frac{bc}{12x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/x^5,x]
```


```
[Out] -(b*c)/(12*x^3) - (b*c^3)/(4*x) + (b*c^4*ArcTanh[c*x])/4 - (a + b*ArcTanh[c
*x])/(4*x^4)
```


Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}(cx)}{x^5}\,dx &= -\frac{a+b\tanh^{-1}(cx)}{4x^4}+\frac{1}{4}(bc)\int \frac{1}{x^4\left(1-c^2x^2\right)}\,dx\\ &= -\frac{bc}{12x^3}-\frac{a+b\tanh^{-1}(cx)}{4x^4}+\frac{1}{4}\left(bc^3\right)\int \frac{1}{x^2\left(1-c^2x^2\right)}\,dx\\ &= -\frac{bc}{12x^3}-\frac{bc^3}{4x}-\frac{a+b\tanh^{-1}(cx)}{4x^4}+\frac{1}{4}\left(bc^5\right)\int \frac{1}{1-c^2x^2}\,dx\\ &= -\frac{bc}{12x^3}-\frac{bc^3}{4x}+\frac{1}{4}bc^4\tanh^{-1}(cx)-\frac{a+b\tanh^{-1}(cx)}{4x^4}\end{aligned}$$

Mathematica [A] time = 0.0082598, size = 70, normalized size = 1.46

$$-\frac{a}{4x^4}-\frac{bc^3}{4x}-\frac{1}{8}bc^4\log(1-cx)+\frac{1}{8}bc^4\log(cx+1)-\frac{bc}{12x^3}-\frac{b\tanh^{-1}(cx)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])/x^5,x]
```

```
[Out] -a/(4*x^4) - (b*c)/(12*x^3) - (b*c^3)/(4*x) - (b*ArcTanh[c*x])/(4*x^4) - (b
*c^4*Log[1 - c*x])/8 + (b*c^4*Log[1 + c*x])/8
```

Maple [A] time = 0.01, size = 58, normalized size = 1.2

$$-\frac{a}{4\,x^4}-\frac{b\mathrm{Artanh}\,(cx)}{4\,x^4}-\frac{c^4b\ln\,(cx-1)}{8}-\frac{bc}{12\,x^3}-\frac{bc^3}{4\,x}+\frac{c^4b\ln\,(cx+1)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/x^5,x)
```

```
[Out] -1/4*a/x^4-1/4*b/x^4*arctanh(c*x)-1/8*c^4*b*ln(c*x-1)-1/12*b*c/x^3-1/4*b*c^
3/x+1/8*c^4*b*ln(c*x+1)
```

Maxima [A] time = 0.987132, size = 81, normalized size = 1.69

$$\frac{1}{24}\left(\left(3\,c^3\log\,(cx+1)-3\,c^3\log\,(cx-1)-\frac{2\left(3\,c^2x^2+1\right)}{x^3}\right)c-\frac{6\,\mathrm{artanh}\,(cx)}{x^4}\right)b-\frac{a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^5,x, algorithm="maxima")
```

```
[Out] 1/24*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1) - 2*(3*c^2*x^2 + 1)/x^3)*c -
 6*arctanh(c*x)/x^4)*b - 1/4*a/x^4
```

Fricas [A] time = 1.92559, size = 117, normalized size = 2.44

$$-\frac{6\,bc^3x^3+2\,bcx-3\left(bc^4x^4-b\right)\log\left(-\frac{cx+1}{cx-1}\right)+6\,a}{24\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^5,x, algorithm="fricas")
```

```
[Out] -1/24*(6*b*c^3*x^3 + 2*b*c*x - 3*(b*c^4*x^4 - b)*log(-(c*x + 1)/(c*x - 1))
+ 6*a)/x^4
```

Sympy [A] time = 1.16585, size = 46, normalized size = 0.96

$$-\frac{a}{4x^4} + \frac{bc^4 \operatorname{atanh}(cx)}{4} - \frac{bc^3}{4x} - \frac{bc}{12x^3} - \frac{b \operatorname{atanh}(cx)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/x**5,x)
```

```
[Out] -a/(4*x**4) + b*c**4*atanh(c*x)/4 - b*c**3/(4*x) - b*c/(12*x**3) - b*atanh(
c*x)/(4*x**4)
```

Giac [A] time = 1.21371, size = 92, normalized size = 1.92

$$\frac{1}{8} bc^4 \log(cx + 1) - \frac{1}{8} bc^4 \log(cx - 1) - \frac{b \log\left(-\frac{cx+1}{cx-1}\right)}{8 x^4} - \frac{3 bc^3 x^3 + bcx + 3 a}{12 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^5,x, algorithm="giac")
```

```
[Out] 1/8*b*c^4*log(c*x + 1) - 1/8*b*c^4*log(c*x - 1) - 1/8*b*log(-(c*x + 1)/(c*x
 - 1))/x^4 - 1/12*(3*b*c^3*x^3 + b*c*x + 3*a)/x^4
```

3.12 $\int \frac{a+b\tanh^{-1}(cx)}{x^6}\,dx$

Optimal. Leaf size=65

$$-\frac{a+b\tanh^{-1}(cx)}{5x^5}-\frac{bc^3}{10x^2}-\frac{1}{10}bc^5\log\left(1-c^2x^2\right)+\frac{1}{5}bc^5\log(x)-\frac{bc}{20x^4}$$

```
[Out] -(b*c)/(20*x^4) - (b*c^3)/(10*x^2) - (a + b*ArcTanh[c*x])/(5*x^5) + (b*c^5*
Log[x])/5 - (b*c^5*Log[1 - c^2*x^2])/10
```

Rubi [A] time = 0.0413205, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5916, 266, 44}

$$-\frac{a+b\tanh^{-1}(cx)}{5x^5}-\frac{bc^3}{10x^2}-\frac{1}{10}bc^5\log\left(1-c^2x^2\right)+\frac{1}{5}bc^5\log(x)-\frac{bc}{20x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/x^6,x]
```

```
[Out] -(b*c)/(20*x^4) - (b*c^3)/(10*x^2) - (a + b*ArcTanh[c*x])/(5*x^5) + (b*c^5*
Log[x])/5 - (b*c^5*Log[1 - c^2*x^2])/10
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}(cx)}{x^6}\,dx = -\frac{a+b\tanh^{-1}(cx)}{5x^5}+\frac{1}{5}(bc)\int \frac{1}{x^5\left(1-c^2x^2\right)}\,dx$$
$$= -\frac{a+b\tanh^{-1}(cx)}{5x^5}+\frac{1}{10}(bc)\,\text{Subst}\left(\int \frac{1}{x^3\left(1-c^2x\right)}\,dx,x,x^2\right)$$
$$= -\frac{a+b\tanh^{-1}(cx)}{5x^5}+\frac{1}{10}(bc)\,\text{Subst}\left(\int \left(\frac{1}{x^3}+\frac{c^2}{x^2}+\frac{c^4}{x}-\frac{c^6}{-1+c^2x}\right)dx,x,x^2\right)$$
$$= -\frac{bc}{20x^4}-\frac{bc^3}{10x^2}-\frac{a+b\tanh^{-1}(cx)}{5x^5}+\frac{1}{5}bc^5\log(x)-\frac{1}{10}bc^5\log\left(1-c^2x^2\right)$$

Mathematica [A] time = 0.0089695, size = 70, normalized size = 1.08

$$-\frac{a}{5x^5}-\frac{bc^3}{10x^2}-\frac{1}{10}bc^5\log\left(1-c^2x^2\right)+\frac{1}{5}bc^5\log(x)-\frac{bc}{20x^4}-\frac{b\tanh^{-1}(cx)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])/x^6,x]
```

```
[Out] -a/(5*x^5) - (b*c)/(20*x^4) - (b*c^3)/(10*x^2) - (b*ArcTanh[c*x])/(5*x^5) +
 (b*c^5*Log[x])/5 - (b*c^5*Log[1 - c^2*x^2])/10
```

Maple [A] time = 0.01, size = 68, normalized size = 1.1

$$-\frac{a}{5\,x^5}-\frac{b\text{Artanh}\,(cx)}{5\,x^5}-\frac{c^5b\ln(cx-1)}{10}-\frac{bc}{20\,x^4}-\frac{bc^3}{10\,x^2}+\frac{c^5b\ln(cx)}{5}-\frac{c^5b\ln(cx+1)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/x^6,x)
```

```
[Out] -1/5*a/x^5-1/5*b/x^5*arctanh(c*x)-1/10*c^5*b*ln(c*x-1)-1/20*b*c/x^4-1/10*b*
c^3/x^2+1/5*c^5*b*ln(c*x)-1/10*c^5*b*ln(c*x+1)
```

Maxima [A] time = 0.972758, size = 82, normalized size = 1.26

$$-\frac{1}{20}\left(\left(2\,c^4\log\left(c^2x^2-1\right)-2\,c^4\log\left(x^2\right)+\frac{2\,c^2x^2+1}{x^4}\right)c+\frac{4\,\text{artanh}\,(cx)}{x^5}\right)b-\frac{a}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^6,x, algorithm="maxima")
```

```
[Out] -1/20*((2*c^4*log(c^2*x^2 - 1) - 2*c^4*log(x^2) + (2*c^2*x^2 + 1)/x^4)*c +
4*arctanh(c*x)/x^5)*b - 1/5*a/x^5
```

Fricas [A] time = 1.97896, size = 166, normalized size = 2.55

$$-\frac{2\,bc^5x^5\log\left(c^2x^2-1\right)-4\,bc^5x^5\log(x)+2\,bc^3x^3+bcx+2\,b\log\left(-\frac{cx+1}{cx-1}\right)+4\,a}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^6,x, algorithm="fricas")
```

```
[Out] -1/20*(2*b*c^5*x^5*log(c^2*x^2 - 1) - 4*b*c^5*x^5*log(x) + 2*b*c^3*x^3 + b*
c*x + 2*b*log(-(c*x + 1)/(c*x - 1)) + 4*a)/x^5
```

Sympy [A] time = 2.78227, size = 80, normalized size = 1.23

$$\begin{cases} -\frac{a}{5x^5} + \frac{bc^5 \log(x)}{5} - \frac{bc^5 \log\left(x - \frac{1}{c}\right)}{5} - \frac{bc^5 \operatorname{atanh}(cx)}{5} - \frac{bc^3}{10x^2} - \frac{bc}{20x^4} - \frac{b \operatorname{atanh}(cx)}{5x^5} & \text{for}\ c \neq 0 \\ -\frac{a}{5x^5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/x**6,x)
```

```
[Out] Piecewise((-a/(5*x**5) + b*c**5*log(x)/5 - b*c**5*log(x - 1/c)/5 - b*c**5*a
tanh(c*x)/5 - b*c**3/(10*x**2) - b*c/(20*x**4) - b*atanh(c*x)/(5*x**5), Ne(
c, 0)), (-a/(5*x**5), True))
```

Giac [A] time = 1.22231, size = 92, normalized size = 1.42

$$-\frac{1}{10}\, bc^5 \log\left(c^2x^2 - 1\right) + \frac{1}{5}\, bc^5 \log(x) - \frac{b \log\left(-\frac{cx+1}{cx-1}\right)}{10\, x^5} - \frac{2\, bc^3x^3 + bcx + 4\, a}{20\, x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^6,x, algorithm="giac")
```

```
[Out] -1/10*b*c^5*log(c^2*x^2 - 1) + 1/5*b*c^5*log(x) - 1/10*b*log(-(c*x + 1)/(c*
x - 1))/x^5 - 1/20*(2*b*c^3*x^3 + b*c*x + 4*a)/x^5
```

3.13 $\int x^5 \left(a+b\tanh^{-1}(cx)\right)^2 dx$

Optimal. Leaf size=145

$$\frac{bx^3\left(a+b\tanh^{-1}(cx)\right)}{9c^3}+\frac{abx}{3c^5}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{6c^6}+\frac{1}{6}x^6\left(a+b\tanh^{-1}(cx)\right)^2+\frac{bx^5\left(a+b\tanh^{-1}(cx)\right)}{15c}+\frac{b^2x^4}{60c^2}+\frac{4b^2x}{45c}$$

```
[Out] (a*b*x)/(3*c^5) + (4*b^2*x^2)/(45*c^4) + (b^2*x^4)/(60*c^2) + (b^2*x*ArcTan
h[c*x])/(3*c^5) + (b*x^3*(a + b*ArcTanh[c*x]))/(9*c^3) + (b*x^5*(a + b*ArcT
anh[c*x]))/(15*c) - (a + b*ArcTanh[c*x])^2/(6*c^6) + (x^6*(a + b*ArcTanh[c*
x])^2)/6 + (23*b^2*Log[1 - c^2*x^2])/(90*c^6)
```

Rubi [A] time = 0.326833, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5916, 5980, 266, 43, 5910, 260, 5948}

$$\frac{bx^3\left(a+b\tanh^{-1}(cx)\right)}{9c^3}+\frac{abx}{3c^5}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{6c^6}+\frac{1}{6}x^6\left(a+b\tanh^{-1}(cx)\right)^2+\frac{bx^5\left(a+b\tanh^{-1}(cx)\right)}{15c}+\frac{b^2x^4}{60c^2}+\frac{4b^2x}{45c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*ArcTanh[c*x])^2,x]
```


```
[Out] (a*b*x)/(3*c^5) + (4*b^2*x^2)/(45*c^4) + (b^2*x^4)/(60*c^2) + (b^2*x*ArcTan
h[c*x])/(3*c^5) + (b*x^3*(a + b*ArcTanh[c*x]))/(9*c^3) + (b*x^5*(a + b*ArcT
anh[c*x]))/(15*c) - (a + b*ArcTanh[c*x])^2/(6*c^6) + (x^6*(a + b*ArcTanh[c*
x])^2)/6 + (23*b^2*Log[1 - c^2*x^2])/(90*c^6)
```


Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int x^5 \left(a+b\tanh^{-1}(cx)\right)^2 dx = \frac{1}{6}x^6\left(a+b\tanh^{-1}(cx)\right)^2-\frac{1}{3}(bc)\int \frac{x^6\left(a+b\tanh^{-1}(cx)\right)}{1-c^2x^2}dx$$

$$=\frac{1}{6}x^6\left(a+b\tanh^{-1}(cx)\right)^2+\frac{b\int x^4\left(a+b\tanh^{-1}(cx)\right)dx}{3c}-\frac{b\int \frac{x^4\left(a+b\tanh^{-1}(cx)\right)}{1-c^2x^2}dx}{3c}$$

$$=\frac{bx^5\left(a+b\tanh^{-1}(cx)\right)}{15c}+\frac{1}{6}x^6\left(a+b\tanh^{-1}(cx)\right)^2-\frac{1}{15}b^2\int\frac{x^5}{1-c^2x^2}dx+\frac{b\int x^2\left(a+\right.}{}$$

$$=\frac{bx^3\left(a+b\tanh^{-1}(cx)\right)}{9c^3}+\frac{bx^5\left(a+b\tanh^{-1}(cx)\right)}{15c}+\frac{1}{6}x^6\left(a+b\tanh^{-1}(cx)\right)^2-\frac{1}{30}b^2\,\mathrm{S}$$

$$=\frac{abx}{3c^5}+\frac{bx^3\left(a+b\tanh^{-1}(cx)\right)}{9c^3}+\frac{bx^5\left(a+b\tanh^{-1}(cx)\right)}{15c}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{6c^6}+\frac{1}{6}x^6$$

$$=\frac{abx}{3c^5}+\frac{b^2x^2}{30c^4}+\frac{b^2x^4}{60c^2}+\frac{b^2x\tanh^{-1}(cx)}{3c^5}+\frac{bx^3\left(a+b\tanh^{-1}(cx)\right)}{9c^3}+\frac{bx^5\left(a+b\tanh^{-1}(\right.}{15c}$$

$$=\frac{abx}{3c^5}+\frac{4b^2x^2}{45c^4}+\frac{b^2x^4}{60c^2}+\frac{b^2x\tanh^{-1}(cx)}{3c^5}+\frac{bx^3\left(a+b\tanh^{-1}(cx)\right)}{9c^3}+\frac{bx^5\left(a+b\tanh^{-1}\right.}{15c}$$

Mathematica [A] time = 0.0678579, size = 164, normalized size = 1.13

$$\frac{30a^2c^6x^6+12abc^5x^5+20abc^3x^3+4bcx\tanh^{-1}(cx)\left(15ac^5x^5+b\left(3c^4x^4+5c^2x^2+15\right)\right)+60abcx+2b(15a+23b)\log}{180c^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (60*a*b*c*x + 16*b^2*c^2*x^2 + 20*a*b*c^3*x^3 + 3*b^2*c^4*x^4 + 12*a*b*c^5*
x^5 + 30*a^2*c^6*x^6 + 4*b*c*x*(15*a*c^5*x^5 + b*(15 + 5*c^2*x^2 + 3*c^4*x^
4))*ArcTanh[c*x] + 30*b^2*(-1 + c^6*x^6)*ArcTanh[c*x]^2 + 2*b*(15*a + 23*b)
*Log[1 - c*x] - 30*a*b*Log[1 + c*x] + 46*b^2*Log[1 + c*x])/(180*c^6)
```

Maple [B] time = 0.023, size = 314, normalized size = 2.2

$$\frac{x^6a^2}{6}+\frac{b^2x^6(\text{Artanh}(cx))^2}{6}+\frac{b^2\text{Artanh}(cx)\,x^5}{15c}+\frac{b^2\text{Artanh}(cx)\,x^3}{9c^3}+\frac{b^2x\text{Artanh}(cx)}{3c^5}+\frac{b^2\text{Artanh}(cx)\ln(cx-1)}{6c^6}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arctanh(c*x))^2,x)
```

```
[Out] 1/6*x^6*a^2+1/6*b^2*x^6*arctanh(c*x)^2+1/15/c*b^2*arctanh(c*x)*x^5+1/9/c^3*
b^2*arctanh(c*x)*x^3+1/3*b^2*x*arctanh(c*x)/c^5+1/6/c^6*b^2*arctanh(c*x)*ln
(c*x-1)-1/6/c^6*b^2*arctanh(c*x)*ln(c*x+1)+1/24/c^6*b^2*ln(c*x-1)^2-1/12/c^
6*b^2*ln(c*x-1)*ln(1/2+1/2*c*x)-1/12/c^6*b^2*ln(-1/2*c*x+1/2)*ln(c*x+1)+1/1
2/c^6*b^2*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)+1/24/c^6*b^2*ln(c*x+1)^2+1/60*b^
2*x^4/c^2+4/45*b^2*x^2/c^4+23/90/c^6*b^2*ln(c*x-1)+23/90/c^6*b^2*ln(c*x+1)+
1/3*a*b*x^6*arctanh(c*x)+1/15/c*x^5*a*b+1/9*a*b*x^3/c^3+1/3*a*b*x/c^5+1/6/c
^6*a*b*ln(c*x-1)-1/6/c^6*a*b*ln(c*x+1)
```

Maxima [A] time = 1.01312, size = 290, normalized size = 2.

$$\frac{1}{6}b^2x^6\,\text{artanh}(cx)^2+\frac{1}{6}a^2x^6+\frac{1}{90}\left(30x^6\,\text{artanh}(cx)+c\left(\frac{2\left(3c^4x^5+5c^2x^3+15x\right)}{c^6}-\frac{15\log(cx+1)}{c^7}+\frac{15\log(cx-1)}{c^7}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/6*b^2*x^6*arctanh(c*x)^2 + 1/6*a^2*x^6 + 1/90*(30*x^6*arctanh(c*x) + c*(2
*(3*c^4*x^5 + 5*c^2*x^3 + 15*x)/c^6 - 15*log(c*x + 1)/c^7 + 15*log(c*x - 1)
/c^7))*a*b + 1/360*(4*c*(2*(3*c^4*x^5 + 5*c^2*x^3 + 15*x)/c^6 - 15*log(c*x
+ 1)/c^7 + 15*log(c*x - 1)/c^7)*arctanh(c*x) + (6*c^4*x^4 + 32*c^2*x^2 - 2*
(15*log(c*x - 1) - 46)*log(c*x + 1) + 15*log(c*x + 1)^2 + 15*log(c*x - 1)^2
 + 92*log(c*x - 1))/c^6)*b^2
```

Fricas [A] time = 2.18829, size = 437, normalized size = 3.01

$$\frac{60a^2c^6x^6+24abc^5x^5+6b^2c^4x^4+40abc^3x^3+32b^2c^2x^2+120abcx+15\left(b^2c^6x^6-b^2\right)\log\left(-\frac{cx+1}{cx-1}\right)^2-4\left(15ab-23b^2\right)\text{l}}{360c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/360*(60*a^2*c^6*x^6 + 24*a*b*c^5*x^5 + 6*b^2*c^4*x^4 + 40*a*b*c^3*x^3 + 3
2*b^2*c^2*x^2 + 120*a*b*c*x + 15*(b^2*c^6*x^6 - b^2)*log(-(c*x + 1)/(c*x -
1))^2 - 4*(15*a*b - 23*b^2)*log(c*x + 1) + 4*(15*a*b + 23*b^2)*log(c*x - 1)
 + 4*(15*a*b*c^6*x^6 + 3*b^2*c^5*x^5 + 5*b^2*c^3*x^3 + 15*b^2*c*x)*log(-(c*
x + 1)/(c*x - 1)))/c^6
```

Sympy [A] time = 4.11455, size = 211, normalized size = 1.46

$$\begin{cases} \frac{a^2x^6}{6} + \frac{abx^6\operatorname{atanh}(cx)}{3} + \frac{abx^5}{15c} + \frac{abx^3}{9c^3} + \frac{abx}{3c^5} - \frac{ab\operatorname{atanh}(cx)}{3c^6} + \frac{b^2x^6\operatorname{atanh}^2(cx)}{6} + \frac{b^2x^5\operatorname{atanh}(cx)}{15c} + \frac{b^2x^4}{60c^2} + \frac{b^2x^3\operatorname{atanh}(cx)}{9c^3} + \frac{4b^2x^2}{45c^4} + \cdots \\ \frac{a^2x^6}{6} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*atanh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*x**6/6 + a*b*x**6*atanh(c*x)/3 + a*b*x**5/(15*c) + a*b*x**3
/(9*c**3) + a*b*x/(3*c**5) - a*b*atanh(c*x)/(3*c**6) + b**2*x**6*atanh(c*x)
**2/6 + b**2*x**5*atanh(c*x)/(15*c) + b**2*x**4/(60*c**2) + b**2*x**3*atanh
(c*x)/(9*c**3) + 4*b**2*x**2/(45*c**4) + b**2*x*atanh(c*x)/(3*c**5) + 23*b*
*2*log(x - 1/c)/(45*c**6) - b**2*atanh(c*x)**2/(6*c**6) + 23*b**2*atanh(c*x
)/(45*c**6), Ne(c, 0)), (a**2*x**6/6, True))
```

Giac [A] time = 1.30206, size = 259, normalized size = 1.79

$$\frac{1}{6}a^2x^6 + \frac{abx^5}{15c} + \frac{b^2x^4}{60c^2} + \frac{1}{24}\left(b^2x^6 - \frac{b^2}{c^6}\right)\log\left(-\frac{cx+1}{cx-1}\right)^2 + \frac{abx^3}{9c^3} + \frac{1}{90}\left(15abx^6 + \frac{3b^2x^5}{c} + \frac{5b^2x^3}{c^3} + \frac{15b^2x}{c^5}\right)\log\left(-\frac{cx \cdots}{cx \cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

```
[Out] 1/6*a^2*x^6 + 1/15*a*b*x^5/c + 1/60*b^2*x^4/c^2 + 1/24*(b^2*x^6 - b^2/c^6)*
log(-(c*x + 1)/(c*x - 1))^2 + 1/9*a*b*x^3/c^3 + 1/90*(15*a*b*x^6 + 3*b^2*x^
5/c + 5*b^2*x^3/c^3 + 15*b^2*x/c^5)*log(-(c*x + 1)/(c*x - 1)) + 4/45*b^2*x^
2/c^4 + 1/3*a*b*x/c^5 - 1/90*(15*a*b - 23*b^2)*log(c*x + 1)/c^6 + 1/90*(15*
a*b + 23*b^2)*log(c*x - 1)/c^6
```

3.14 $\int x^4 \left(a+b\tanh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=162

$$-\frac{b^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{5c^5}+\frac{bx^2\left(a+b\tanh^{-1}(cx)\right)}{5c^3}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{5c^5}-\frac{2b\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{5c^5}+\frac{1}{5}x^5\left(a\right.$$

```
[Out] (3*b^2*x)/(10*c^4) + (b^2*x^3)/(30*c^2) - (3*b^2*ArcTanh[c*x])/(10*c^5) + (
b*x^2*(a + b*ArcTanh[c*x]))/(5*c^3) + (b*x^4*(a + b*ArcTanh[c*x]))/(10*c) +
 (a + b*ArcTanh[c*x])^2/(5*c^5) + (x^5*(a + b*ArcTanh[c*x])^2)/5 - (2*b*(a
+ b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(5*c^5) - (b^2*PolyLog[2, 1 - 2/(1 - c*
x)])/(5*c^5)
```

Rubi [A] time = 0.302228, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {5916, 5980, 302, 206, 321, 5984, 5918, 2402, 2315}

$$-\frac{b^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{5c^5}+\frac{bx^2\left(a+b\tanh^{-1}(cx)\right)}{5c^3}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{5c^5}-\frac{2b\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{5c^5}+\frac{1}{5}x^5\left(a\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*ArcTanh[c*x])^2,x]
```


```
[Out] (3*b^2*x)/(10*c^4) + (b^2*x^3)/(30*c^2) - (3*b^2*ArcTanh[c*x])/(10*c^5) + (
b*x^2*(a + b*ArcTanh[c*x]))/(5*c^3) + (b*x^4*(a + b*ArcTanh[c*x]))/(10*c) +
 (a + b*ArcTanh[c*x])^2/(5*c^5) + (x^5*(a + b*ArcTanh[c*x])^2)/5 - (2*b*(a
+ b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(5*c^5) - (b^2*PolyLog[2, 1 - 2/(1 - c*
x)])/(5*c^5)
```


Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int x^4 \left(a+b\tanh^{-1}(cx)\right)^2 dx = \frac{1}{5}x^5\left(a+b\tanh^{-1}(cx)\right)^2 - \frac{1}{5}(2bc)\int \frac{x^5\left(a+b\tanh^{-1}(cx)\right)}{1-c^2x^2}dx$$

$$= \frac{1}{5}x^5\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{(2b)\int x^3\left(a+b\tanh^{-1}(cx)\right)dx}{5c} - \frac{(2b)\int \frac{x^3\left(a+b\tanh^{-1}(cx)\right)}{1-c^2x^2}d}{5c}$$

$$= \frac{bx^4\left(a+b\tanh^{-1}(cx)\right)}{10c} + \frac{1}{5}x^5\left(a+b\tanh^{-1}(cx)\right)^2 - \frac{1}{10}b^2\int\frac{x^4}{1-c^2x^2}dx + \frac{(2b)\int x\,(a}{}$$

$$= \frac{bx^2\left(a+b\tanh^{-1}(cx)\right)}{5c^3} + \frac{bx^4\left(a+b\tanh^{-1}(cx)\right)}{10c} + \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{5c^5} + \frac{1}{5}x^5\left(a+b\right.$$

$$= \frac{3b^2x}{10c^4} + \frac{b^2x^3}{30c^2} + \frac{bx^2\left(a+b\tanh^{-1}(cx)\right)}{5c^3} + \frac{bx^4\left(a+b\tanh^{-1}(cx)\right)}{10c} + \frac{\left(a+b\tanh^{-1}(cx)\right.}{5c^5}$$

$$= \frac{3b^2x}{10c^4} + \frac{b^2x^3}{30c^2} - \frac{3b^2\tanh^{-1}(cx)}{10c^5} + \frac{bx^2\left(a+b\tanh^{-1}(cx)\right)}{5c^3} + \frac{bx^4\left(a+b\tanh^{-1}(cx)\right)}{10c} +$$

$$= \frac{3b^2x}{10c^4} + \frac{b^2x^3}{30c^2} - \frac{3b^2\tanh^{-1}(cx)}{10c^5} + \frac{bx^2\left(a+b\tanh^{-1}(cx)\right)}{5c^3} + \frac{bx^4\left(a+b\tanh^{-1}(cx)\right)}{10c} +$$

Mathematica [A] time = 0.451515, size = 161, normalized size = 0.99

$$\frac{6b^2\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)+6a^2c^5x^5+3abc^4x^4+6abc^2x^2+6ab\log\left(c^2x^2-1\right)+3b\tanh^{-1}(cx)\left(4ac^5x^5+b\left(c^4x^4+\right.\right.}{30c^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (-9*a*b + 9*b^2*c*x + 6*a*b*c^2*x^2 + b^2*c^3*x^3 + 3*a*b*c^4*x^4 + 6*a^2*c
^5*x^5 + 6*b^2*(-1 + c^5*x^5)*ArcTanh[c*x]^2 + 3*b*ArcTanh[c*x]*(4*a*c^5*x^
5 + b*(-3 + 2*c^2*x^2 + c^4*x^4) - 4*b*Log[1 + E^(-2*ArcTanh[c*x])]) + 6*a*
b*Log[-1 + c^2*x^2] + 6*b^2*PolyLog[2, -E^(-2*ArcTanh[c*x])])/(30*c^5)
```

Maple [B] time = 0.016, size = 306, normalized size = 1.9

$$\frac{x^5a^2}{5}+\frac{x^5b^2\left(\text{Artanh}\left(cx\right)\right)^2}{5}+\frac{b^2\text{Artanh}\left(cx\right)x^4}{10c}+\frac{b^2\text{Artanh}\left(cx\right)x^2}{5c^3}+\frac{b^2\text{Artanh}\left(cx\right)\ln\left(cx-1\right)}{5c^5}+\frac{b^2\text{Artanh}\left(cx\right)\ln\left(c\right.}{5c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arctanh(c*x))^2,x)
```

```
[Out] 1/5*x^5*a^2+1/5*x^5*b^2*arctanh(c*x)^2+1/10/c*b^2*arctanh(c*x)*x^4+1/5/c^3*
b^2*arctanh(c*x)*x^2+1/5/c^5*b^2*arctanh(c*x)*ln(c*x-1)+1/5/c^5*b^2*arctanh
(c*x)*ln(c*x+1)+1/30*b^2*x^3/c^2+3/10*b^2*x/c^4+3/20/c^5*b^2*ln(c*x-1)-3/20
/c^5*b^2*ln(c*x+1)+1/20/c^5*b^2*ln(c*x-1)^2-1/5/c^5*b^2*dilog(1/2+1/2*c*x)-
1/10/c^5*b^2*ln(c*x-1)*ln(1/2+1/2*c*x)+1/10/c^5*b^2*ln(-1/2*c*x+1/2)*ln(c*x
+1)-1/10/c^5*b^2*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)-1/20/c^5*b^2*ln(c*x+1)^2+
2/5*x^5*a*b*arctanh(c*x)+1/10/c*x^4*a*b+1/5*a*b*x^2/c^3+1/5/c^5*a*b*ln(c*x-
1)+1/5/c^5*a*b*ln(c*x+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/5*a^2*x^5 + 1/10*(4*x^5*arctanh(c*x) + c*((c^2*x^4 + 2*x^2)/c^4 + 2*log(c
^2*x^2 - 1)/c^6))*a*b - 1/36000*(24*c^6*(2*(3*c^4*x^5 + 5*c^2*x^3 + 15*x)/c
^10 - 15*log(c*x + 1)/c^11 + 15*log(c*x - 1)/c^11) - 45*c^5*((c^2*x^4 + 2*x
^2)/c^8 + 2*log(c^2*x^2 - 1)/c^10) - 1080000*c^5*integrate(1/150*x^5*log(c*
x + 1)/(c^6*x^2 - c^4), x) + 50*c^4*(2*(c^2*x^3 + 3*x)/c^8 - 3*log(c*x + 1)
/c^9 + 3*log(c*x - 1)/c^9) - 300*c^3*(x^2/c^6 + log(c^2*x^2 - 1)/c^8) + 900
*c^2*(2*x/c^6 - log(c*x + 1)/c^7 + log(c*x - 1)/c^7) - 540000*c*integrate(1
/150*x*log(c*x + 1)/(c^6*x^2 - c^4), x) - 60*(30*c^5*x^5*log(c*x + 1)^2 + (
12*c^5*x^5 - 15*c^4*x^4 + 20*c^3*x^3 - 30*c^2*x^2 + 60*c*x - 60*(c^5*x^5 +
1)*log(c*x + 1))*log(-c*x + 1))/c^5 - (72*(c*x - 1)^5*(25*log(-c*x + 1)^2 -
 10*log(-c*x + 1) + 2) + 1125*(c*x - 1)^4*(8*log(-c*x + 1)^2 - 4*log(-c*x +
 1) + 1) + 2000*(c*x - 1)^3*(9*log(-c*x + 1)^2 - 6*log(-c*x + 1) + 2) + 900
0*(c*x - 1)^2*(2*log(-c*x + 1)^2 - 2*log(-c*x + 1) + 1) + 9000*(c*x - 1)*(l
```

```
og(-c*x + 1)^2 - 2*log(-c*x + 1) + 2))/c^5 + 1800*log(150*c^6*x^2 - 150*c^4
)/c^5 - 540000*integrate(1/150*log(c*x + 1)/(c^6*x^2 - c^4), x))*b^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(b^2x^4\,\text{artanh}\,(cx)^2 + 2\,abx^4\,\text{artanh}\,(cx) + a^2x^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^4*arctanh(c*x)^2 + 2*a*b*x^4*arctanh(c*x) + a^2*x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^4\,(a + b\,\text{atanh}\,(cx))^2\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*atanh(c*x))**2,x)
```

```
[Out] Integral(x**4*(a + b*atanh(c*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\,\text{artanh}\,(cx) + a)^2 x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2*x^4, x)
```

3.15 $\int x^3 \left(a+b\tanh^{-1}(cx)\right)^2 dx$

Optimal. Leaf size=113

$$\frac{abx}{2c^3}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{4c^4}+\frac{1}{4}x^4\left(a+b\tanh^{-1}(cx)\right)^2+\frac{bx^3\left(a+b\tanh^{-1}(cx)\right)}{6c}+\frac{b^2x^2}{12c^2}+\frac{b^2\log\left(1-c^2x^2\right)}{3c^4}+\frac{b^2x\tanh^{-1}(cx)}{2c^3}$$

```
[Out] (a*b*x)/(2*c^3) + (b^2*x^2)/(12*c^2) + (b^2*x*ArcTanh[c*x])/(2*c^3) + (b*x^
3*(a + b*ArcTanh[c*x]))/(6*c) - (a + b*ArcTanh[c*x])^2/(4*c^4) + (x^4*(a +
b*ArcTanh[c*x])^2)/4 + (b^2*Log[1 - c^2*x^2])/(3*c^4)
```

Rubi [A] time = 0.222507, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5916, 5980, 266, 43, 5910, 260, 5948}

$$\frac{abx}{2c^3}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{4c^4}+\frac{1}{4}x^4\left(a+b\tanh^{-1}(cx)\right)^2+\frac{bx^3\left(a+b\tanh^{-1}(cx)\right)}{6c}+\frac{b^2x^2}{12c^2}+\frac{b^2\log\left(1-c^2x^2\right)}{3c^4}+\frac{b^2x\tanh^{-1}(cx)}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (a*b*x)/(2*c^3) + (b^2*x^2)/(12*c^2) + (b^2*x*ArcTanh[c*x])/(2*c^3) + (b*x^
3*(a + b*ArcTanh[c*x]))/(6*c) - (a + b*ArcTanh[c*x])^2/(4*c^4) + (x^4*(a +
b*ArcTanh[c*x])^2)/4 + (b^2*Log[1 - c^2*x^2])/(3*c^4)
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int x^3\left(a+b\tanh^{-1}(cx)\right)^2 dx = \frac{1}{4}x^4\left(a+b\tanh^{-1}(cx)\right)^2 - \frac{1}{2}(bc)\int \frac{x^4\left(a+b\tanh^{-1}(cx)\right)}{1-c^2x^2}dx$$

$$= \frac{1}{4}x^4\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{b\int x^2\left(a+b\tanh^{-1}(cx)\right)dx}{2c} - \frac{b\int \frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{1-c^2x^2}dx}{2c}$$

$$= \frac{bx^3\left(a+b\tanh^{-1}(cx)\right)}{6c} + \frac{1}{4}x^4\left(a+b\tanh^{-1}(cx)\right)^2 - \frac{1}{6}b^2\int \frac{x^3}{1-c^2x^2}dx + \frac{b\int\left(a+b\,\mathrm{t}\right.}{}$$

$$= \frac{abx}{2c^3} + \frac{bx^3\left(a+b\tanh^{-1}(cx)\right)}{6c} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{4c^4} + \frac{1}{4}x^4\left(a+b\tanh^{-1}(cx)\right)^2 - \frac{1}{12}$$

$$= \frac{abx}{2c^3} + \frac{b^2x\tanh^{-1}(cx)}{2c^3} + \frac{bx^3\left(a+b\tanh^{-1}(cx)\right)}{6c} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{4c^4} + \frac{1}{4}x^4\left(a+b\,\mathrm{t}\right.$$

$$= \frac{abx}{2c^3} + \frac{b^2x^2}{12c^2} + \frac{b^2x\tanh^{-1}(cx)}{2c^3} + \frac{bx^3\left(a+b\tanh^{-1}(cx)\right)}{6c} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{4c^4} + \frac{1}{4}x^4$$

Mathematica [A] time = 0.0588042, size = 132, normalized size = 1.17

$$\frac{3a^2c^4x^4 + 2abc^3x^3 + 2bcx\tanh^{-1}(cx)\left(3ac^3x^3 + b\left(c^2x^2+3\right)\right) + 6abcx + b(3a+4b)\log(1-cx) - 3ab\log(cx+1) + b^2}{12c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (6*a*b*c*x + b^2*c^2*x^2 + 2*a*b*c^3*x^3 + 3*a^2*c^4*x^4 + 2*b*c*x*(3*a*c^3
*x^3 + b*(3 + c^2*x^2))*ArcTanh[c*x] + 3*b^2*(-1 + c^4*x^4)*ArcTanh[c*x]^2
+ b*(3*a + 4*b)*Log[1 - c*x] - 3*a*b*Log[1 + c*x] + 4*b^2*Log[1 + c*x])/(12
*c^4)
```

Maple [B] time = 0.015, size = 278, normalized size = 2.5

$$\frac{a^2x^4}{4} + \frac{b^2x^4\left(\mathrm{Artanh}\,(cx)\right)^2}{4} + \frac{b^2\mathrm{Artanh}\,(cx)\,x^3}{6\,c} + \frac{b^2x\mathrm{Artanh}\,(cx)}{2\,c^3} + \frac{b^2\mathrm{Artanh}\,(cx)\ln(cx-1)}{4\,c^4} - \frac{b^2\mathrm{Artanh}\,(cx)\ln}{4\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c*x))^2,x)
```

```
[Out] 1/4*a^2*x^4+1/4*b^2*x^4*arctanh(c*x)^2+1/6/c*b^2*arctanh(c*x)*x^3+1/2*b^2*x
*arctanh(c*x)/c^3+1/4/c^4*b^2*arctanh(c*x)*ln(c*x-1)-1/4/c^4*b^2*arctanh(c*
x)*ln(c*x+1)+1/16/c^4*b^2*ln(c*x-1)^2-1/8/c^4*b^2*ln(c*x-1)*ln(1/2+1/2*c*x)
-1/8/c^4*b^2*ln(-1/2*c*x+1/2)*ln(c*x+1)+1/8/c^4*b^2*ln(-1/2*c*x+1/2)*ln(1/2
+1/2*c*x)+1/16/c^4*b^2*ln(c*x+1)^2+1/12*b^2*x^2/c^2+1/3/c^4*b^2*ln(c*x-1)+1
/3/c^4*b^2*ln(c*x+1)+1/2*x^4*a*b*arctanh(c*x)+1/6*a*b*x^3/c+1/2*a*b*x/c^3+1
/4/c^4*a*b*ln(c*x-1)-1/4/c^4*a*b*ln(c*x+1)
```

Maxima [A] time = 0.976638, size = 255, normalized size = 2.26

$$\frac{1}{4} b^2 x^4 \operatorname{artanh}(cx)^2 + \frac{1}{4} a^2 x^4 + \frac{1}{12}\left(6 x^4 \operatorname{artanh}(cx) + c\left(\frac{2\left(c^2x^3 + 3x\right)}{c^4} - \frac{3\log(cx+1)}{c^5} + \frac{3\log(cx-1)}{c^5}\right)\right)ab + \frac{1}{48}\left(4c\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*x^4*arctanh(c*x)^2 + 1/4*a^2*x^4 + 1/12*(6*x^4*arctanh(c*x) + c*(2*
(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*a*b + 1/48*
(4*c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5)*arct
anh(c*x) + (4*c^2*x^2 - 2*(3*log(c*x - 1) - 8)*log(c*x + 1) + 3*log(c*x + 1
)^2 + 3*log(c*x - 1)^2 + 16*log(c*x - 1))/c^4)*b^2
```

Fricas [A] time = 2.09657, size = 354, normalized size = 3.13

$$\frac{12\,a^2c^4x^4 + 8\,abc^3x^3 + 4\,b^2c^2x^2 + 24\,abcx + 3\left(b^2c^4x^4 - b^2\right)\log\left(-\frac{cx+1}{cx-1}\right)^2 - 4\left(3\,ab - 4\,b^2\right)\log(cx+1) + 4\left(3\,ab + 4\,b^2\right)\text{lo}}{48\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/48*(12*a^2*c^4*x^4 + 8*a*b*c^3*x^3 + 4*b^2*c^2*x^2 + 24*a*b*c*x + 3*(b^2*
c^4*x^4 - b^2)*log(-(c*x + 1)/(c*x - 1))^2 - 4*(3*a*b - 4*b^2)*log(c*x + 1)
 + 4*(3*a*b + 4*b^2)*log(c*x - 1) + 4*(3*a*b*c^4*x^4 + b^2*c^3*x^3 + 3*b^2*
c*x)*log(-(c*x + 1)/(c*x - 1)))/c^4
```

Sympy [A] time = 2.24423, size = 168, normalized size = 1.49

$$\begin{cases} \frac{a^2x^4}{4} + \frac{abx^4\operatorname{atanh}(cx)}{2} + \frac{abx^3}{6c} + \frac{abx}{2c^3} - \frac{ab\operatorname{atanh}(cx)}{2c^4} + \frac{b^2x^4\operatorname{atanh}^2(cx)}{4} + \frac{b^2x^3\operatorname{atanh}(cx)}{6c} + \frac{b^2x^2}{12c^2} + \frac{b^2x\operatorname{atanh}(cx)}{2c^3} + \frac{2b^2\log\left(x-\frac{1}{c}\right)}{3c^4} - \frac{b^2\operatorname{ata}}{4} \\ \frac{a^2x^4}{4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*x**4/4 + a*b*x**4*atanh(c*x)/2 + a*b*x**3/(6*c) + a*b*x/(2*
c**3) - a*b*atanh(c*x)/(2*c**4) + b**2*x**4*atanh(c*x)**2/4 + b**2*x**3*ata
nh(c*x)/(6*c) + b**2*x**2/(12*c**2) + b**2*x*atanh(c*x)/(2*c**3) + 2*b**2*l
og(x - 1/c)/(3*c**4) - b**2*atanh(c*x)**2/(4*c**4) + 2*b**2*atanh(c*x)/(3*c
**4), Ne(c, 0)), (a**2*x**4/4, True))
```

Giac [A] time = 1.25719, size = 215, normalized size = 1.9

$$\frac{1}{4}\,a^2x^4 + \frac{abx^3}{6\,c} + \frac{1}{16}\left(b^2x^4 - \frac{b^2}{c^4}\right)\log\left(-\frac{cx+1}{cx-1}\right)^2 + \frac{b^2x^2}{12\,c^2} + \frac{1}{12}\left(3\,abx^4 + \frac{b^2x^3}{c} + \frac{3\,b^2x}{c^3}\right)\log\left(-\frac{cx+1}{cx-1}\right) + \frac{abx}{2\,c^3} - (3\,ab -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

```
[Out] 1/4*a^2*x^4 + 1/6*a*b*x^3/c + 1/16*(b^2*x^4 - b^2/c^4)*log(-(c*x + 1)/(c*x
- 1))^2 + 1/12*b^2*x^2/c^2 + 1/12*(3*a*b*x^4 + b^2*x^3/c + 3*b^2*x/c^3)*log
(-(c*x + 1)/(c*x - 1)) + 1/2*a*b*x/c^3 - 1/12*(3*a*b - 4*b^2)*log(c*x + 1)/
c^4 + 1/12*(3*a*b + 4*b^2)*log(c*x - 1)/c^4
```

3.16 $\int x^2 \left(a + b \tanh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=130

$$-\frac{b^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{3c^3}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{3c^3}-\frac{2b\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{3c^3}+\frac{1}{3}x^3\left(a+b\tanh^{-1}(cx)\right)^2+\frac{bx^2\left(a\right.}{}$$

```
[Out] (b^2*x)/(3*c^2) - (b^2*ArcTanh[c*x])/(3*c^3) + (b*x^2*(a + b*ArcTanh[c*x]))
/(3*c) + (a + b*ArcTanh[c*x])^2/(3*c^3) + (x^3*(a + b*ArcTanh[c*x])^2)/3 -
(2*b*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(3*c^3) - (b^2*PolyLog[2, 1 - 2
/(1 - c*x)])/(3*c^3)
```

Rubi [A] time = 0.200816, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5916, 5980, 321, 206, 5984, 5918, 2402, 2315}

$$-\frac{b^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{3c^3}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{3c^3}-\frac{2b\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{3c^3}+\frac{1}{3}x^3\left(a+b\tanh^{-1}(cx)\right)^2+\frac{bx^2\left(a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (b^2*x)/(3*c^2) - (b^2*ArcTanh[c*x])/(3*c^3) + (b*x^2*(a + b*ArcTanh[c*x]))
/(3*c) + (a + b*ArcTanh[c*x])^2/(3*c^3) + (x^3*(a + b*ArcTanh[c*x])^2)/3 -
(2*b*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(3*c^3) - (b^2*PolyLog[2, 1 - 2
/(1 - c*x)])/(3*c^3)
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int x^2\left(a+b\tanh^{-1}(cx)\right)^2dx=\frac{1}{3}x^3\left(a+b\tanh^{-1}(cx)\right)^2-\frac{1}{3}(2bc)\int\frac{x^3\left(a+b\tanh^{-1}(cx)\right)}{1-c^2x^2}dx$$

$$=\frac{1}{3}x^3\left(a+b\tanh^{-1}(cx)\right)^2+\frac{(2b)\int x\left(a+b\tanh^{-1}(cx)\right)dx}{3c}-\frac{(2b)\int\frac{x\left(a+b\tanh^{-1}(cx)\right)}{1-c^2x^2}dx}{3c}$$

$$=\frac{bx^2\left(a+b\tanh^{-1}(cx)\right)}{3c}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{3c^3}+\frac{1}{3}x^3\left(a+b\tanh^{-1}(cx)\right)^2-\frac{1}{3}b^2\int\frac{}{1-}$$

$$=\frac{b^2x}{3c^2}+\frac{bx^2\left(a+b\tanh^{-1}(cx)\right)}{3c}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{3c^3}+\frac{1}{3}x^3\left(a+b\tanh^{-1}(cx)\right)^2-\frac{2b}{}$$

$$=\frac{b^2x}{3c^2}-\frac{b^2\tanh^{-1}(cx)}{3c^3}+\frac{bx^2\left(a+b\tanh^{-1}(cx)\right)}{3c}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{3c^3}+\frac{1}{3}x^3\left(a+b\text{ta}\right.$$

$$=\frac{b^2x}{3c^2}-\frac{b^2\tanh^{-1}(cx)}{3c^3}+\frac{bx^2\left(a+b\tanh^{-1}(cx)\right)}{3c}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{3c^3}+\frac{1}{3}x^3\left(a+b\text{ta}\right.$$

Mathematica [A] time = 0.257234, size = 122, normalized size = 0.94

$$\frac{b^2\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)+a^2c^3x^3+abc^2x^2+ab\log\left(c^2x^2-1\right)+b\tanh^{-1}(cx)\left(2ac^3x^3+bc^2x^2-2b\log\left(e^{-2\tanh}\right.\right.}{3c^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (b^2*c*x + a*b*c^2*x^2 + a^2*c^3*x^3 + b^2*(-1 + c^3*x^3)*ArcTanh[c*x]^2 +
b*ArcTanh[c*x]*(-b + b*c^2*x^2 + 2*a*c^3*x^3 - 2*b*Log[1 + E^(-2*ArcTanh[c*
x])]) + a*b*Log[-1 + c^2*x^2] + b^2*PolyLog[2, -E^(-2*ArcTanh[c*x])])/(3*c^
3)
```

Maple [B] time = 0.013, size = 270, normalized size = 2.1

$$\frac{x^3a^2}{3} + \frac{b^2x^3(\mathrm{Artanh}(cx))^2}{3} + \frac{b^2\mathrm{Artanh}(cx)\,x^2}{3c} + \frac{b^2\mathrm{Artanh}(cx)\ln(cx-1)}{3c^3} + \frac{b^2\mathrm{Artanh}(cx)\ln(cx+1)}{3c^3} + \frac{b^2x}{3c^2} + \frac{b^2\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c*x))^2,x)
```

```
[Out] 1/3*x^3*a^2+1/3*b^2*x^3*arctanh(c*x)^2+1/3/c*b^2*arctanh(c*x)*x^2+1/3/c^3*b
^2*arctanh(c*x)*ln(c*x-1)+1/3/c^3*b^2*arctanh(c*x)*ln(c*x+1)+1/3*b^2*x/c^2+
1/6/c^3*b^2*ln(c*x-1)-1/6/c^3*b^2*ln(c*x+1)+1/12/c^3*b^2*ln(c*x-1)^2-1/3/c^
3*b^2*dilog(1/2+1/2*c*x)-1/6/c^3*b^2*ln(c*x-1)*ln(1/2+1/2*c*x)-1/6/c^3*b^2*
ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)+1/6/c^3*b^2*ln(-1/2*c*x+1/2)*ln(c*x+1)-1/1
2/c^3*b^2*ln(c*x+1)^2+2/3*a*b*x^3*arctanh(c*x)+1/3*a*b*x^2/c+1/3/c^3*a*b*ln
(c*x-1)+1/3/c^3*a*b*ln(c*x+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{3}a^2x^3 + \frac{1}{3}\left(2\,x^3\,\mathrm{artanh}\,(cx) + c\left(\frac{x^2}{c^2} + \frac{\log\left(c^2x^2-1\right)}{c^4}\right)\right)ab - \frac{1}{216}\left(2\,c^4\left(\frac{2\left(c^2x^3+3\,x\right)}{c^6} - \frac{3\,\log\,(cx+1)}{c^7} + \frac{3\,\log\,(cx-1)}{c^7}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*x^3 + 1/3*(2*x^3*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x^2 - 1)/c^4))
*a*b - 1/216*(2*c^4*(2*(c^2*x^3 + 3*x)/c^6 - 3*log(c*x + 1)/c^7 + 3*log(c*x
 - 1)/c^7) - 3*c^3*(x^2/c^4 + log(c^2*x^2 - 1)/c^6) - 648*c^3*integrate(1/9
*x^3*log(c*x + 1)/(c^4*x^2 - c^2), x) + 9*c^2*(2*x/c^4 - log(c*x + 1)/c^5 +
 log(c*x - 1)/c^5) - 324*c*integrate(1/9*x*log(c*x + 1)/(c^4*x^2 - c^2), x)
 - 6*(3*c^3*x^3*log(c*x + 1)^2 + (2*c^3*x^3 - 3*c^2*x^2 + 6*c*x - 6*(c^3*x^
3 + 1)*log(c*x + 1))*log(-c*x + 1))/c^3 - (2*(c*x - 1)^3*(9*log(-c*x + 1)^2
 - 6*log(-c*x + 1) + 2) + 27*(c*x - 1)^2*(2*log(-c*x + 1)^2 - 2*log(-c*x +
1) + 1) + 54*(c*x - 1)*(log(-c*x + 1)^2 - 2*log(-c*x + 1) + 2))/c^3 + 18*lo
g(9*c^4*x^2 - 9*c^2)/c^3 - 324*integrate(1/9*log(c*x + 1)/(c^4*x^2 - c^2),
x))*b^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(b^2x^2\,\mathrm{artanh}\,(cx)^2 + 2\,abx^2\,\mathrm{artanh}\,(cx) + a^2x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^2*arctanh(c*x)^2 + 2*a*b*x^2*arctanh(c*x) + a^2*x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 (a + b \operatorname{atanh}(cx))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c*x))**2,x)
```

```
[Out] Integral(x**2*(a + b*atanh(c*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{artanh}(cx) + a)^2 x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2*x^2, x)
```

3.17 $\int x\left(a+b\tanh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=75

$$-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2c^2}+\frac{1}{2}x^2\left(a+b\tanh^{-1}(cx)\right)^2+\frac{abx}{c}+\frac{b^2\log\left(1-c^2x^2\right)}{2c^2}+\frac{b^2x\tanh^{-1}(cx)}{c}$$

```
[Out] (a*b*x)/c + (b^2*x*ArcTanh[c*x])/c - (a + b*ArcTanh[c*x])^2/(2*c^2) + (x^2*
(a + b*ArcTanh[c*x])^2)/2 + (b^2*Log[1 - c^2*x^2])/(2*c^2)
```

Rubi [A] time = 0.114009, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5916, 5980, 5910, 260, 5948}

$$-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2c^2}+\frac{1}{2}x^2\left(a+b\tanh^{-1}(cx)\right)^2+\frac{abx}{c}+\frac{b^2\log\left(1-c^2x^2\right)}{2c^2}+\frac{b^2x\tanh^{-1}(cx)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (a*b*x)/c + (b^2*x*ArcTanh[c*x])/c - (a + b*ArcTanh[c*x])^2/(2*c^2) + (x^2*
(a + b*ArcTanh[c*x])^2)/2 + (b^2*Log[1 - c^2*x^2])/(2*c^2)
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
```

```
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int x\left(a+b\tanh^{-1}(cx)\right)^2\,dx &= \frac{1}{2}x^2\left(a+b\tanh^{-1}(cx)\right)^2-(bc)\int\frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{1-c^2x^2}\,dx\\
&= \frac{1}{2}x^2\left(a+b\tanh^{-1}(cx)\right)^2+\frac{b\int\left(a+b\tanh^{-1}(cx)\right)dx}{c}-\frac{b\int\frac{a+b\tanh^{-1}(cx)}{1-c^2x^2}dx}{c}\\
&= \frac{abx}{c}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2c^2}+\frac{1}{2}x^2\left(a+b\tanh^{-1}(cx)\right)^2+\frac{b^2\int\tanh^{-1}(cx)\,dx}{c}\\
&= \frac{abx}{c}+\frac{b^2x\tanh^{-1}(cx)}{c}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2c^2}+\frac{1}{2}x^2\left(a+b\tanh^{-1}(cx)\right)^2-b^2\int\frac{x}{1-c^2\ldots}\\
&= \frac{abx}{c}+\frac{b^2x\tanh^{-1}(cx)}{c}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2c^2}+\frac{1}{2}x^2\left(a+b\tanh^{-1}(cx)\right)^2+\frac{b^2\log\left(1-\ldots\right.}{2c^2}
\end{aligned}
$$

Mathematica [A] time = 0.0623569, size = 90, normalized size = 1.2

$$\frac{a^2c^2x^2+2abcx+b(a+b)\log(1-cx)-ab\log(cx+1)+2bcx\tanh^{-1}(cx)(acx+b)+b^2\left(c^2x^2-1\right)\tanh^{-1}(cx)^2+b^2\log\ldots}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (2*a*b*c*x + a^2*c^2*x^2 + 2*b*c*x*(b + a*c*x)*ArcTanh[c*x] + b^2*(-1 + c^2
*x^2)*ArcTanh[c*x]^2 + b*(a + b)*Log[1 - c*x] - a*b*Log[1 + c*x] + b^2*Log[
1 + c*x])/(2*c^2)
```

Maple [B] time = 0.016, size = 239, normalized size = 3.2

$$\frac{a^2x^2}{2}+\frac{x^2b^2\left(\mathrm{Artanh}\,(cx)\right)^2}{2}+\frac{b^2x\mathrm{Artanh}\,(cx)}{c}+\frac{b^2\mathrm{Artanh}\,(cx)\ln(cx-1)}{2\,c^2}-\frac{b^2\mathrm{Artanh}\,(cx)\ln(cx+1)}{2\,c^2}+\frac{b^2\left(\ln(cx\ldots\right.}{8\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c*x))^2,x)
```

```
[Out] 1/2*a^2*x^2+1/2*x^2*b^2*arctanh(c*x)^2+b^2*x*arctanh(c*x)/c+1/2/c^2*b^2*arc
tanh(c*x)*ln(c*x-1)-1/2/c^2*b^2*arctanh(c*x)*ln(c*x+1)+1/8/c^2*b^2*ln(c*x-1
)^2-1/4/c^2*b^2*ln(c*x-1)*ln(1/2+1/2*c*x)+1/2/c^2*b^2*ln(c*x-1)+1/2/c^2*b^2
*ln(c*x+1)-1/4/c^2*b^2*ln(-1/2*c*x+1/2)*ln(c*x+1)+1/4/c^2*b^2*ln(-1/2*c*x+1
/2)*ln(1/2+1/2*c*x)+1/8/c^2*b^2*ln(c*x+1)^2+b*x^2*a*arctanh(c*x)+a*b*x/c+1/
2/c^2*a*b*ln(c*x-1)-1/2/c^2*a*b*ln(c*x+1)
```

Maxima [B] time = 0.99353, size = 213, normalized size = 2.84

$$\frac{1}{2}\,b^2x^2\,\mathrm{artanh}\,(cx)^2+\frac{1}{2}\,a^2x^2+\frac{1}{2}\left(2\,x^2\,\mathrm{artanh}\,(cx)+c\left(\frac{2\,x}{c^2}-\frac{\log(cx+1)}{c^3}+\frac{\log(cx-1)}{c^3}\right)\right)ab+\frac{1}{8}\left(4\,c\left(\frac{2\,x}{c^2}-\frac{\log(cx\ldots}{c^3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^2*arctanh(c*x)^2 + 1/2*a^2*x^2 + 1/2*(2*x^2*arctanh(c*x) + c*(2*x
/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3))*a*b + 1/8*(4*c*(2*x/c^2 - log(
c*x + 1)/c^3 + log(c*x - 1)/c^3)*arctanh(c*x) - (2*(log(c*x - 1) - 2)*log(c
*x + 1) - log(c*x + 1)^2 - log(c*x - 1)^2 - 4*log(c*x - 1))/c^2)*b^2
```

Fricas [A] time = 2.23298, size = 269, normalized size = 3.59

$$\frac{4\,a^2c^2x^2 + 8\,abcx + \left(b^2c^2x^2 - b^2\right)\log\left(-\frac{cx+1}{cx-1}\right)^2 - 4\left(ab - b^2\right)\log\left(cx+1\right) + 4\left(ab + b^2\right)\log\left(cx-1\right) + 4\left(abc^2x^2 + b^2cx\right)\log}{8\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*a^2*c^2*x^2 + 8*a*b*c*x + (b^2*c^2*x^2 - b^2)*log(-(c*x + 1)/(c*x -
1))^2 - 4*(a*b - b^2)*log(c*x + 1) + 4*(a*b + b^2)*log(c*x - 1) + 4*(a*b*c^
2*x^2 + b^2*c*x)*log(-(c*x + 1)/(c*x - 1)))/c^2
```

Sympy [A] time = 1.08891, size = 114, normalized size = 1.52

$$\begin{cases} \frac{a^2x^2}{2} + abx^2\operatorname{atanh}(cx) + \frac{abx}{c} - \frac{ab\operatorname{atanh}(cx)}{c^2} + \frac{b^2x^2\operatorname{atanh}^2(cx)}{2} + \frac{b^2x\operatorname{atanh}(cx)}{c} + \frac{b^2\log\left(x-\frac{1}{c}\right)}{c^2} - \frac{b^2\operatorname{atanh}^2(cx)}{2c^2} + \frac{b^2\operatorname{atanh}(cx)}{c^2} & \text{for} \\ \frac{a^2x^2}{2} & \text{oth} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*x**2/2 + a*b*x**2*atanh(c*x) + a*b*x/c - a*b*atanh(c*x)/c**
2 + b**2*x**2*atanh(c*x)**2/2 + b**2*x*atanh(c*x)/c + b**2*log(x - 1/c)/c**
2 - b**2*atanh(c*x)**2/(2*c**2) + b**2*atanh(c*x)/c**2, Ne(c, 0)), (a**2*x*
*2/2, True))
```

Giac [A] time = 1.21261, size = 163, normalized size = 2.17

$$\frac{1}{2}\,a^2x^2 + \frac{1}{8}\left(b^2x^2 - \frac{b^2}{c^2}\right)\log\left(-\frac{cx+1}{cx-1}\right)^2 + \frac{abx}{c} + \frac{1}{2}\left(abx^2 + \frac{b^2x}{c}\right)\log\left(-\frac{cx+1}{cx-1}\right) - \frac{\left(ab - b^2\right)\log\left(cx+1\right)}{2\,c^2} + \frac{\left(ab + b^2\right)\log}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

```
[Out] 1/2*a^2*x^2 + 1/8*(b^2*x^2 - b^2/c^2)*log(-(c*x + 1)/(c*x - 1))^2 + a*b*x/c
 + 1/2*(a*b*x^2 + b^2*x/c)*log(-(c*x + 1)/(c*x - 1)) - 1/2*(a*b - b^2)*log(
c*x + 1)/c^2 + 1/2*(a*b + b^2)*log(c*x - 1)/c^2
```

3.18 $\int \left(a+b\tanh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=74

$$-\frac{b^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{c}+x\left(a+b\tanh^{-1}(cx)\right)^2+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c}-\frac{2b\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{c}$$

```
[Out] (a + b*ArcTanh[c*x])^2/c + x*(a + b*ArcTanh[c*x])^2 - (2*b*(a + b*ArcTanh[c
*x])*Log[2/(1 - c*x)])/c - (b^2*PolyLog[2, 1 - 2/(1 - c*x)])/c
```

Rubi [A] time = 0.101124, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5910, 5984, 5918, 2402, 2315}

$$-\frac{b^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{c}+x\left(a+b\tanh^{-1}(cx)\right)^2+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c}-\frac{2b\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (a + b*ArcTanh[c*x])^2/c + x*(a + b*ArcTanh[c*x])^2 - (2*b*(a + b*ArcTanh[c
*x])*Log[2/(1 - c*x)])/c - (b^2*PolyLog[2, 1 - 2/(1 - c*x)])/c
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\tanh^{-1}(cx)\right)^2 dx &= x\left(a+b\tanh^{-1}(cx)\right)^2-(2bc)\int \frac{x\left(a+b\tanh^{-1}(cx)\right)}{1-c^2x^2}dx\\ &= \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c}+x\left(a+b\tanh^{-1}(cx)\right)^2-(2b)\int \frac{a+b\tanh^{-1}(cx)}{1-cx}dx\\ &= \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c}+x\left(a+b\tanh^{-1}(cx)\right)^2-\frac{2b\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1-cx}\right)}{c}+\left(2b^2\right)\int\\ &= \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c}+x\left(a+b\tanh^{-1}(cx)\right)^2-\frac{2b\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1-cx}\right)}{c}-\frac{\left(2b^2\right)\text{Su}}{}\\ &= \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c}+x\left(a+b\tanh^{-1}(cx)\right)^2-\frac{2b\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1-cx}\right)}{c}-\frac{b^2\text{Li}_2\left(1\right.}{c}\end{aligned}$$

Mathematica [A] time = 0.142661, size = 82, normalized size = 1.11

$$\frac{b^2\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)+a\left(acx+b\log\left(1-c^2x^2\right)\right)+2b\tanh^{-1}(cx)\left(acx-b\log\left(e^{-2\tanh^{-1}(cx)}+1\right)\right)+b^2(cx-1)\,\text{t}}{c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (b^2*(-1 + c*x)*ArcTanh[c*x]^2 + 2*b*ArcTanh[c*x]*(a*c*x - b*Log[1 + E^(-2*
ArcTanh[c*x])]) + a*(a*c*x + b*Log[1 - c^2*x^2]) + b^2*PolyLog[2, -E^(-2*Ar
cTanh[c*x])])/c
```

Maple [A] time = 0.049, size = 123, normalized size = 1.7

$$xb^2\left(\text{Artanh}(cx)\right)^2+2xab\text{Artanh}(cx)+\frac{b^2\left(\text{Artanh}(cx)\right)^2}{c}-2\frac{\text{Artanh}(cx)\,b^2}{c}\ln\left(\frac{(cx+1)^2}{-c^2x^2+1}+1\right)+a^2x-\frac{b^2}{c}\text{polylog}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^2,x)
```

```
[Out] x*b^2*arctanh(c*x)^2+2*x*a*b*arctanh(c*x)+1/c*b^2*arctanh(c*x)^2-2/c*arctan
h(c*x)*ln((c*x+1)^2/(-c^2*x^2+1)+1)*b^2+a^2*x-1/c*polylog(2,-(c*x+1)^2/(-c^
2*x^2+1))*b^2+1/c*a*b*ln(-c^2*x^2+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{4}\left(c^2\left(\frac{2x}{c^2}-\frac{\log(cx+1)}{c^3}+\frac{\log(cx-1)}{c^3}\right)-6c\int\frac{x\log(cx+1)}{c^2x^2-1}dx-\frac{(cx-1)\left(\log(-cx+1)^2-2\log(-cx+1)+2\right)}{c}-\frac{cx}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -1/4*(c^2*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3) - 6*c*integrate(x
*log(c*x + 1)/(c^2*x^2 - 1), x) - (c*x - 1)*(log(-c*x + 1)^2 - 2*log(-c*x +
 1) + 2)/c - (c*x*log(c*x + 1)^2 + 2*(c*x - (c*x + 1)*log(c*x + 1))*log(-c*
x + 1))/c + log(c^2*x^2 - 1)/c - 2*integrate(log(c*x + 1)/(c^2*x^2 - 1), x)
)*b^2 + a^2*x + (2*c*x*arctanh(c*x) + log(-c^2*x^2 + 1))*a*b/c
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(b^2 \operatorname{artanh}(cx)^2 + 2\,ab \operatorname{artanh}(cx) + a^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \operatorname{atanh}(cx))^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**2,x)
```

```
[Out] Integral((a + b*atanh(c*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{artanh}(cx) + a)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2, x)
```

3.19 $\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x}\,dx$

Optimal. Leaf size=117

$$-b\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)+b\text{PolyLog}\left(2,\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{2}b^2\text{PolyLog}\left(3,1-\frac{2}{1-\right.$$

```
[Out] 2*(a + b*ArcTanh[c*x])^2*ArcTanh[1 - 2/(1 - c*x)] - b*(a + b*ArcTanh[c*x])*
PolyLog[2, 1 - 2/(1 - c*x)] + b*(a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 -
 c*x)] + (b^2*PolyLog[3, 1 - 2/(1 - c*x)])/2 - (b^2*PolyLog[3, -1 + 2/(1 -
c*x)])/2
```

Rubi [A] time = 0.263671, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5914, 6052, 5948, 6058, 6610}

$$-b\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)+b\text{PolyLog}\left(2,\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{2}b^2\text{PolyLog}\left(3,1-\frac{2}{1-\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^2/x,x]
```


```
[Out] 2*(a + b*ArcTanh[c*x])^2*ArcTanh[1 - 2/(1 - c*x)] - b*(a + b*ArcTanh[c*x])*
PolyLog[2, 1 - 2/(1 - c*x)] + b*(a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 -
 c*x)] + (b^2*PolyLog[3, 1 - 2/(1 - c*x)])/2 - (b^2*PolyLog[3, -1 + 2/(1 -
c*x)])/2
```


Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
 e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
```

```
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x}\,dx = 2\left(a+b\tanh^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx}\right)-(4bc)\int \frac{\left(a+b\tanh^{-1}(cx)\right)\tanh^{-1}\left(1-\frac{2}{1-c}\right.}{1-c^2x^2}$$

$$= 2\left(a+b\tanh^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx}\right)+(2bc)\int \frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1-cx}\right)}{1-c^2x^2}\,dx-$$

$$= 2\left(a+b\tanh^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx}\right)-b\left(a+b\tanh^{-1}(cx)\right)\mathrm{Li}_2\left(1-\frac{2}{1-cx}\right)+b\left(a\right.$$

$$= 2\left(a+b\tanh^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx}\right)-b\left(a+b\tanh^{-1}(cx)\right)\mathrm{Li}_2\left(1-\frac{2}{1-cx}\right)+b\left(a\right.$$

Mathematica [A] time = 0.0715302, size = 120, normalized size = 1.03

$$\frac{1}{2}b\left(2\text{PolyLog}\left(2,\frac{cx+1}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)-2\text{PolyLog}\left(2,\frac{cx+1}{cx-1}\right)\left(a+b\tanh^{-1}(cx)\right)+b\left(\text{PolyLog}\left(3,\frac{cx+1}{cx-1}\right)-\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])^2/x,x]
```

```
[Out] 2*(a + b*ArcTanh[c*x])^2*ArcTanh[(1 + c*x)/(-1 + c*x)] + (b*(2*(a + b*ArcTa
nh[c*x])*PolyLog[2, (1 + c*x)/(1 - c*x)] - 2*(a + b*ArcTanh[c*x])*PolyLog[2
, (1 + c*x)/(-1 + c*x)] + b*(-PolyLog[3, (1 + c*x)/(1 - c*x)] + PolyLog[3,
(1 + c*x)/(-1 + c*x)])))/2
```

Maple [C] time = 0.214, size = 701, normalized size = 6.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^2/x,x)
```

```
[Out] a^2*ln(c*x)+b^2*ln(c*x)*arctanh(c*x)^2-b^2*arctanh(c*x)*polylog(2,-(c*x+1)^
2/(-c^2*x^2+1))+1/2*b^2*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))-b^2*arctanh(c*x)
^2*ln((c*x+1)^2/(-c^2*x^2+1)-1)+b^2*arctanh(c*x)^2*ln(1-(c*x+1)/(-c^2*x^2+1
)^(1/2))+2*b^2*arctanh(c*x)*polylog(2,(c*x+1)/(-c^2*x^2+1)^(1/2))-2*b^2*pol
ylog(3,(c*x+1)/(-c^2*x^2+1)^(1/2))+b^2*arctanh(c*x)^2*ln(1+(c*x+1)/(-c^2*x^
2+1)^(1/2))+2*b^2*arctanh(c*x)*polylog(2,-(c*x+1)/(-c^2*x^2+1)^(1/2))-2*b^2
*polylog(3,-(c*x+1)/(-c^2*x^2+1)^(1/2))+1/2*I*b^2*Pi*csgn(I*((c*x+1)^2/(-c^
2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^3*arctanh(c*x)^2-1/2*I*b^2*Pi*csgn(
I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/
(-c^2*x^2+1)+1))^2*arctanh(c*x)^2+1/2*I*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+
```

```
1)-1))*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)
/((c*x+1)^2/(-c^2*x^2+1)+1))*arctanh(c*x)^2-1/2*I*b^2*Pi*csgn(I*((c*x+1)^2/
(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+
1))^2*arctanh(c*x)^2+2*a*b*ln(c*x)*arctanh(c*x)-a*b*ln(c*x)*ln(c*x+1)-a*b*d
ilog(c*x)-a*b*dilog(c*x+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$a^2 \log(x) + \int \frac{b^2(\log(cx+1) - \log(-cx+1))^2}{4x} + \frac{ab(\log(cx+1) - \log(-cx+1))}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x,x, algorithm="maxima")
```

```
[Out] a^2*log(x) + integrate(1/4*b^2*(log(c*x + 1) - log(-c*x + 1))^2/x + a*b*(lo
g(c*x + 1) - log(-c*x + 1))/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2 \operatorname{artanh}(cx)^2 + 2\,ab \operatorname{artanh}(cx) + a^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{atanh}(cx))^2}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**2/x,x)
```

```
[Out] Integral((a + b*atanh(c*x))**2/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{artanh}(cx) + a)^2}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2/x, x)
```

3.20 $\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^2}\,dx$

Optimal. Leaf size=71

$$b^2(-c)\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)+c\left(a+b\tanh^{-1}(cx)\right)^2-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x}+2bc\log\left(2-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(c\right.$$

```
[Out] c*(a + b*ArcTanh[c*x])^2 - (a + b*ArcTanh[c*x])^2/x + 2*b*c*(a + b*ArcTanh[
c*x])*Log[2 - 2/(1 + c*x)] - b^2*c*PolyLog[2, -1 + 2/(1 + c*x)]
```

Rubi [A] time = 0.145297, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5916, 5988, 5932, 2447}

$$b^2(-c)\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)+c\left(a+b\tanh^{-1}(cx)\right)^2-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x}+2bc\log\left(2-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(c\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^2/x^2,x]
```

```
[Out] c*(a + b*ArcTanh[c*x])^2 - (a + b*ArcTanh[c*x])^2/x + 2*b*c*(a + b*ArcTanh[
c*x])*Log[2 - 2/(1 + c*x)] - b^2*c*PolyLog[2, -1 + 2/(1 + c*x)]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^2}\,dx = -\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x} + (2bc)\int \frac{a+b\tanh^{-1}(cx)}{x\left(1-c^2x^2\right)}\,dx$$
$$= c\left(a+b\tanh^{-1}(cx)\right)^2 - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x} + (2bc)\int \frac{a+b\tanh^{-1}(cx)}{x(1+cx)}\,dx$$
$$= c\left(a+b\tanh^{-1}(cx)\right)^2 - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x} + 2bc\left(a+b\tanh^{-1}(cx)\right)\log\left(2-\frac{2}{1+cx}\right) - (2$$
$$= c\left(a+b\tanh^{-1}(cx)\right)^2 - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x} + 2bc\left(a+b\tanh^{-1}(cx)\right)\log\left(2-\frac{2}{1+cx}\right) - b^2$$

Mathematica [A] time = 0.143522, size = 94, normalized size = 1.32

$$\frac{-b^2cx\text{PolyLog}\left(2,e^{-2\tanh^{-1}(cx)}\right) - a\left(a+bcx\log\left(1-c^2x^2\right)-2bcx\log(cx)\right) + 2b\tanh^{-1}(cx)\left(bcx\log\left(1-e^{-2\tanh^{-1}(cx)}\right) - \right.}{x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])^2/x^2,x]
```

```
[Out] (b^2*(-1 + c*x)*ArcTanh[c*x]^2 + 2*b*ArcTanh[c*x]*(-a + b*c*x*Log[1 - E^(-2
*ArcTanh[c*x])]) - a*(a - 2*b*c*x*Log[c*x] + b*c*x*Log[1 - c^2*x^2]) - b^2*
c*x*PolyLog[2, E^(-2*ArcTanh[c*x])])/x
```

Maple [B] time = 0.021, size = 248, normalized size = 3.5

$$-\frac{a^2}{x} - \frac{b^2\left(\text{Artanh}(cx)\right)^2}{x} - cb^2\text{Artanh}(cx)\ln(cx-1) + 2cb^2\ln(cx)\text{Artanh}(cx) - cb^2\text{Artanh}(cx)\ln(cx+1) - \frac{cb^2(\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^2/x^2,x)
```

```
[Out] -a^2/x-b^2/x*arctanh(c*x)^2-c*b^2*arctanh(c*x)*ln(c*x-1)+2*c*b^2*ln(c*x)*ar
ctanh(c*x)-c*b^2*arctanh(c*x)*ln(c*x+1)-1/4*c*b^2*ln(c*x-1)^2+c*b^2*dilog(1
/2+1/2*c*x)+1/2*c*b^2*ln(c*x-1)*ln(1/2+1/2*c*x)+1/2*c*b^2*ln(-1/2*c*x+1/2)*
ln(1/2+1/2*c*x)-1/2*c*b^2*ln(-1/2*c*x+1/2)*ln(c*x+1)+1/4*c*b^2*ln(c*x+1)^2-
c*b^2*dilog(c*x)-c*b^2*dilog(c*x+1)-c*b^2*ln(c*x)*ln(c*x+1)-2*a*b/x*arctanh
(c*x)-c*a*b*ln(c*x-1)+2*c*a*b*ln(c*x)-c*a*b*ln(c*x+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\left(c\left(\log\left(c^2x^2-1\right)-\log\left(x^2\right)\right)+\frac{2\,\text{artanh}(cx)}{x}\right)ab - \frac{1}{4}b^2\left(\frac{\log(-cx+1)^2}{x} + \int -\frac{(cx-1)\log(cx+1)^2 + 2(cx-(cx-1)\log}{cx^3-x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^2,x, algorithm="maxima")
```

```
[Out] -(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*a*b - 1/4*b^2*(log(-c
*x + 1)^2/x + integrate(-((c*x - 1)*log(c*x + 1)^2 + 2*(c*x - (c*x - 1)*log
(c*x + 1))*log(-c*x + 1))/(c*x^3 - x^2), x)) - a^2/x
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2\,\text{artanh}\,(cx)^2 + 2\,ab\,\text{artanh}\,(cx) + a^2}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\,\text{atanh}\,(cx))^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**2/x**2,x)
```

```
[Out] Integral((a + b*atanh(c*x))**2/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\text{artanh}\,(cx) + a)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2/x^2, x)
```

3.21 $\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^3}\,dx$

Optimal. Leaf size=80

$$\frac{1}{2}c^2\left(a+b\tanh^{-1}(cx)\right)^2-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2}-\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{x}-\frac{1}{2}b^2c^2\log\left(1-c^2x^2\right)+b^2c^2\log(x)$$

```
[Out] -((b*c*(a + b*ArcTanh[c*x]))/x) + (c^2*(a + b*ArcTanh[c*x])^2)/2 - (a + b*A
rcTanh[c*x])^2/(2*x^2) + b^2*c^2*Log[x] - (b^2*c^2*Log[1 - c^2*x^2])/2
```

Rubi [A] time = 0.13302, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5916, 5982, 266, 36, 29, 31, 5948}

$$\frac{1}{2}c^2\left(a+b\tanh^{-1}(cx)\right)^2-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2}-\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{x}-\frac{1}{2}b^2c^2\log\left(1-c^2x^2\right)+b^2c^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^2/x^3,x]
```

```
[Out] -((b*c*(a + b*ArcTanh[c*x]))/x) + (c^2*(a + b*ArcTanh[c*x])^2)/2 - (a + b*A
rcTanh[c*x])^2/(2*x^2) + b^2*c^2*Log[x] - (b^2*c^2*Log[1 - c^2*x^2])/2
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^3}\,dx = -\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2}+(bc)\int \frac{a+b\tanh^{-1}(cx)}{x^2\left(1-c^2x^2\right)}\,dx$$

$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2}+(bc)\int \frac{a+b\tanh^{-1}(cx)}{x^2}\,dx+\left(bc^3\right)\int \frac{a+b\tanh^{-1}(cx)}{1-c^2x^2}\,dx$$

$$= -\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{x}+\frac{1}{2}c^2\left(a+b\tanh^{-1}(cx)\right)^2-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2}+\left(b^2c^2\right)\int \frac{}{x(}$$

$$= -\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{x}+\frac{1}{2}c^2\left(a+b\tanh^{-1}(cx)\right)^2-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2}+\frac{1}{2}\left(b^2c^2\right)\text{Sul}$$

$$= -\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{x}+\frac{1}{2}c^2\left(a+b\tanh^{-1}(cx)\right)^2-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2}+\frac{1}{2}\left(b^2c^2\right)\text{Sul}$$

$$= -\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{x}+\frac{1}{2}c^2\left(a+b\tanh^{-1}(cx)\right)^2-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2}+b^2c^2\log(x)-$$

Mathematica [A] time = 0.0676631, size = 101, normalized size = 1.26

$$-\frac{a^2+bc^2x^2(a+b)\log(1-cx)-bc^2x^2(a-b)\log(cx+1)+2abcx+2b\tanh^{-1}(cx)(a+bcx)-2b^2c^2x^2\log(x)-b^2\left(c^2x\right.}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])^2/x^3,x]
```

```
[Out] -(a^2 + 2*a*b*c*x + 2*b*(a + b*c*x)*ArcTanh[c*x] - b^2*(-1 + c^2*x^2)*ArcTa
nh[c*x]^2 - 2*b^2*c^2*x^2*Log[x] + b*(a + b)*c^2*x^2*Log[1 - c*x] - (a - b)
*b*c^2*x^2*Log[1 + c*x])/(2*x^2)
```

Maple [B] time = 0.019, size = 253, normalized size = 3.2

$$-\frac{a^2}{2x^2}-\frac{b^2(\text{Artanh}(cx))^2}{2x^2}-\frac{cb^2\text{Artanh}(cx)}{x}-\frac{c^2b^2\text{Artanh}(cx)\ln(cx-1)}{2}+\frac{c^2b^2\text{Artanh}(cx)\ln(cx+1)}{2}-\frac{c^2b^2(\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^2/x^3,x)
```

```
[Out] -1/2*a^2/x^2-1/2*b^2/x^2*arctanh(c*x)^2-c*b^2*arctanh(c*x)/x-1/2*c^2*b^2*ar
ctanh(c*x)*ln(c*x-1)+1/2*c^2*b^2*arctanh(c*x)*ln(c*x+1)-1/8*c^2*b^2*ln(c*x-
```

```
1)^2+1/4*c^2*b^2*ln(c*x-1)*ln(1/2+1/2*c*x)-1/2*c^2*b^2*ln(c*x-1)+c^2*b^2*ln
(c*x)-1/2*c^2*b^2*ln(c*x+1)-1/4*c^2*b^2*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)+1/
4*c^2*b^2*ln(-1/2*c*x+1/2)*ln(c*x+1)-1/8*c^2*b^2*ln(c*x+1)^2-a*b/x^2*arctan
h(c*x)-a*b*c/x-1/2*c^2*a*b*ln(c*x-1)+1/2*c^2*a*b*ln(c*x+1)
```

Maxima [B] time = 1.00399, size = 204, normalized size = 2.55

$$\frac{1}{2}\left(\left(c\log\left(cx+1\right)-c\log\left(cx-1\right)-\frac{2}{x}\right)c-\frac{2\,\operatorname{artanh}\left(cx\right)}{x^2}\right)ab+\frac{1}{8}\left(\left(2\left(\log\left(cx-1\right)-2\right)\log\left(cx+1\right)-\log\left(cx+1\right)^2-\log\left(c\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*a*b +
1/8*((2*(log(c*x - 1) - 2)*log(c*x + 1) - log(c*x + 1)^2 - log(c*x - 1)^2 -
 4*log(c*x - 1) + 8*log(x))*c^2 + 4*(c*log(c*x + 1) - c*log(c*x - 1) - 2/x)
*c*arctanh(c*x))*b^2 - 1/2*b^2*arctanh(c*x)^2/x^2 - 1/2*a^2/x^2
```

Fricas [A] time = 2.36725, size = 300, normalized size = 3.75

$$\frac{8\,b^2c^2x^2\log\left(x\right)+4\left(ab-b^2\right)c^2x^2\log\left(cx+1\right)-4\left(ab+b^2\right)c^2x^2\log\left(cx-1\right)-8\,abcx+\left(b^2c^2x^2-b^2\right)\log\left(-\frac{cx+1}{cx-1}\right)^2-4\,a^2-}{8\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/8*(8*b^2*c^2*x^2*log(x) + 4*(a*b - b^2)*c^2*x^2*log(c*x + 1) - 4*(a*b + b
^2)*c^2*x^2*log(c*x - 1) - 8*a*b*c*x + (b^2*c^2*x^2 - b^2)*log(-(c*x + 1)/(
c*x - 1))^2 - 4*a^2 - 4*(b^2*c*x + a*b)*log(-(c*x + 1)/(c*x - 1)))/x^2
```

Sympy [A] time = 1.45457, size = 126, normalized size = 1.58

$$\begin{cases} -\frac{a^2}{2x^2}+abc^2\operatorname{atanh}\left(cx\right)-\frac{abc}{x}-\frac{ab\operatorname{atanh}\left(cx\right)}{x^2}+b^2c^2\log\left(x\right)-b^2c^2\log\left(x-\frac{1}{c}\right)+\frac{b^2c^2\operatorname{atanh}^2\left(cx\right)}{2}-b^2c^2\operatorname{atanh}\left(cx\right)-\frac{b^2c\operatorname{atanh}}{x} \\ -\frac{a^2}{2x^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**2/x**3,x)
```

```
[Out] Piecewise((-a**2/(2*x**2) + a*b*c**2*atanh(c*x) - a*b*c/x - a*b*atanh(c*x)/
x**2 + b**2*c**2*log(x) - b**2*c**2*log(x - 1/c) + b**2*c**2*atanh(c*x)**2/
2 - b**2*c**2*atanh(c*x) - b**2*c*atanh(c*x)/x - b**2*atanh(c*x)**2/(2*x**2
), Ne(c, 0)), (-a**2/(2*x**2), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(b\operatorname{artanh}\left(cx\right)+a\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2/x^3, x)
```

3.22 $\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^4}\,dx$

Optimal. Leaf size=130

$$-\frac{1}{3}b^2c^3\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)+\frac{1}{3}c^3\left(a+b\tanh^{-1}(cx)\right)^2+\frac{2}{3}bc^3\log\left(2-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)-\frac{bc\left(a+b\tanh^{-1}\right.}{3x^2}$$

```
[Out] -(b^2*c^2)/(3*x) + (b^2*c^3*ArcTanh[c*x])/3 - (b*c*(a + b*ArcTanh[c*x]))/(3
*x^2) + (c^3*(a + b*ArcTanh[c*x])^2)/3 - (a + b*ArcTanh[c*x])^2/(3*x^3) + (
2*b*c^3*(a + b*ArcTanh[c*x])*Log[2 - 2/(1 + c*x)])/3 - (b^2*c^3*PolyLog[2,
-1 + 2/(1 + c*x)])/3
```

Rubi [A] time = 0.230829, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5916, 5982, 325, 206, 5988, 5932, 2447}

$$-\frac{1}{3}b^2c^3\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)+\frac{1}{3}c^3\left(a+b\tanh^{-1}(cx)\right)^2+\frac{2}{3}bc^3\log\left(2-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)-\frac{bc\left(a+b\tanh^{-1}\right.}{3x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^2/x^4,x]
```

```
[Out] -(b^2*c^2)/(3*x) + (b^2*c^3*ArcTanh[c*x])/3 - (b*c*(a + b*ArcTanh[c*x]))/(3
*x^2) + (c^3*(a + b*ArcTanh[c*x])^2)/3 - (a + b*ArcTanh[c*x])^2/(3*x^3) + (
2*b*c^3*(a + b*ArcTanh[c*x])*Log[2 - 2/(1 + c*x)])/3 - (b^2*c^3*PolyLog[2,
-1 + 2/(1 + c*x)])/3
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^4}\,dx &= -\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{3x^3}+\frac{1}{3}(2bc)\int \frac{a+b\tanh^{-1}(cx)}{x^3\left(1-c^2x^2\right)}\,dx \\
&= -\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{3x^3}+\frac{1}{3}(2bc)\int \frac{a+b\tanh^{-1}(cx)}{x^3}\,dx+\frac{1}{3}\left(2bc^3\right)\int \frac{a+b\tanh^{-1}(cx)}{x\left(1-c^2x^2\right)} \\
&= -\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{3x^2}+\frac{1}{3}c^3\left(a+b\tanh^{-1}(cx)\right)^2-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{3x^3}+\frac{1}{3}\left(b^2c^2\right)\int \\
&= -\frac{b^2c^2}{3x}-\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{3x^2}+\frac{1}{3}c^3\left(a+b\tanh^{-1}(cx)\right)^2-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{3x^3}+\frac{2}{3}bc^3 \\
&= -\frac{b^2c^2}{3x}+\frac{1}{3}b^2c^3\tanh^{-1}(cx)-\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{3x^2}+\frac{1}{3}c^3\left(a+b\tanh^{-1}(cx)\right)^2-
\end{aligned}$$

Mathematica [A] time = 0.34386, size = 145, normalized size = 1.12

$$-\frac{b^2c^3x^3\text{PolyLog}\left(2,e^{-2\tanh^{-1}(cx)}\right)+a^2-2abc^3x^3\log(cx)+abc^3x^3\log\left(1-c^2x^2\right)+b\tanh^{-1}(cx)\left(2a-bc^3x^3-2bc^3x^3\right.}{3x^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])^2/x^4,x]
```

```
[Out] -(a^2 + a*b*c*x + b^2*c^2*x^2 + b^2*(1 - c^3*x^3)*ArcTanh[c*x]^2 + b*ArcTan
h[c*x]*(2*a + b*c*x - b*c^3*x^3 - 2*b*c^3*x^3*Log[1 - E^(-2*ArcTanh[c*x])])
 - 2*a*b*c^3*x^3*Log[c*x] + a*b*c^3*x^3*Log[1 - c^2*x^2] + b^2*c^3*x^3*Poly
Log[2, E^(-2*ArcTanh[c*x])])/(3*x^3)
```

Maple [B] time = 0.02, size = 339, normalized size = 2.6

$$-\frac{a^2}{3x^3}-\frac{b^2(\operatorname{Artanh}(cx))^2}{3x^3}-\frac{c^3b^2\operatorname{Artanh}(cx)\ln(cx-1)}{3}-\frac{cb^2\operatorname{Artanh}(cx)}{3x^2}+\frac{2c^3b^2\ln(cx)\operatorname{Artanh}(cx)}{3}-\frac{c^3b^2\operatorname{Artanh}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^2/x^4,x)
```

```
[Out] -1/3*a^2/x^3-1/3*b^2/x^3*arctanh(c*x)^2-1/3*c^3*b^2*arctanh(c*x)*ln(c*x-1)-
1/3*c*b^2*arctanh(c*x)/x^2+2/3*c^3*b^2*ln(c*x)*arctanh(c*x)-1/3*c^3*b^2*arc
tanh(c*x)*ln(c*x+1)-1/3*b^2*c^2/x-1/6*c^3*b^2*ln(c*x-1)+1/6*c^3*b^2*ln(c*x+
1)-1/12*c^3*b^2*ln(c*x-1)^2+1/3*c^3*b^2*dilog(1/2+1/2*c*x)+1/6*c^3*b^2*ln(c
*x-1)*ln(1/2+1/2*c*x)+1/6*c^3*b^2*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)-1/6*c^3*
b^2*ln(-1/2*c*x+1/2)*ln(c*x+1)+1/12*c^3*b^2*ln(c*x+1)^2-1/3*c^3*b^2*dilog(c
*x)-1/3*c^3*b^2*dilog(c*x+1)-1/3*c^3*b^2*ln(c*x)*ln(c*x+1)-2/3*a*b/x^3*arct
anh(c*x)-1/3*c^3*a*b*ln(c*x-1)-1/3*c*a*b/x^2+2/3*c^3*a*b*ln(c*x)-1/3*c^3*a*
b*ln(c*x+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{3}\left(\left(c^2\log\left(c^2x^2-1\right)-c^2\log\left(x^2\right)+\frac{1}{x^2}\right)c+\frac{2\operatorname{artanh}(cx)}{x^3}\right)ab-\frac{1}{12}b^2\left(\frac{\log(-cx+1)^2}{x^3}+3\int-\frac{3(cx-1)\log(cx+1)^2+}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/x^3)
*a*b - 1/12*b^2*(log(-c*x + 1)^2/x^3 + 3*integrate(-1/3*(3*(c*x - 1)*log(c*
x + 1)^2 + 2*(c*x - 3*(c*x - 1)*log(c*x + 1))*log(-c*x + 1))/(c*x^5 - x^4),
 x)) - 1/3*a^2/x^3
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{b^2\operatorname{artanh}(cx)^2+2ab\operatorname{artanh}(cx)+a^2}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^4,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)/x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(a+b\operatorname{atanh}(cx))^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**2/x**4,x)
```

```
[Out] Integral((a + b*atanh(c*x))**2/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{artanh}(cx) + a)^2}{x^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2/x^4, x)
```

3.23 $\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^5}\,dx$

Optimal. Leaf size=117

$$\frac{1}{4}c^4\left(a+b\tanh^{-1}(cx)\right)^2-\frac{bc^3\left(a+b\tanh^{-1}(cx)\right)}{2x}-\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{6x^3}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{4x^4}-\frac{b^2c^2}{12x^2}-\frac{1}{3}b^2c^4\log\left(1-\right.$$

```
[Out] -(b^2*c^2)/(12*x^2) - (b*c*(a + b*ArcTanh[c*x]))/(6*x^3) - (b*c^3*(a + b*Ar
cTanh[c*x]))/(2*x) + (c^4*(a + b*ArcTanh[c*x])^2)/4 - (a + b*ArcTanh[c*x])^
2/(4*x^4) + (2*b^2*c^4*Log[x])/3 - (b^2*c^4*Log[1 - c^2*x^2])/3
```

Rubi [A] time = 0.228329, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5916, 5982, 266, 44, 36, 29, 31, 5948}

$$\frac{1}{4}c^4\left(a+b\tanh^{-1}(cx)\right)^2-\frac{bc^3\left(a+b\tanh^{-1}(cx)\right)}{2x}-\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{6x^3}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{4x^4}-\frac{b^2c^2}{12x^2}-\frac{1}{3}b^2c^4\log\left(1-\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^2/x^5,x]
```


```
[Out] -(b^2*c^2)/(12*x^2) - (b*c*(a + b*ArcTanh[c*x]))/(6*x^3) - (b*c^3*(a + b*Ar
cTanh[c*x]))/(2*x) + (c^4*(a + b*ArcTanh[c*x])^2)/4 - (a + b*ArcTanh[c*x])^
2/(4*x^4) + (2*b^2*c^4*Log[x])/3 - (b^2*c^4*Log[1 - c^2*x^2])/3
```


Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^5}\,dx = -\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{4x^4}+\frac{1}{2}(bc)\int \frac{a+b\tanh^{-1}(cx)}{x^4\left(1-c^2x^2\right)}\,dx$$

$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{4x^4}+\frac{1}{2}(bc)\int \frac{a+b\tanh^{-1}(cx)}{x^4}\,dx+\frac{1}{2}\left(bc^3\right)\int \frac{a+b\tanh^{-1}(cx)}{x^2\left(1-c^2x^2\right)}\,dx$$

$$= -\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{6x^3}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{4x^4}+\frac{1}{6}\left(b^2c^2\right)\int \frac{1}{x^3\left(1-c^2x^2\right)}\,dx+\frac{1}{2}\left(bc^3\right)$$

$$= -\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{6x^3}-\frac{bc^3\left(a+b\tanh^{-1}(cx)\right)}{2x}+\frac{1}{4}c^4\left(a+b\tanh^{-1}(cx)\right)^2-\frac{\left(a+b\tan\right.}{4}$$

$$= -\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{6x^3}-\frac{bc^3\left(a+b\tanh^{-1}(cx)\right)}{2x}+\frac{1}{4}c^4\left(a+b\tanh^{-1}(cx)\right)^2-\frac{\left(a+b\tan\right.}{4}$$

$$= -\frac{b^2c^2}{12x^2}-\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{6x^3}-\frac{bc^3\left(a+b\tanh^{-1}(cx)\right)}{2x}+\frac{1}{4}c^4\left(a+b\tanh^{-1}(cx)\right)^2-\frac{(a}{}$$

$$= -\frac{b^2c^2}{12x^2}-\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{6x^3}-\frac{bc^3\left(a+b\tanh^{-1}(cx)\right)}{2x}+\frac{1}{4}c^4\left(a+b\tanh^{-1}(cx)\right)^2-\frac{(a}{}$$

Mathematica [A] time = 0.0691125, size = 164, normalized size = 1.4

$$-\frac{3a^2+6abc^3x^3+3abc^4x^4\log(1-cx)-3abc^4x^4\log(cx+1)+2b\tanh^{-1}(cx)\left(3a+3bc^3x^3+bcx\right)+2abcx+b^2c^2x^2-}{12x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])^2/x^5,x]
```

```
[Out] -(3*a^2 + 2*a*b*c*x + b^2*c^2*x^2 + 6*a*b*c^3*x^3 + 2*b*(3*a + b*c*x + 3*b*
c^3*x^3)*ArcTanh[c*x] - 3*b^2*(-1 + c^4*x^4)*ArcTanh[c*x]^2 - 8*b^2*c^4*x^4
*Log[x] + 3*a*b*c^4*x^4*Log[1 - c*x] + 4*b^2*c^4*x^4*Log[1 - c*x] - 3*a*b*c
```

```
^4*x^4*Log[1 + c*x] + 4*b^2*c^4*x^4*Log[1 + c*x])/(12*x^4)
```

Maple [B] time = 0.026, size = 290, normalized size = 2.5

$$-\frac{a^2}{4x^4}-\frac{b^2(\mathrm{Artanh}(cx))^2}{4x^4}-\frac{c^4b^2\mathrm{Artanh}(cx)\ln(cx-1)}{4}-\frac{cb^2\mathrm{Artanh}(cx)}{6x^3}-\frac{c^3b^2\mathrm{Artanh}(cx)}{2x}+\frac{c^4b^2\mathrm{Artanh}(cx)\ln(c}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^2/x^5,x)
```

```
[Out] -1/4*a^2/x^4-1/4*b^2/x^4*arctanh(c*x)^2-1/4*c^4*b^2*arctanh(c*x)*ln(c*x-1)-
1/6*c*b^2*arctanh(c*x)/x^3-1/2*c^3*b^2*arctanh(c*x)/x+1/4*c^4*b^2*arctanh(c
*x)*ln(c*x+1)-1/16*c^4*b^2*ln(c*x-1)^2+1/8*c^4*b^2*ln(c*x-1)*ln(1/2+1/2*c*x
)-1/8*c^4*b^2*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)+1/8*c^4*b^2*ln(-1/2*c*x+1/2)
*ln(c*x+1)-1/16*c^4*b^2*ln(c*x+1)^2-1/3*c^4*b^2*ln(c*x-1)-1/12*b^2*c^2/x^2+
2/3*c^4*b^2*ln(c*x)-1/3*c^4*b^2*ln(c*x+1)-1/2*a*b/x^4*arctanh(c*x)-1/4*c^4*
a*b*ln(c*x-1)-1/6*a*b*c/x^3-1/2*c^3*a*b/x+1/4*c^4*a*b*ln(c*x+1)
```

Maxima [B] time = 0.997564, size = 302, normalized size = 2.58

$$\frac{1}{12}\left(\left(3c^3\log(cx+1)-3c^3\log(cx-1)-\frac{2\left(3c^2x^2+1\right)}{x^3}\right)c-\frac{6\ \mathrm{artanh}(cx)}{x^4}\right)ab+\frac{1}{48}\left(\left(32c^2\log(x)-\frac{3c^2x^2\log(cx+1)^2}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^5,x, algorithm="maxima")
```

```
[Out] 1/12*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1) - 2*(3*c^2*x^2 + 1)/x^3)*c -
 6*arctanh(c*x)/x^4)*a*b + 1/48*((32*c^2*log(x) - (3*c^2*x^2*log(c*x + 1)^2
 + 3*c^2*x^2*log(c*x - 1)^2 + 16*c^2*x^2*log(c*x - 1) - 2*(3*c^2*x^2*log(c*
x - 1) - 8*c^2*x^2)*log(c*x + 1) + 4)/x^2)*c^2 + 4*(3*c^3*log(c*x + 1) - 3*
c^3*log(c*x - 1) - 2*(3*c^2*x^2 + 1)/x^3)*c*arctanh(c*x))*b^2 - 1/4*b^2*arc
tanh(c*x)^2/x^4 - 1/4*a^2/x^4
```

Fricas [A] time = 2.37999, size = 386, normalized size = 3.3

$$\frac{32b^2c^4x^4\log(x)+4\left(3ab-4b^2\right)c^4x^4\log(cx+1)-4\left(3ab+4b^2\right)c^4x^4\log(cx-1)-24abc^3x^3-4b^2c^2x^2-8abcx+3\left(b\right.}{48x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/48*(32*b^2*c^4*x^4*log(x) + 4*(3*a*b - 4*b^2)*c^4*x^4*log(c*x + 1) - 4*(3
*a*b + 4*b^2)*c^4*x^4*log(c*x - 1) - 24*a*b*c^3*x^3 - 4*b^2*c^2*x^2 - 8*a*b
*c*x + 3*(b^2*c^4*x^4 - b^2)*log(-(c*x + 1)/(c*x - 1))^2 - 12*a^2 - 4*(3*b^
2*c^3*x^3 + b^2*c*x + 3*a*b)*log(-(c*x + 1)/(c*x - 1)))/x^4
```

Sympy [A] time = 2.60062, size = 184, normalized size = 1.57

$$\begin{cases} -\frac{a^2}{4x^4} + \frac{abc^4 \operatorname{atanh}(cx)}{2} - \frac{abc^3}{2x} - \frac{abc}{6x^3} - \frac{ab \operatorname{atanh}(cx)}{2x^4} + \frac{2b^2c^4 \log(x)}{3} - \frac{2b^2c^4 \log\left(x-\frac{1}{c}\right)}{3} + \frac{b^2c^4 \operatorname{atanh}^2(cx)}{4} - \frac{2b^2c^4 \operatorname{atanh}(cx)}{3} - \frac{b^2c^3 \operatorname{atanh}}{2x} \\ -\frac{a^2}{4x^4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**2/x**5,x)
```

```
[Out] Piecewise((-a**2/(4*x**4) + a*b*c**4*atanh(c*x)/2 - a*b*c**3/(2*x) - a*b*c/
(6*x**3) - a*b*atanh(c*x)/(2*x**4) + 2*b**2*c**4*log(x)/3 - 2*b**2*c**4*log
(x - 1/c)/3 + b**2*c**4*atanh(c*x)**2/4 - 2*b**2*c**4*atanh(c*x)/3 - b**2*c
**3*atanh(c*x)/(2*x) - b**2*c**2/(12*x**2) - b**2*c*atanh(c*x)/(6*x**3) - b
**2*atanh(c*x)**2/(4*x**4), Ne(c, 0)), (-a**2/(4*x**4), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{artanh}(cx) + a)^2}{x^5} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^5,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2/x^5, x)
```

3.24 $\int x^5 \left(a+b\tanh^{-1}(cx)\right)^3\,dx$

Optimal. Leaf size=247

$$-\frac{23b^3\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{30c^6}+\frac{b^2x^4\left(a+b\tanh^{-1}(cx)\right)}{20c^2}+\frac{4b^2x^2\left(a+b\tanh^{-1}(cx)\right)}{15c^4}-\frac{23b^2\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{15c^6}$$

```
[Out] (19*b^3*x)/(60*c^5) + (b^3*x^3)/(60*c^3) - (19*b^3*ArcTanh[c*x])/(60*c^6) +
 (4*b^2*x^2*(a + b*ArcTanh[c*x]))/(15*c^4) + (b^2*x^4*(a + b*ArcTanh[c*x]))
/(20*c^2) + (23*b*(a + b*ArcTanh[c*x])^2)/(30*c^6) + (b*x*(a + b*ArcTanh[c*
x])^2)/(2*c^5) + (b*x^3*(a + b*ArcTanh[c*x])^2)/(6*c^3) + (b*x^5*(a + b*Arc
Tanh[c*x])^2)/(10*c) - (a + b*ArcTanh[c*x])^3/(6*c^6) + (x^6*(a + b*ArcTanh
[c*x])^3)/6 - (23*b^2*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(15*c^6) - (23
*b^3*PolyLog[2, 1 - 2/(1 - c*x)])/(30*c^6)
```

Rubi [A] time = 0.963608, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 33, number of rules used = 11, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {5916, 5980, 302, 206, 321, 5984, 5918, 2402, 2315, 5910, 5948}

$$-\frac{23b^3\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{30c^6}+\frac{b^2x^4\left(a+b\tanh^{-1}(cx)\right)}{20c^2}+\frac{4b^2x^2\left(a+b\tanh^{-1}(cx)\right)}{15c^4}-\frac{23b^2\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{15c^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*ArcTanh[c*x])^3,x]
```


```
[Out] (19*b^3*x)/(60*c^5) + (b^3*x^3)/(60*c^3) - (19*b^3*ArcTanh[c*x])/(60*c^6) +
 (4*b^2*x^2*(a + b*ArcTanh[c*x]))/(15*c^4) + (b^2*x^4*(a + b*ArcTanh[c*x]))
/(20*c^2) + (23*b*(a + b*ArcTanh[c*x])^2)/(30*c^6) + (b*x*(a + b*ArcTanh[c*
x])^2)/(2*c^5) + (b*x^3*(a + b*ArcTanh[c*x])^2)/(6*c^3) + (b*x^5*(a + b*Arc
Tanh[c*x])^2)/(10*c) - (a + b*ArcTanh[c*x])^3/(6*c^6) + (x^6*(a + b*ArcTanh
[c*x])^3)/6 - (23*b^2*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(15*c^6) - (23
*b^3*PolyLog[2, 1 - 2/(1 - c*x)])/(30*c^6)
```


Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int x^5\left(a+b\tanh^{-1}(cx)\right)^3dx=\frac{1}{6}x^6\left(a+b\tanh^{-1}(cx)\right)^3-\frac{1}{2}(bc)\int\frac{x^6\left(a+b\tanh^{-1}(cx)\right)^2}{1-c^2x^2}dx$$

$$=\frac{1}{6}x^6\left(a+b\tanh^{-1}(cx)\right)^3+\frac{b\int x^4\left(a+b\tanh^{-1}(cx)\right)^2dx}{2c}-\frac{b\int\frac{x^4\left(a+b\tanh^{-1}(cx)\right)^2}{1-c^2x^2}dx}{2c}$$

$$=\frac{bx^5\left(a+b\tanh^{-1}(cx)\right)^2}{10c}+\frac{1}{6}x^6\left(a+b\tanh^{-1}(cx)\right)^3-\frac{1}{5}b^2\int\frac{x^5\left(a+b\tanh^{-1}(cx)\right)}{1-c^2x^2}dx+$$

$$=\frac{bx^3\left(a+b\tanh^{-1}(cx)\right)^2}{6c^3}+\frac{bx^5\left(a+b\tanh^{-1}(cx)\right)^2}{10c}+\frac{1}{6}x^6\left(a+b\tanh^{-1}(cx)\right)^3+\frac{b\int(a+}{}$$

$$=\frac{b^2x^4\left(a+b\tanh^{-1}(cx)\right)}{20c^2}+\frac{bx\left(a+b\tanh^{-1}(cx)\right)^2}{2c^5}+\frac{bx^3\left(a+b\tanh^{-1}(cx)\right)^2}{6c^3}+\frac{bx^5(a+}{}$$

$$=\frac{4b^2x^2\left(a+b\tanh^{-1}(cx)\right)}{15c^4}+\frac{b^2x^4\left(a+b\tanh^{-1}(cx)\right)}{20c^2}+\frac{23b\left(a+b\tanh^{-1}(cx)\right)^2}{30c^6}+\frac{bx(a+}{}$$

$$=\frac{19b^3x}{60c^5}+\frac{b^3x^3}{60c^3}+\frac{4b^2x^2\left(a+b\tanh^{-1}(cx)\right)}{15c^4}+\frac{b^2x^4\left(a+b\tanh^{-1}(cx)\right)}{20c^2}+\frac{23b\left(a+b\tanh^{-1}\right.}{30c^6}$$

$$=\frac{19b^3x}{60c^5}+\frac{b^3x^3}{60c^3}-\frac{19b^3\tanh^{-1}(cx)}{60c^6}+\frac{4b^2x^2\left(a+b\tanh^{-1}(cx)\right)}{15c^4}+\frac{b^2x^4\left(a+b\tanh^{-1}(cx)\right)}{20c^2}$$

$$=\frac{19b^3x}{60c^5}+\frac{b^3x^3}{60c^3}-\frac{19b^3\tanh^{-1}(cx)}{60c^6}+\frac{4b^2x^2\left(a+b\tanh^{-1}(cx)\right)}{15c^4}+\frac{b^2x^4\left(a+b\tanh^{-1}(cx)\right)}{20c^2}$$

Mathematica [A] time = 0.703991, size = 305, normalized size = 1.23

$$46b^3\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)+b\tanh^{-1}(cx)\left(30a^2c^6x^6+4abcx\left(3c^4x^4+5c^2x^2+15\right)+b^2\left(3c^4x^4+16c^2x^2-19\right)-92\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^5*(a + b*ArcTanh[c*x])^3,x]
```

```
[Out] (-19*a*b^2 + 30*a^2*b*c*x + 19*b^3*c*x + 16*a*b^2*c^2*x^2 + 10*a^2*b*c^3*x^
3 + b^3*c^3*x^3 + 3*a*b^2*c^4*x^4 + 6*a^2*b*c^5*x^5 + 10*a^3*c^6*x^6 + 2*b^
2*(b*(-23 + 15*c*x + 5*c^3*x^3 + 3*c^5*x^5) + 15*a*(-1 + c^6*x^6))*ArcTanh[
c*x]^2 + 10*b^3*(-1 + c^6*x^6)*ArcTanh[c*x]^3 + b*ArcTanh[c*x]*(30*a^2*c^6*
x^6 + 4*a*b*c*x*(15 + 5*c^2*x^2 + 3*c^4*x^4) + b^2*(-19 + 16*c^2*x^2 + 3*c^
4*x^4) - 92*b^2*Log[1 + E^(-2*ArcTanh[c*x])]) + 15*a^2*b*Log[1 - c*x] - 15*
a^2*b*Log[1 + c*x] + 46*a*b^2*Log[1 - c^2*x^2] + 46*b^3*PolyLog[2, -E^(-2*A
rcTanh[c*x])])/(60*c^6)
```

Maple [C] time = 0.754, size = 1330, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arctanh(c*x))^3,x)
```

```
[Out] 1/4*I/c^6*b^3*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x
^2-1))^2*arctanh(c*x)^2+1/8*I/c^6*b^3*Pi*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))
*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*arctanh(c*x)^2-
1/8*I/c^6*b^3*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)
/((c*x+1)^2/(-c^2*x^2+1)+1))^2*arctanh(c*x)^2+1/8*I/c^6*b^3*Pi*csgn(I*(c*x+
1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*arctanh(c*x)^2+1/6/c
^3*b^3*arctanh(c*x)^2*x^3+1/2/c^5*b^3*arctanh(c*x)^2*x+1/2*a*b^2*x^6*arctan
h(c*x)^2+1/2*a^2*b*x^6*arctanh(c*x)+23/30/c^6*a*b^2*ln(c*x-1)+23/30/c^6*a*b
^2*ln(c*x+1)+1/4/c^6*a^2*b*ln(c*x-1)-1/4/c^6*a^2*b*ln(c*x+1)+1/8/c^6*a*b^2*
ln(c*x-1)^2+19/60*b^3*x/c^5+1/60*b^3*x^3/c^3-19/60*b^3*arctanh(c*x)/c^6+1/6
*x^6*a^3-1/3/c^6*b^3+1/10/c*x^5*a^2*b+4/15/c^4*x^2*a*b^2+1/2/c^5*x*a^2*b+1/
6/c^3*a^2*b*x^3+1/20/c^2*a*b^2*x^4+1/8/c^6*a*b^2*ln(c*x+1)^2+1/4/c^6*b^3*ar
ctanh(c*x)^2*ln(c*x-1)-1/4/c^6*b^3*arctanh(c*x)^2*ln(c*x+1)-23/15/c^6*b^3*a
rctanh(c*x)*ln(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))-23/15/c^6*b^3*arctanh(c*x)*l
n(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))+1/2/c^6*b^3*arctanh(c*x)^2*ln((c*x+1)/(-c
^2*x^2+1)^(1/2))+4/15/c^4*b^3*arctanh(c*x)*x^2+1/20/c^2*b^3*arctanh(c*x)*x^
4+1/10/c*b^3*arctanh(c*x)^2*x^5-1/6/c^6*b^3*arctanh(c*x)^3+23/30/c^6*b^3*ar
ctanh(c*x)^2-23/15/c^6*b^3*dilog(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))-23/15/c^6*
b^3*dilog(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))+1/6*b^3*x^6*arctanh(c*x)^3-1/8*I/
c^6*b^3*Pi*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*(c*x+1)^2/(c^2*x^2-1))
*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))*arctanh(c*x)^2+1/
5/c*a*b^2*x^5*arctanh(c*x)+1/3/c^3*a*b^2*arctanh(c*x)*x^3+1/c^5*a*b^2*arcta
nh(c*x)*x+1/2/c^6*a*b^2*arctanh(c*x)*ln(c*x-1)-1/2/c^6*a*b^2*arctanh(c*x)*l
n(c*x+1)-1/4/c^6*a*b^2*ln(c*x-1)*ln(1/2+1/2*c*x)-1/4/c^6*a*b^2*ln(-1/2*c*x+
1/2)*ln(c*x+1)+1/4/c^6*a*b^2*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)-1/4*I/c^6*b^3
*Pi*arctanh(c*x)^2+1/8*I/c^6*b^3*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*arctanh
(c*x)^2-1/4*I/c^6*b^3*Pi*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))^3*arctanh(c*x)^
2+1/8*I/c^6*b^3*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))
^3*arctanh(c*x)^2+1/4*I/c^6*b^3*Pi*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))^2*arc
tanh(c*x)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x))^3,x, algorithm="maxima")
```

```
[Out] 1/2*a*b^2*x^6*arctanh(c*x)^2 + 1/6*a^3*x^6 + 1/60*(30*x^6*arctanh(c*x) + c*
(2*(3*c^4*x^5 + 5*c^2*x^3 + 15*x)/c^6 - 15*log(c*x + 1)/c^7 + 15*log(c*x -
1)/c^7))*a^2*b + 1/120*(4*c*(2*(3*c^4*x^5 + 5*c^2*x^3 + 15*x)/c^6 - 15*log(
c*x + 1)/c^7 + 15*log(c*x - 1)/c^7)*arctanh(c*x) + (6*c^4*x^4 + 32*c^2*x^2
- 2*(15*log(c*x - 1) - 46)*log(c*x + 1) + 15*log(c*x + 1)^2 + 15*log(c*x -
1)^2 + 92*log(c*x - 1))/c^6)*a*b^2 - 1/1728000*(500*c^7*((2*c^4*x^6 + 3*c^2
*x^4 + 6*x^2)/c^11 + 6*log(c^2*x^2 - 1)/c^13) + 728*c^6*(2*(3*c^4*x^5 + 5*c
^2*x^3 + 15*x)/c^11 - 15*log(c*x + 1)/c^12 + 15*log(c*x - 1)/c^12) + 1485*c
^5*((c^2*x^4 + 2*x^2)/c^9 + 2*log(c^2*x^2 - 1)/c^11) - 622080000*c^5*integr
ate(1/3600*x^5*log(c*x + 1)/(c^7*x^2 - c^5), x) + 9750*c^4*(2*(c^2*x^3 + 3*
x)/c^9 - 3*log(c*x + 1)/c^10 + 3*log(c*x - 1)/c^10) - 2700*c^3*(x^2/c^7 + l
og(c^2*x^2 - 1)/c^9) - 1036800000*c^3*integrate(1/3600*x^3*log(c*x + 1)/(c^
7*x^2 - c^5), x) + 227700*c^2*(2*x/c^7 - log(c*x + 1)/c^8 + log(c*x - 1)/c^
8) - 5495040000*c*integrate(1/3600*x*log(c*x + 1)/(c^7*x^2 - c^5), x) + (10
00*(36*log(-c*x + 1)^3 - 18*log(-c*x + 1)^2 + 6*log(-c*x + 1) - 1)*(c*x - 1
)^6 + 1728*(125*log(-c*x + 1)^3 - 75*log(-c*x + 1)^2 + 30*log(-c*x + 1) - 6
)*(c*x - 1)^5 + 16875*(32*log(-c*x + 1)^3 - 24*log(-c*x + 1)^2 + 12*log(-c*
x + 1) - 3)*(c*x - 1)^4 + 80000*(9*log(-c*x + 1)^3 - 9*log(-c*x + 1)^2 + 6*
```

```
log(-c*x + 1) - 2)*(c*x - 1)^3 + 135000*(4*log(-c*x + 1)^3 - 6*log(-c*x + 1
)^2 + 6*log(-c*x + 1) - 3)*(c*x - 1)^2 + 216000*(log(-c*x + 1)^3 - 3*log(-c
*x + 1)^2 + 6*log(-c*x + 1) - 6)*(c*x - 1))/c^6 - 60*(600*(c^6*x^6 - 1)*log
(c*x + 1)^3 + 240*(3*c^5*x^5 + 5*c^3*x^3 + 15*c*x)*log(c*x + 1)^2 - 30*(10*
c^6*x^6 - 12*c^5*x^5 + 15*c^4*x^4 - 20*c^3*x^3 + 30*c^2*x^2 - 60*c*x - 60*(
c^6*x^6 - 1)*log(c*x + 1) + 37)*log(-c*x + 1)^2 + (100*c^6*x^6 + 264*c^5*x^
5 - 165*c^4*x^4 + 1140*c^3*x^3 - 1230*c^2*x^2 - 1800*(c^6*x^6 - 1)*log(c*x
+ 1)^2 + 8820*c*x - 480*(3*c^5*x^5 + 5*c^3*x^3 + 15*c*x + 23)*log(c*x + 1))
*log(-c*x + 1))/c^6 + 264600*log(3600*c^7*x^2 - 3600*c^5)/c^6 - 2384640000*
integrate(1/3600*log(c*x + 1)/(c^7*x^2 - c^5), x))*b^3
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(b^3x^5\,\text{artanh}\,(cx)^3 + 3\,ab^2x^5\,\text{artanh}\,(cx)^2 + 3\,a^2bx^5\,\text{artanh}\,(cx) + a^3x^5, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^5*arctanh(c*x)^3 + 3*a*b^2*x^5*arctanh(c*x)^2 + 3*a^2*b*x^5*
arctanh(c*x) + a^3*x^5, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^5\,(a + b\,\text{atanh}\,(cx))^3\,\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*atanh(c*x))**3,x)
```

```
[Out] Integral(x**5*(a + b*atanh(c*x))**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\,\text{artanh}\,(cx) + a)^3 x^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^3*x^5, x)
```

3.25 $\int x^4 \left(a+b\tanh^{-1}(cx)\right)^3\,dx$

Optimal. Leaf size=262

$$-\frac{3b^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{5c^5}+\frac{3b^3\text{PolyLog}\left(3,1-\frac{2}{1-cx}\right)}{10c^5}+\frac{b^2x^3\left(a+b\tanh^{-1}(cx)\right)}{10c^2}+\frac{9ab^2x}{10c^4}+\frac{3b}{}$$

```
[Out] (9*a*b^2*x)/(10*c^4) + (b^3*x^2)/(20*c^3) + (9*b^3*x*ArcTanh[c*x])/(10*c^4)
 + (b^2*x^3*(a + b*ArcTanh[c*x]))/(10*c^2) - (9*b*(a + b*ArcTanh[c*x])^2)/(
20*c^5) + (3*b*x^2*(a + b*ArcTanh[c*x])^2)/(10*c^3) + (3*b*x^4*(a + b*ArcTa
nh[c*x])^2)/(20*c) + (a + b*ArcTanh[c*x])^3/(5*c^5) + (x^5*(a + b*ArcTanh[c
*x])^3)/5 - (3*b*(a + b*ArcTanh[c*x])^2*Log[2/(1 - c*x)])/(5*c^5) + (b^3*Lo
g[1 - c^2*x^2])/(2*c^5) - (3*b^2*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 -
 c*x)])/(5*c^5) + (3*b^3*PolyLog[3, 1 - 2/(1 - c*x)])/(10*c^5)
```

Rubi [A] time = 0.773954, antiderivative size = 262, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 11, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {5916, 5980, 266, 43, 5910, 260, 5948, 5984, 5918, 6058, 6610}

$$-\frac{3b^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{5c^5}+\frac{3b^3\text{PolyLog}\left(3,1-\frac{2}{1-cx}\right)}{10c^5}+\frac{b^2x^3\left(a+b\tanh^{-1}(cx)\right)}{10c^2}+\frac{9ab^2x}{10c^4}+\frac{3b}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*ArcTanh[c*x])^3,x]
```

```
[Out] (9*a*b^2*x)/(10*c^4) + (b^3*x^2)/(20*c^3) + (9*b^3*x*ArcTanh[c*x])/(10*c^4)
 + (b^2*x^3*(a + b*ArcTanh[c*x]))/(10*c^2) - (9*b*(a + b*ArcTanh[c*x])^2)/(
20*c^5) + (3*b*x^2*(a + b*ArcTanh[c*x])^2)/(10*c^3) + (3*b*x^4*(a + b*ArcTa
nh[c*x])^2)/(20*c) + (a + b*ArcTanh[c*x])^3/(5*c^5) + (x^5*(a + b*ArcTanh[c
*x])^3)/5 - (3*b*(a + b*ArcTanh[c*x])^2*Log[2/(1 - c*x)])/(5*c^5) + (b^3*Lo
g[1 - c^2*x^2])/(2*c^5) - (3*b^2*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 -
 c*x)])/(5*c^5) + (3*b^3*PolyLog[3, 1 - 2/(1 - c*x)])/(10*c^5)
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int x^4\left(a+b\tanh^{-1}(cx)\right)^3\,dx=\frac{1}{5}x^5\left(a+b\tanh^{-1}(cx)\right)^3-\frac{1}{5}(3bc)\int\frac{x^5\left(a+b\tanh^{-1}(cx)\right)^2}{1-c^2x^2}\,dx$$

$$=\frac{1}{5}x^5\left(a+b\tanh^{-1}(cx)\right)^3+\frac{(3b)\int x^3\left(a+b\tanh^{-1}(cx)\right)^2\,dx}{5c}-\frac{(3b)\int\frac{x^3\left(a+b\tanh^{-1}(cx)\right)^2}{1-c^2x^2}}{5c}$$

$$=\frac{3bx^4\left(a+b\tanh^{-1}(cx)\right)^2}{20c}+\frac{1}{5}x^5\left(a+b\tanh^{-1}(cx)\right)^3-\frac{1}{10}\left(3b^2\right)\int\frac{x^4\left(a+b\tanh^{-1}(c\right.}{1-c^2x^2}$$

$$=\frac{3bx^2\left(a+b\tanh^{-1}(cx)\right)^2}{10c^3}+\frac{3bx^4\left(a+b\tanh^{-1}(cx)\right)^2}{20c}+\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{5c^5}+\frac{1}{5}x^5\left(a\right.$$

$$=\frac{b^2x^3\left(a+b\tanh^{-1}(cx)\right)}{10c^2}+\frac{3bx^2\left(a+b\tanh^{-1}(cx)\right)^2}{10c^3}+\frac{3bx^4\left(a+b\tanh^{-1}(cx)\right)^2}{20c}+\left(a\right.$$

$$=\frac{9ab^2x}{10c^4}+\frac{b^2x^3\left(a+b\tanh^{-1}(cx)\right)}{10c^2}-\frac{9b\left(a+b\tanh^{-1}(cx)\right)^2}{20c^5}+\frac{3bx^2\left(a+b\tanh^{-1}(cx)\right)}{10c^3}$$

$$=\frac{9ab^2x}{10c^4}+\frac{9b^3x\tanh^{-1}(cx)}{10c^4}+\frac{b^2x^3\left(a+b\tanh^{-1}(cx)\right)}{10c^2}-\frac{9b\left(a+b\tanh^{-1}(cx)\right)^2}{20c^5}+3bx$$

$$=\frac{9ab^2x}{10c^4}+\frac{b^3x^2}{20c^3}+\frac{9b^3x\tanh^{-1}(cx)}{10c^4}+\frac{b^2x^3\left(a+b\tanh^{-1}(cx)\right)}{10c^2}-\frac{9b\left(a+b\tanh^{-1}(cx)\right)^2}{20c^5}$$

Mathematica [A] time = 0.735381, size = 383, normalized size = 1.46

$$12b^2\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)\left(a+b\tanh^{-1}(cx)\right)+6b^3\text{PolyLog}\left(3,-e^{-2\tanh^{-1}(cx)}\right)+3a^2bc^4x^4+6a^2bc^2x^2+6a^2b\,l$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4*(a + b*ArcTanh[c*x])^3,x]
```

```
[Out] (-b^3 + 18*a*b^2*c*x + 6*a^2*b*c^2*x^2 + b^3*c^2*x^2 + 2*a*b^2*c^3*x^3 + 3*
a^2*b*c^4*x^4 + 4*a^3*c^5*x^5 - 18*a*b^2*ArcTanh[c*x] + 18*b^3*c*x*ArcTanh[
c*x] + 12*a*b^2*c^2*x^2*ArcTanh[c*x] + 2*b^3*c^3*x^3*ArcTanh[c*x] + 6*a*b^2
*c^4*x^4*ArcTanh[c*x] + 12*a^2*b*c^5*x^5*ArcTanh[c*x] - 12*a*b^2*ArcTanh[c*
x]^2 - 9*b^3*ArcTanh[c*x]^2 + 6*b^3*c^2*x^2*ArcTanh[c*x]^2 + 3*b^3*c^4*x^4*
ArcTanh[c*x]^2 + 12*a*b^2*c^5*x^5*ArcTanh[c*x]^2 - 4*b^3*ArcTanh[c*x]^3 + 4
*b^3*c^5*x^5*ArcTanh[c*x]^3 - 24*a*b^2*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c
*x])] - 12*b^3*ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTanh[c*x])] + 6*a^2*b*Log[1
- c^2*x^2] + 10*b^3*Log[1 - c^2*x^2] + 12*b^2*(a + b*ArcTanh[c*x])*PolyLog[
2, -E^(-2*ArcTanh[c*x])] + 6*b^3*PolyLog[3, -E^(-2*ArcTanh[c*x])])/(20*c^5)
```

Maple [C] time = 0.823, size = 1275, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arctanh(c*x))^3,x)
```

```
[Out] 3/20*I/c^5*b^3*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2
*x^2+1)+1))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*Pi-3/20*I/c^5*b^3*arctanh(c*x)^
```

```
2*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*Pi-1/2
0/c^5*b^3+1/5*x^5*a^3+1/20*b^3*x^2/c^3+3/10/c^5*b^3*arctanh(c*x)^2*ln(c*x+1
)-3/5/c^5*b^3*arctanh(c*x)^2*ln((c*x+1)/(-c^2*x^2+1)^(1/2))-3/5/c^5*b^3*arc
tanh(c*x)*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))+3/5*x^5*a^2*b*arctanh(c*x)+3/5
*x^5*a*b^2*arctanh(c*x)^2+1/10*a*b^2*x^3/c^2+9/10*a*b^2*x/c^4+9/10*b^3*x*ar
ctanh(c*x)/c^4+3/20*I/c^5*b^3*arctanh(c*x)^2*csgn(I/((c*x+1)^2/(-c^2*x^2+1)
+1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*(c*x+1
)^2/(c^2*x^2-1))*Pi-3/10*I/c^5*b^3*arctanh(c*x)^2*csgn(I*(c*x+1)/(-c^2*x^2+
1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*Pi-3/20*I/c^5*b^3*arctanh(c*x)^2*
csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/
(-c^2*x^2+1)+1))^2*Pi-3/10*I/c^5*b^3*arctanh(c*x)^2*Pi+3/5/c^5*a*b^2*arctan
h(c*x)*ln(c*x-1)+3/5/c^5*a*b^2*arctanh(c*x)*ln(c*x+1)-3/10/c^5*a*b^2*ln(c*x
-1)*ln(1/2+1/2*c*x)+3/10/c^5*a*b^2*ln(-1/2*c*x+1/2)*ln(c*x+1)-3/10/c^5*a*b^
2*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)+3/10/c*a*b^2*x^4*arctanh(c*x)+3/5/c^3*a*
b^2*arctanh(c*x)*x^2+3/20/c*x^4*a^2*b+3/10/c^3*a^2*b*x^2+3/20/c*b^3*arctanh
(c*x)^2*x^4+3/10/c^3*b^3*arctanh(c*x)^2*x^2+1/10/c^2*b^3*arctanh(c*x)*x^3+9
/20/c^5*a*b^2*ln(c*x-1)-9/20/c^5*a*b^2*ln(c*x+1)+3/10/c^5*a^2*b*ln(c*x-1)+3
/10/c^5*a^2*b*ln(c*x+1)+3/20/c^5*a*b^2*ln(c*x-1)^2-3/20/c^5*a*b^2*ln(c*x+1)
^2-3/5/c^5*b^3*arctanh(c*x)^2*ln(2)-3/5/c^5*a*b^2*dilog(1/2+1/2*c*x)+3/10/c
^5*b^3*arctanh(c*x)^2*ln(c*x-1)+1/5*x^5*b^3*arctanh(c*x)^3+1/5/c^5*b^3*arct
anh(c*x)^3-9/20/c^5*b^3*arctanh(c*x)^2+3/10/c^5*b^3*polylog(3,-(c*x+1)^2/(-
c^2*x^2+1))-1/c^5*b^3*ln((c*x+1)^2/(-c^2*x^2+1)+1)+1/c^5*b^3*arctanh(c*x)-3
/10*I/c^5*b^3*arctanh(c*x)^2*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))^3*Pi-3/20*I
/c^5*b^3*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1
)+1))^3*Pi-3/20*I/c^5*b^3*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*Pi
+3/10*I/c^5*b^3*arctanh(c*x)^2*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))^2*Pi
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{5}a^3x^5 + \frac{3}{20}\left(4x^5\operatorname{artanh}(cx) + c\left(\frac{c^2x^4 + 2x^2}{c^4} + \frac{2\log\left(c^2x^2 - 1\right)}{c^6}\right)\right)a^2b - \frac{2\left(b^3c^5x^5 - b^3\right)\log\left(-cx + 1\right)^3 - 3\left(4ab^2c^5x^5 + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x))^3,x, algorithm="maxima")
```

```
[Out] 1/5*a^3*x^5 + 3/20*(4*x^5*arctanh(c*x) + c*((c^2*x^4 + 2*x^2)/c^4 + 2*log(c
^2*x^2 - 1)/c^6))*a^2*b - 1/80*(2*(b^3*c^5*x^5 - b^3)*log(-c*x + 1)^3 - 3*(
4*a*b^2*c^5*x^5 + b^3*c^4*x^4 + 2*b^3*c^2*x^2 + 2*(b^3*c^5*x^5 + b^3)*log(c
*x + 1))*log(-c*x + 1)^2)/c^5 - integrate(-1/40*(5*(b^3*c^5*x^5 - b^3*c^4*x
^4)*log(c*x + 1)^3 + 30*(a*b^2*c^5*x^5 - a*b^2*c^4*x^4)*log(c*x + 1)^2 - 3*
(4*a*b^2*c^5*x^5 + b^3*c^4*x^4 + 2*b^3*c^2*x^2 + 5*(b^3*c^5*x^5 - b^3*c^4*x
^4)*log(c*x + 1)^2 - 2*(10*a*b^2*c^4*x^4 - (10*a*b^2*c^5 + b^3*c^5)*x^5 - b
^3)*log(c*x + 1))*log(-c*x + 1))/(c^5*x - c^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(b^3x^4\operatorname{artanh}(cx)^3 + 3\,ab^2x^4\operatorname{artanh}(cx)^2 + 3\,a^2bx^4\operatorname{artanh}(cx) + a^3x^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^4*arctanh(c*x)^3 + 3*a*b^2*x^4*arctanh(c*x)^2 + 3*a^2*b*x^4*
arctanh(c*x) + a^3*x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 (a + b\,\mathrm{atanh}\,(cx))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*atanh(c*x))**3,x)
```

```
[Out] Integral(x**4*(a + b*atanh(c*x))**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\,\mathrm{artanh}\,(cx) + a)^3 x^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^3*x^4, x)
```

3.26 $\int x^3 \left(a + b \tanh^{-1}(cx)\right)^3 \, dx$

Optimal. Leaf size=185

$$-\frac{b^3 \text{PolyLog}\left(2, 1 - \frac{2}{1-cx}\right)}{c^4} + \frac{b^2 x^2 \left(a + b \tanh^{-1}(cx)\right)}{4c^2} - \frac{2b^2 \log\left(\frac{2}{1-cx}\right)\left(a + b \tanh^{-1}(cx)\right)}{c^4} + \frac{3bx\left(a + b \tanh^{-1}(cx)\right)^2}{4c^3} - \frac{(}{}$$

```
[Out] (b^3*x)/(4*c^3) - (b^3*ArcTanh[c*x])/(4*c^4) + (b^2*x^2*(a + b*ArcTanh[c*x]
))/(4*c^2) + (b*(a + b*ArcTanh[c*x])^2)/c^4 + (3*b*x*(a + b*ArcTanh[c*x])^2
)/(4*c^3) + (b*x^3*(a + b*ArcTanh[c*x])^2)/(4*c) - (a + b*ArcTanh[c*x])^3/(
4*c^4) + (x^4*(a + b*ArcTanh[c*x])^3)/4 - (2*b^2*(a + b*ArcTanh[c*x])*Log[2
/(1 - c*x)])/c^4 - (b^3*PolyLog[2, 1 - 2/(1 - c*x)])/c^4
```

Rubi [A] time = 0.569391, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {5916, 5980, 321, 206, 5984, 5918, 2402, 2315, 5910, 5948}

$$-\frac{b^3 \text{PolyLog}\left(2, 1 - \frac{2}{1-cx}\right)}{c^4} + \frac{b^2 x^2 \left(a + b \tanh^{-1}(cx)\right)}{4c^2} - \frac{2b^2 \log\left(\frac{2}{1-cx}\right)\left(a + b \tanh^{-1}(cx)\right)}{c^4} + \frac{3bx\left(a + b \tanh^{-1}(cx)\right)^2}{4c^3} - \frac{(}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcTanh[c*x])^3,x]
```



```
[Out] (b^3*x)/(4*c^3) - (b^3*ArcTanh[c*x])/(4*c^4) + (b^2*x^2*(a + b*ArcTanh[c*x]
))/(4*c^2) + (b*(a + b*ArcTanh[c*x])^2)/c^4 + (3*b*x*(a + b*ArcTanh[c*x])^2
)/(4*c^3) + (b*x^3*(a + b*ArcTanh[c*x])^2)/(4*c) - (a + b*ArcTanh[c*x])^3/(
4*c^4) + (x^4*(a + b*ArcTanh[c*x])^3)/4 - (2*b^2*(a + b*ArcTanh[c*x])*Log[2
/(1 - c*x)])/c^4 - (b^3*PolyLog[2, 1 - 2/(1 - c*x)])/c^4
```


Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int x^3\left(a+b\tanh^{-1}(cx)\right)^3 dx = \frac{1}{4}x^4\left(a+b\tanh^{-1}(cx)\right)^3 - \frac{1}{4}(3bc)\int \frac{x^4\left(a+b\tanh^{-1}(cx)\right)^2}{1-c^2x^2}dx$$

$$= \frac{1}{4}x^4\left(a+b\tanh^{-1}(cx)\right)^3 + \frac{(3b)\int x^2\left(a+b\tanh^{-1}(cx)\right)^2 dx}{4c} - \frac{(3b)\int \frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{1-c^2x^2}dx}{4c}$$

$$= \frac{bx^3\left(a+b\tanh^{-1}(cx)\right)^2}{4c} + \frac{1}{4}x^4\left(a+b\tanh^{-1}(cx)\right)^3 - \frac{1}{2}b^2\int \frac{x^3\left(a+b\tanh^{-1}(cx)\right)}{1-c^2x^2}dx +$$

$$= \frac{3bx\left(a+b\tanh^{-1}(cx)\right)^2}{4c^3} + \frac{bx^3\left(a+b\tanh^{-1}(cx)\right)^2}{4c} - \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{4c^4} + \frac{1}{4}x^4\left(a+b\mathrm{t}\right.$$

$$= \frac{b^2x^2\left(a+b\tanh^{-1}(cx)\right)}{4c^2} + \frac{b\left(a+b\tanh^{-1}(cx)\right)^2}{c^4} + \frac{3bx\left(a+b\tanh^{-1}(cx)\right)^2}{4c^3} + \frac{bx^3\left(a+b\right.}{}$$

$$= \frac{b^3x}{4c^3} + \frac{b^2x^2\left(a+b\tanh^{-1}(cx)\right)}{4c^2} + \frac{b\left(a+b\tanh^{-1}(cx)\right)^2}{c^4} + \frac{3bx\left(a+b\tanh^{-1}(cx)\right)^2}{4c^3} + \frac{bx^3}{}$$

$$= \frac{b^3x}{4c^3} - \frac{b^3\tanh^{-1}(cx)}{4c^4} + \frac{b^2x^2\left(a+b\tanh^{-1}(cx)\right)}{4c^2} + \frac{b\left(a+b\tanh^{-1}(cx)\right)^2}{c^4} + \frac{3bx\left(a+b\tan\right.}{4c^3}$$

$$= \frac{b^3x}{4c^3} - \frac{b^3\tanh^{-1}(cx)}{4c^4} + \frac{b^2x^2\left(a+b\tanh^{-1}(cx)\right)}{4c^2} + \frac{b\left(a+b\tanh^{-1}(cx)\right)^2}{c^4} + \frac{3bx\left(a+b\tan\right.}{4c^3}$$

Mathematica [A] time = 0.49171, size = 245, normalized size = 1.32

$$8b^3\mathrm{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right) + 2b\tanh^{-1}(cx)\left(3a^2c^4x^4 + 2abcx\left(c^2x^2+3\right) + b^2\left(c^2x^2-1\right) - 8b^2\log\left(e^{-2\tanh^{-1}(cx)}+1\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*(a + b*ArcTanh[c*x])^3,x]
```

```
[Out] (-2*a*b^2 + 6*a^2*b*c*x + 2*b^3*c*x + 2*a*b^2*c^2*x^2 + 2*a^2*b*c^3*x^3 + 2
*a^3*c^4*x^4 + 2*b^2*(b*(-4 + 3*c*x + c^3*x^3) + 3*a*(-1 + c^4*x^4))*ArcTan
h[c*x]^2 + 2*b^3*(-1 + c^4*x^4)*ArcTanh[c*x]^3 + 2*b*ArcTanh[c*x]*(3*a^2*c^
4*x^4 + b^2*(-1 + c^2*x^2) + 2*a*b*c*x*(3 + c^2*x^2) - 8*b^2*Log[1 + E^(-2*
ArcTanh[c*x])]) + 3*a^2*b*Log[1 - c*x] - 3*a^2*b*Log[1 + c*x] + 8*a*b^2*Log
[1 - c^2*x^2] + 8*b^3*PolyLog[2, -E^(-2*ArcTanh[c*x])])/(8*c^4)
```

Maple [C] time = 0.602, size = 1245, normalized size = 6.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c*x))^3,x)
```

```
[Out] -1/4/c^4*b^3*arctanh(c*x)^3+1/c^4*b^3*arctanh(c*x)^2-2/c^4*b^3*dilog(1-I*(c
*x+1)/(-c^2*x^2+1)^(1/2))-2/c^4*b^3*dilog(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))+1
/4*x^4*b^3*arctanh(c*x)^3-3/16*I/c^4*b^3*Pi*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+
1))*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(
-c^2*x^2+1)+1))*arctanh(c*x)^2+1/4*b^3*x/c^3-1/4*b^3*arctanh(c*x)/c^4+1/4*x
^4*a^3-3/16*I/c^4*b^3*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^
```

```
2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*arctanh(c*x)^2+3/16*I/c^4*b^3*Pi*csg
n(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*arctanh(c*x
)^2+3/16*I/c^4*b^3*Pi*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*(c*x+1)^2/(
c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*arctanh(c*x)^2+3/8*I/c^4*b^3*Pi*cs
gn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*arctanh(c*
x)^2+1/2/c*a*b^2*arctanh(c*x)*x^3+3/2/c^3*a*b^2*arctanh(c*x)*x+3/4/c^4*a*b^
2*arctanh(c*x)*ln(c*x-1)-3/4/c^4*a*b^2*arctanh(c*x)*ln(c*x+1)-3/8/c^4*a*b^2
*ln(c*x-1)*ln(1/2+1/2*c*x)-3/8/c^4*a*b^2*ln(-1/2*c*x+1/2)*ln(c*x+1)+3/8/c^4
*a*b^2*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)-3/8*I/c^4*b^3*Pi*arctanh(c*x)^2+1/4
/c^2*x^2*a*b^2+3/4/c^3*x*a^2*b+1/4/c*a^2*b*x^3+3/4*a*b^2*x^4*arctanh(c*x)^2
+3/4*x^4*a^2*b*arctanh(c*x)+3/8/c^4*b^3*arctanh(c*x)^2*ln(c*x-1)-3/8/c^4*b^
3*arctanh(c*x)^2*ln(c*x+1)-2/c^4*b^3*arctanh(c*x)*ln(1+I*(c*x+1)/(-c^2*x^2+
1)^(1/2))-2/c^4*b^3*arctanh(c*x)*ln(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))+3/4/c^4
*b^3*arctanh(c*x)^2*ln((c*x+1)/(-c^2*x^2+1)^(1/2))+1/c^4*a*b^2*ln(c*x-1)+1/
c^4*a*b^2*ln(c*x+1)+3/8/c^4*a^2*b*ln(c*x-1)-3/8/c^4*a^2*b*ln(c*x+1)+3/16/c^
4*a*b^2*ln(c*x-1)^2+3/16/c^4*a*b^2*ln(c*x+1)^2+1/4/c^2*b^3*arctanh(c*x)*x^2
+1/4/c*b^3*arctanh(c*x)^2*x^3+3/4/c^3*b^3*arctanh(c*x)^2*x-1/4/c^4*b^3+3/8*
I/c^4*b^3*Pi*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))^2*arctanh(c*x)^2+3/16*I/c^4
*b^3*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^3*arctanh(
c*x)^2+3/16*I/c^4*b^3*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*arctanh(c*x)^2-3/8
*I/c^4*b^3*Pi*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))^3*arctanh(c*x)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))^3,x, algorithm="maxima")
```

```
[Out] 3/4*a*b^2*x^4*arctanh(c*x)^2 + 1/4*a^3*x^4 + 1/8*(6*x^4*arctanh(c*x) + c*(2
*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*a^2*b + 1/
16*(4*c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5)*a
rctanh(c*x) + (4*c^2*x^2 - 2*(3*log(c*x - 1) - 8)*log(c*x + 1) + 3*log(c*x
+ 1)^2 + 3*log(c*x - 1)^2 + 16*log(c*x - 1))/c^4)*a*b^2 - 1/9216*(27*c^5*((
c^2*x^4 + 2*x^2)/c^7 + 2*log(c^2*x^2 - 1)/c^9) + 74*c^4*(2*(c^2*x^3 + 3*x)/
c^7 - 3*log(c*x + 1)/c^8 + 3*log(c*x - 1)/c^8) + 60*c^3*(x^2/c^5 + log(c^2*
x^2 - 1)/c^7) - 221184*c^3*integrate(1/96*x^3*log(c*x + 1)/(c^5*x^2 - c^3),
 x) + 1692*c^2*(2*x/c^5 - log(c*x + 1)/c^6 + log(c*x - 1)/c^6) - 1105920*c*
integrate(1/96*x*log(c*x + 1)/(c^5*x^2 - c^3), x) + (9*(32*log(-c*x + 1)^3
- 24*log(-c*x + 1)^2 + 12*log(-c*x + 1) - 3)*(c*x - 1)^4 + 128*(9*log(-c*x
+ 1)^3 - 9*log(-c*x + 1)^2 + 6*log(-c*x + 1) - 2)*(c*x - 1)^3 + 432*(4*log(
-c*x + 1)^3 - 6*log(-c*x + 1)^2 + 6*log(-c*x + 1) - 3)*(c*x - 1)^2 + 1152*(
log(-c*x + 1)^3 - 3*log(-c*x + 1)^2 + 6*log(-c*x + 1) - 6)*(c*x - 1))/c^4 -
 12*(24*(c^4*x^4 - 1)*log(c*x + 1)^3 + 48*(c^3*x^3 + 3*c*x)*log(c*x + 1)^2
- 6*(3*c^4*x^4 - 4*c^3*x^3 + 6*c^2*x^2 - 12*c*x - 12*(c^4*x^4 - 1)*log(c*x
+ 1) + 7)*log(-c*x + 1)^2 + (9*c^4*x^4 + 28*c^3*x^3 - 18*c^2*x^2 - 72*(c^4*
x^4 - 1)*log(c*x + 1)^2 + 300*c*x - 96*(c^3*x^3 + 3*c*x + 4)*log(c*x + 1))*
log(-c*x + 1))/c^4 + 1800*log(96*c^5*x^2 - 96*c^3)/c^4 - 442368*integrate(1
/96*log(c*x + 1)/(c^5*x^2 - c^3), x))*b^3
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(b^3x^3\,\text{artanh}\left(cx\right)^3+3\,ab^2x^3\,\text{artanh}\left(cx\right)^2+3\,a^2bx^3\,\text{artanh}\left(cx\right)+a^3x^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^3*arctanh(c*x)^3 + 3*a*b^2*x^3*arctanh(c*x)^2 + 3*a^2*b*x^3*
arctanh(c*x) + a^3*x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 (a + b \operatorname{atanh}(cx))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c*x))**3,x)
```

```
[Out] Integral(x**3*(a + b*atanh(c*x))**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{artanh}(cx) + a)^3 x^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^3*x^3, x)
```

3.27 $\int x^2 \left(a+b\tanh^{-1}(cx)\right)^3\,dx$

Optimal. Leaf size=197

$$-\frac{b^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^3}+\frac{b^3\text{PolyLog}\left(3,1-\frac{2}{1-cx}\right)}{2c^3}+\frac{ab^2x}{c^2}-\frac{b\left(a+b\tanh^{-1}(cx)\right)^2}{2c^3}+\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{3\ldots}$$

```
[Out] (a*b^2*x)/c^2 + (b^3*x*ArcTanh[c*x])/c^2 - (b*(a + b*ArcTanh[c*x])^2)/(2*c^
3) + (b*x^2*(a + b*ArcTanh[c*x])^2)/(2*c) + (a + b*ArcTanh[c*x])^3/(3*c^3)
+ (x^3*(a + b*ArcTanh[c*x])^3)/3 - (b*(a + b*ArcTanh[c*x])^2*Log[2/(1 - c*x
)])/c^3 + (b^3*Log[1 - c^2*x^2])/(2*c^3) - (b^2*(a + b*ArcTanh[c*x])*PolyLo
g[2, 1 - 2/(1 - c*x)])/c^3 + (b^3*PolyLog[3, 1 - 2/(1 - c*x)])/(2*c^3)
```

Rubi [A] time = 0.449067, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {5916, 5980, 5910, 260, 5948, 5984, 5918, 6058, 6610}

$$-\frac{b^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^3}+\frac{b^3\text{PolyLog}\left(3,1-\frac{2}{1-cx}\right)}{2c^3}+\frac{ab^2x}{c^2}-\frac{b\left(a+b\tanh^{-1}(cx)\right)^2}{2c^3}+\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{3\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcTanh[c*x])^3,x]
```


```
[Out] (a*b^2*x)/c^2 + (b^3*x*ArcTanh[c*x])/c^2 - (b*(a + b*ArcTanh[c*x])^2)/(2*c^
3) + (b*x^2*(a + b*ArcTanh[c*x])^2)/(2*c) + (a + b*ArcTanh[c*x])^3/(3*c^3)
+ (x^3*(a + b*ArcTanh[c*x])^3)/3 - (b*(a + b*ArcTanh[c*x])^2*Log[2/(1 - c*x
)])/c^3 + (b^3*Log[1 - c^2*x^2])/(2*c^3) - (b^2*(a + b*ArcTanh[c*x])*PolyLo
g[2, 1 - 2/(1 - c*x)])/c^3 + (b^3*PolyLog[3, 1 - 2/(1 - c*x)])/(2*c^3)
```


Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int x^2 \left(a+b\tanh^{-1}(cx)\right)^3 dx = \frac{1}{3}x^3 \left(a+b\tanh^{-1}(cx)\right)^3 - (bc)\int \frac{x^3 \left(a+b\tanh^{-1}(cx)\right)^2}{1-c^2x^2} dx$$

$$= \frac{1}{3}x^3 \left(a+b\tanh^{-1}(cx)\right)^3 + \frac{b\int x \left(a+b\tanh^{-1}(cx)\right)^2 dx}{c} - \frac{b\int \frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{1-c^2x^2} dx}{c}$$

$$= \frac{bx^2 \left(a+b\tanh^{-1}(cx)\right)^2}{2c} + \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{3c^3} + \frac{1}{3}x^3 \left(a+b\tanh^{-1}(cx)\right)^3 - b^2 \int \frac{x^2 (}{}$$

$$= \frac{bx^2 \left(a+b\tanh^{-1}(cx)\right)^2}{2c} + \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{3c^3} + \frac{1}{3}x^3 \left(a+b\tanh^{-1}(cx)\right)^3 - \frac{b\left(a+b\,t\right.}{}$$

$$= \frac{ab^2x}{c^2} - \frac{b\left(a+b\tanh^{-1}(cx)\right)^2}{2c^3} + \frac{bx^2 \left(a+b\tanh^{-1}(cx)\right)^2}{2c} + \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{3c^3} + \frac{1}{3}x^{}$$

$$= \frac{ab^2x}{c^2} + \frac{b^3x\tanh^{-1}(cx)}{c^2} - \frac{b\left(a+b\tanh^{-1}(cx)\right)^2}{2c^3} + \frac{bx^2 \left(a+b\tanh^{-1}(cx)\right)^2}{2c} + \frac{\left(a+b\,t\right.}{}$$

$$= \frac{ab^2x}{c^2} + \frac{b^3x\tanh^{-1}(cx)}{c^2} - \frac{b\left(a+b\tanh^{-1}(cx)\right)^2}{2c^3} + \frac{bx^2 \left(a+b\tanh^{-1}(cx)\right)^2}{2c} + \frac{\left(a+b\,t\right.}{}$$

Mathematica [A] time = 0.520302, size = 250, normalized size = 1.27

$$\frac{6ab^2\left(\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right) + \left(c^3x^3-1\right)\tanh^{-1}(cx)^2 + \tanh^{-1}(cx)\left(c^2x^2 - 2\log\left(e^{-2\tanh^{-1}(cx)}+1\right)-1\right)+cx\right)+}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(a + b*ArcTanh[c*x])^3,x]
```

```
[Out] (3*a^2*b*c^2*x^2 + 2*a^3*c^3*x^3 + 6*a^2*b*c^3*x^3*ArcTanh[c*x] + 3*a^2*b*L
og[1 - c^2*x^2] + 6*a*b^2*(c*x + (-1 + c^3*x^3)*ArcTanh[c*x]^2 + ArcTanh[c*
x]*(-1 + c^2*x^2 - 2*Log[1 + E^(-2*ArcTanh[c*x])]) + PolyLog[2, -E^(-2*ArcT
anh[c*x])]) + b^3*(6*c*x*ArcTanh[c*x] - 3*ArcTanh[c*x]^2 + 3*c^2*x^2*ArcTan
h[c*x]^2 - 2*ArcTanh[c*x]^3 + 2*c^3*x^3*ArcTanh[c*x]^3 - 6*ArcTanh[c*x]^2*L
og[1 + E^(-2*ArcTanh[c*x])] + 3*Log[1 - c^2*x^2] + 6*ArcTanh[c*x]*PolyLog[2
, -E^(-2*ArcTanh[c*x])] + 3*PolyLog[3, -E^(-2*ArcTanh[c*x])]))/(6*c^3)
```

Maple [C] time = 0.477, size = 1177, normalized size = 6.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c*x))^3,x)
```

```
[Out] 1/c^3*b^3*arctanh(c*x)+1/3/c^3*b^3*arctanh(c*x)^3-1/2/c^3*b^3*arctanh(c*x)^
2+1/2/c^3*b^3*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))-1/c^3*b^3*ln((c*x+1)^2/(-c
^2*x^2+1)+1)+1/3*x^3*b^3*arctanh(c*x)^3+a*b^2*x/c^2+b^3*x*arctanh(c*x)/c^2+
1/c*a*b^2*arctanh(c*x)*x^2+1/c^3*a*b^2*arctanh(c*x)*ln(c*x-1)+1/c^3*a*b^2*a
rctanh(c*x)*ln(c*x+1)-1/2/c^3*a*b^2*ln(c*x-1)*ln(1/2+1/2*c*x)+1/2/c^3*a*b^2
*ln(-1/2*c*x+1/2)*ln(c*x+1)-1/2/c^3*a*b^2*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)-
1/2*I/c^3*b^3*arctanh(c*x)^2*Pi+1/3*x^3*a^3+1/4*I/c^3*b^3*arctanh(c*x)^2*cs
```

```
gn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x
^2+1)+1))*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*Pi+1/2/c*a^2*b*x^2+a^2*b*x^3*a
rctanh(c*x)+a*b^2*x^3*arctanh(c*x)^2+1/2/c^3*a*b^2*ln(c*x-1)-1/2/c^3*a*b^2*
ln(c*x+1)+1/2/c^3*a^2*b*ln(c*x-1)+1/2/c^3*a^2*b*ln(c*x+1)+1/4/c^3*a*b^2*ln(
c*x-1)^2-1/4/c^3*a*b^2*ln(c*x+1)^2-1/c^3*b^3*arctanh(c*x)^2*ln(2)-1/c^3*a*b
^2*dilog(1/2+1/2*c*x)+1/2/c^3*b^3*arctanh(c*x)^2*ln(c*x-1)+1/2/c^3*b^3*arct
anh(c*x)^2*ln(c*x+1)-1/c^3*b^3*arctanh(c*x)^2*ln((c*x+1)/(-c^2*x^2+1)^(1/2)
)-1/c^3*b^3*arctanh(c*x)*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))+1/2/c*b^3*arcta
nh(c*x)^2*x^2-1/4*I/c^3*b^3*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*
Pi-1/4*I/c^3*b^3*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c
^2*x^2+1)+1))^3*Pi-1/2*I/c^3*b^3*arctanh(c*x)^2*csgn(I/((c*x+1)^2/(-c^2*x^2
+1)+1))^3*Pi+1/2*I/c^3*b^3*arctanh(c*x)^2*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1)
)^2*Pi-1/2*I/c^3*b^3*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*csgn(I*
(c*x+1)/(-c^2*x^2+1)^(1/2))*Pi+1/4*I/c^3*b^3*arctanh(c*x)^2*csgn(I*(c*x+1)^
2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*P
i-1/4*I/c^3*b^3*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)
/(-c^2*x^2+1)^(1/2))^2*Pi-1/4*I/c^3*b^3*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^
2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*P
i
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{3}a^3x^3+\frac{1}{2}\left(2x^3\operatorname{artanh}(cx)+c\left(\frac{x^2}{c^2}+\frac{\log\left(c^2x^2-1\right)}{c^4}\right)\right)a^2b-\frac{\left(b^3c^3x^3-b^3\right)\log(-cx+1)^3-3\left(2ab^2c^3x^3+b^3c^2x^2+\left(b^3c^3\right.\right.}{24c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))^3,x, algorithm="maxima")
```

```
[Out] 1/3*a^3*x^3 + 1/2*(2*x^3*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x^2 - 1)/c^4))
*a^2*b - 1/24*((b^3*c^3*x^3 - b^3)*log(-c*x + 1)^3 - 3*(2*a*b^2*c^3*x^3 + b
^3*c^2*x^2 + (b^3*c^3*x^3 + b^3)*log(c*x + 1))*log(-c*x + 1)^2)/c^3 - integ
rate(-1/8*((b^3*c^3*x^3 - b^3*c^2*x^2)*log(c*x + 1)^3 + 6*(a*b^2*c^3*x^3 -
a*b^2*c^2*x^2)*log(c*x + 1)^2 - (4*a*b^2*c^3*x^3 + 2*b^3*c^2*x^2 + 3*(b^3*c
^3*x^3 - b^3*c^2*x^2)*log(c*x + 1)^2 - 2*(6*a*b^2*c^2*x^2 - (6*a*b^2*c^3 +
b^3*c^3)*x^3 - b^3)*log(c*x + 1))*log(-c*x + 1))/(c^3*x - c^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(b^3x^2\operatorname{artanh}(cx)^3+3\,ab^2x^2\operatorname{artanh}(cx)^2+3\,a^2bx^2\operatorname{artanh}(cx)+a^3x^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^2*arctanh(c*x)^3 + 3*a*b^2*x^2*arctanh(c*x)^2 + 3*a^2*b*x^2*
arctanh(c*x) + a^3*x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\,(a+b\operatorname{atanh}(cx))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c*x))**3,x)
```

```
[Out] Integral(x**2*(a + b*atanh(c*x))**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{artanh}(cx) + a)^3 x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^3*x^2, x)
```

3.28 $\int x\left(a+b\tanh^{-1}(cx)\right)^3\,dx$

Optimal. Leaf size=123

$$-\frac{3b^3\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{2c^2}-\frac{3b^2\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^2}+\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{2c^2}-\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{2c^2}+\frac{1}{2}x^2\left(\right.$$

```
[Out] (3*b*(a + b*ArcTanh[c*x])^2)/(2*c^2) + (3*b*x*(a + b*ArcTanh[c*x])^2)/(2*c)
 - (a + b*ArcTanh[c*x])^3/(2*c^2) + (x^2*(a + b*ArcTanh[c*x])^3)/2 - (3*b^2
*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/c^2 - (3*b^3*PolyLog[2, 1 - 2/(1 -
c*x)])/(2*c^2)
```

Rubi [A] time = 0.250315, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5916, 5980, 5910, 5984, 5918, 2402, 2315, 5948}

$$-\frac{3b^3\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{2c^2}-\frac{3b^2\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^2}+\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{2c^2}-\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{2c^2}+\frac{1}{2}x^2\left(\right.$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcTanh[c*x])^3,x]
```


```
[Out] (3*b*(a + b*ArcTanh[c*x])^2)/(2*c^2) + (3*b*x*(a + b*ArcTanh[c*x])^2)/(2*c)
 - (a + b*ArcTanh[c*x])^3/(2*c^2) + (x^2*(a + b*ArcTanh[c*x])^3)/2 - (3*b^2
*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/c^2 - (3*b^3*PolyLog[2, 1 - 2/(1 -
c*x)])/(2*c^2)
```


Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
```

```
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int x\left(a+b\tanh^{-1}(cx)\right)^3dx=\frac{1}{2}x^2\left(a+b\tanh^{-1}(cx)\right)^3-\frac{1}{2}(3bc)\int\frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{1-c^2x^2}dx$$

$$=\frac{1}{2}x^2\left(a+b\tanh^{-1}(cx)\right)^3+\frac{(3b)\int\left(a+b\tanh^{-1}(cx)\right)^2dx}{2c}-\frac{(3b)\int\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{1-c^2x^2}dx}{2c}$$

$$=\frac{3bx\left(a+b\tanh^{-1}(cx)\right)^2}{2c}-\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{2c^2}+\frac{1}{2}x^2\left(a+b\tanh^{-1}(cx)\right)^3-\left(3b^2\right)\int\frac{x}{}$$

$$=\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{2c^2}+\frac{3bx\left(a+b\tanh^{-1}(cx)\right)^2}{2c}-\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{2c^2}+\frac{1}{2}x^2\left(a+b\right.$$

$$=\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{2c^2}+\frac{3bx\left(a+b\tanh^{-1}(cx)\right)^2}{2c}-\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{2c^2}+\frac{1}{2}x^2\left(a+b\right.$$

$$=\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{2c^2}+\frac{3bx\left(a+b\tanh^{-1}(cx)\right)^2}{2c}-\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{2c^2}+\frac{1}{2}x^2\left(a+b\right.$$

$$=\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{2c^2}+\frac{3bx\left(a+b\tanh^{-1}(cx)\right)^2}{2c}-\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{2c^2}+\frac{1}{2}x^2\left(a+b\right.$$

Mathematica [A] time = 0.28394, size = 161, normalized size = 1.31

$$\frac{6b^3\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)+a\left(2a^2c^2x^2+6abcx+3ab\log(1-cx)-3ab\log(cx+1)+6b^2\log\left(1-c^2x^2\right)\right)+6b^2(c}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(a + b*ArcTanh[c*x])^3,x]
```

```
[Out] (6*b^2*(-1 + c*x)*(a + b + a*c*x)*ArcTanh[c*x]^2 + 2*b^3*(-1 + c^2*x^2)*Arc
Tanh[c*x]^3 + 6*b*ArcTanh[c*x]*(a*c*x*(2*b + a*c*x) - 2*b^2*Log[1 + E^(-2*A
rcTanh[c*x])]) + a*(6*a*b*c*x + 2*a^2*c^2*x^2 + 3*a*b*Log[1 - c*x] - 3*a*b*
Log[1 + c*x] + 6*b^2*Log[1 - c^2*x^2]) + 6*b^3*PolyLog[2, -E^(-2*ArcTanh[c*
x])])/(4*c^2)
```

Maple [C] time = 0.342, size = 6097, normalized size = 49.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c*x))^3,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))^3,x, algorithm="maxima")
```

```
[Out] 3/2*a*b^2*x^2*arctanh(c*x)^2 + 1/2*a^3*x^2 + 3/4*(2*x^2*arctanh(c*x) + c*(2
*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3))*a^2*b + 3/8*(4*c*(2*x/c^2 -
log(c*x + 1)/c^3 + log(c*x - 1)/c^3)*arctanh(c*x) - (2*(log(c*x - 1) - 2)*l
og(c*x + 1) - log(c*x + 1)^2 - log(c*x - 1)^2 - 4*log(c*x - 1))/c^2)*a*b^2
- 1/64*(3*c^3*(x^2/c^3 + log(c^2*x^2 - 1)/c^5) + 21*c^2*(2*x/c^3 - log(c*x
+ 1)/c^4 + log(c*x - 1)/c^4) - 576*c*integrate(1/4*x*log(c*x + 1)/(c^3*x^2
- c), x) - 2*(12*c*x*log(c*x + 1)^2 + 2*(c^2*x^2 - 1)*log(c*x + 1)^3 - 3*(c
^2*x^2 - 2*c*x - 2*(c^2*x^2 - 1)*log(c*x + 1) + 1)*log(-c*x + 1)^2 + 3*(c^2
*x^2 - 2*(c^2*x^2 - 1)*log(c*x + 1)^2 + 6*c*x - 8*(c*x + 1)*log(c*x + 1))*l
og(-c*x + 1))/c^2 + ((4*log(-c*x + 1)^3 - 6*log(-c*x + 1)^2 + 6*log(-c*x +
1) - 3)*(c*x - 1)^2 + 8*(log(-c*x + 1)^3 - 3*log(-c*x + 1)^2 + 6*log(-c*x +
 1) - 6)*(c*x - 1))/c^2 + 18*log(4*c^3*x^2 - 4*c)/c^2 - 192*integrate(1/4*l
og(c*x + 1)/(c^3*x^2 - c), x))*b^3
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(b^3x\,\mathrm{artanh}\,(cx)^3+3\,ab^2x\,\mathrm{artanh}\,(cx)^2+3\,a^2bx\,\mathrm{artanh}\,(cx)+a^3x,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x*arctanh(c*x)^3 + 3*a*b^2*x*arctanh(c*x)^2 + 3*a^2*b*x*arctan
h(c*x) + a^3*x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(a+b\,\mathrm{atanh}\left(cx\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c*x))**3,x)
```

```
[Out] Integral(x*(a + b*atanh(c*x))**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\,\mathrm{artanh}\left(cx\right)+a)^{3}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^3*x, x)
```

3.29 $\int \left(a+b\tanh^{-1}(cx)\right)^3\,dx$

Optimal. Leaf size=108

$$-\frac{3b^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{c}+\frac{3b^3\text{PolyLog}\left(3,1-\frac{2}{1-cx}\right)}{2c}+x\left(a+b\tanh^{-1}(cx)\right)^3+\frac{\left(a+b\tanh^{-1}(cx)\right)}{c}$$

```
[Out] (a + b*ArcTanh[c*x])^3/c + x*(a + b*ArcTanh[c*x])^3 - (3*b*(a + b*ArcTanh[c
*x])^2*Log[2/(1 - c*x)])/c - (3*b^2*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(
1 - c*x)])/c + (3*b^3*PolyLog[3, 1 - 2/(1 - c*x)])/(2*c)
```

Rubi [A] time = 0.217234, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {5910, 5984, 5918, 5948, 6058, 6610}

$$-\frac{3b^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{c}+\frac{3b^3\text{PolyLog}\left(3,1-\frac{2}{1-cx}\right)}{2c}+x\left(a+b\tanh^{-1}(cx)\right)^3+\frac{\left(a+b\tanh^{-1}(cx)\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^3,x]
```

```
[Out] (a + b*ArcTanh[c*x])^3/c + x*(a + b*ArcTanh[c*x])^3 - (3*b*(a + b*ArcTanh[c
*x])^2*Log[2/(1 - c*x)])/c - (3*b^2*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(
1 - c*x)])/c + (3*b^3*PolyLog[3, 1 - 2/(1 - c*x)])/(2*c)
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \left(a+b\tanh^{-1}(cx)\right)^3\,dx = x\left(a+b\tanh^{-1}(cx)\right)^3-(3bc)\int \frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{1-c^2x^2}\,dx$$

$$=\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{c}+x\left(a+b\tanh^{-1}(cx)\right)^3-(3b)\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{1-cx}\,dx$$

$$=\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{c}+x\left(a+b\tanh^{-1}(cx)\right)^3-\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1-cx}\right)}{c}+\left(6b^2\right.$$

$$=\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{c}+x\left(a+b\tanh^{-1}(cx)\right)^3-\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1-cx}\right)}{c}-\frac{3b^2\left(\right.}{}$$

$$=\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{c}+x\left(a+b\tanh^{-1}(cx)\right)^3-\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1-cx}\right)}{c}-\frac{3b^2\left(\right.}{}$$

Mathematica [A] time = 0.260217, size = 161, normalized size = 1.49

$$\frac{6ab^2\left(\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)+\tanh^{-1}(cx)\left((cx-1)\tanh^{-1}(cx)-2\log\left(e^{-2\tanh^{-1}(cx)}+1\right)\right)\right)+b^3\left(6\tanh^{-1}(cx)\text{P}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])^3,x]
```

```
[Out] (2*a^3*c*x + 6*a^2*b*c*x*ArcTanh[c*x] + 3*a^2*b*Log[1 - c^2*x^2] + 6*a*b^2*
(ArcTanh[c*x]*((-1 + c*x)*ArcTanh[c*x] - 2*Log[1 + E^(-2*ArcTanh[c*x])]) +
PolyLog[2, -E^(-2*ArcTanh[c*x])]) + b^3*(2*ArcTanh[c*x]^2*((-1 + c*x)*ArcTa
nh[c*x] - 3*Log[1 + E^(-2*ArcTanh[c*x])]) + 6*ArcTanh[c*x]*PolyLog[2, -E^(-
2*ArcTanh[c*x])] + 3*PolyLog[3, -E^(-2*ArcTanh[c*x])]))/(2*c)
```

Maple [B] time = 0.082, size = 261, normalized size = 2.4

$$xa^3+b^3x\left(\text{Artanh}\left(cx\right)\right)^3+\frac{b^3\left(\text{Artanh}\left(cx\right)\right)^3}{c}-3\frac{b^3\left(\text{Artanh}\left(cx\right)\right)^2}{c}\ln\left(\frac{(cx+1)^2}{-c^2x^2+1}+1\right)-3\frac{b^3\text{Artanh}\left(cx\right)}{c}\text{polylog}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^3,x)
```

```
[Out] x*a^3+b^3*x*arctanh(c*x)^3+1/c*b^3*arctanh(c*x)^3-3/c*b^3*arctanh(c*x)^2*ln
((c*x+1)^2/(-c^2*x^2+1)+1)-3/c*b^3*arctanh(c*x)*polylog(2,-(c*x+1)^2/(-c^2*
```

```
x^2+1))+3/2/c*b^3*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))+3*x*a*b^2*arctanh(c*x)
^2+3/c*a*b^2*arctanh(c*x)^2-6/c*arctanh(c*x)*ln((c*x+1)^2/(-c^2*x^2+1)+1)*a
*b^2-3/c*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))*a*b^2+3*x*a^2*b*arctanh(c*x)+3/
2/c*a^2*b*ln(-c^2*x^2+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$a^3x + \frac{3\left(2\,cx\,\mathrm{artanh}\,(cx) + \log\left(-c^2x^2+1\right)\right)a^2b}{2\,c} - \frac{\left(b^3cx - b^3\right)\log\left(-cx+1\right)^3 - 3\left(2\,ab^2cx + \left(b^3cx + b^3\right)\log\left(cx+1\right)\right)\log}{8\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3,x, algorithm="maxima")
```

```
[Out] a^3*x + 3/2*(2*c*x*arctanh(c*x) + log(-c^2*x^2 + 1))*a^2*b/c - 1/8*((b^3*c*
x - b^3)*log(-c*x + 1)^3 - 3*(2*a*b^2*c*x + (b^3*c*x + b^3)*log(c*x + 1))*l
og(-c*x + 1)^2)/c - integrate(-1/8*((b^3*c*x - b^3)*log(c*x + 1)^3 + 6*(a*b
^2*c*x - a*b^2)*log(c*x + 1)^2 - 3*(4*a*b^2*c*x + (b^3*c*x - b^3)*log(c*x +
 1)^2 - 2*(2*a*b^2 - b^3 - (2*a*b^2*c + b^3*c)*x)*log(c*x + 1))*log(-c*x +
1))/(c*x - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(b^3\,\mathrm{artanh}\,(cx)^3 + 3\,ab^2\,\mathrm{artanh}\,(cx)^2 + 3\,a^2b\,\mathrm{artanh}\,(cx) + a^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*arctanh(c*x)^3 + 3*a*b^2*arctanh(c*x)^2 + 3*a^2*b*arctanh(c*x)
 + a^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\,\mathrm{atanh}\,(cx)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**3,x)
```

```
[Out] Integral((a + b*atanh(c*x))**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\,\mathrm{artanh}\,(cx) + a\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^3, x)
```

3.30 $\int \frac{(a+b\tanh^{-1}(cx))^3}{x}\,dx$

Optimal. Leaf size=184

$$\frac{3}{2}b^2\text{PolyLog}\left(3,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)-\frac{3}{2}b^2\text{PolyLog}\left(3,\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)-\frac{3}{2}b\text{PolyLog}\left(2,1-\right.$$

```
[Out] 2*(a + b*ArcTanh[c*x])^3*ArcTanh[1 - 2/(1 - c*x)] - (3*b*(a + b*ArcTanh[c*x
])^2*PolyLog[2, 1 - 2/(1 - c*x)])/2 + (3*b*(a + b*ArcTanh[c*x])^2*PolyLog[2
, -1 + 2/(1 - c*x)])/2 + (3*b^2*(a + b*ArcTanh[c*x])*PolyLog[3, 1 - 2/(1 -
c*x)])/2 - (3*b^2*(a + b*ArcTanh[c*x])*PolyLog[3, -1 + 2/(1 - c*x)])/2 - (3
*b^3*PolyLog[4, 1 - 2/(1 - c*x)])/4 + (3*b^3*PolyLog[4, -1 + 2/(1 - c*x)])/
4
```

Rubi [A] time = 0.448718, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5914, 6052, 5948, 6058, 6062, 6610}

$$\frac{3}{2}b^2\text{PolyLog}\left(3,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)-\frac{3}{2}b^2\text{PolyLog}\left(3,\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)-\frac{3}{2}b\text{PolyLog}\left(2,1-\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^3/x,x]
```

```
[Out] 2*(a + b*ArcTanh[c*x])^3*ArcTanh[1 - 2/(1 - c*x)] - (3*b*(a + b*ArcTanh[c*x
])^2*PolyLog[2, 1 - 2/(1 - c*x)])/2 + (3*b*(a + b*ArcTanh[c*x])^2*PolyLog[2
, -1 + 2/(1 - c*x)])/2 + (3*b^2*(a + b*ArcTanh[c*x])*PolyLog[3, 1 - 2/(1 -
c*x)])/2 - (3*b^2*(a + b*ArcTanh[c*x])*PolyLog[3, -1 + 2/(1 - c*x)])/2 - (3
*b^3*PolyLog[4, 1 - 2/(1 - c*x)])/4 + (3*b^3*PolyLog[4, -1 + 2/(1 - c*x)])/
4
```

Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
 e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6062

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[k + 1, u])/(2
*c*d), x] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[k + 1,
u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && Eq
Q[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{x}\,dx = 2\left(a+b\tanh^{-1}(cx)\right)^3\tanh^{-1}\left(1-\frac{2}{1-cx}\right)-(6bc)\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1-\ldots}\right)}{1-c^2x^2}$$

$$= 2\left(a+b\tanh^{-1}(cx)\right)^3\tanh^{-1}\left(1-\frac{2}{1-cx}\right)+(3bc)\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1-cx}\right)}{1-c^2x^2}\,dx-\ldots$$

$$= 2\left(a+b\tanh^{-1}(cx)\right)^3\tanh^{-1}\left(1-\frac{2}{1-cx}\right)-\frac{3}{2}b\left(a+b\tanh^{-1}(cx)\right)^2\operatorname{Li}_2\left(1-\frac{2}{1-cx}\right)+\ldots$$

$$= 2\left(a+b\tanh^{-1}(cx)\right)^3\tanh^{-1}\left(1-\frac{2}{1-cx}\right)-\frac{3}{2}b\left(a+b\tanh^{-1}(cx)\right)^2\operatorname{Li}_2\left(1-\frac{2}{1-cx}\right)+\ldots$$

$$= 2\left(a+b\tanh^{-1}(cx)\right)^3\tanh^{-1}\left(1-\frac{2}{1-cx}\right)-\frac{3}{2}b\left(a+b\tanh^{-1}(cx)\right)^2\operatorname{Li}_2\left(1-\frac{2}{1-cx}\right)+\ldots$$

Mathematica [A] time = 0.127977, size = 178, normalized size = 0.97

$$\frac{3}{4}b\left(2\mathrm{PolyLog}\left(2,\frac{cx+1}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)^2-2\mathrm{PolyLog}\left(2,\frac{cx+1}{cx-1}\right)\left(a+b\tanh^{-1}(cx)\right)^2+b\left(-2\mathrm{PolyLog}\left(3,\frac{cx+\ldots}{1-\ldots}\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])^3/x,x]
```

```
[Out] 2*(a + b*ArcTanh[c*x])^3*ArcTanh[(1 + c*x)/(-1 + c*x)] + (3*b*(2*(a + b*Arc
Tanh[c*x])^2*PolyLog[2, (1 + c*x)/(1 - c*x)] - 2*(a + b*ArcTanh[c*x])^2*Pol
yLog[2, (1 + c*x)/(-1 + c*x)] + b*(-2*(a + b*ArcTanh[c*x])*PolyLog[3, (1 +
c*x)/(1 - c*x)] + 2*(a + b*ArcTanh[c*x])*PolyLog[3, (1 + c*x)/(-1 + c*x)] +
 b*(PolyLog[4, (1 + c*x)/(1 - c*x)] - PolyLog[4, (1 + c*x)/(-1 + c*x)]))))/
4
```

Maple [C] time = 0.113, size = 1470, normalized size = 8.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^3/x,x)
```

```
[Out] 3/2*a*b^2*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))-6*a*b^2*polylog(3,(c*x+1)/(-c^
2*x^2+1)^(1/2))-6*a*b^2*polylog(3,-(c*x+1)/(-c^2*x^2+1)^(1/2))+b^3*ln(c*x)*
arctanh(c*x)^3-b^3*arctanh(c*x)^3*ln((c*x+1)^2/(-c^2*x^2+1)-1)-3/2*b^3*arct
anh(c*x)^2*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))-3/2*a^2*b*dilog(c*x)+3/2*b^3*
arctanh(c*x)*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))+b^3*arctanh(c*x)^3*ln(1-(c*
x+1)/(-c^2*x^2+1)^(1/2))+3*b^3*arctanh(c*x)^2*polylog(2,(c*x+1)/(-c^2*x^2+1
)^(1/2))-6*b^3*arctanh(c*x)*polylog(3,(c*x+1)/(-c^2*x^2+1)^(1/2))+b^3*arcta
nh(c*x)^3*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+3*b^3*arctanh(c*x)^2*polylog(2,-
(c*x+1)/(-c^2*x^2+1)^(1/2))-6*b^3*arctanh(c*x)*polylog(3,-(c*x+1)/(-c^2*x^2
+1)^(1/2))-3/2*a^2*b*dilog(c*x+1)+3/2*I*a*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^
2+1)-1))*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-
1)/((c*x+1)^2/(-c^2*x^2+1)+1))*arctanh(c*x)^2-1/2*I*b^3*Pi*csgn(I*((c*x+1)^
2/(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1
)+1))^2*arctanh(c*x)^3+3/2*I*a*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c
*x+1)^2/(-c^2*x^2+1)+1))^3*arctanh(c*x)^2-1/2*I*b^3*Pi*csgn(I/((c*x+1)^2/(-
c^2*x^2+1)+1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1)
)^2*arctanh(c*x)^3-3/2*I*a*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I
*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*arctanh(c*x)^2-3/
2*I*a*b^2*Pi*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*((c*x+1)^2/(-c^2*x^2
+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*arctanh(c*x)^2+1/2*I*b^3*Pi*csgn(I*((c
*x+1)^2/(-c^2*x^2+1)-1))*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*((c*x+1)
^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))*arctanh(c*x)^3-3*a*b^2*arcta
nh(c*x)*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))-3*a*b^2*arctanh(c*x)^2*ln((c*x+1
)^2/(-c^2*x^2+1)-1)+3*a*b^2*arctanh(c*x)^2*ln(1-(c*x+1)/(-c^2*x^2+1)^(1/2))
+6*a*b^2*arctanh(c*x)*polylog(2,(c*x+1)/(-c^2*x^2+1)^(1/2))+3*a*b^2*arctanh
(c*x)^2*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+6*a*b^2*arctanh(c*x)*polylog(2,-(c
*x+1)/(-c^2*x^2+1)^(1/2))+3*a^2*b*ln(c*x)*arctanh(c*x)+3*a*b^2*ln(c*x)*arct
anh(c*x)^2+a^3*ln(c*x)-3/4*b^3*polylog(4,-(c*x+1)^2/(-c^2*x^2+1))+6*b^3*pol
ylog(4,(c*x+1)/(-c^2*x^2+1)^(1/2))+6*b^3*polylog(4,-(c*x+1)/(-c^2*x^2+1)^(1
/2))+1/2*I*b^3*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)
+1))^3*arctanh(c*x)^3-3/2*a^2*b*ln(c*x)*ln(c*x+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$a^3 \log(x) + \int \frac{b^3(\log(cx+1) - \log(-cx+1))^3}{8x} + \frac{3ab^2(\log(cx+1) - \log(-cx+1))^2}{4x} + \frac{3a^2b(\log(cx+1) - \log(-cx+1}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/x,x, algorithm="maxima")
```

```
[Out] a^3*log(x) + integrate(1/8*b^3*(log(c*x + 1) - log(-c*x + 1))^3/x + 3/4*a*b
^2*(log(c*x + 1) - log(-c*x + 1))^2/x + 3/2*a^2*b*(log(c*x + 1) - log(-c*x
+ 1))/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3 \operatorname{artanh}(cx)^3 + 3ab^2 \operatorname{artanh}(cx)^2 + 3a^2b \operatorname{artanh}(cx) + a^3}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/x,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctanh(c*x)^3 + 3*a*b^2*arctanh(c*x)^2 + 3*a^2*b*arctanh(c*x
) + a^3)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\,\mathrm{atanh}\,(cx))^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**3/x,x)
```

```
[Out] Integral((a + b*atanh(c*x))**3/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\mathrm{artanh}\,(cx) + a)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^3/x, x)
```

3.31 $\int \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{x^2}\,dx$

Optimal. Leaf size=102

$$-3b^2c\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)\left(a+b\tanh^{-1}(cx)\right)-\frac{3}{2}b^3c\text{PolyLog}\left(3,\frac{2}{cx+1}-1\right)+c\left(a+b\tanh^{-1}(cx)\right)^3-\frac{\left(a+b\tanh\right.}{x}$$

```
[Out] c*(a + b*ArcTanh[c*x])^3 - (a + b*ArcTanh[c*x])^3/x + 3*b*c*(a + b*ArcTanh[
c*x])^2*Log[2 - 2/(1 + c*x)] - 3*b^2*c*(a + b*ArcTanh[c*x])*PolyLog[2, -1 +
 2/(1 + c*x)] - (3*b^3*c*PolyLog[3, -1 + 2/(1 + c*x)])/2
```

Rubi [A] time = 0.268323, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5916, 5988, 5932, 5948, 6056, 6610}

$$-3b^2c\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)\left(a+b\tanh^{-1}(cx)\right)-\frac{3}{2}b^3c\text{PolyLog}\left(3,\frac{2}{cx+1}-1\right)+c\left(a+b\tanh^{-1}(cx)\right)^3-\frac{\left(a+b\tanh\right.}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^3/x^2,x]
```

```
[Out] c*(a + b*ArcTanh[c*x])^3 - (a + b*ArcTanh[c*x])^3/x + 3*b*c*(a + b*ArcTanh[
c*x])^2*Log[2 - 2/(1 + c*x)] - 3*b^2*c*(a + b*ArcTanh[c*x])*PolyLog[2, -1 +
 2/(1 + c*x)] - (3*b^3*c*PolyLog[3, -1 + 2/(1 + c*x)])/2
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
```

```
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{x^2}\,dx &= -\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{x}+(3bc)\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x\left(1-c^2x^2\right)}\,dx\\ &= c\left(a+b\tanh^{-1}(cx)\right)^3-\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{x}+(3bc)\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x(1+cx)}\,dx\\ &= c\left(a+b\tanh^{-1}(cx)\right)^3-\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{x}+3bc\left(a+b\tanh^{-1}(cx)\right)^2\log\left(2-\frac{2}{1+cx}\right)\\ &= c\left(a+b\tanh^{-1}(cx)\right)^3-\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{x}+3bc\left(a+b\tanh^{-1}(cx)\right)^2\log\left(2-\frac{2}{1+cx}\right)\\ &= c\left(a+b\tanh^{-1}(cx)\right)^3-\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{x}+3bc\left(a+b\tanh^{-1}(cx)\right)^2\log\left(2-\frac{2}{1+cx}\right)\end{aligned}$$

Mathematica [C] time = 0.320924, size = 196, normalized size = 1.92

$$3ab^2c\left(\tanh^{-1}(cx)\left(-\frac{\tanh^{-1}(cx)}{cx}+\tanh^{-1}(cx)+2\log\left(1-e^{-2\tanh^{-1}(cx)}\right)\right)-\text{PolyLog}\left(2,e^{-2\tanh^{-1}(cx)}\right)\right)+b^3c\left(3\tanh\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])^3/x^2,x]
```

```
[Out] -(a^3/x) - (3*a^2*b*ArcTanh[c*x])/x + 3*a^2*b*c*Log[x] - (3*a^2*b*c*Log[1 -
 c^2*x^2])/2 + 3*a*b^2*c*(ArcTanh[c*x]*(ArcTanh[c*x] - ArcTanh[c*x]/(c*x) +
 2*Log[1 - E^(-2*ArcTanh[c*x])]) - PolyLog[2, E^(-2*ArcTanh[c*x])]) + b^3*c
*((I/8)*Pi^3 - ArcTanh[c*x]^3 - ArcTanh[c*x]^3/(c*x) + 3*ArcTanh[c*x]^2*Log
[1 - E^(2*ArcTanh[c*x])] + 3*ArcTanh[c*x]*PolyLog[2, E^(2*ArcTanh[c*x])] -
(3*PolyLog[3, E^(2*ArcTanh[c*x])])/2)
```

Maple [C] time = 0.183, size = 1583, normalized size = 15.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^3/x^2,x)
```

```
[Out] 6*c*a*b^2*arctanh(c*x)*ln(c*x)-3*c*a*b^2*arctanh(c*x)*ln(c*x-1)-3*c*a*b^2*l
n(c*x)*ln(c*x+1)-3*c*b^3*arctanh(c*x)^2*ln((c*x+1)^2/(-c^2*x^2+1)-1)+6*c*b^
3*arctanh(c*x)*polylog(2,-(c*x+1)/(-c^2*x^2+1)^(1/2))+3*c*b^3*arctanh(c*x)^
2*ln(1-(c*x+1)/(-c^2*x^2+1)^(1/2))+6*c*b^3*arctanh(c*x)*polylog(2,(c*x+1)/(
-c^2*x^2+1)^(1/2))-3/2*c*b^3*arctanh(c*x)^2*ln(c*x-1)-3/2*c*b^3*arctanh(c*x
)^2*ln(c*x+1)+3*c*b^3*arctanh(c*x)^2*ln((c*x+1)/(-c^2*x^2+1)^(1/2))-3*c*a*b
^2*dilog(c*x)-3*c*a*b^2*dilog(c*x+1)-3*a^2*b/x*arctanh(c*x)-3*a*b^2/x*arcta
nh(c*x)^2+3*c*a^2*b*ln(c*x)-3/2*c*a^2*b*ln(c*x-1)-3/2*c*a^2*b*ln(c*x+1)-3/4
*c*a*b^2*ln(c*x-1)^2+3/4*c*a*b^2*ln(c*x+1)^2+3*c*b^3*arctanh(c*x)^2*ln(2)+3
*c*a*b^2*dilog(1/2+1/2*c*x)+3*c*b^3*arctanh(c*x)^2*ln(1+(c*x+1)/(-c^2*x^2+1
)^(1/2))+3*c*b^3*ln(c*x)*arctanh(c*x)^2-b^3/x*arctanh(c*x)^3-6*c*b^3*polylo
g(3,-(c*x+1)/(-c^2*x^2+1)^(1/2))-6*c*b^3*polylog(3,(c*x+1)/(-c^2*x^2+1)^(1/
2))-c*b^3*arctanh(c*x)^3+3/2*I*c*b^3*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*
csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+
1)^2/(-c^2*x^2+1)+1))*arctanh(c*x)^2-3/4*I*c*b^3*Pi*csgn(I/((c*x+1)^2/(-c^2
*x^2+1)+1))*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*
x+1)^2/(-c^2*x^2+1)+1))*arctanh(c*x)^2-a^3/x+3/4*I*c*b^3*Pi*csgn(I*(c*x+1)/
(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*arctanh(c*x)^2-3/4*I*c*
b^3*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^
2/(-c^2*x^2+1)+1))^2*arctanh(c*x)^2+3/2*I*c*b^3*Pi*csgn(I*((c*x+1)^2/(-c^2*
x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^3*arctanh(c*x)^2+3/4*I*c*b^3*Pi*csgn(
I*(c*x+1)^2/(c^2*x^2-1))^3*arctanh(c*x)^2+3/4*I*c*b^3*Pi*csgn(I*(c*x+1)^2/(
c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^3*arctanh(c*x)^2+3/2*I*c*b^3*Pi*csgn
(I/((c*x+1)^2/(-c^2*x^2+1)+1))^3*arctanh(c*x)^2-3/2*I*c*b^3*Pi*csgn(I/((c*x
+1)^2/(-c^2*x^2+1)+1))^2*arctanh(c*x)^2-3*c*a*b^2*arctanh(c*x)*ln(c*x+1)+3/
2*c*a*b^2*ln(c*x-1)*ln(1/2+1/2*c*x)-3/2*c*a*b^2*ln(-1/2*c*x+1/2)*ln(c*x+1)+
3/2*c*a*b^2*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)+3/2*I*c*b^3*Pi*arctanh(c*x)^2+
3/2*I*c*b^3*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2
-1))^2*arctanh(c*x)^2-3/2*I*c*b^3*Pi*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csg
n(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*arctanh(c*x)^2
-3/2*I*c*b^3*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^2/(-c^2*
x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*arctanh(c*x)^2+3/4*I*c*b^3*Pi*csgn(
I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2
*x^2+1)+1))^2*arctanh(c*x)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{3}{2}\left(c\left(\log\left(c^2x^2-1\right)-\log\left(x^2\right)\right)+\frac{2\,\mathrm{artanh}\,(cx)}{x}\right)a^2b-\frac{a^3}{x}-\frac{\left(b^3cx-b^3\right)\log\left(-cx+1\right)^3+3\left(2\,ab^2+\left(b^3cx+b^3\right)\log\left(cx\right.\right.}{8\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/x^2,x, algorithm="maxima")
```

```
[Out] -3/2*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*a^2*b - a^3/x - 1
/8*((b^3*c*x - b^3)*log(-c*x + 1)^3 + 3*(2*a*b^2 + (b^3*c*x + b^3)*log(c*x
+ 1))*log(-c*x + 1)^2)/x - integrate(-1/8*((b^3*c*x - b^3)*log(c*x + 1)^3 +
 6*(a*b^2*c*x - a*b^2)*log(c*x + 1)^2 + 3*(4*a*b^2*c*x - (b^3*c*x - b^3)*lo
g(c*x + 1)^2 + 2*(b^3*c^2*x^2 + 2*a*b^2 - (2*a*b^2*c - b^3*c)*x)*log(c*x +
1))*log(-c*x + 1))/(c*x^3 - x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3 \operatorname{artanh}(cx)^3 + 3\,ab^2 \operatorname{artanh}(cx)^2 + 3\,a^2 b \operatorname{artanh}(cx) + a^3}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctanh(c*x)^3 + 3*a*b^2*arctanh(c*x)^2 + 3*a^2*b*arctanh(c*x
) + a^3)/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{atanh}(cx))^3}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**3/x**2,x)
```

```
[Out] Integral((a + b*atanh(c*x))**3/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{artanh}(cx) + a)^3}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^3/x^2, x)
```

3.32 $\int \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{x^3}\,dx$

Optimal. Leaf size=123

$$-\frac{3}{2}b^3c^2\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)+3b^2c^2\log\left(2-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)+\frac{3}{2}bc^2\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{2}c^2\left(a+b\tan\right.$$

```
[Out] (3*b*c^2*(a + b*ArcTanh[c*x])^2)/2 - (3*b*c*(a + b*ArcTanh[c*x])^2)/(2*x) +
 (c^2*(a + b*ArcTanh[c*x])^3)/2 - (a + b*ArcTanh[c*x])^3/(2*x^2) + 3*b^2*c^
2*(a + b*ArcTanh[c*x])*Log[2 - 2/(1 + c*x)] - (3*b^3*c^2*PolyLog[2, -1 + 2/
(1 + c*x)])/2
```

Rubi [A] time = 0.294355, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5916, 5982, 5988, 5932, 2447, 5948}

$$-\frac{3}{2}b^3c^2\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)+3b^2c^2\log\left(2-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)+\frac{3}{2}bc^2\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{2}c^2\left(a+b\tan\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^3/x^3,x]
```


```
[Out] (3*b*c^2*(a + b*ArcTanh[c*x])^2)/2 - (3*b*c*(a + b*ArcTanh[c*x])^2)/(2*x) +
 (c^2*(a + b*ArcTanh[c*x])^3)/2 - (a + b*ArcTanh[c*x])^3/(2*x^2) + 3*b^2*c^
2*(a + b*ArcTanh[c*x])*Log[2 - 2/(1 + c*x)] - (3*b^3*c^2*PolyLog[2, -1 + 2/
(1 + c*x)])/2
```


Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
```

```
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{x^3}\,dx = -\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{2x^2}+\frac{1}{2}(3bc)\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^2\left(1-c^2x^2\right)}\,dx$$

$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{2x^2}+\frac{1}{2}(3bc)\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^2}\,dx+\frac{1}{2}\left(3bc^3\right)\int \frac{\left(a+b\tanh^{-1}(c\right.}{1-c^2x^2}$$

$$= -\frac{3bc\left(a+b\tanh^{-1}(cx)\right)^2}{2x}+\frac{1}{2}c^2\left(a+b\tanh^{-1}(cx)\right)^3-\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{2x^2}+\left(3b^2c^2\right)\int$$

$$= \frac{3}{2}bc^2\left(a+b\tanh^{-1}(cx)\right)^2-\frac{3bc\left(a+b\tanh^{-1}(cx)\right)^2}{2x}+\frac{1}{2}c^2\left(a+b\tanh^{-1}(cx)\right)^3-\frac{\left(a+b\right.}{}$$

$$= \frac{3}{2}bc^2\left(a+b\tanh^{-1}(cx)\right)^2-\frac{3bc\left(a+b\tanh^{-1}(cx)\right)^2}{2x}+\frac{1}{2}c^2\left(a+b\tanh^{-1}(cx)\right)^3-\frac{\left(a+b\right.}{}$$

$$= \frac{3}{2}bc^2\left(a+b\tanh^{-1}(cx)\right)^2-\frac{3bc\left(a+b\tanh^{-1}(cx)\right)^2}{2x}+\frac{1}{2}c^2\left(a+b\tanh^{-1}(cx)\right)^3-\frac{\left(a+b\right.}{}$$

Mathematica [A] time = 0.271884, size = 192, normalized size = 1.56

$$-6b^3c^2x^2\text{PolyLog}\left(2,e^{-2\tanh^{-1}(cx)}\right)+a\left(-2a^2-3abc^2x^2\log(1-cx)+3abc^2x^2\log(cx+1)-6abcx+12b^2c^2x^2\log\left(\frac{}{\sqrt{1}}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])^3/x^3,x]
```

```
[Out] (6*b^2*(-1 + c*x)*(a + a*c*x + b*c*x)*ArcTanh[c*x]^2 + 2*b^3*(-1 + c^2*x^2)
*ArcTanh[c*x]^3 - 6*b*ArcTanh[c*x]*(a^2 + 2*a*b*c*x - 2*b^2*c^2*x^2*Log[1 -
 E^(-2*ArcTanh[c*x])]) + a*(-2*a^2 - 6*a*b*c*x - 3*a*b*c^2*x^2*Log[1 - c*x]
 + 3*a*b*c^2*x^2*Log[1 + c*x] + 12*b^2*c^2*x^2*Log[(c*x)/Sqrt[1 - c^2*x^2]]
) - 6*b^3*c^2*x^2*PolyLog[2, E^(-2*ArcTanh[c*x])]))/(4*x^2)
```

Maple [C] time = 0.316, size = 5098, normalized size = 41.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^3/x^3,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{3}{4}\left(\left(c\log\left(cx+1\right)-c\log\left(cx-1\right)-\frac{2}{x}\right)c-\frac{2\,\operatorname{artanh}\left(cx\right)}{x^2}\right)a^2b+\frac{3}{8}\left(\left(2\left(\log\left(cx-1\right)-2\right)\log\left(cx+1\right)-\log\left(cx+1\right)^2-\log\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/x^3,x, algorithm="maxima")
```

```
[Out] 3/4*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*a^2*b
+ 3/8*((2*(log(c*x - 1) - 2)*log(c*x + 1) - log(c*x + 1)^2 - log(c*x - 1)^2
 - 4*log(c*x - 1) + 8*log(x))*c^2 + 4*(c*log(c*x + 1) - c*log(c*x - 1) - 2/
x)*c*arctanh(c*x))*a*b^2 - 1/16*b^3*(((c^2*x^2 - 1)*log(-c*x + 1)^3 + 3*(2*
c*x - (c^2*x^2 - 1)*log(c*x + 1))*log(-c*x + 1)^2)/x^2 + 2*integrate(-((c*x
 - 1)*log(c*x + 1)^3 + 3*(2*c^2*x^2 - (c*x - 1)*log(c*x + 1)^2 - (c^3*x^3 -
 c*x)*log(c*x + 1))*log(-c*x + 1))/(c*x^4 - x^3), x)) - 3/2*a*b^2*arctanh(c
*x)^2/x^2 - 1/2*a^3/x^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3\operatorname{artanh}\left(cx\right)^3+3\,ab^2\operatorname{artanh}\left(cx\right)^2+3\,a^2b\operatorname{artanh}\left(cx\right)+a^3}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/x^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctanh(c*x)^3 + 3*a*b^2*arctanh(c*x)^2 + 3*a^2*b*arctanh(c*x
) + a^3)/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+b\operatorname{atanh}\left(cx\right)\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**3/x**3,x)
```

```
[Out] Integral((a + b*atanh(c*x))**3/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(b\operatorname{artanh}\left(cx\right)+a\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^3/x^3, x)
```

3.33 $\int \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{x^4}\,dx$

Optimal. Leaf size=200

$$-b^2c^3\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)\left(a+b\tanh^{-1}(cx)\right)-\frac{1}{2}b^3c^3\text{PolyLog}\left(3,\frac{2}{cx+1}-1\right)-\frac{b^2c^2\left(a+b\tanh^{-1}(cx)\right)}{x}+\frac{1}{3}c^3\left(a+\right.$$

```
[Out] -((b^2*c^2*(a + b*ArcTanh[c*x]))/x) + (b*c^3*(a + b*ArcTanh[c*x])^2)/2 - (b
*c*(a + b*ArcTanh[c*x])^2)/(2*x^2) + (c^3*(a + b*ArcTanh[c*x])^3)/3 - (a +
b*ArcTanh[c*x])^3/(3*x^3) + b^3*c^3*Log[x] - (b^3*c^3*Log[1 - c^2*x^2])/2 +
 b*c^3*(a + b*ArcTanh[c*x])^2*Log[2 - 2/(1 + c*x)] - b^2*c^3*(a + b*ArcTanh
[c*x])*PolyLog[2, -1 + 2/(1 + c*x)] - (b^3*c^3*PolyLog[3, -1 + 2/(1 + c*x)]
)/2
```

Rubi [A] time = 0.498341, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {5916, 5982, 266, 36, 29, 31, 5948, 5988, 5932, 6056, 6610}

$$-b^2c^3\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)\left(a+b\tanh^{-1}(cx)\right)-\frac{1}{2}b^3c^3\text{PolyLog}\left(3,\frac{2}{cx+1}-1\right)-\frac{b^2c^2\left(a+b\tanh^{-1}(cx)\right)}{x}+\frac{1}{3}c^3\left(a+\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^3/x^4,x]
```


```
[Out] -((b^2*c^2*(a + b*ArcTanh[c*x]))/x) + (b*c^3*(a + b*ArcTanh[c*x])^2)/2 - (b
*c*(a + b*ArcTanh[c*x])^2)/(2*x^2) + (c^3*(a + b*ArcTanh[c*x])^3)/3 - (a +
b*ArcTanh[c*x])^3/(3*x^3) + b^3*c^3*Log[x] - (b^3*c^3*Log[1 - c^2*x^2])/2 +
 b*c^3*(a + b*ArcTanh[c*x])^2*Log[2 - 2/(1 + c*x)] - b^2*c^3*(a + b*ArcTanh
[c*x])*PolyLog[2, -1 + 2/(1 + c*x)] - (b^3*c^3*PolyLog[3, -1 + 2/(1 + c*x)]
)/2
```


Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(a+b\tanh^{-1}(cx))^3}{x^4}\,dx = -\frac{(a+b\tanh^{-1}(cx))^3}{3x^3} + (bc)\int \frac{(a+b\tanh^{-1}(cx))^2}{x^3(1-c^2x^2)}\,dx$$

$$= -\frac{(a+b\tanh^{-1}(cx))^3}{3x^3} + (bc)\int \frac{(a+b\tanh^{-1}(cx))^2}{x^3}\,dx + (bc^3)\int \frac{(a+b\tanh^{-1}(cx))^2}{x(1-c^2x^2)}\,dx$$

$$= -\frac{bc(a+b\tanh^{-1}(cx))^2}{2x^2} + \frac{1}{3}c^3(a+b\tanh^{-1}(cx))^3 - \frac{(a+b\tanh^{-1}(cx))^3}{3x^3} + (b^2c^2)\int \frac{a+b}{x^2}$$

$$= -\frac{bc(a+b\tanh^{-1}(cx))^2}{2x^2} + \frac{1}{3}c^3(a+b\tanh^{-1}(cx))^3 - \frac{(a+b\tanh^{-1}(cx))^3}{3x^3} + bc^3(a+b\tanh$$

$$= -\frac{b^2c^2(a+b\tanh^{-1}(cx))}{x} + \frac{1}{2}bc^3(a+b\tanh^{-1}(cx))^2 - \frac{bc(a+b\tanh^{-1}(cx))^2}{2x^2} + \frac{1}{3}c^3(a+b$$

$$= -\frac{b^2c^2(a+b\tanh^{-1}(cx))}{x} + \frac{1}{2}bc^3(a+b\tanh^{-1}(cx))^2 - \frac{bc(a+b\tanh^{-1}(cx))^2}{2x^2} + \frac{1}{3}c^3(a+b$$

$$= -\frac{b^2c^2(a+b\tanh^{-1}(cx))}{x} + \frac{1}{2}bc^3(a+b\tanh^{-1}(cx))^2 - \frac{bc(a+b\tanh^{-1}(cx))^2}{2x^2} + \frac{1}{3}c^3(a+b$$

$$= -\frac{b^2c^2(a+b\tanh^{-1}(cx))}{x} + \frac{1}{2}bc^3(a+b\tanh^{-1}(cx))^2 - \frac{bc(a+b\tanh^{-1}(cx))^2}{2x^2} + \frac{1}{3}c^3(a+b$$

Mathematica [C] time = 0.861358, size = 323, normalized size = 1.62

$$-\frac{24ab^2\left(c^3x^3\mathrm{PolyLog}\left(2, e^{-2\tanh^{-1}(cx)}\right) + c^2x^2 + (1-c^3x^3)\tanh^{-1}(cx)^2 - cx\tanh^{-1}(cx)\left(c^2x^2 + 2c^2x^2\log\left(1-e^{-2\tanh^{-1}\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])^3/x^4,x]
```

```
[Out] -(8*a^3 + 12*a^2*b*c*x + 24*a^2*b*ArcTanh[c*x] - 24*a^2*b*c^3*x^3*Log[x] +
12*a^2*b*c^3*x^3*Log[1 - c^2*x^2] + 24*a*b^2*(c^2*x^2 + (1 - c^3*x^3)*ArcTa
nh[c*x]^2 - c*x*ArcTanh[c*x]*(-1 + c^2*x^2 + 2*c^2*x^2*Log[1 - E^(-2*ArcTan
h[c*x])]) + c^3*x^3*PolyLog[2, E^(-2*ArcTanh[c*x])]) + b^3*((-I)*c^3*Pi^3*x
^3 + 24*c^2*x^2*ArcTanh[c*x] + 12*c*x*ArcTanh[c*x]^2 - 12*c^3*x^3*ArcTanh[c
*x]^2 + 8*ArcTanh[c*x]^3 + 8*c^3*x^3*ArcTanh[c*x]^3 - 24*c^3*x^3*ArcTanh[c*
x]^2*Log[1 - E^(2*ArcTanh[c*x])] - 24*c^3*x^3*Log[(c*x)/Sqrt[1 - c^2*x^2]]
- 24*c^3*x^3*ArcTanh[c*x]*PolyLog[2, E^(2*ArcTanh[c*x])] + 12*c^3*x^3*PolyL
og[3, E^(2*ArcTanh[c*x])]))/(24*x^3)
```

Maple [C] time = 0.849, size = 1838, normalized size = 9.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^3/x^4,x)
```

```
[Out] -1/2*c^3*a*b^2*ln(c*x-1)+1/2*c^3*a*b^2*ln(c*x+1)-1/2*c^3*a^2*b*ln(c*x-1)-1/
2*c^3*a^2*b*ln(c*x+1)-1/4*c^3*a*b^2*ln(c*x-1)^2+1/4*c^3*a*b^2*ln(c*x+1)^2-a
```

```
^2*b/x^3*arctanh(c*x)-a*b^2/x^3*arctanh(c*x)^2-1/4*I*c^3*b^3*arctanh(c*x)^2
*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*(c
*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))*Pi+1/2*I*c^3*b^3*arctanh(c*
x)^2*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*
csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))*Pi-1/3*a^3/x^
3-1/3*b^3/x^3*arctanh(c*x)^3+c^3*b^3*ln((c*x+1)/(-c^2*x^2+1)^(1/2)-1)+c^3*b
^3*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))-2*c^3*b^3*polylog(3,-(c*x+1)/(-c^2*x^2+
1)^(1/2))-2*c^3*b^3*polylog(3,(c*x+1)/(-c^2*x^2+1)^(1/2))-c^3*b^3*arctanh(c
*x)-1/3*c^3*b^3*arctanh(c*x)^3+1/2*c^3*b^3*arctanh(c*x)^2+1/4*I*c^3*b^3*arc
tanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*Pi+1/2*I*c^3*b^3*arctanh(c*x)^2
*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))^3*Pi+1/2*I*c^3*b^3*arctanh(c*x)^2*csgn(
I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^3*Pi+1/4*I*c^3*b^3
*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^3*
Pi-1/2*I*c^3*b^3*arctanh(c*x)^2*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))^2*Pi+1/4
*I*c^3*b^3*arctanh(c*x)^2*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*(c*x+1)
^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*Pi-c^2*a*b^2/x-1/2*c*a^2*b/x^2
-c^2*b^3*arctanh(c*x)/x-1/2*c*b^3*arctanh(c*x)^2/x^2+c^3*b^3*arctanh(c*x)^2
*ln(2)+c^3*a*b^2*dilog(1/2+1/2*c*x)+c^3*b^3*arctanh(c*x)^2*ln(1+(c*x+1)/(-c
^2*x^2+1)^(1/2))+c^3*b^3*ln(c*x)*arctanh(c*x)^2-c^3*b^3*arctanh(c*x)^2*ln((
c*x+1)^2/(-c^2*x^2+1)-1)+2*c^3*b^3*arctanh(c*x)*polylog(2,-(c*x+1)/(-c^2*x^
2+1)^(1/2))+c^3*b^3*arctanh(c*x)^2*ln(1-(c*x+1)/(-c^2*x^2+1)^(1/2))+2*c^3*b
^3*arctanh(c*x)*polylog(2,(c*x+1)/(-c^2*x^2+1)^(1/2))-1/2*c^3*b^3*arctanh(c
*x)^2*ln(c*x-1)-1/2*c^3*b^3*arctanh(c*x)^2*ln(c*x+1)+c^3*b^3*arctanh(c*x)^2
*ln((c*x+1)/(-c^2*x^2+1)^(1/2))-c^3*a*b^2*dilog(c*x)-c^3*a*b^2*dilog(c*x+1)
+c^3*a^2*b*ln(c*x)-c^3*a*b^2*arctanh(c*x)*ln(c*x-1)+2*c^3*a*b^2*arctanh(c*x
)*ln(c*x)-c^3*a*b^2*ln(c*x)*ln(c*x+1)-c^3*a*b^2*arctanh(c*x)*ln(c*x+1)+1/2*
c^3*a*b^2*ln(c*x-1)*ln(1/2+1/2*c*x)-1/2*c^3*a*b^2*ln(-1/2*c*x+1/2)*ln(c*x+1
)+1/2*c^3*a*b^2*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)-c*a*b^2*arctanh(c*x)/x^2+1
/2*I*c^3*b^3*arctanh(c*x)^2*Pi-1/2*I*c^3*b^3*arctanh(c*x)^2*csgn(I*((c*x+1)
^2/(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+
1)+1))^2*Pi+1/4*I*c^3*b^3*arctanh(c*x)^2*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))
^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*Pi+1/2*I*c^3*b^3*arctanh(c*x)^2*csgn(I*(c*
x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*Pi-1/4*I*c^3*b^3*a
rctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((
c*x+1)^2/(-c^2*x^2+1)+1))^2*Pi-1/2*I*c^3*b^3*arctanh(c*x)^2*csgn(I/((c*x+1)
^2/(-c^2*x^2+1)+1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+
1)+1))^2*Pi
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}\left(\left(c^2\log\left(c^2x^2-1\right)-c^2\log\left(x^2\right)+\frac{1}{x^2}\right)c+\frac{2\,\mathrm{artanh}\,(cx)}{x^3}\right)a^2b-\frac{a^3}{3\,x^3}-\frac{\left(b^3c^3x^3-b^3\right)\log\left(-cx+1\right)^3+3\left(b^3cx+2\,a\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/x^4,x, algorithm="maxima")
```

```
[Out] -1/2*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/x^3)
*a^2*b - 1/3*a^3/x^3 - 1/24*((b^3*c^3*x^3 - b^3)*log(-c*x + 1)^3 + 3*(b^3*c
*x + 2*a*b^2 + (b^3*c^3*x^3 + b^3)*log(c*x + 1))*log(-c*x + 1)^2)/x^3 - int
egrate(-1/8*((b^3*c*x - b^3)*log(c*x + 1)^3 + 6*(a*b^2*c*x - a*b^2)*log(c*x
 + 1)^2 + (2*b^3*c^2*x^2 + 4*a*b^2*c*x - 3*(b^3*c*x - b^3)*log(c*x + 1)^2 +
 2*(b^3*c^4*x^4 + 6*a*b^2 - (6*a*b^2*c - b^3*c)*x)*log(c*x + 1))*log(-c*x +
 1))/(c*x^5 - x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^3 \operatorname{artanh}(cx)^3 + 3\,ab^2 \operatorname{artanh}(cx)^2 + 3\,a^2 b \operatorname{artanh}(cx) + a^3}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/x^4,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctanh(c*x)^3 + 3*a*b^2*arctanh(c*x)^2 + 3*a^2*b*arctanh(c*x
) + a^3)/x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{atanh}(cx))^3}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**3/x**4,x)
```

```
[Out] Integral((a + b*atanh(c*x))**3/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{artanh}(cx) + a)^3}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^3/x^4, x)
```

3.34 $\int \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{x^5}\,dx$

Optimal. Leaf size=187

$$-b^3c^4\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)-\frac{b^2c^2\left(a+b\tanh^{-1}(cx)\right)}{4x^2}+2b^2c^4\log\left(2-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{4}c^4\left(a+b\tan\right.$$

```
[Out] -(b^3*c^3)/(4*x) + (b^3*c^4*ArcTanh[c*x])/4 - (b^2*c^2*(a + b*ArcTanh[c*x])
)/(4*x^2) + b*c^4*(a + b*ArcTanh[c*x])^2 - (b*c*(a + b*ArcTanh[c*x])^2)/(4*
x^3) - (3*b*c^3*(a + b*ArcTanh[c*x])^2)/(4*x) + (c^4*(a + b*ArcTanh[c*x])^3
)/4 - (a + b*ArcTanh[c*x])^3/(4*x^4) + 2*b^2*c^4*(a + b*ArcTanh[c*x])*Log[2
 - 2/(1 + c*x)] - b^3*c^4*PolyLog[2, -1 + 2/(1 + c*x)]
```

Rubi [A] time = 0.62579, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5916, 5982, 325, 206, 5988, 5932, 2447, 5948}

$$-b^3c^4\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)-\frac{b^2c^2\left(a+b\tanh^{-1}(cx)\right)}{4x^2}+2b^2c^4\log\left(2-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{4}c^4\left(a+b\tan\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^3/x^5,x]
```


```
[Out] -(b^3*c^3)/(4*x) + (b^3*c^4*ArcTanh[c*x])/4 - (b^2*c^2*(a + b*ArcTanh[c*x])
)/(4*x^2) + b*c^4*(a + b*ArcTanh[c*x])^2 - (b*c*(a + b*ArcTanh[c*x])^2)/(4*
x^3) - (3*b*c^3*(a + b*ArcTanh[c*x])^2)/(4*x) + (c^4*(a + b*ArcTanh[c*x])^3
)/4 - (a + b*ArcTanh[c*x])^3/(4*x^4) + 2*b^2*c^4*(a + b*ArcTanh[c*x])*Log[2
 - 2/(1 + c*x)] - b^3*c^4*PolyLog[2, -1 + 2/(1 + c*x)]
```


Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{x^5}\,dx &= -\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{4x^4}+\frac{1}{4}(3bc)\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^4\left(1-c^2x^2\right)}\,dx\\
&= -\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{4x^4}+\frac{1}{4}(3bc)\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^4}\,dx+\frac{1}{4}\left(3bc^3\right)\int \frac{\left(a+b\tanh^{-1}(cx)\right)}{x^2\left(1-c^2x^2\right)}\\
&= -\frac{bc\left(a+b\tanh^{-1}(cx)\right)^2}{4x^3}-\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{4x^4}+\frac{1}{2}\left(b^2c^2\right)\int \frac{a+b\tanh^{-1}(cx)}{x^3\left(1-c^2x^2\right)}\,dx+\frac{1}{4}\left(3bc\right.\\
&= -\frac{bc\left(a+b\tanh^{-1}(cx)\right)^2}{4x^3}-\frac{3bc^3\left(a+b\tanh^{-1}(cx)\right)^2}{4x}+\frac{1}{4}c^4\left(a+b\tanh^{-1}(cx)\right)^3-\frac{\left(a+b\tan\right.}{4}\\
&= -\frac{b^2c^2\left(a+b\tanh^{-1}(cx)\right)}{4x^2}+bc^4\left(a+b\tanh^{-1}(cx)\right)^2-\frac{bc\left(a+b\tanh^{-1}(cx)\right)^2}{4x^3}-\frac{3bc^3\left(a+b\right.}{4}\\
&= -\frac{b^3c^3}{4x}-\frac{b^2c^2\left(a+b\tanh^{-1}(cx)\right)}{4x^2}+bc^4\left(a+b\tanh^{-1}(cx)\right)^2-\frac{bc\left(a+b\tanh^{-1}(cx)\right)^2}{4x^3}-\frac{3bc^3}{}\\
&= -\frac{b^3c^3}{4x}+\frac{1}{4}b^3c^4\tanh^{-1}(cx)-\frac{b^2c^2\left(a+b\tanh^{-1}(cx)\right)}{4x^2}+bc^4\left(a+b\tanh^{-1}(cx)\right)^2-\frac{bc\left(a+b\right.}{}
\end{aligned}$$

Mathematica [A] time = 0.638678, size = 295, normalized size = 1.58

$$-\frac{8b^3c^4x^4\text{PolyLog}\left(2,e^{-2\tanh^{-1}(cx)}\right)+2b\tanh^{-1}(cx)\left(3a^2+2abcx\left(3c^2x^2+1\right)+b^2c^2x^2\left(1-c^2x^2\right)-8b^2c^4x^4\log\left(1-e\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])^3/x^5,x]
```

```
[Out] -(2*a^3 + 2*a^2*b*c*x + 2*a*b^2*c^2*x^2 + 6*a^2*b*c^3*x^3 + 2*b^3*c^3*x^3 -
 2*a*b^2*c^4*x^4 + 2*b^2*(b*c*x*(1 + 3*c^2*x^2 - 4*c^3*x^3) + a*(3 - 3*c^4*
x^4))*ArcTanh[c*x]^2 - 2*b^3*(-1 + c^4*x^4)*ArcTanh[c*x]^3 + 2*b*ArcTanh[c*
x]*(3*a^2 + b^2*c^2*x^2*(1 - c^2*x^2) + 2*a*b*c*x*(1 + 3*c^2*x^2) - 8*b^2*c
^4*x^4*Log[1 - E^(-2*ArcTanh[c*x])]) + 3*a^2*b*c^4*x^4*Log[1 - c*x] - 3*a^2
*b*c^4*x^4*Log[1 + c*x] - 16*a*b^2*c^4*x^4*Log[(c*x)/Sqrt[1 - c^2*x^2]] + 8
*b^3*c^4*x^4*PolyLog[2, E^(-2*ArcTanh[c*x])])/(8*x^4)
```

Maple [C] time = 0.873, size = 1281, normalized size = 6.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^3/x^5,x)
```

```
[Out] 1/4*c^4*b^3*arctanh(c*x)^3-c^4*b^3*arctanh(c*x)^2+2*c^4*b^3*dilog(1+(c*x+1)
/(-c^2*x^2+1)^(1/2))-2*c^4*b^3*dilog((c*x+1)/(-c^2*x^2+1)^(1/2))-1/4*b^3/x^
4*arctanh(c*x)^3-1/2*c*a*b^2*arctanh(c*x)/x^3-3/8*I*c^4*b^3*Pi*csgn(I*(c*x+
1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*arctanh(c*x)^2-3/16*
I*c^4*b^3*Pi*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*(c*x+1)^2/(c^2*x^2-1
)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*arctanh(c*x)^2-3/16*I*c^4*b^3*Pi*csgn(I*(c*
x+1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*arctanh(c*x)^2+3/1
6*I*c^4*b^3*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(
(c*x+1)^2/(-c^2*x^2+1)+1))^2*arctanh(c*x)^2+1/4*b^3*c^4*arctanh(c*x)-1/4*a^
3/x^4+3/16*I*c^4*b^3*Pi*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*(c*x+1)^2
/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))*arct
anh(c*x)^2-3/8*c^4*a^2*b*ln(c*x-1)+3/8*c^4*a^2*b*ln(c*x+1)-3/16*c^4*a*b^2*l
n(c*x-1)^2-3/16*c^4*a*b^2*ln(c*x+1)^2-3/8*c^4*b^3*arctanh(c*x)^2*ln(c*x-1)+
3/8*c^4*b^3*arctanh(c*x)^2*ln(c*x+1)-3/4*c^4*b^3*arctanh(c*x)^2*ln((c*x+1)/
(-c^2*x^2+1)^(1/2))+2*c^4*b^3*arctanh(c*x)*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))
-1/4*c^4*b^3/(c*x+1-(-c^2*x^2+1)^(1/2))*(-c^2*x^2+1)^(1/2)+1/4*c^4*b^3/((-c
^2*x^2+1)^(1/2)+c*x+1)*(-c^2*x^2+1)^(1/2)-1/4*c*b^3*arctanh(c*x)^2/x^3-1/4*
c^2*b^3*arctanh(c*x)/x^2-3/4*c^3*b^3*arctanh(c*x)^2/x+3/8*I*c^4*b^3*Pi*csgn
(I/((c*x+1)^2/(-c^2*x^2+1)+1))^3*arctanh(c*x)^2-3/4*c^3*a^2*b/x-1/4*c^2*a*b
^2/x^2-1/4*c*a^2*b/x^3-3/4*a*b^2/x^4*arctanh(c*x)^2-3/4*a^2*b/x^4*arctanh(c
*x)+2*c^4*a*b^2*ln(c*x)-c^4*a*b^2*ln(c*x-1)-c^4*a*b^2*ln(c*x+1)-3/16*I*c^4*
b^3*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*arctanh(c*x)^2-3/8*I*c^4*b^3*Pi*csgn
(I/((c*x+1)^2/(-c^2*x^2+1)+1))^2*arctanh(c*x)^2-3/16*I*c^4*b^3*Pi*csgn(I*(c
*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^3*arctanh(c*x)^2-3/2*c^3*a*
b^2/x*arctanh(c*x)-3/4*c^4*a*b^2*arctanh(c*x)*ln(c*x-1)+3/4*c^4*a*b^2*arcta
nh(c*x)*ln(c*x+1)+3/8*c^4*a*b^2*ln(c*x-1)*ln(1/2+1/2*c*x)+3/8*c^4*a*b^2*ln(
-1/2*c*x+1/2)*ln(c*x+1)-3/8*c^4*a*b^2*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)+3/8*
I*c^4*b^3*Pi*arctanh(c*x)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{8}\left(\left(3\,c^3\log\left(cx+1\right)-3\,c^3\log\left(cx-1\right)-\frac{2\left(3\,c^2x^2+1\right)}{x^3}\right)c-\frac{6\,\text{artanh}\left(cx\right)}{x^4}\right)a^2b+\frac{1}{16}\left(\left(32\,c^2\log\left(x\right)-\frac{3\,c^2x^2\log\left(cx+1\right)^2}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/x^5,x, algorithm="maxima")
```

```
[Out] 1/8*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1) - 2*(3*c^2*x^2 + 1)/x^3)*c -
6*arctanh(c*x)/x^4)*a^2*b + 1/16*((32*c^2*log(x) - (3*c^2*x^2*log(c*x + 1)^
2 + 3*c^2*x^2*log(c*x - 1)^2 + 16*c^2*x^2*log(c*x - 1) - 2*(3*c^2*x^2*log(c
*x - 1) - 8*c^2*x^2)*log(c*x + 1) + 4)/x^2)*c^2 + 4*(3*c^3*log(c*x + 1) - 3
*c^3*log(c*x - 1) - 2*(3*c^2*x^2 + 1)/x^3)*c*arctanh(c*x))*a*b^2 - 1/32*b^3
*(((c^4*x^4 - 1)*log(-c*x + 1)^3 + (6*c^3*x^3 + 2*c*x - 3*(c^4*x^4 - 1)*log
(c*x + 1))*log(-c*x + 1)^2)/x^4 + 4*integrate(-1/2*(2*(c*x - 1)*log(c*x + 1
)^3 + (6*c^4*x^4 + 2*c^2*x^2 - 6*(c*x - 1)*log(c*x + 1)^2 - 3*(c^5*x^5 - c*
x)*log(c*x + 1))*log(-c*x + 1))/(c*x^6 - x^5), x)) - 3/4*a*b^2*arctanh(c*x)
^2/x^4 - 1/4*a^3/x^4
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3\,\text{artanh}\left(cx\right)^3+3\,ab^2\,\text{artanh}\left(cx\right)^2+3\,a^2b\,\text{artanh}\left(cx\right)+a^3}{x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/x^5,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctanh(c*x)^3 + 3*a*b^2*arctanh(c*x)^2 + 3*a^2*b*arctanh(c*x
) + a^3)/x^5, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+b\,\text{atanh}\left(cx\right)\right)^3}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**3/x**5,x)
```

```
[Out] Integral((a + b*atanh(c*x))**3/x**5, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(b\,\text{artanh}\left(cx\right)+a\right)^3}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/x^5,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^3/x^5, x)
```

3.35 $\int (dx)^{5/2} \left(a + b \tanh^{-1}(cx)\right) dx$

Optimal. Leaf size=124

$$\frac{2(dx)^{7/2}\left(a+b\tanh^{-1}(cx)\right)}{7d}+\frac{4bd^2\sqrt{dx}}{7c^3}-\frac{2bd^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{7c^{7/2}}-\frac{2bd^{5/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{7c^{7/2}}+\frac{4b(dx)^{5/2}}{35c}$$

```
[Out] (4*b*d^2*Sqrt[d*x])/(7*c^3) + (4*b*(d*x)^(5/2))/(35*c) - (2*b*d^(5/2)*ArcTa
n[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]])/(7*c^(7/2)) + (2*(d*x)^(7/2)*(a + b*ArcTanh
[c*x]))/(7*d) - (2*b*d^(5/2)*ArcTanh[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]])/(7*c^(7/
2))
```

Rubi [A] time = 0.0864655, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5916, 321, 329, 212, 208, 205}

$$\frac{2(dx)^{7/2}\left(a+b\tanh^{-1}(cx)\right)}{7d}+\frac{4bd^2\sqrt{dx}}{7c^3}-\frac{2bd^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{7c^{7/2}}-\frac{2bd^{5/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{7c^{7/2}}+\frac{4b(dx)^{5/2}}{35c}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(5/2)*(a + b*ArcTanh[c*x]),x]
```


```
[Out] (4*b*d^2*Sqrt[d*x])/(7*c^3) + (4*b*(d*x)^(5/2))/(35*c) - (2*b*d^(5/2)*ArcTa
n[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]])/(7*c^(7/2)) + (2*(d*x)^(7/2)*(a + b*ArcTanh
[c*x]))/(7*d) - (2*b*d^(5/2)*ArcTanh[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]])/(7*c^(7/
2))
```


Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
```

```
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int (dx)^{5/2}\left(a+b\tanh^{-1}(cx)\right)dx=\frac{2(dx)^{7/2}\left(a+b\tanh^{-1}(cx)\right)}{7d}-\frac{(2bc)\int\frac{(dx)^{7/2}}{1-c^2x^2}dx}{7d}$$

$$=\frac{4b(dx)^{5/2}}{35c}+\frac{2(dx)^{7/2}\left(a+b\tanh^{-1}(cx)\right)}{7d}-\frac{(2bd)\int\frac{(dx)^{3/2}}{1-c^2x^2}dx}{7c}$$

$$=\frac{4bd^2\sqrt{dx}}{7c^3}+\frac{4b(dx)^{5/2}}{35c}+\frac{2(dx)^{7/2}\left(a+b\tanh^{-1}(cx)\right)}{7d}-\frac{\left(2bd^3\right)\int\frac{1}{\sqrt{dx}\left(1-c^2x^2\right)}dx}{7c^3}$$

$$=\frac{4bd^2\sqrt{dx}}{7c^3}+\frac{4b(dx)^{5/2}}{35c}+\frac{2(dx)^{7/2}\left(a+b\tanh^{-1}(cx)\right)}{7d}-\frac{\left(4bd^2\right)\text{Subst}\left(\int\frac{1}{1-\frac{c^2x^4}{d^2}}dx,x,\sqrt{\ldots}\right.}{7c^3}$$

$$=\frac{4bd^2\sqrt{dx}}{7c^3}+\frac{4b(dx)^{5/2}}{35c}+\frac{2(dx)^{7/2}\left(a+b\tanh^{-1}(cx)\right)}{7d}-\frac{\left(2bd^3\right)\text{Subst}\left(\int\frac{1}{d-cx^2}dx,x,\sqrt{d\ldots}\right.}{7c^3}$$

$$=\frac{4bd^2\sqrt{dx}}{7c^3}+\frac{4b(dx)^{5/2}}{35c}-\frac{2bd^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{7c^{7/2}}+\frac{2(dx)^{7/2}\left(a+b\tanh^{-1}(cx)\right)}{7d}-\frac{2bd^{5/2}\,\mathrm{t}\ldots}{}$$

Mathematica [A] time = 0.0731334, size = 128, normalized size = 1.03

$$\frac{(dx)^{5/2}\left(10ac^{7/2}x^{7/2}+4bc^{5/2}x^{5/2}+10bc^{7/2}x^{7/2}\tanh^{-1}(cx)+20b\sqrt{c}\sqrt{x}+5b\log\left(1-\sqrt{c}\sqrt{x}\right)-5b\log\left(\sqrt{c}\sqrt{x}+1\right)-10b\tan\ldots\right.}{35c^{7/2}x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(5/2)*(a + b*ArcTanh[c*x]),x]
```

```
[Out] ((d*x)^(5/2)*(20*b*Sqrt[c]*Sqrt[x] + 4*b*c^(5/2)*x^(5/2) + 10*a*c^(7/2)*x^(
7/2) - 10*b*ArcTan[Sqrt[c]*Sqrt[x]] + 10*b*c^(7/2)*x^(7/2)*ArcTanh[c*x] + 5
*b*Log[1 - Sqrt[c]*Sqrt[x]] - 5*b*Log[1 + Sqrt[c]*Sqrt[x]]))/(35*c^(7/2)*x^
(5/2))
```

Maple [A] time = 0.036, size = 107, normalized size = 0.9

$$\frac{2a}{7d}(dx)^{\frac{7}{2}}+\frac{2b\mathrm{Artanh}(cx)}{7d}(dx)^{\frac{7}{2}}+\frac{4b}{35c}(dx)^{\frac{5}{2}}+\frac{4bd^2}{7c^3}\sqrt{dx}-\frac{2d^3b}{7c^3}\arctan\left(c\sqrt{dx}\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{2d^3b}{7c^3}\mathrm{Artanh}\left(c\sqrt{dx}\frac{1}{\sqrt{c\ldots}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)*(a+b*arctanh(c*x)),x)
```

```
[Out] 2/7/d*(d*x)^(7/2)*a+2/7/d*b*(d*x)^(7/2)*arctanh(c*x)+4/35*b*(d*x)^(5/2)/c+4
/7*b*d^2*(d*x)^(1/2)/c^3-2/7*d^3*b/c^3/(c*d)^(1/2)*arctan(c*(d*x)^(1/2)/(c*
d)^(1/2))-2/7*d^3*b/c^3/(c*d)^(1/2)*arctanh(c*(d*x)^(1/2)/(c*d)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.33861, size = 660, normalized size = 5.32

$$\left[-\frac{10\,bd^2\sqrt{\frac{d}{c}}\arctan\left(\frac{\sqrt{dx}c\sqrt{\frac{d}{c}}}{d}\right)-5\,bd^2\sqrt{\frac{d}{c}}\log\left(\frac{cdx-2\,\sqrt{dx}c\sqrt{\frac{d}{c}}+d}{cx-1}\right)-\left(5\,bc^3d^2x^3\log\left(-\frac{cx+1}{cx-1}\right)+10\,ac^3d^2x^3+4\,bc^2d^2x^2+20\right.}{35\,c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] [-1/35*(10*b*d^2*sqrt(d/c)*arctan(sqrt(d*x)*c*sqrt(d/c)/d) - 5*b*d^2*sqrt(d
/c)*log((c*d*x - 2*sqrt(d*x)*c*sqrt(d/c) + d)/(c*x - 1)) - (5*b*c^3*d^2*x^3
*log(-(c*x + 1)/(c*x - 1)) + 10*a*c^3*d^2*x^3 + 4*b*c^2*d^2*x^2 + 20*b*d^2)
*sqrt(d*x))/c^3, 1/35*(10*b*d^2*sqrt(-d/c)*arctan(sqrt(d*x)*c*sqrt(-d/c)/d)
 + 5*b*d^2*sqrt(-d/c)*log((c*d*x - 2*sqrt(d*x)*c*sqrt(-d/c) - d)/(c*x + 1))
 + (5*b*c^3*d^2*x^3*log(-(c*x + 1)/(c*x - 1)) + 10*a*c^3*d^2*x^3 + 4*b*c^2*
d^2*x^2 + 20*b*d^2)*sqrt(d*x))/c^3]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(5/2)*(a+b*atanh(c*x)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^{\frac{5}{2}}\,(b\,\mathrm{artanh}\,(cx)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((d*x)^(5/2)*(b*arctanh(c*x) + a), x)
```

3.36 $\int (dx)^{3/2} \left(a + b \tanh^{-1}(cx)\right) dx$

Optimal. Leaf size=106

$$\frac{2(dx)^{5/2}\left(a+b\tanh^{-1}(cx)\right)}{5d}+\frac{2bd^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{5c^{5/2}}-\frac{2bd^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{5c^{5/2}}+\frac{4b(dx)^{3/2}}{15c}$$

```
[Out] (4*b*(d*x)^(3/2))/(15*c) + (2*b*d^(3/2)*ArcTan[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]]
)/(5*c^(5/2)) + (2*(d*x)^(5/2)*(a + b*ArcTanh[c*x]))/(5*d) - (2*b*d^(3/2)*A
rcTanh[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]])/(5*c^(5/2))
```

Rubi [A] time = 0.0672078, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5916, 321, 329, 298, 205, 208}

$$\frac{2(dx)^{5/2}\left(a+b\tanh^{-1}(cx)\right)}{5d}+\frac{2bd^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{5c^{5/2}}-\frac{2bd^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{5c^{5/2}}+\frac{4b(dx)^{3/2}}{15c}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)*(a + b*ArcTanh[c*x]),x]
```


```
[Out] (4*b*(d*x)^(3/2))/(15*c) + (2*b*d^(3/2)*ArcTan[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]]
)/(5*c^(5/2)) + (2*(d*x)^(5/2)*(a + b*ArcTanh[c*x]))/(5*d) - (2*b*d^(3/2)*A
rcTanh[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]])/(5*c^(5/2))
```


Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int (dx)^{3/2}\left(a+b\tanh^{-1}(cx)\right)dx &= \frac{2(dx)^{5/2}\left(a+b\tanh^{-1}(cx)\right)}{5d}-\frac{(2bc)\int\frac{(dx)^{5/2}}{1-c^2x^2}\,dx}{5d}\\
&= \frac{4b(dx)^{3/2}}{15c}+\frac{2(dx)^{5/2}\left(a+b\tanh^{-1}(cx)\right)}{5d}-\frac{(2bd)\int\frac{\sqrt{dx}}{1-c^2x^2}\,dx}{5c}\\
&= \frac{4b(dx)^{3/2}}{15c}+\frac{2(dx)^{5/2}\left(a+b\tanh^{-1}(cx)\right)}{5d}-\frac{(4b)\,\text{Subst}\left(\int\frac{x^2}{1-\frac{c^2x^4}{d^2}}\,dx,x,\sqrt{dx}\right)}{5c}\\
&= \frac{4b(dx)^{3/2}}{15c}+\frac{2(dx)^{5/2}\left(a+b\tanh^{-1}(cx)\right)}{5d}-\frac{\left(2bd^2\right)\text{Subst}\left(\int\frac{1}{d-cx^2}\,dx,x,\sqrt{dx}\right)}{5c^2}+\frac{\left(2bd^2\right)\cdots}{\cdots}\\
&= \frac{4b(dx)^{3/2}}{15c}+\frac{2bd^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{5c^{5/2}}+\frac{2(dx)^{5/2}\left(a+b\tanh^{-1}(cx)\right)}{5d}-\frac{2bd^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\cdots}}{\sqrt{d}}\right)}{5c^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.055221, size = 115, normalized size = 1.08

$$\frac{(dx)^{3/2}\left(6ac^{5/2}x^{5/2}+4bc^{3/2}x^{3/2}+6bc^{5/2}x^{5/2}\tanh^{-1}(cx)+3b\log\left(1-\sqrt{c}\sqrt{x}\right)-3b\log\left(\sqrt{c}\sqrt{x}+1\right)+6b\tan^{-1}\left(\sqrt{c}\sqrt{x}\right)\right)}{15c^{5/2}x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)*(a + b*ArcTanh[c*x]),x]
```

```
[Out] ((d*x)^(3/2)*(4*b*c^(3/2)*x^(3/2) + 6*a*c^(5/2)*x^(5/2) + 6*b*ArcTan[Sqrt[c
]*Sqrt[x]] + 6*b*c^(5/2)*x^(5/2)*ArcTanh[c*x] + 3*b*Log[1 - Sqrt[c]*Sqrt[x]
] - 3*b*Log[1 + Sqrt[c]*Sqrt[x]]))/(15*c^(5/2)*x^(3/2))
```

Maple [A] time = 0.011, size = 93, normalized size = 0.9

$$\frac{2a}{5d}(dx)^{\frac{5}{2}}+\frac{2b\mathrm{Artanh}(cx)}{5d}(dx)^{\frac{5}{2}}+\frac{4b}{15c}(dx)^{\frac{3}{2}}+\frac{2d^2b}{5c^2}\arctan\left(c\sqrt{dx}\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{2d^2b}{5c^2}\mathrm{Artanh}\left(c\sqrt{dx}\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*(a+b*arctanh(c*x)),x)
```

```
[Out] 2/5/d*(d*x)^(5/2)*a+2/5/d*b*(d*x)^(5/2)*arctanh(c*x)+4/15*b*(d*x)^(3/2)/c+2
/5*d^2*b/c^2/(c*d)^(1/2)*arctan(c*(d*x)^(1/2)/(c*d)^(1/2))-2/5*d^2*b/c^2/(c
*d)^(1/2)*arctanh(c*(d*x)^(1/2)/(c*d)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.2526, size = 586, normalized size = 5.53

$$\left[\frac{6\,bd\sqrt{\frac{d}{c}}\arctan\left(\frac{\sqrt{dx}c\sqrt{\frac{d}{c}}}{d}\right)+3\,bd\sqrt{\frac{d}{c}}\log\left(\frac{cdx-2\,\sqrt{dx}c\sqrt{\frac{d}{c}}+d}{cx-1}\right)+\left(3\,bc^2dx^2\log\left(-\frac{cx+1}{cx-1}\right)+6\,ac^2dx^2+4\,bcdx\right)\sqrt{dx}}{15\,c^2}, \frac{6\,bd\sqrt{-\frac{d}{c}}\ldots}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] [1/15*(6*b*d*sqrt(d/c)*arctan(sqrt(d*x)*c*sqrt(d/c)/d) + 3*b*d*sqrt(d/c)*lo
g((c*d*x - 2*sqrt(d*x)*c*sqrt(d/c) + d)/(c*x - 1)) + (3*b*c^2*d*x^2*log(-(c
*x + 1)/(c*x - 1)) + 6*a*c^2*d*x^2 + 4*b*c*d*x)*sqrt(d*x))/c^2, 1/15*(6*b*d
*sqrt(-d/c)*arctan(sqrt(d*x)*c*sqrt(-d/c)/d) + 3*b*d*sqrt(-d/c)*log((c*d*x
+ 2*sqrt(d*x)*c*sqrt(-d/c) - d)/(c*x + 1)) + (3*b*c^2*d*x^2*log(-(c*x + 1)/
(c*x - 1)) + 6*a*c^2*d*x^2 + 4*b*c*d*x)*sqrt(d*x))/c^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^{\frac{3}{2}}\,(a+b\operatorname{atanh}(cx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)*(a+b*atanh(c*x)),x)
```

```
[Out] Integral((d*x)**(3/2)*(a + b*atanh(c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^{\frac{3}{2}}\,(b\operatorname{artanh}(cx)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((d*x)^(3/2)*(b*arctanh(c*x) + a), x)
```

3.37 $\int \sqrt{dx}\left(a+b\tanh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=106

$$\frac{2(dx)^{3/2}\left(a+b\tanh^{-1}(cx)\right)}{3d}-\frac{2b\sqrt{d}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{3c^{3/2}}-\frac{2b\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{3c^{3/2}}+\frac{4b\sqrt{dx}}{3c}$$

```
[Out] (4*b*Sqrt[d*x])/(3*c) - (2*b*Sqrt[d]*ArcTan[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]])/(
3*c^(3/2)) + (2*(d*x)^(3/2)*(a + b*ArcTanh[c*x]))/(3*d) - (2*b*Sqrt[d]*ArcT
anh[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]])/(3*c^(3/2))
```

Rubi [A] time = 0.0612043, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5916, 321, 329, 212, 208, 205}

$$\frac{2(dx)^{3/2}\left(a+b\tanh^{-1}(cx)\right)}{3d}-\frac{2b\sqrt{d}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{3c^{3/2}}-\frac{2b\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{3c^{3/2}}+\frac{4b\sqrt{dx}}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (4*b*Sqrt[d*x])/(3*c) - (2*b*Sqrt[d]*ArcTan[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]])/(
3*c^(3/2)) + (2*(d*x)^(3/2)*(a + b*ArcTanh[c*x]))/(3*d) - (2*b*Sqrt[d]*ArcT
anh[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]])/(3*c^(3/2))
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{dx}\left(a+b\tanh^{-1}(cx)\right)dx &= \frac{2(dx)^{3/2}\left(a+b\tanh^{-1}(cx)\right)}{3d}-\frac{(2bc)\int\frac{(dx)^{3/2}}{1-c^2x^2}\,dx}{3d}\\
&= \frac{4b\sqrt{dx}}{3c}+\frac{2(dx)^{3/2}\left(a+b\tanh^{-1}(cx)\right)}{3d}-\frac{(2bd)\int\frac{1}{\sqrt{dx}(1-c^2x^2)}\,dx}{3c}\\
&= \frac{4b\sqrt{dx}}{3c}+\frac{2(dx)^{3/2}\left(a+b\tanh^{-1}(cx)\right)}{3d}-\frac{(4b)\,\text{Subst}\left(\int\frac{1}{1-\frac{c^2x^4}{d^2}}\,dx,x,\sqrt{dx}\right)}{3c}\\
&= \frac{4b\sqrt{dx}}{3c}+\frac{2(dx)^{3/2}\left(a+b\tanh^{-1}(cx)\right)}{3d}-\frac{(2bd)\,\text{Subst}\left(\int\frac{1}{d-cx^2}\,dx,x,\sqrt{dx}\right)}{3c}-\frac{(2bd)\,\text{Su}}{}\\
&= \frac{4b\sqrt{dx}}{3c}-\frac{2b\sqrt{d}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{3c^{3/2}}+\frac{2(dx)^{3/2}\left(a+b\tanh^{-1}(cx)\right)}{3d}-\frac{2b\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right.}{3c^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.0502362, size = 114, normalized size = 1.08

$$\frac{\sqrt{dx}\left(2ac^{3/2}x^{3/2}+2bc^{3/2}x^{3/2}\tanh^{-1}(cx)+4b\sqrt{c}\sqrt{x}+b\log\left(1-\sqrt{c}\sqrt{x}\right)-b\log\left(\sqrt{c}\sqrt{x}+1\right)-2b\tan^{-1}\left(\sqrt{c}\sqrt{x}\right)\right)}{3c^{3/2}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (Sqrt[d*x]*(4*b*Sqrt[c]*Sqrt[x] + 2*a*c^(3/2)*x^(3/2) - 2*b*ArcTan[Sqrt[c]*
Sqrt[x]] + 2*b*c^(3/2)*x^(3/2)*ArcTanh[c*x] + b*Log[1 - Sqrt[c]*Sqrt[x]] -
b*Log[1 + Sqrt[c]*Sqrt[x]]))/(3*c^(3/2)*Sqrt[x])
```

Maple [A] time = 0.012, size = 89, normalized size = 0.8

$$\frac{2a}{3d}(dx)^{\frac{3}{2}}+\frac{2b\,\text{Artanh}(cx)}{3d}(dx)^{\frac{3}{2}}+\frac{4b}{3c}\sqrt{dx}-\frac{2db}{3c}\arctan\left(c\sqrt{dx}\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{2db}{3c}\text{Artanh}\left(c\sqrt{dx}\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)*(a+b*arctanh(c*x)),x)
```

```
[Out] 2/3/d*(d*x)^(3/2)*a+2/3/d*b*(d*x)^(3/2)*arctanh(c*x)+4/3*b*(d*x)^(1/2)/c-2/
3*d*b/c/(c*d)^(1/2)*arctan(c*(d*x)^(1/2)/(c*d)^(1/2))-2/3*d*b/c/(c*d)^(1/2)
*arctanh(c*(d*x)^(1/2)/(c*d)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.22862, size = 509, normalized size = 4.8

$$\left[-\frac{2\,b\sqrt{\frac{d}{c}}\arctan\left(\frac{\sqrt{dx}c\sqrt{\frac{d}{c}}}{d}\right)-b\sqrt{\frac{d}{c}}\log\left(\frac{cdx-2\,\sqrt{dx}c\sqrt{\frac{d}{c}}+d}{cx-1}\right)-\left(bcx\log\left(-\frac{cx+1}{cx-1}\right)+2\,acx+4\,b\right)\sqrt{dx}}{3\,c},\ \frac{2\,b\sqrt{-\frac{d}{c}}\arctan\left(\frac{\sqrt{dx}c\sqrt{-\frac{d}{c}}}{d}\right)}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] [-1/3*(2*b*sqrt(d/c)*arctan(sqrt(d*x)*c*sqrt(d/c)/d) - b*sqrt(d/c)*log((c*d
*x - 2*sqrt(d*x)*c*sqrt(d/c) + d)/(c*x - 1)) - (b*c*x*log(-(c*x + 1)/(c*x -
 1)) + 2*a*c*x + 4*b)*sqrt(d*x))/c, 1/3*(2*b*sqrt(-d/c)*arctan(sqrt(d*x)*c*
sqrt(-d/c)/d) + b*sqrt(-d/c)*log((c*d*x - 2*sqrt(d*x)*c*sqrt(-d/c) - d)/(c*
x + 1)) + (b*c*x*log(-(c*x + 1)/(c*x - 1)) + 2*a*c*x + 4*b)*sqrt(d*x))/c]
```

Sympy [C] time = 20.0057, size = 586, normalized size = 5.53

$$\frac{2a\,(dx)^{\frac{3}{2}}}{3d}+\frac{2b\left(\begin{cases}-\frac{icd^{\frac{7}{2}}\left(\frac{1}{c}\right)^{\frac{3}{2}}\log\left(-\sqrt{d}\sqrt{\frac{1}{c}}+\sqrt{dx}\right)}{-\frac{3ic^3d^2}{c^2}+\frac{15ic^2d^2}{c}}-\frac{cd^{\frac{7}{2}}\left(\frac{1}{c}\right)^{\frac{3}{2}}\log\left(i\sqrt{d}\sqrt{\frac{1}{c}}+\sqrt{dx}\right)}{-\frac{3ic^3d^2}{c^2}+\frac{15ic^2d^2}{c}}+\frac{icd^{\frac{7}{2}}\left(\frac{1}{c}\right)^{\frac{3}{2}}\log\left(i\sqrt{d}\sqrt{\frac{1}{c}}+\sqrt{dx}\right)}{-\frac{3ic^3d^2}{c^2}+\frac{15ic^2d^2}{c}}+\frac{5id^{\frac{7}{2}}\sqrt{\frac{1}{c}}\log\left(-\sqrt{d}\sqrt{\frac{1}{c}}+\sqrt{dx}\right)}{-\frac{3ic^3d^2}{c^2}+\frac{15ic^2d^2}{c}}- \\ 0\end{cases}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)*(a+b*atanh(c*x)),x)
```

```
[Out] 2*a*(d*x)**(3/2)/(3*d) + 2*b*Piecewise((-I*c*d**(7/2)*(1/c)**(3/2)*log(-sqr
t(d)*sqrt(1/c) + sqrt(d*x))/(-3*I*c**3*d**2/c**2 + 15*I*c**2*d**2/c) - c*d*
*(7/2)*(1/c)**(3/2)*log(I*sqrt(d)*sqrt(1/c) + sqrt(d*x))/(-3*I*c**3*d**2/c*
*2 + 15*I*c**2*d**2/c) + I*c*d**(7/2)*(1/c)**(3/2)*log(I*sqrt(d)*sqrt(1/c)
+ sqrt(d*x))/(-3*I*c**3*d**2/c**2 + 15*I*c**2*d**2/c) + 5*I*d**(7/2)*sqrt(1
/c)*log(-sqrt(d)*sqrt(1/c) + sqrt(d*x))/(-3*I*c**3*d**2/c**2 + 15*I*c**2*d*
*2/c) - 2*d**(7/2)*sqrt(1/c)*log(-I*sqrt(d)*sqrt(1/c) + sqrt(d*x))/(-3*I*c*
*3*d**2/c**2 + 15*I*c**2*d**2/c) - 2*I*d**(7/2)*sqrt(1/c)*log(-I*sqrt(d)*sq
rt(1/c) + sqrt(d*x))/(-3*I*c**3*d**2/c**2 + 15*I*c**2*d**2/c) + 3*d**(7/2)*
sqrt(1/c)*log(I*sqrt(d)*sqrt(1/c) + sqrt(d*x))/(-3*I*c**3*d**2/c**2 + 15*I*
c**2*d**2/c) - 3*I*d**(7/2)*sqrt(1/c)*log(I*sqrt(d)*sqrt(1/c) + sqrt(d*x))/
(-3*I*c**3*d**2/c**2 + 15*I*c**2*d**2/c) + 4*I*d**(7/2)*sqrt(1/c)*atanh(c*x
```

```
)/(-3*I*c**3*d**2/c**2 + 15*I*c**2*d**2/c) + (d*x)**(3/2)*atanh(c*x)/3 + 2*
d*sqrt(d*x)/(3*c), Ne(c, 0)), (0, True))/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dx}(b \operatorname{artanh}(cx) + a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x)*(b*arctanh(c*x) + a), x)
```

3.38 $\int \frac{a+b\tanh^{-1}(cx)}{\sqrt{dx}}\,dx$

Optimal. Leaf size=85

$$\frac{2\sqrt{dx}\left(a+b\tanh^{-1}(cx)\right)}{d}+\frac{2b\tan^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{c}\sqrt{d}}-\frac{2b\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{c}\sqrt{d}}$$

```
[Out] (2*b*ArcTan[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]])/(Sqrt[c]*Sqrt[d]) + (2*Sqrt[d*x]*
(a + b*ArcTanh[c*x]))/d - (2*b*ArcTanh[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]])/(Sqrt[
c]*Sqrt[d])
```

Rubi [A] time = 0.0515333, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5916, 329, 298, 205, 208}

$$\frac{2\sqrt{dx}\left(a+b\tanh^{-1}(cx)\right)}{d}+\frac{2b\tan^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{c}\sqrt{d}}-\frac{2b\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{c}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/Sqrt[d*x],x]
```

```
[Out] (2*b*ArcTan[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]])/(Sqrt[c]*Sqrt[d]) + (2*Sqrt[d*x]*
(a + b*ArcTanh[c*x]))/d - (2*b*ArcTanh[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]])/(Sqrt[
c]*Sqrt[d])
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\tanh^{-1}(cx)}{\sqrt{dx}}\,dx &= \frac{2\sqrt{dx}\left(a+b\tanh^{-1}(cx)\right)}{d}-\frac{(2bc)\int \frac{\sqrt{dx}}{1-c^2x^2}\,dx}{d}\\
&= \frac{2\sqrt{dx}\left(a+b\tanh^{-1}(cx)\right)}{d}-\frac{(4bc)\operatorname{Subst}\left(\int \frac{x^2}{1-\frac{c^2x^4}{d^2}}\,dx,x,\sqrt{dx}\right)}{d^2}\\
&= \frac{2\sqrt{dx}\left(a+b\tanh^{-1}(cx)\right)}{d}-(2b)\operatorname{Subst}\left(\int \frac{1}{d-cx^2}\,dx,x,\sqrt{dx}\right)+(2b)\operatorname{Subst}\left(\int \frac{1}{d+cx^2}\,dx\right.\\
&= \frac{2b\tan^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{c}\sqrt{d}}+\frac{2\sqrt{dx}\left(a+b\tanh^{-1}(cx)\right)}{d}-\frac{2b\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{c}\sqrt{d}}
\end{aligned}$$

Mathematica [A] time = 0.031621, size = 98, normalized size = 1.15

$$\frac{\sqrt{x}\left(2a\sqrt{c}\sqrt{x}+b\log\left(1-\sqrt{c}\sqrt{x}\right)-b\log\left(\sqrt{c}\sqrt{x}+1\right)+2b\tan^{-1}\left(\sqrt{c}\sqrt{x}\right)+2b\sqrt{c}\sqrt{x}\tanh^{-1}(cx)\right)}{\sqrt{c}\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])/Sqrt[d*x],x]
```

```
[Out] (Sqrt[x]*(2*a*Sqrt[c]*Sqrt[x] + 2*b*ArcTan[Sqrt[c]*Sqrt[x]] + 2*b*Sqrt[c]*S
qrt[x]*ArcTanh[c*x] + b*Log[1 - Sqrt[c]*Sqrt[x]] - b*Log[1 + Sqrt[c]*Sqrt[x
]]))/(Sqrt[c]*Sqrt[d*x])
```

Maple [A] time = 0.013, size = 70, normalized size = 0.8

$$2\,\frac{a\sqrt{dx}}{d}+2\,\frac{b\sqrt{dx}\operatorname{Artanh}(cx)}{d}+2\,\frac{b}{\sqrt{cd}}\arctan\left(\frac{c\sqrt{dx}}{\sqrt{cd}}\right)-2\,\frac{b}{\sqrt{cd}}\operatorname{Artanh}\left(\frac{c\sqrt{dx}}{\sqrt{cd}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/(d*x)^(1/2),x)
```

```
[Out] 2/d*a*(d*x)^(1/2)+2/d*b*(d*x)^(1/2)*arctanh(c*x)+2*b/(c*d)^(1/2)*arctan(c*(
d*x)^(1/2)/(c*d)^(1/2))-2*b/(c*d)^(1/2)*arctanh(c*(d*x)^(1/2)/(c*d)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.32443, size = 487, normalized size = 5.73

$$\left[-\frac{2\sqrt{cd}b\arctan\left(\frac{\sqrt{cd}\sqrt{dx}}{cdx}\right)-\sqrt{cd}b\log\left(\frac{cdx-2\sqrt{cd}\sqrt{dx}+d}{cx-1}\right)-\left(bc\log\left(-\frac{cx+1}{cx-1}\right)+2ac\right)\sqrt{dx}}{cd}, \frac{2\sqrt{-cd}b\arctan\left(\frac{\sqrt{-cd}\sqrt{dx}}{cdx}\right)-\sqrt{-cd}\ldots}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [-(2*sqrt(c*d)*b*arctan(sqrt(c*d)*sqrt(d*x)/(c*d*x)) - sqrt(c*d)*b*log((c*d
*x - 2*sqrt(c*d)*sqrt(d*x) + d)/(c*x - 1)) - (b*c*log(-(c*x + 1)/(c*x - 1))
 + 2*a*c)*sqrt(d*x))/(c*d), (2*sqrt(-c*d)*b*arctan(sqrt(-c*d)*sqrt(d*x)/(c*
d*x)) - sqrt(-c*d)*b*log((c*d*x - 2*sqrt(-c*d)*sqrt(d*x) - d)/(c*x + 1)) +
(b*c*log(-(c*x + 1)/(c*x - 1)) + 2*a*c)*sqrt(d*x))/(c*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+b\,\mathrm{atanh}\,(cx)}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/(d*x)**(1/2),x)
```

```
[Out] Integral((a + b*atanh(c*x))/sqrt(d*x), x)
```

Giac [A] time = 1.16206, size = 119, normalized size = 1.4

$$\frac{\left(2\,cd\left(\frac{\arctan\left(\frac{\sqrt{dx}c}{\sqrt{cd}}\right)}{\sqrt{cd}c}+\frac{\arctan\left(\frac{\sqrt{dx}c}{\sqrt{-cd}}\right)}{\sqrt{-cd}c}\right)+\sqrt{dx}\log\left(-\frac{cx+1}{cx-1}\right)\right)b+2\sqrt{dx}a}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] ((2*c*d*(arctan(sqrt(d*x)*c/sqrt(c*d))/(sqrt(c*d)*c) + arctan(sqrt(d*x)*c/s
qrt(-c*d))/(sqrt(-c*d)*c)) + sqrt(d*x)*log(-(c*x + 1)/(c*x - 1)))*b + 2*sqr
t(d*x)*a)/d
```

3.39 $\int \frac{a+b\tanh^{-1}(cx)}{(dx)^{3/2}} \, dx$

Optimal. Leaf size=85

$$-\frac{2\left(a+b\tanh^{-1}(cx)\right)}{d\sqrt{dx}}+\frac{2b\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{2b\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}$$

```
[Out] (2*b*Sqrt[c]*ArcTan[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]])/d^(3/2) - (2*(a + b*ArcTa
nh[c*x]))/(d*Sqrt[d*x]) + (2*b*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]]
)/d^(3/2)
```

Rubi [A] time = 0.0546279, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5916, 329, 212, 208, 205}

$$-\frac{2\left(a+b\tanh^{-1}(cx)\right)}{d\sqrt{dx}}+\frac{2b\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{2b\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/(d*x)^(3/2),x]
```

```
[Out] (2*b*Sqrt[c]*ArcTan[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]])/d^(3/2) - (2*(a + b*ArcTa
nh[c*x]))/(d*Sqrt[d*x]) + (2*b*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]]
)/d^(3/2)
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}(cx)}{(dx)^{3/2}}\,dx &= -\frac{2\left(a+b\tanh^{-1}(cx)\right)}{d\sqrt{dx}}+\frac{(2bc)\int \frac{1}{\sqrt{dx}\left(1-c^2x^2\right)}\,dx}{d}\\ &= -\frac{2\left(a+b\tanh^{-1}(cx)\right)}{d\sqrt{dx}}+\frac{(4bc)\operatorname{Subst}\left(\int \frac{1}{1-\frac{c^2x^4}{d^2}}\,dx,x,\sqrt{dx}\right)}{d^2}\\ &= -\frac{2\left(a+b\tanh^{-1}(cx)\right)}{d\sqrt{dx}}+\frac{(2bc)\operatorname{Subst}\left(\int \frac{1}{d-cx^2}\,dx,x,\sqrt{dx}\right)}{d}+\frac{(2bc)\operatorname{Subst}\left(\int \frac{1}{d+cx^2}\,dx,x,\sqrt{dx}\right)}{d}\\ &= \frac{2b\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{2\left(a+b\tanh^{-1}(cx)\right)}{d\sqrt{dx}}+\frac{2b\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0429033, size = 99, normalized size = 1.16

$$\frac{x\left(-2a-b\sqrt{c}\sqrt{x}\log\left(1-\sqrt{c}\sqrt{x}\right)+b\sqrt{c}\sqrt{x}\log\left(\sqrt{c}\sqrt{x}+1\right)+2b\sqrt{c}\sqrt{x}\tan^{-1}\left(\sqrt{c}\sqrt{x}\right)-2b\tanh^{-1}(cx)\right)}{(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])/(d*x)^(3/2),x]
```

```
[Out] (x*(-2*a + 2*b*Sqrt[c]*Sqrt[x]*ArcTan[Sqrt[c]*Sqrt[x]] - 2*b*ArcTanh[c*x] -
 b*Sqrt[c]*Sqrt[x]*Log[1 - Sqrt[c]*Sqrt[x]] + b*Sqrt[c]*Sqrt[x]*Log[1 + Sqr
t[c]*Sqrt[x]]))/(d*x)^(3/2)
```

Maple [A] time = 0.013, size = 78, normalized size = 0.9

$$-2\,{\frac{a}{d\sqrt{dx}}}-2\,{\frac{b{\rm Artanh}\left(cx\right)}{d\sqrt{dx}}}+2\,{\frac{bc}{d\sqrt{cd}}\arctan\left({\frac{c\sqrt{dx}}{\sqrt{cd}}}\right)}+2\,{\frac{bc}{d\sqrt{cd}}{\rm Artanh}\left({\frac{c\sqrt{dx}}{\sqrt{cd}}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/(d*x)^(3/2),x)
```

```
[Out] -2/d*a/(d*x)^(1/2)-2/d*b/(d*x)^(1/2)*arctanh(c*x)+2/d*b*c/(c*d)^(1/2)*arcta
n(c*(d*x)^(1/2)/(c*d)^(1/2))+2/d*b*c/(c*d)^(1/2)*arctanh(c*(d*x)^(1/2)/(c*d
)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(d*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.16046, size = 494, normalized size = 5.81

$$\left[-\frac{2\,bdx\sqrt{\frac{c}{d}}\arctan\left(\frac{\sqrt{dx}\sqrt{\frac{c}{d}}}{cx}\right)-bdx\sqrt{\frac{c}{d}}\log\left(\frac{cx+2\sqrt{dx}\sqrt{\frac{c}{d}}+1}{cx-1}\right)+\sqrt{dx}\left(b\log\left(-\frac{cx+1}{cx-1}\right)+2\,a\right)}{d^2x},-\frac{2\,bdx\sqrt{-\frac{c}{d}}\arctan\left(\frac{\sqrt{dx}\sqrt{-\frac{c}{d}}}{cx}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] [-(2*b*d*x*sqrt(c/d)*arctan(sqrt(d*x)*sqrt(c/d)/(c*x)) - b*d*x*sqrt(c/d)*lo
g((c*x + 2*sqrt(d*x)*sqrt(c/d) + 1)/(c*x - 1)) + sqrt(d*x)*(b*log(-(c*x + 1
)/(c*x - 1)) + 2*a))/(d^2*x), -(2*b*d*x*sqrt(-c/d)*arctan(sqrt(d*x)*sqrt(-c
/d)/(c*x)) - b*d*x*sqrt(-c/d)*log((c*x + 2*sqrt(d*x)*sqrt(-c/d) - 1)/(c*x +
 1)) + sqrt(d*x)*(b*log(-(c*x + 1)/(c*x - 1)) + 2*a))/(d^2*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+b\,\mathrm{atanh}\,(cx)}{(dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/(d*x)**(3/2),x)
```

```
[Out] Integral((a + b*atanh(c*x))/(d*x)**(3/2), x)
```

Giac [A] time = 1.27011, size = 127, normalized size = 1.49

$$2\,bcd\left(\frac{\arctan\left(\frac{\sqrt{dx}c}{\sqrt{cd}}\right)}{\sqrt{cd}d^2}-\frac{\arctan\left(\frac{\sqrt{dx}c}{\sqrt{-cd}}\right)}{\sqrt{-cd}d^2}\right)-\frac{\frac{b\log\left(-\frac{cdx+d}{cdx-d}\right)}{\sqrt{dx}}+\frac{2\,a}{\sqrt{dx}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] 2*b*c*d*(arctan(sqrt(d*x)*c/sqrt(c*d))/(sqrt(c*d)*d^2) - arctan(sqrt(d*x)*c
/sqrt(-c*d))/(sqrt(-c*d)*d^2)) - (b*log(-(c*d*x + d)/(c*d*x - d))/sqrt(d*x)
 + 2*a/sqrt(d*x))/d
```

3.40 $\int \frac{a+b\tanh^{-1}(cx)}{(dx)^{5/2}}\,dx$

Optimal. Leaf size=107

$$-\frac{2\left(a+b\tanh^{-1}(cx)\right)}{3d(dx)^{3/2}}-\frac{2bc^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}}+\frac{2bc^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}}-\frac{4bc}{3d^2\sqrt{dx}}$$

```
[Out] (-4*b*c)/(3*d^2*Sqrt[d*x]) - (2*b*c^(3/2)*ArcTan[(Sqrt[c]*Sqrt[d*x])/Sqrt[d
]])/(3*d^(5/2)) - (2*(a + b*ArcTanh[c*x]))/(3*d*(d*x)^(3/2)) + (2*b*c^(3/2)
*ArcTanh[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]])/(3*d^(5/2))
```

Rubi [A] time = 0.0633332, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5916, 325, 329, 298, 205, 208}

$$-\frac{2\left(a+b\tanh^{-1}(cx)\right)}{3d(dx)^{3/2}}-\frac{2bc^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}}+\frac{2bc^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}}-\frac{4bc}{3d^2\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/(d*x)^(5/2),x]
```

```
[Out] (-4*b*c)/(3*d^2*Sqrt[d*x]) - (2*b*c^(3/2)*ArcTan[(Sqrt[c]*Sqrt[d*x])/Sqrt[d
]])/(3*d^(5/2)) - (2*(a + b*ArcTanh[c*x]))/(3*d*(d*x)^(3/2)) + (2*b*c^(3/2)
*ArcTanh[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]])/(3*d^(5/2))
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
```

```
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}(cx)}{(dx)^{5/2}}\,dx &= -\frac{2\left(a+b\tanh^{-1}(cx)\right)}{3d(dx)^{3/2}}+\frac{(2bc)\int \frac{1}{(dx)^{3/2}\left(1-c^2x^2\right)}\,dx}{3d}\\
&= -\frac{4bc}{3d^2\sqrt{dx}}-\frac{2\left(a+b\tanh^{-1}(cx)\right)}{3d(dx)^{3/2}}+\frac{\left(2bc^3\right)\int \frac{\sqrt{dx}}{1-c^2x^2}\,dx}{3d^3}\\
&= -\frac{4bc}{3d^2\sqrt{dx}}-\frac{2\left(a+b\tanh^{-1}(cx)\right)}{3d(dx)^{3/2}}+\frac{\left(4bc^3\right)\text{Subst}\left(\int \frac{x^2}{1-\frac{c^2x^4}{d^2}}\,dx,x,\sqrt{dx}\right)}{3d^4}\\
&= -\frac{4bc}{3d^2\sqrt{dx}}-\frac{2\left(a+b\tanh^{-1}(cx)\right)}{3d(dx)^{3/2}}+\frac{\left(2bc^2\right)\text{Subst}\left(\int \frac{1}{d-cx^2}\,dx,x,\sqrt{dx}\right)}{3d^2}-\frac{\left(2bc^2\right)\text{Subst}\left(\int \frac{}{d}\right.}{3d^2}\\
&= -\frac{4bc}{3d^2\sqrt{dx}}-\frac{2bc^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}}-\frac{2\left(a+b\tanh^{-1}(cx)\right)}{3d(dx)^{3/2}}+\frac{2bc^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.0604537, size = 107, normalized size = 1.

$$-\frac{x\left(2a+bc^{3/2}x^{3/2}\log\left(1-\sqrt{c}\sqrt{x}\right)-bc^{3/2}x^{3/2}\log\left(\sqrt{c}\sqrt{x}+1\right)+2bc^{3/2}x^{3/2}\tan^{-1}\left(\sqrt{c}\sqrt{x}\right)+4bcx+2b\tanh^{-1}(cx)\right)}{3(dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])/(d*x)^(5/2),x]
```

```
[Out] -(x*(2*a + 4*b*c*x + 2*b*c^(3/2)*x^(3/2)*ArcTan[Sqrt[c]*Sqrt[x]] + 2*b*ArcT
anh[c*x] + b*c^(3/2)*x^(3/2)*Log[1 - Sqrt[c]*Sqrt[x]] - b*c^(3/2)*x^(3/2)*L
og[1 + Sqrt[c]*Sqrt[x]]))/(3*(d*x)^(5/2))
```

Maple [A] time = 0.014, size = 94, normalized size = 0.9

$$-\frac{2a}{3d}(dx)^{-\frac{3}{2}}-\frac{2b\mathrm{Artanh}(cx)}{3d}(dx)^{-\frac{3}{2}}-\frac{4bc}{3d^2}\frac{1}{\sqrt{dx}}-\frac{2bc^2}{3d^2}\arctan\left(c\sqrt{dx}\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+\frac{2bc^2}{3d^2}\mathrm{Artanh}\left(c\sqrt{dx}\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/(d*x)^(5/2),x)
```

```
[Out] -2/3/d*a/(d*x)^(3/2)-2/3/d*b/(d*x)^(3/2)*arctanh(c*x)-4/3*b*c/d^2/(d*x)^(1/
2)-2/3/d^2*b*c^2/(c*d)^(1/2)*arctan(c*(d*x)^(1/2)/(c*d)^(1/2))+2/3/d^2*b*c^
2/(c*d)^(1/2)*arctanh(c*(d*x)^(1/2)/(c*d)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(d*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.2123, size = 558, normalized size = 5.21

$$\left[\frac{2\,bcdx^2\sqrt{\frac{c}{d}}\arctan\left(\frac{\sqrt{dx}\sqrt{\frac{c}{d}}}{cx}\right)+bcdx^2\sqrt{\frac{c}{d}}\log\left(\frac{cx+2\,\sqrt{dx}\sqrt{\frac{c}{d}}+1}{cx-1}\right)-\left(4\,bcx+b\log\left(-\frac{cx+1}{cx-1}\right)+2\,a\right)\sqrt{dx}}{3\,d^3x^2},-\frac{2\,bcdx^2\sqrt{-\frac{c}{d}}\arctan\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(d*x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(2*b*c*d*x^2*sqrt(c/d)*arctan(sqrt(d*x)*sqrt(c/d)/(c*x)) + b*c*d*x^2*s
qrt(c/d)*log((c*x + 2*sqrt(d*x)*sqrt(c/d) + 1)/(c*x - 1)) - (4*b*c*x + b*lo
g(-(c*x + 1)/(c*x - 1)) + 2*a)*sqrt(d*x))/(d^3*x^2), -1/3*(2*b*c*d*x^2*sqrt
(-c/d)*arctan(sqrt(d*x)*sqrt(-c/d)/(c*x)) - b*c*d*x^2*sqrt(-c/d)*log((c*x -
 2*sqrt(d*x)*sqrt(-c/d) - 1)/(c*x + 1)) + (4*b*c*x + b*log(-(c*x + 1)/(c*x
- 1)) + 2*a)*sqrt(d*x))/(d^3*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+b\operatorname{atanh}(cx)}{(dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/(d*x)**(5/2),x)
```

```
[Out] Integral((a + b*atanh(c*x))/(d*x)**(5/2), x)
```

Giac [A] time = 1.34339, size = 163, normalized size = 1.52

$$-\frac{2}{3}\,bc^3\left(\frac{\arctan\left(\frac{\sqrt{dx}c}{\sqrt{cd}}\right)}{\sqrt{cd}cd^2}+\frac{\arctan\left(\frac{\sqrt{dx}c}{\sqrt{-cd}}\right)}{\sqrt{-cd}cd^2}\right)-\frac{\frac{b\log\left(-\frac{cdx+d}{cdx-d}\right)}{\sqrt{dx}dx}+\frac{2\,(2\,bcdx+ad)}{\sqrt{dx}d^2x}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(d*x)^(5/2),x, algorithm="giac")
```

```
[Out] -2/3*b*c^3*(arctan(sqrt(d*x)*c/sqrt(c*d))/(sqrt(c*d)*c*d^2) + arctan(sqrt(d
*x)*c/sqrt(-c*d))/(sqrt(-c*d)*c*d^2)) - 1/3*(b*log(-(c*d*x + d)/(c*d*x - d)
)/(sqrt(d*x)*d*x) + 2*(2*b*c*d*x + a*d)/(sqrt(d*x)*d^2*x))/d
```

3.41 $\int \frac{a+b\tanh^{-1}(cx)}{(dx)^{7/2}}\,dx$

Optimal. Leaf size=107

$$-\frac{2\left(a+b\tanh^{-1}(cx)\right)}{5d(dx)^{5/2}}+\frac{2bc^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{5d^{7/2}}+\frac{2bc^{5/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{5d^{7/2}}-\frac{4bc}{15d^2(dx)^{3/2}}$$

```
[Out] (-4*b*c)/(15*d^2*(d*x)^(3/2)) + (2*b*c^(5/2)*ArcTan[(Sqrt[c]*Sqrt[d*x])/Sqr
t[d]])/(5*d^(7/2)) - (2*(a + b*ArcTanh[c*x]))/(5*d*(d*x)^(5/2)) + (2*b*c^(5
/2)*ArcTanh[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]])/(5*d^(7/2))
```

Rubi [A] time = 0.0662016, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5916, 325, 329, 212, 208, 205}

$$-\frac{2\left(a+b\tanh^{-1}(cx)\right)}{5d(dx)^{5/2}}+\frac{2bc^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{5d^{7/2}}+\frac{2bc^{5/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{5d^{7/2}}-\frac{4bc}{15d^2(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/(d*x)^(7/2),x]
```

```
[Out] (-4*b*c)/(15*d^2*(d*x)^(3/2)) + (2*b*c^(5/2)*ArcTan[(Sqrt[c]*Sqrt[d*x])/Sqr
t[d]])/(5*d^(7/2)) - (2*(a + b*ArcTanh[c*x]))/(5*d*(d*x)^(5/2)) + (2*b*c^(5
/2)*ArcTanh[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]])/(5*d^(7/2))
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
```

```
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\tanh^{-1}(cx)}{(dx)^{7/2}}\,dx &= -\frac{2\left(a+b\tanh^{-1}(cx)\right)}{5d(dx)^{5/2}}+\frac{(2bc)\int \frac{1}{(dx)^{5/2}\left(1-c^2x^2\right)}\,dx}{5d}\\
&= -\frac{4bc}{15d^2(dx)^{3/2}}-\frac{2\left(a+b\tanh^{-1}(cx)\right)}{5d(dx)^{5/2}}+\frac{\left(2bc^3\right)\int \frac{1}{\sqrt{dx}\left(1-c^2x^2\right)}\,dx}{5d^3}\\
&= -\frac{4bc}{15d^2(dx)^{3/2}}-\frac{2\left(a+b\tanh^{-1}(cx)\right)}{5d(dx)^{5/2}}+\frac{\left(4bc^3\right)\mathrm{Subst}\left(\int \frac{1}{1-\frac{c^2x^4}{d^2}}\,dx,x,\sqrt{dx}\right)}{5d^4}\\
&= -\frac{4bc}{15d^2(dx)^{3/2}}-\frac{2\left(a+b\tanh^{-1}(cx)\right)}{5d(dx)^{5/2}}+\frac{\left(2bc^3\right)\mathrm{Subst}\left(\int \frac{1}{d-cx^2}\,dx,x,\sqrt{dx}\right)}{5d^3}+\frac{\left(2bc^3\right)\mathrm{Subst}\left(\right.}{}\\
&= -\frac{4bc}{15d^2(dx)^{3/2}}+\frac{2bc^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{5d^{7/2}}-\frac{2\left(a+b\tanh^{-1}(cx)\right)}{5d(dx)^{5/2}}+\frac{2bc^{5/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{5d^{7/2}}
\end{aligned}$$

Mathematica [A] time = 0.0480971, size = 108, normalized size = 1.01

$$\frac{x\left(-6a-3bc^{5/2}x^{5/2}\log\left(1-\sqrt{c}\sqrt{x}\right)+3bc^{5/2}x^{5/2}\log\left(\sqrt{c}\sqrt{x}+1\right)+6bc^{5/2}x^{5/2}\tan^{-1}\left(\sqrt{c}\sqrt{x}\right)-4bcx-6b\tanh^{-1}(cx)\right)}{15(dx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])/(d*x)^(7/2),x]
```

```
[Out] (x*(-6*a - 4*b*c*x + 6*b*c^(5/2)*x^(5/2)*ArcTan[Sqrt[c]*Sqrt[x]] - 6*b*ArcT
anh[c*x] - 3*b*c^(5/2)*x^(5/2)*Log[1 - Sqrt[c]*Sqrt[x]] + 3*b*c^(5/2)*x^(5/
2)*Log[1 + Sqrt[c]*Sqrt[x]]))/(15*(d*x)^(7/2))
```

Maple [A] time = 0.014, size = 94, normalized size = 0.9

$$-\frac{2a}{5d}(dx)^{-\frac{5}{2}}-\frac{2b\mathrm{Artanh}(cx)}{5d}(dx)^{-\frac{5}{2}}+\frac{2bc^3}{5d^3}\arctan\left(c\sqrt{dx}\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+\frac{2bc^3}{5d^3}\mathrm{Artanh}\left(c\sqrt{dx}\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{4bc}{15d^2}(dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/(d*x)^(7/2),x)
```

```
[Out] -2/5/d*a/(d*x)^(5/2)-2/5/d*b/(d*x)^(5/2)*arctanh(c*x)+2/5/d^3*b*c^3/(c*d)^(
1/2)*arctan(c*(d*x)^(1/2)/(c*d)^(1/2))+2/5/d^3*b*c^3/(c*d)^(1/2)*arctanh(c*
(d*x)^(1/2)/(c*d)^(1/2))-4/15*b*c/d^2/(d*x)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(d*x)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.27957, size = 583, normalized size = 5.45

$$\left[-\frac{6\,bc^2dx^3\sqrt{\frac{c}{d}}\arctan\left(\frac{\sqrt{dx}\sqrt{\frac{c}{d}}}{cx}\right)-3\,bc^2dx^3\sqrt{\frac{c}{d}}\log\left(\frac{cx+2\,\sqrt{dx}\sqrt{\frac{c}{d}}+1}{cx-1}\right)+\left(4\,bcx+3\,b\log\left(-\frac{cx+1}{cx-1}\right)+6\,a\right)\sqrt{dx}}{15\,d^4x^3},-\frac{6\,bc^2dx^3\sqrt{-\frac{c}{d}}\,\mathrm{a}\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(d*x)^(7/2),x, algorithm="fricas")
```

```
[Out] [-1/15*(6*b*c^2*d*x^3*sqrt(c/d)*arctan(sqrt(d*x)*sqrt(c/d)/(c*x)) - 3*b*c^2
*d*x^3*sqrt(c/d)*log((c*x + 2*sqrt(d*x)*sqrt(c/d) + 1)/(c*x - 1)) + (4*b*c*
x + 3*b*log(-(c*x + 1)/(c*x - 1)) + 6*a)*sqrt(d*x))/(d^4*x^3), -1/15*(6*b*c
^2*d*x^3*sqrt(-c/d)*arctan(sqrt(d*x)*sqrt(-c/d)/(c*x)) - 3*b*c^2*d*x^3*sqrt
(-c/d)*log((c*x + 2*sqrt(d*x)*sqrt(-c/d) - 1)/(c*x + 1)) + (4*b*c*x + 3*b*l
og(-(c*x + 1)/(c*x - 1)) + 6*a)*sqrt(d*x))/(d^4*x^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/(d*x)**(7/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.26523, size = 159, normalized size = 1.49

$$\frac{2}{5}\,bc^3\left(\frac{\arctan\left(\frac{\sqrt{dx}c}{\sqrt{cd}}\right)}{\sqrt{cd}d^3}-\frac{\arctan\left(\frac{\sqrt{dx}c}{\sqrt{-cd}}\right)}{\sqrt{-cd}d^3}\right)-\frac{\frac{3\,b\log\left(-\frac{cdx+d}{cdx-d}\right)}{\sqrt{dx}d^2x^2}+\frac{2\,(2\,bcdx+3\,ad)}{\sqrt{dx}d^3x^2}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(d*x)^(7/2),x, algorithm="giac")
```

```
[Out] 2/5*b*c^3*(arctan(sqrt(d*x)*c/sqrt(c*d))/(sqrt(c*d)*d^3) - arctan(sqrt(d*x)
*c/sqrt(-c*d))/(sqrt(-c*d)*d^3)) - 1/15*(3*b*log(-(c*d*x + d)/(c*d*x - d))/
(sqrt(d*x)*d^2*x^2) + 2*(2*b*c*d*x + 3*a*d)/(sqrt(d*x)*d^3*x^2))/d
```

3.42 $\int \frac{a+b\tanh^{-1}(cx)}{(dx)^{9/2}}\,dx$

Optimal. Leaf size=125

$$-\frac{2\left(a+b\tanh^{-1}(cx)\right)}{7d(dx)^{7/2}}-\frac{4bc^3}{7d^4\sqrt{dx}}-\frac{2bc^{7/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{7d^{9/2}}+\frac{2bc^{7/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{7d^{9/2}}-\frac{4bc}{35d^2(dx)^{5/2}}$$

```
[Out] (-4*b*c)/(35*d^2*(d*x)^(5/2)) - (4*b*c^3)/(7*d^4*Sqrt[d*x]) - (2*b*c^(7/2)*
ArcTan[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]])/(7*d^(9/2)) - (2*(a + b*ArcTanh[c*x]))
/(7*d*(d*x)^(7/2)) + (2*b*c^(7/2)*ArcTanh[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]])/(7*
d^(9/2))
```

Rubi [A] time = 0.078228, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5916, 325, 329, 298, 205, 208}

$$-\frac{2\left(a+b\tanh^{-1}(cx)\right)}{7d(dx)^{7/2}}-\frac{4bc^3}{7d^4\sqrt{dx}}-\frac{2bc^{7/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{7d^{9/2}}+\frac{2bc^{7/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{7d^{9/2}}-\frac{4bc}{35d^2(dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/(d*x)^(9/2),x]
```


```
[Out] (-4*b*c)/(35*d^2*(d*x)^(5/2)) - (4*b*c^3)/(7*d^4*Sqrt[d*x]) - (2*b*c^(7/2)*
ArcTan[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]])/(7*d^(9/2)) - (2*(a + b*ArcTanh[c*x]))
/(7*d*(d*x)^(7/2)) + (2*b*c^(7/2)*ArcTanh[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]])/(7*
d^(9/2))
```


Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
```

```
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\tanh^{-1}(cx)}{(dx)^{9/2}}\,dx &= -\frac{2\left(a+b\tanh^{-1}(cx)\right)}{7d(dx)^{7/2}}+\frac{(2bc)\int \frac{1}{(dx)^{7/2}\left(1-c^2x^2\right)}\,dx}{7d}\\
&= -\frac{4bc}{35d^2(dx)^{5/2}}-\frac{2\left(a+b\tanh^{-1}(cx)\right)}{7d(dx)^{7/2}}+\frac{\left(2bc^3\right)\int \frac{1}{(dx)^{3/2}\left(1-c^2x^2\right)}\,dx}{7d^3}\\
&= -\frac{4bc}{35d^2(dx)^{5/2}}-\frac{4bc^3}{7d^4\sqrt{dx}}-\frac{2\left(a+b\tanh^{-1}(cx)\right)}{7d(dx)^{7/2}}+\frac{\left(2bc^5\right)\int \frac{\sqrt{dx}}{1-c^2x^2}\,dx}{7d^5}\\
&= -\frac{4bc}{35d^2(dx)^{5/2}}-\frac{4bc^3}{7d^4\sqrt{dx}}-\frac{2\left(a+b\tanh^{-1}(cx)\right)}{7d(dx)^{7/2}}+\frac{\left(4bc^5\right)\text{Subst}\left(\int \frac{x^2}{1-\frac{c^2x^4}{d^2}}\,dx,x,\sqrt{dx}\right)}{7d^6}\\
&= -\frac{4bc}{35d^2(dx)^{5/2}}-\frac{4bc^3}{7d^4\sqrt{dx}}-\frac{2\left(a+b\tanh^{-1}(cx)\right)}{7d(dx)^{7/2}}+\frac{\left(2bc^4\right)\text{Subst}\left(\int \frac{1}{d-cx^2}\,dx,x,\sqrt{dx}\right)}{7d^4}-\frac{(2b
\end{aligned}$$

$$= -\frac{4bc}{35d^2(dx)^{5/2}}-\frac{4bc^3}{7d^4\sqrt{dx}}-\frac{2bc^{7/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{7d^{9/2}}-\frac{2\left(a+b\tanh^{-1}(cx)\right)}{7d(dx)^{7/2}}+\frac{2bc^{7/2}\tanh^{-1}\left(\sqrt{\right.}{7d^{9/2}}$$

Mathematica [A] time = 0.0661833, size = 122, normalized size = 0.98

$$-\frac{\sqrt{dx}\left(10a+20bc^3x^3+5bc^{7/2}x^{7/2}\log\left(1-\sqrt{c}\sqrt{x}\right)-5bc^{7/2}x^{7/2}\log\left(\sqrt{c}\sqrt{x}+1\right)+10bc^{7/2}x^{7/2}\tan^{-1}\left(\sqrt{c}\sqrt{x}\right)+4bcx+1}{35d^5x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])/(d*x)^(9/2),x]
```

```
[Out] -(Sqrt[d*x]*(10*a + 4*b*c*x + 20*b*c^3*x^3 + 10*b*c^(7/2)*x^(7/2)*ArcTan[Sq
rt[c]*Sqrt[x]] + 10*b*ArcTanh[c*x] + 5*b*c^(7/2)*x^(7/2)*Log[1 - Sqrt[c]*Sq
rt[x]] - 5*b*c^(7/2)*x^(7/2)*Log[1 + Sqrt[c]*Sqrt[x]]))/(35*d^5*x^4)
```

Maple [A] time = 0.015, size = 108, normalized size = 0.9

$$-\frac{2a}{7d}(dx)^{-\frac{7}{2}}-\frac{2b\text{Artanh}(cx)}{7d}(dx)^{-\frac{7}{2}}-\frac{2bc^4}{7d^4}\arctan\left(c\sqrt{dx}\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{4bc}{35d^2}(dx)^{-\frac{5}{2}}-\frac{4bc^3}{7d^4}\frac{1}{\sqrt{dx}}+\frac{2bc^4}{7d^4}\text{Artanh}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/(d*x)^(9/2),x)
```

```
[Out] -2/7/d*a/(d*x)^(7/2)-2/7/d*b/(d*x)^(7/2)*arctanh(c*x)-2/7/d^4*b*c^4/(c*d)^(
1/2)*arctan(c*(d*x)^(1/2)/(c*d)^(1/2))-4/35*b*c/d^2/(d*x)^(5/2)-4/7*b*c^3/d
^4/(d*x)^(1/2)+2/7/d^4*b*c^4/(c*d)^(1/2)*arctanh(c*(d*x)^(1/2)/(c*d)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(d*x)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.45178, size = 628, normalized size = 5.02

$$\left[\frac{10\,bc^3dx^4\sqrt{\frac{c}{d}}\arctan\left(\frac{\sqrt{dx}\sqrt{\frac{c}{d}}}{cx}\right)+5\,bc^3dx^4\sqrt{\frac{c}{d}}\log\left(\frac{cx+2\,\sqrt{dx}\sqrt{\frac{c}{d}}+1}{cx-1}\right)-\left(20\,bc^3x^3+4\,bcx+5\,b\log\left(-\frac{cx+1}{cx-1}\right)+10\,a\right)\sqrt{dx}}{35\,d^5x^4}, -\frac{10\,b\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(d*x)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/35*(10*b*c^3*d*x^4*sqrt(c/d)*arctan(sqrt(d*x)*sqrt(c/d)/(c*x)) + 5*b*c^3
*d*x^4*sqrt(c/d)*log((c*x + 2*sqrt(d*x)*sqrt(c/d) + 1)/(c*x - 1)) - (20*b*c
^3*x^3 + 4*b*c*x + 5*b*log(-(c*x + 1)/(c*x - 1)) + 10*a)*sqrt(d*x))/(d^5*x^
4), -1/35*(10*b*c^3*d*x^4*sqrt(-c/d)*arctan(sqrt(d*x)*sqrt(-c/d)/(c*x)) - 5
*b*c^3*d*x^4*sqrt(-c/d)*log((c*x - 2*sqrt(d*x)*sqrt(-c/d) - 1)/(c*x + 1)) +
 (20*b*c^3*x^3 + 4*b*c*x + 5*b*log(-(c*x + 1)/(c*x - 1)) + 10*a)*sqrt(d*x))
/(d^5*x^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/(d*x)**(9/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.4231, size = 188, normalized size = 1.5

$$-\frac{2}{7}\,bc^5\left(\frac{\arctan\left(\frac{\sqrt{dx}c}{\sqrt{cd}}\right)}{\sqrt{cd}cd^4}+\frac{\arctan\left(\frac{\sqrt{dx}c}{\sqrt{-cd}}\right)}{\sqrt{-cd}cd^4}\right)-\frac{\frac{5\,b\log\left(-\frac{cdx+d}{cdx-d}\right)}{\sqrt{dx}d^3x^3}+\frac{2\left(10\,bc^3d^3x^3+2\,bcd^3x+5\,ad^3\right)}{\sqrt{dx}d^6x^3}}{35\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(d*x)^(9/2),x, algorithm="giac")
```

```
[Out] -2/7*b*c^5*(arctan(sqrt(d*x)*c/sqrt(c*d))/(sqrt(c*d)*c*d^4) + arctan(sqrt(d
*x)*c/sqrt(-c*d))/(sqrt(-c*d)*c*d^4)) - 1/35*(5*b*log(-(c*d*x + d)/(c*d*x -
 d))/(sqrt(d*x)*d^3*x^3) + 2*(10*b*c^3*d^3*x^3 + 2*b*c*d^3*x + 5*a*d^3)/(sq
rt(d*x)*d^6*x^3))/d
```

3.43 $\int (dx)^m \left(a+b\tanh^{-1}(cx)\right)^3\,dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left((dx)^m \left(a+b\tanh^{-1}(cx)\right)^3,x\right)$$

```
[Out] Unintegrable[(d*x)^m*(a + b*ArcTanh[c*x])^3, x]
```

Rubi [A] time = 0.0244612, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (dx)^m \left(a+b\tanh^{-1}(cx)\right)^3\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m*(a + b*ArcTanh[c*x])^3,x]
```

```
[Out] Defer[Int][(d*x)^m*(a + b*ArcTanh[c*x])^3, x]
```

Rubi steps

$$\int (dx)^m \left(a+b\tanh^{-1}(cx)\right)^3\,dx=\int (dx)^m \left(a+b\tanh^{-1}(cx)\right)^3\,dx$$

Mathematica [A] time = 3.51876, size = 0, normalized size = 0.

$$\int (dx)^m \left(a+b\tanh^{-1}(cx)\right)^3\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m*(a + b*ArcTanh[c*x])^3,x]
```

```
[Out] Integrate[(d*x)^m*(a + b*ArcTanh[c*x])^3, x]
```

Maple [A] time = 1.024, size = 0, normalized size = 0.

$$\int (dx)^m (a+b\text{Artanh}(cx))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctanh(c*x))^3,x)
```

```
[Out] int((d*x)^m*(a+b*arctanh(c*x))^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^3 \operatorname{artanh}(cx)^3 + 3\,ab^2 \operatorname{artanh}(cx)^2 + 3\,a^2b \operatorname{artanh}(cx) + a^3\right)(dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctanh(c*x)^3 + 3*a*b^2*arctanh(c*x)^2 + 3*a^2*b*arctanh(c*x
) + a^3)*(d*x)^m, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int (dx)^m (a + b \operatorname{atanh}(cx))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atanh(c*x))**3,x)
```

```
[Out] Integral((d*x)**m*(a + b*atanh(c*x))**3, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{artanh}(cx) + a)^3 (dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^3*(d*x)^m, x)
```

3.44 $\int (dx)^m \left(a+b\tanh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left((dx)^m \left(a+b\tanh^{-1}(cx)\right)^2,x\right)$$

```
[Out] Unintegrable[(d*x)^m*(a + b*ArcTanh[c*x])^2, x]
```

Rubi [A] time = 0.0234099, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (dx)^m \left(a+b\tanh^{-1}(cx)\right)^2 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] Defer[Int][(d*x)^m*(a + b*ArcTanh[c*x])^2, x]
```

Rubi steps

$$\int (dx)^m \left(a+b\tanh^{-1}(cx)\right)^2 \, dx=\int (dx)^m \left(a+b\tanh^{-1}(cx)\right)^2 \, dx$$

Mathematica [A] time = 2.32125, size = 0, normalized size = 0.

$$\int (dx)^m \left(a+b\tanh^{-1}(cx)\right)^2 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] Integrate[(d*x)^m*(a + b*ArcTanh[c*x])^2, x]
```

Maple [A] time = 0.914, size = 0, normalized size = 0.

$$\int (dx)^m (a+b\text{Artanh}\,(cx))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctanh(c*x))^2,x)
```

```
[Out] int((d*x)^m*(a+b*arctanh(c*x))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2 \operatorname{artanh}(cx)^2 + 2\,ab \operatorname{artanh}(cx) + a^2\right)(dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)*(d*x)^m, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int (dx)^m \left(a + b \operatorname{atanh}(cx)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atanh(c*x))**2,x)
```

```
[Out] Integral((d*x)**m*(a + b*atanh(c*x))**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b \operatorname{artanh}(cx) + a\right)^2 (dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2*(d*x)^m, x)
```

3.45 $\int (dx)^m \left(a+b\tanh^{-1}(cx)\right) dx$

Optimal. Leaf size=72

$$\frac{(dx)^{m+1}\left(a+b\tanh^{-1}(cx)\right)}{d(m+1)}-\frac{bc(dx)^{m+2}\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},c^2x^2\right)}{d^2(m+1)(m+2)}$$

```
[Out] ((d*x)^(1 + m)*(a + b*ArcTanh[c*x]))/(d*(1 + m)) - (b*c*(d*x)^(2 + m)*Hyper
geometric2F1[1, (2 + m)/2, (4 + m)/2, c^2*x^2])/(d^2*(1 + m)*(2 + m))
```

Rubi [A] time = 0.032328, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5916, 364}

$$\frac{(dx)^{m+1}\left(a+b\tanh^{-1}(cx)\right)}{d(m+1)}-\frac{bc(dx)^{m+2}\,{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};c^2x^2\right)}{d^2(m+1)(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*ArcTanh[c*x]),x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*ArcTanh[c*x]))/(d*(1 + m)) - (b*c*(d*x)^(2 + m)*Hyper
geometric2F1[1, (2 + m)/2, (4 + m)/2, c^2*x^2])/(d^2*(1 + m)*(2 + m))
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (dx)^m\left(a+b\tanh^{-1}(cx)\right)dx &= \frac{(dx)^{1+m}\left(a+b\tanh^{-1}(cx)\right)}{d(1+m)}-\frac{(bc)\int\frac{(dx)^{1+m}}{1-c^2x^2}\,dx}{d(1+m)}\\ &= \frac{(dx)^{1+m}\left(a+b\tanh^{-1}(cx)\right)}{d(1+m)}-\frac{bc(dx)^{2+m}\,{}_2F_1\left(1,\frac{2+m}{2};\frac{4+m}{2};c^2x^2\right)}{d^2(1+m)(2+m)}\end{aligned}$$

Mathematica [A] time = 0.0667753, size = 59, normalized size = 0.82

$$-\frac{x(dx)^m\left(bcx\text{Hypergeometric2F1}\left(1,\frac{m}{2}+1,\frac{m}{2}+2,c^2x^2\right)-(m+2)\left(a+b\tanh^{-1}(cx)\right)\right)}{(m+1)(m+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a + b*ArcTanh[c*x]),x]
```

```
[Out] -((x*(d*x)^m*(-((2 + m)*(a + b*ArcTanh[c*x])) + b*c*x*Hypergeometric2F1[1,
1 + m/2, 2 + m/2, c^2*x^2]))/((1 + m)*(2 + m)))
```

Maple [F] time = 0.925, size = 0, normalized size = 0.

$$\int (dx)^m (a + b\text{Artanh}(cx))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctanh(c*x)),x)
```

```
[Out] int((d*x)^m*(a+b*arctanh(c*x)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\,\text{artanh}(cx) + a)(dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*x) + a)*(d*x)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^m (a + b\,\text{atanh}(cx))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atanh(c*x)),x)
```

```
[Out] Integral((d*x)**m*(a + b*atanh(c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{artanh}(cx) + a)(dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)*(d*x)^m, x)
```

3.46 $\int \frac{(dx)^m}{a+b\tanh^{-1}(cx)}\,dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{(dx)^m}{a+b\tanh^{-1}(cx)},x\right)$$

```
[Out] Unintegrable[(d*x)^m/(a + b*ArcTanh[c*x]), x]
```

Rubi [A] time = 0.0271058, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(dx)^m}{a+b\tanh^{-1}(cx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m/(a + b*ArcTanh[c*x]),x]
```

```
[Out] Defer[Int][(d*x)^m/(a + b*ArcTanh[c*x]), x]
```

Rubi steps

$$\int \frac{(dx)^m}{a+b\tanh^{-1}(cx)}\,dx=\int \frac{(dx)^m}{a+b\tanh^{-1}(cx)}\,dx$$

Mathematica [A] time = 0.253179, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{a+b\tanh^{-1}(cx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m/(a + b*ArcTanh[c*x]),x]
```

```
[Out] Integrate[(d*x)^m/(a + b*ArcTanh[c*x]), x]
```

Maple [A] time = 0.654, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{a+b\text{Artanh}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a+b*arctanh(c*x)),x)
```

```
[Out] int((d*x)^m/(a+b*arctanh(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{b \operatorname{artanh}(cx) + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/(b*arctanh(c*x) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx)^m}{b \operatorname{artanh}(cx) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(b*arctanh(c*x) + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{a + b \operatorname{atanh}(cx)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*atanh(c*x)),x)
```

```
[Out] Integral((d*x)**m/(a + b*atanh(c*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{b \operatorname{artanh}(cx) + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b*arctanh(c*x) + a), x)
```

3.47 $\int \frac{(dx)^m}{\left(a+b\tanh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{(dx)^m}{\left(a+b\tanh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable[(d*x)^m/(a + b*ArcTanh[c*x])^2, x]
```

Rubi [A] time = 0.0263568, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(dx)^m}{\left(a+b\tanh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m/(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] Defer[Int][(d*x)^m/(a + b*ArcTanh[c*x])^2, x]
```

Rubi steps

$$\int \frac{(dx)^m}{\left(a+b\tanh^{-1}(cx)\right)^2}\,dx=\int \frac{(dx)^m}{\left(a+b\tanh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A] time = 0.499509, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left(a+b\tanh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m/(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] Integrate[(d*x)^m/(a + b*ArcTanh[c*x])^2, x]
```

Maple [A] time = 0.658, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{(a+b\text{Artanh}\,(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a+b*arctanh(c*x))^2,x)
```

```
[Out] int((d*x)^m/(a+b*arctanh(c*x))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{2\left(c^2d^mx^2 - d^m\right)x^m}{b^2c\log\left(cx+1\right) - b^2c\log\left(-cx+1\right) + 2\,abc} + \int -\frac{2\left(c^2d^m(m+2)x^2 - d^mm\right)x^m}{b^2cx\log\left(cx+1\right) - b^2cx\log\left(-cx+1\right) + 2\,abcx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 2*(c^2*d^m*x^2 - d^m)*x^m/(b^2*c*log(c*x + 1) - b^2*c*log(-c*x + 1) + 2*a*b
*c) + integrate(-2*(c^2*d^m*(m + 2)*x^2 - d^m*m)*x^m/(b^2*c*x*log(c*x + 1)
- b^2*c*x*log(-c*x + 1) + 2*a*b*c*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(dx)^m}{b^2\,\mathrm{artanh}\left(cx\right)^2 + 2\,ab\,\mathrm{artanh}\left(cx\right) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left(a + b\,\mathrm{atanh}\left(cx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*atanh(c*x))**2,x)
```

```
[Out] Integral((d*x)**m/(a + b*atanh(c*x))**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left(b\,\mathrm{artanh}\left(cx\right) + a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b*arctanh(c*x) + a)^2, x)
```

3.48 $\int \left(a+b\tanh^{-1}(cx)\right)^p\,dx$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(\left(a+b\tanh^{-1}(cx)\right)^p,x\right)$$

```
[Out] Unintegrable[(a + b*ArcTanh[c*x])^p, x]
```

Rubi [A] time = 0.0053112, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \left(a+b\tanh^{-1}(cx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcTanh[c*x])^p,x]
```

```
[Out] Defer[Int][(a + b*ArcTanh[c*x])^p, x]
```

Rubi steps

$$\int \left(a+b\tanh^{-1}(cx)\right)^p\,dx=\int \left(a+b\tanh^{-1}(cx)\right)^p\,dx$$

Mathematica [A] time = 0.441212, size = 0, normalized size = 0.

$$\int \left(a+b\tanh^{-1}(cx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcTanh[c*x])^p,x]
```

```
[Out] Integrate[(a + b*ArcTanh[c*x])^p, x]
```

Maple [A] time = 0.201, size = 0, normalized size = 0.

$$\int (a+b\text{Artanh}\,(cx))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^p,x)
```

```
[Out] int((a+b*arctanh(c*x))^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{artanh}(cx) + a)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*arctanh(c*x) + a)^p, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left((b \operatorname{artanh}(cx) + a)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^p,x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*x) + a)^p, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int (a + b \operatorname{atanh}(cx))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**p,x)
```

```
[Out] Integral((a + b*atanh(c*x))**p, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{artanh}(cx) + a)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^p,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^p, x)
```

3.49 $\int (dx)^m \left(a+b\tanh^{-1}(cx)\right)^p \, dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left((dx)^m \left(a+b\tanh^{-1}(cx)\right)^p, x\right)$$

```
[Out] Unintegrable[(d*x)^m*(a + b*ArcTanh[c*x])^p, x]
```

Rubi [A] time = 0.0236748, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (dx)^m \left(a+b\tanh^{-1}(cx)\right)^p \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m*(a + b*ArcTanh[c*x])^p,x]

[Out] Defer[Int][(d*x)^m*(a + b*ArcTanh[c*x])^p, x]
```

Rubi steps

$$\int (dx)^m \left(a+b\tanh^{-1}(cx)\right)^p \, dx = \int (dx)^m \left(a+b\tanh^{-1}(cx)\right)^p \, dx$$

Mathematica [A] time = 0.326303, size = 0, normalized size = 0.

$$\int (dx)^m \left(a+b\tanh^{-1}(cx)\right)^p \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m*(a + b*ArcTanh[c*x])^p,x]

[Out] Integrate[(d*x)^m*(a + b*ArcTanh[c*x])^p, x]
```

Maple [A] time = 0.378, size = 0, normalized size = 0.

$$\int (dx)^m (a+b\text{Artanh}(cx))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctanh(c*x))^p,x)

[Out] int((d*x)^m*(a+b*arctanh(c*x))^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx)^m (b \operatorname{artanh}(cx) + a)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x))^p,x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m*(b*arctanh(c*x) + a)^p, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x))^p,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atanh(c*x))**p,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dx)^m (b \operatorname{artanh}(cx) + a)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x))^p,x, algorithm="giac")
```

```
[Out] integrate((d*x)^m*(b*arctanh(c*x) + a)^p, x)
```

3.50 $\int x^7 \left(a+b\tanh^{-1}\left(cx^2\right)\right)\,dx$

Optimal. Leaf size=54

$$\frac{1}{8}x^8\left(a+b\tanh^{-1}\left(cx^2\right)\right)+\frac{bx^2}{8c^3}-\frac{b\tanh^{-1}\left(cx^2\right)}{8c^4}+\frac{bx^6}{24c}$$

```
[Out] (b*x^2)/(8*c^3) + (b*x^6)/(24*c) - (b*ArcTanh[c*x^2])/(8*c^4) + (x^8*(a + b
*ArcTanh[c*x^2]))/8
```

Rubi [A] time = 0.03915, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6097, 275, 302, 206}

$$\frac{1}{8}x^8\left(a+b\tanh^{-1}\left(cx^2\right)\right)+\frac{bx^2}{8c^3}-\frac{b\tanh^{-1}\left(cx^2\right)}{8c^4}+\frac{bx^6}{24c}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*ArcTanh[c*x^2]),x]
```

```
[Out] (b*x^2)/(8*c^3) + (b*x^6)/(24*c) - (b*ArcTanh[c*x^2])/(8*c^4) + (x^8*(a + b
*ArcTanh[c*x^2]))/8
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int x^7\left(a+b\tanh^{-1}\left(cx^2\right)\right)dx &= \frac{1}{8}x^8\left(a+b\tanh^{-1}\left(cx^2\right)\right)-\frac{1}{4}(bc)\int\frac{x^9}{1-c^2x^4}\,dx\\ &= \frac{1}{8}x^8\left(a+b\tanh^{-1}\left(cx^2\right)\right)-\frac{1}{8}(bc)\,\text{Subst}\left(\int\frac{x^4}{1-c^2x^2}\,dx,x,x^2\right)\\ &= \frac{1}{8}x^8\left(a+b\tanh^{-1}\left(cx^2\right)\right)-\frac{1}{8}(bc)\,\text{Subst}\left(\int\left(-\frac{1}{c^4}-\frac{x^2}{c^2}+\frac{1}{c^4\left(1-c^2x^2\right)}\right)dx,x,x^2\right)\\ &= \frac{bx^2}{8c^3}+\frac{bx^6}{24c}+\frac{1}{8}x^8\left(a+b\tanh^{-1}\left(cx^2\right)\right)-\frac{b\,\text{Subst}\left(\int\frac{1}{1-c^2x^2}\,dx,x,x^2\right)}{8c^3}\\ &= \frac{bx^2}{8c^3}+\frac{bx^6}{24c}-\frac{b\tanh^{-1}\left(cx^2\right)}{8c^4}+\frac{1}{8}x^8\left(a+b\tanh^{-1}\left(cx^2\right)\right)\end{aligned}$$

Mathematica [A] time = 0.016156, size = 78, normalized size = 1.44

$$\frac{ax^8}{8}+\frac{bx^2}{8c^3}+\frac{b\log\left(1-cx^2\right)}{16c^4}-\frac{b\log\left(cx^2+1\right)}{16c^4}+\frac{bx^6}{24c}+\frac{1}{8}bx^8\tanh^{-1}\left(cx^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*ArcTanh[c*x^2]),x]
```

```
[Out] (b*x^2)/(8*c^3) + (b*x^6)/(24*c) + (a*x^8)/8 + (b*x^8*ArcTanh[c*x^2])/8 + (
b*Log[1 - c*x^2])/(16*c^4) - (b*Log[1 + c*x^2])/(16*c^4)
```

Maple [A] time = 0.022, size = 66, normalized size = 1.2

$$\frac{x^8a}{8}+\frac{bx^8\text{Artanh}\left(cx^2\right)}{8}+\frac{bx^6}{24\,c}+\frac{bx^2}{8\,c^3}+\frac{b\ln\left(cx^2-1\right)}{16\,c^4}-\frac{b\ln\left(cx^2+1\right)}{16\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a+b*arctanh(c*x^2)),x)
```

```
[Out] 1/8*x^8*a+1/8*b*x^8*arctanh(c*x^2)+1/24*b*x^6/c+1/8*b*x^2/c^3+1/16*b/c^4*ln
(c*x^2-1)-1/16*b/c^4*ln(c*x^2+1)
```

Maxima [A] time = 0.952859, size = 93, normalized size = 1.72

$$\frac{1}{8}\,ax^8+\frac{1}{48}\left(6\,x^8\,\text{artanh}\left(cx^2\right)+c\left(\frac{2\left(c^2x^6+3\,x^2\right)}{c^4}-\frac{3\,\log\left(cx^2+1\right)}{c^5}+\frac{3\,\log\left(cx^2-1\right)}{c^5}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arctanh(c*x^2)),x, algorithm="maxima")
```

```
[Out] 1/8*a*x^8 + 1/48*(6*x^8*arctanh(c*x^2) + c*(2*(c^2*x^6 + 3*x^2)/c^4 - 3*log
(c*x^2 + 1)/c^5 + 3*log(c*x^2 - 1)/c^5))*b
```

Fricas [A] time = 1.99897, size = 135, normalized size = 2.5

$$\frac{6\,ac^4x^8 + 2\,bc^3x^6 + 6\,bcx^2 + 3\left(bc^4x^8 - b\right)\log\left(-\frac{cx^2+1}{cx^2-1}\right)}{48\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arctanh(c*x^2)),x, algorithm="fricas")
```

```
[Out] 1/48*(6*a*c^4*x^8 + 2*b*c^3*x^6 + 6*b*c*x^2 + 3*(b*c^4*x^8 - b)*log(-(c*x^2
 + 1)/(c*x^2 - 1)))/c^4
```

Sympy [A] time = 29.8785, size = 58, normalized size = 1.07

$$\begin{cases} \frac{ax^8}{8} + \frac{bx^8\operatorname{atanh}\left(cx^2\right)}{8} + \frac{bx^6}{24c} + \frac{bx^2}{8c^3} - \frac{b\operatorname{atanh}\left(cx^2\right)}{8c^4} & \text{for}\ c \neq 0 \\ \frac{ax^8}{8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(a+b*atanh(c*x**2)),x)
```

```
[Out] Piecewise((a*x**8/8 + b*x**8*atanh(c*x**2)/8 + b*x**6/(24*c) + b*x**2/(8*c*
*3) - b*atanh(c*x**2)/(8*c**4), Ne(c, 0)), (a*x**8/8, True))
```

Giac [A] time = 1.20634, size = 105, normalized size = 1.94

$$\frac{1}{16}\,bx^8\log\left(-\frac{cx^2+1}{cx^2-1}\right) + \frac{1}{8}\,ax^8 + \frac{bx^6}{24\,c} + \frac{bx^2}{8\,c^3} - \frac{b\log\left(cx^2+1\right)}{16\,c^4} + \frac{b\log\left(cx^2-1\right)}{16\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arctanh(c*x^2)),x, algorithm="giac")
```

```
[Out] 1/16*b*x^8*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 1/8*a*x^8 + 1/24*b*x^6/c + 1/8*b
*x^2/c^3 - 1/16*b*log(c*x^2 + 1)/c^4 + 1/16*b*log(c*x^2 - 1)/c^4
```

3.51 $\int x^5 \left(a+b \tanh^{-1}\left(cx^2\right)\right) dx$

Optimal. Leaf size=48

$$\frac{1}{6}x^6 \left(a+b \tanh^{-1}\left(cx^2\right)\right)+\frac{b \log \left(1-c^2x^4\right)}{12c^3}+\frac{bx^4}{12c}$$

```
[Out] (b*x^4)/(12*c) + (x^6*(a + b*ArcTanh[c*x^2]))/6 + (b*Log[1 - c^2*x^4])/(12*
c^3)
```

Rubi [A] time = 0.0347672, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {6097, 266, 43}

$$\frac{1}{6}x^6 \left(a+b \tanh^{-1}\left(cx^2\right)\right)+\frac{b \log \left(1-c^2x^4\right)}{12c^3}+\frac{bx^4}{12c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*ArcTanh[c*x^2]),x]
```

```
[Out] (b*x^4)/(12*c) + (x^6*(a + b*ArcTanh[c*x^2]))/6 + (b*Log[1 - c^2*x^4])/(12*
c^3)
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x^5 \left(a+b \tanh^{-1}\left(cx^2\right)\right) dx &= \frac{1}{6}x^6 \left(a+b \tanh^{-1}\left(cx^2\right)\right)-\frac{1}{3}(bc) \int \frac{x^7}{1-c^2x^4} dx \\
&= \frac{1}{6}x^6 \left(a+b \tanh^{-1}\left(cx^2\right)\right)-\frac{1}{12}(bc) \text{Subst}\left(\int \frac{x}{1-c^2x} dx, x, x^4\right) \\
&= \frac{1}{6}x^6 \left(a+b \tanh^{-1}\left(cx^2\right)\right)-\frac{1}{12}(bc) \text{Subst}\left(\int \left(-\frac{1}{c^2}-\frac{1}{c^2\left(-1+c^2x\right)}\right) dx, x, x^4\right) \\
&= \frac{bx^4}{12c}+\frac{1}{6}x^6 \left(a+b \tanh^{-1}\left(cx^2\right)\right)+\frac{b \log \left(1-c^2x^4\right)}{12c^3}
\end{aligned}$$

Mathematica [A] time = 0.0139929, size = 53, normalized size = 1.1

$$\frac{ax^6}{6} + \frac{b\log\left(1-c^2x^4\right)}{12c^3} + \frac{bx^4}{12c} + \frac{1}{6}bx^6\tanh^{-1}\left(cx^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*ArcTanh[c*x^2]),x]
```

```
[Out] (b*x^4)/(12*c) + (a*x^6)/6 + (b*x^6*ArcTanh[c*x^2])/6 + (b*Log[1 - c^2*x^4]
)/(12*c^3)
```

Maple [A] time = 0.009, size = 45, normalized size = 0.9

$$\frac{x^6a}{6} + \frac{bx^6\mathrm{Artanh}\left(cx^2\right)}{6} + \frac{bx^4}{12\,c} + \frac{b\ln\left(c^2x^4-1\right)}{12\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arctanh(c*x^2)),x)
```

```
[Out] 1/6*x^6*a+1/6*b*x^6*arctanh(c*x^2)+1/12*b*x^4/c+1/12*b/c^3*ln(c^2*x^4-1)
```

Maxima [A] time = 0.98235, size = 62, normalized size = 1.29

$$\frac{1}{6}\,ax^6 + \frac{1}{12}\left(2\,x^6\,\mathrm{artanh}\left(cx^2\right) + \left(\frac{x^4}{c^2} + \frac{\log\left(c^2x^4-1\right)}{c^4}\right)c\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x^2)),x, algorithm="maxima")
```

```
[Out] 1/6*a*x^6 + 1/12*(2*x^6*arctanh(c*x^2) + (x^4/c^2 + log(c^2*x^4 - 1)/c^4)*c
)*b
```

Fricas [A] time = 2.05896, size = 134, normalized size = 2.79

$$\frac{bc^3x^6\log\left(-\frac{cx^2+1}{cx^2-1}\right) + 2\,ac^3x^6 + bc^2x^4 + b\log\left(c^2x^4-1\right)}{12\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x^2)),x, algorithm="fricas")
```

```
[Out] 1/12*(b*c^3*x^6*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 2*a*c^3*x^6 + b*c^2*x^4 + b
*log(c^2*x^4 - 1))/c^3
```

Sympy [A] time = 21.0352, size = 85, normalized size = 1.77

$$\begin{cases} \frac{ax^6}{6} + \frac{bx^6 \operatorname{atanh}(cx^2)}{6} + \frac{bx^4}{12c} + \frac{b\log\left(x-i\sqrt{\frac{1}{c}}\right)}{6c^3} + \frac{b\log\left(x+i\sqrt{\frac{1}{c}}\right)}{6c^3} - \frac{b\operatorname{atanh}(cx^2)}{6c^3} & \text{for } c \neq 0 \\ \frac{ax^6}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*atanh(c*x**2)),x)
```

```
[Out] Piecewise((a*x**6/6 + b*x**6*atanh(c*x**2)/6 + b*x**4/(12*c) + b*log(x - I*
sqrt(1/c))/(6*c**3) + b*log(x + I*sqrt(1/c))/(6*c**3) - b*atanh(c*x**2)/(6*
c**3), Ne(c, 0)), (a*x**6/6, True))
```

Giac [A] time = 1.17006, size = 77, normalized size = 1.6

$$\frac{1}{12}\,bx^6\log\left(-\frac{cx^2+1}{cx^2-1}\right) + \frac{1}{6}\,ax^6 + \frac{bx^4}{12\,c} + \frac{b\log\left(c^2x^4-1\right)}{12\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x^2)),x, algorithm="giac")
```

```
[Out] 1/12*b*x^6*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 1/6*a*x^6 + 1/12*b*x^4/c + 1/12*
b*log(c^2*x^4 - 1)/c^3
```

3.52 $\int x^3 \left(a+b\tanh^{-1}\left(cx^2\right)\right) dx$

Optimal. Leaf size=43

$$\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(cx^2\right)\right)-\frac{b\tanh^{-1}\left(cx^2\right)}{4c^2}+\frac{bx^2}{4c}$$

```
[Out] (b*x^2)/(4*c) - (b*ArcTanh[c*x^2])/(4*c^2) + (x^4*(a + b*ArcTanh[c*x^2]))/4
```

Rubi [A] time = 0.0297085, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6097, 275, 321, 206}

$$\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(cx^2\right)\right)-\frac{b\tanh^{-1}\left(cx^2\right)}{4c^2}+\frac{bx^2}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcTanh[c*x^2]),x]
```

```
[Out] (b*x^2)/(4*c) - (b*ArcTanh[c*x^2])/(4*c^2) + (x^4*(a + b*ArcTanh[c*x^2]))/4
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^3\left(a+b\tanh^{-1}\left(cx^2\right)\right)dx=\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(cx^2\right)\right)-\frac{1}{2}(bc)\int\frac{x^5}{1-c^2x^4}dx$$
$$=\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(cx^2\right)\right)-\frac{1}{4}(bc)\,\text{Subst}\left(\int\frac{x^2}{1-c^2x^2}dx,x,x^2\right)$$
$$=\frac{bx^2}{4c}+\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(cx^2\right)\right)-\frac{b\,\text{Subst}\left(\int\frac{1}{1-c^2x^2}dx,x,x^2\right)}{4c}$$
$$=\frac{bx^2}{4c}-\frac{b\tanh^{-1}\left(cx^2\right)}{4c^2}+\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(cx^2\right)\right)$$

Mathematica [A] time = 0.0129561, size = 67, normalized size = 1.56

$$\frac{ax^4}{4}+\frac{b\log\left(1-cx^2\right)}{8c^2}-\frac{b\log\left(cx^2+1\right)}{8c^2}+\frac{bx^2}{4c}+\frac{1}{4}bx^4\tanh^{-1}\left(cx^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcTanh[c*x^2]),x]
```

```
[Out] (b*x^2)/(4*c) + (a*x^4)/4 + (b*x^4*ArcTanh[c*x^2])/4 + (b*Log[1 - c*x^2])/(
8*c^2) - (b*Log[1 + c*x^2])/(8*c^2)
```

Maple [A] time = 0.012, size = 57, normalized size = 1.3

$$\frac{x^4a}{4}+\frac{bx^4\text{Artanh}\left(cx^2\right)}{4}+\frac{bx^2}{4c}+\frac{b\ln\left(cx^2-1\right)}{8c^2}-\frac{b\ln\left(cx^2+1\right)}{8c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c*x^2)),x)
```

```
[Out] 1/4*x^4*a+1/4*b*x^4*arctanh(c*x^2)+1/4*b*x^2/c+1/8*b/c^2*ln(c*x^2-1)-1/8*b/
c^2*ln(c*x^2+1)
```

Maxima [A] time = 0.955376, size = 78, normalized size = 1.81

$$\frac{1}{4}ax^4+\frac{1}{8}\left(2x^4\,\text{artanh}\left(cx^2\right)+c\left(\frac{2x^2}{c^2}-\frac{\log\left(cx^2+1\right)}{c^3}+\frac{\log\left(cx^2-1\right)}{c^3}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^2)),x, algorithm="maxima")
```

```
[Out] 1/4*a*x^4 + 1/8*(2*x^4*arctanh(c*x^2) + c*(2*x^2/c^2 - log(c*x^2 + 1)/c^3 +
 log(c*x^2 - 1)/c^3))*b
```

Fricas [A] time = 2.02008, size = 112, normalized size = 2.6

$$\frac{2ac^2x^4+2bcx^2+\left(bc^2x^4-b\right)\log\left(-\frac{cx^2+1}{cx^2-1}\right)}{8c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^2)),x, algorithm="fricas")
```

```
[Out] 1/8*(2*a*c^2*x^4 + 2*b*c*x^2 + (b*c^2*x^4 - b)*log(-(c*x^2 + 1)/(c*x^2 - 1)
))/c^2
```

Sympy [A] time = 14.0799, size = 48, normalized size = 1.12

$$\begin{cases} \frac{ax^4}{4} + \frac{bx^4 \operatorname{atanh}\left(cx^2\right)}{4} + \frac{bx^2}{4c} - \frac{b \operatorname{atanh}\left(cx^2\right)}{4c^2} & \text{for } c \neq 0 \\ \frac{ax^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c*x**2)),x)
```

```
[Out] Piecewise((a*x**4/4 + b*x**4*atanh(c*x**2)/4 + b*x**2/(4*c) - b*atanh(c*x**
2)/(4*c**2), Ne(c, 0)), (a*x**4/4, True))
```

Giac [A] time = 1.18379, size = 93, normalized size = 2.16

$$\frac{1}{8} bx^4 \log\left(-\frac{cx^2+1}{cx^2-1}\right) + \frac{1}{4} ax^4 + \frac{bx^2}{4c} - \frac{b \log\left(cx^2+1\right)}{8c^2} + \frac{b \log\left(cx^2-1\right)}{8c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^2)),x, algorithm="giac")
```

```
[Out] 1/8*b*x^4*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 1/4*a*x^4 + 1/4*b*x^2/c - 1/8*b*l
og(c*x^2 + 1)/c^2 + 1/8*b*log(c*x^2 - 1)/c^2
```

3.53 $\int x\left(a+b\tanh^{-1}\left(cx^2\right)\right)\,dx$

Optimal. Leaf size=37

$$\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(cx^2\right)\right)+\frac{b\log\left(1-c^2x^4\right)}{4c}$$

```
[Out] (x^2*(a + b*ArcTanh[c*x^2]))/2 + (b*Log[1 - c^2*x^4])/(4*c)
```

Rubi [A] time = 0.0149314, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6097, 260}

$$\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(cx^2\right)\right)+\frac{b\log\left(1-c^2x^4\right)}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcTanh[c*x^2]),x]
```

```
[Out] (x^2*(a + b*ArcTanh[c*x^2]))/2 + (b*Log[1 - c^2*x^4])/(4*c)
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int x\left(a+b\tanh^{-1}\left(cx^2\right)\right)\,dx&=\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(cx^2\right)\right)-(bc)\int\frac{x^3}{1-c^2x^4}\,dx\\&=\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(cx^2\right)\right)+\frac{b\log\left(1-c^2x^4\right)}{4c}\end{aligned}$$

Mathematica [A] time = 0.0069701, size = 42, normalized size = 1.14

$$\frac{ax^2}{2}+\frac{b\log\left(1-c^2x^4\right)}{4c}+\frac{1}{2}bx^2\tanh^{-1}\left(cx^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcTanh[c*x^2]),x]
```

```
[Out] (a*x^2)/2 + (b*x^2*ArcTanh[c*x^2])/2 + (b*Log[1 - c^2*x^4])/(4*c)
```

Maple [A] time = 0.003, size = 37, normalized size = 1.

$$\frac{ax^2}{2}+\frac{bx^2\mathrm{Artanh}\left(cx^2\right)}{2}+\frac{b\ln\left(-c^2x^4+1\right)}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c*x^2)),x)
```

```
[Out] 1/2*a*x^2+1/2*b*x^2*arctanh(c*x^2)+1/4*b*ln(-c^2*x^4+1)/c
```

Maxima [A] time = 0.955621, size = 50, normalized size = 1.35

$$\frac{1}{2}ax^2+\frac{\left(2\,cx^2\,\mathrm{artanh}\left(cx^2\right)+\log\left(-c^2x^4+1\right)\right)b}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^2)),x, algorithm="maxima")
```

```
[Out] 1/2*a*x^2 + 1/4*(2*c*x^2*arctanh(c*x^2) + log(-c^2*x^4 + 1))*b/c
```

Fricas [A] time = 2.06531, size = 108, normalized size = 2.92

$$\frac{bcx^2\log\left(-\frac{cx^2+1}{cx^2-1}\right)+2\,acx^2+b\log\left(c^2x^4-1\right)}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^2)),x, algorithm="fricas")
```

```
[Out] 1/4*(b*c*x^2*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 2*a*c*x^2 + b*log(c^2*x^4 - 1)
)/c
```

Sympy [A] time = 10.0682, size = 71, normalized size = 1.92

$$\begin{cases}\frac{ax^2}{2}+\frac{bx^2\,\mathrm{atanh}\left(cx^2\right)}{2}+\frac{b\log\left(x-i\sqrt{\frac{1}{c}}\right)}{2c}+\frac{b\log\left(x+i\sqrt{\frac{1}{c}}\right)}{2c}-\frac{b\,\mathrm{atanh}\left(cx^2\right)}{2c} & \text{for}\;c\neq 0\\ \frac{ax^2}{2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c*x**2)),x)
```

```
[Out] Piecewise((a*x**2/2 + b*x**2*atanh(c*x**2)/2 + b*log(x - I*sqrt(1/c))/(2*c)
 + b*log(x + I*sqrt(1/c))/(2*c) - b*atanh(c*x**2)/(2*c), Ne(c, 0)), (a*x**2
/2, True))
```

Giac [A] time = 1.18776, size = 66, normalized size = 1.78

$$\frac{1}{2}ax^2 + \frac{1}{4}\left(x^2 \log\left(-\frac{cx^2+1}{cx^2-1}\right) + \frac{\log\left(\left|c^2x^4-1\right|\right)}{c}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^2)),x, algorithm="giac")
```

```
[Out] 1/2*a*x^2 + 1/4*(x^2*log(-(c*x^2 + 1)/(c*x^2 - 1)) + log(abs(c^2*x^4 - 1))/
c)*b
```

3.54 $\int \frac{a+b\tanh^{-1}(cx^2)}{x}\,dx$

Optimal. Leaf size=30

$$-\frac{1}{4}b\text{PolyLog}\left(2,-cx^2\right)+\frac{1}{4}b\text{PolyLog}\left(2,cx^2\right)+a\log(x)$$

```
[Out] a*Log[x] - (b*PolyLog[2, -(c*x^2)])/4 + (b*PolyLog[2, c*x^2])/4
```

Rubi [A] time = 0.0331072, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6095, 5912}

$$-\frac{1}{4}b\text{PolyLog}\left(2,-cx^2\right)+\frac{1}{4}b\text{PolyLog}\left(2,cx^2\right)+a\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^2])/x,x]
```

```
[Out] a*Log[x] - (b*PolyLog[2, -(c*x^2)])/4 + (b*PolyLog[2, c*x^2])/4
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[
1/n, Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
 n}, x] && IGtQ[p, 0]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}\left(cx^2\right)}{x}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx,x,x^2\right)\\ &= a\log(x)-\frac{1}{4}b\text{Li}_2\left(-cx^2\right)+\frac{1}{4}b\text{Li}_2\left(cx^2\right)\end{aligned}$$

Mathematica [A] time = 0.0133833, size = 28, normalized size = 0.93

$$\frac{1}{4}b\left(\text{PolyLog}\left(2,cx^2\right)-\text{PolyLog}\left(2,-cx^2\right)\right)+a\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])/x,x]
```

```
[Out] a*Log[x] + (b*(-PolyLog[2, -(c*x^2)] + PolyLog[2, c*x^2]))/4
```

Maple [B] time = 0.029, size = 124, normalized size = 4.1

$$a\ln(x) + b\ln(x)\operatorname{Artanh}\left(cx^2\right) - \frac{b\ln(x)}{2}\ln\left(1 + x\sqrt{-c}\right) - \frac{b\ln(x)}{2}\ln\left(1 - x\sqrt{-c}\right) - \frac{b}{2}\operatorname{dilog}\left(1 + x\sqrt{-c}\right) - \frac{b}{2}\operatorname{dilog}\left(1 - x\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))/x,x)
```

```
[Out] a*ln(x)+b*ln(x)*arctanh(c*x^2)-1/2*b*ln(x)*ln(1+x*(-c)^(1/2))-1/2*b*ln(x)*l
n(1-x*(-c)^(1/2))-1/2*b*dilog(1+x*(-c)^(1/2))-1/2*b*dilog(1-x*(-c)^(1/2))+1
/2*b*ln(x)*ln(1-x*c^(1/2))+1/2*b*ln(x)*ln(1+x*c^(1/2))+1/2*b*dilog(1-x*c^(1
/2))+1/2*b*dilog(1+x*c^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}b\int\frac{\log\left(cx^2+1\right)-\log\left(-cx^2+1\right)}{x}\,dx + a\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/x,x, algorithm="maxima")
```

```
[Out] 1/2*b*integrate((log(c*x^2 + 1) - log(-c*x^2 + 1))/x, x) + a*log(x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{b\operatorname{artanh}\left(cx^2\right)+a}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/x,x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*x^2) + a)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{a+b\operatorname{atanh}\left(cx^2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))/x,x)
```

```
[Out] Integral((a + b*atanh(c*x**2))/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{b\operatorname{artanh}\left(cx^2\right)+a}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^2) + a)/x, x)
```

3.55 $\int \frac{a+b\tanh^{-1}(cx^2)}{x^3}\,dx$

Optimal. Leaf size=40

$$-\frac{a+b\tanh^{-1}\left(cx^2\right)}{2x^2}-\frac{1}{4}bc\log\left(1-c^2x^4\right)+bc\log(x)$$

```
[Out] -(a + b*ArcTanh[c*x^2])/(2*x^2) + b*c*Log[x] - (b*c*Log[1 - c^2*x^4])/4
```

Rubi [A] time = 0.0255877, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6097, 266, 36, 29, 31}

$$-\frac{a+b\tanh^{-1}\left(cx^2\right)}{2x^2}-\frac{1}{4}bc\log\left(1-c^2x^4\right)+bc\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^2])/x^3,x]
```

```
[Out] -(a + b*ArcTanh[c*x^2])/(2*x^2) + b*c*Log[x] - (b*c*Log[1 - c^2*x^4])/4
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}\left(cx^2\right)}{x^3}\,dx = -\frac{a+b\tanh^{-1}\left(cx^2\right)}{2x^2} + (bc)\int \frac{1}{x\left(1-c^2x^4\right)}\,dx$$
$$= -\frac{a+b\tanh^{-1}\left(cx^2\right)}{2x^2} + \frac{1}{4}(bc)\,\text{Subst}\left(\int \frac{1}{x\left(1-c^2x\right)}\,dx, x, x^4\right)$$
$$= -\frac{a+b\tanh^{-1}\left(cx^2\right)}{2x^2} + \frac{1}{4}(bc)\,\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^4\right) + \frac{1}{4}\left(bc^3\right)\text{Subst}\left(\int \frac{1}{1-c^2x}\,dx, x, x^4\right)$$
$$= -\frac{a+b\tanh^{-1}\left(cx^2\right)}{2x^2} + bc\log(x) - \frac{1}{4}bc\log\left(1-c^2x^4\right)$$

Mathematica [A] time = 0.0110482, size = 45, normalized size = 1.12

$$-\frac{a}{2x^2} - \frac{1}{4}bc\log\left(1-c^2x^4\right) - \frac{b\tanh^{-1}\left(cx^2\right)}{2x^2} + bc\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])/x^3,x]
```

```
[Out] -a/(2*x^2) - (b*ArcTanh[c*x^2])/(2*x^2) + b*c*Log[x] - (b*c*Log[1 - c^2*x^4
])/4
```

Maple [A] time = 0.013, size = 49, normalized size = 1.2

$$-\frac{a}{2\,x^2} - \frac{b\text{Artanh}\left(cx^2\right)}{2\,x^2} - \frac{bc\ln\left(cx^2+1\right)}{4} + bc\ln(x) - \frac{bc\ln\left(cx^2-1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))/x^3,x)
```

```
[Out] -1/2*a/x^2-1/2*b/x^2*arctanh(c*x^2)-1/4*b*c*ln(c*x^2+1)+b*c*ln(x)-1/4*b*c*l
n(c*x^2-1)
```

Maxima [A] time = 0.980565, size = 55, normalized size = 1.38

$$-\frac{1}{4}\left(c\left(\log\left(c^2x^4-1\right) - \log\left(x^4\right)\right) + \frac{2\,\text{artanh}\left(cx^2\right)}{x^2}\right)b - \frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/x^3,x, algorithm="maxima")
```

```
[Out] -1/4*(c*(log(c^2*x^4 - 1) - log(x^4)) + 2*arctanh(c*x^2)/x^2)*b - 1/2*a/x^2
```

Fricas [A] time = 2.00197, size = 130, normalized size = 3.25

$$-\frac{bcx^2\log\left(c^2x^4-1\right) - 4\,bcx^2\log(x) + b\log\left(-\frac{cx^2+1}{cx^2-1}\right) + 2\,a}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(b*c*x^2*log(c^2*x^4 - 1) - 4*b*c*x^2*log(x) + b*log(-(c*x^2 + 1)/(c*x
^2 - 1)) + 2*a)/x^2
```

Sympy [A] time = 17.2699, size = 80, normalized size = 2.

$$\begin{cases} -\frac{a}{2x^2} + bc\log(x) - \frac{bc\log\left(x - i\sqrt{\frac{1}{c}}\right)}{2} - \frac{bc\log\left(x + i\sqrt{\frac{1}{c}}\right)}{2} + \frac{bc\,\mathrm{atanh}\left(cx^2\right)}{2} - \frac{b\,\mathrm{atanh}\left(cx^2\right)}{2x^2} & \text{for}\ c \neq 0 \\ -\frac{a}{2x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))/x**3,x)
```

```
[Out] Piecewise((-a/(2*x**2) + b*c*log(x) - b*c*log(x - I*sqrt(1/c))/2 - b*c*log(
x + I*sqrt(1/c))/2 + b*c*atanh(c*x**2)/2 - b*atanh(c*x**2)/(2*x**2), Ne(c,
0)), (-a/(2*x**2), True))
```

Giac [A] time = 1.1719, size = 69, normalized size = 1.72

$$-\frac{1}{4}\,bc\log\left(c^2x^4 - 1\right) + bc\log(x) - \frac{b\log\left(-\frac{cx^2+1}{cx^2-1}\right)}{4\,x^2} - \frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/x^3,x, algorithm="giac")
```

```
[Out] -1/4*b*c*log(c^2*x^4 - 1) + b*c*log(x) - 1/4*b*log(-(c*x^2 + 1)/(c*x^2 - 1)
)/x^2 - 1/2*a/x^2
```

3.56 $\int \frac{a+b\tanh^{-1}(cx^2)}{x^5}\,dx$

Optimal. Leaf size=41

$$-\frac{a+b\tanh^{-1}\left(cx^2\right)}{4x^4}+\frac{1}{4}bc^2\tanh^{-1}\left(cx^2\right)-\frac{bc}{4x^2}$$

```
[Out] -(b*c)/(4*x^2) + (b*c^2*ArcTanh[c*x^2])/4 - (a + b*ArcTanh[c*x^2])/(4*x^4)
```

Rubi [A] time = 0.0266015, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6097, 275, 325, 206}

$$-\frac{a+b\tanh^{-1}\left(cx^2\right)}{4x^4}+\frac{1}{4}bc^2\tanh^{-1}\left(cx^2\right)-\frac{bc}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^2])/x^5,x]
```

```
[Out] -(b*c)/(4*x^2) + (b*c^2*ArcTanh[c*x^2])/4 - (a + b*ArcTanh[c*x^2])/(4*x^4)
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}\left(cx^2\right)}{x^5}\,dx = -\frac{a+b\tanh^{-1}\left(cx^2\right)}{4x^4} + \frac{1}{2}(bc)\int \frac{1}{x^3\left(1-c^2x^4\right)}\,dx$$
$$= -\frac{a+b\tanh^{-1}\left(cx^2\right)}{4x^4} + \frac{1}{4}(bc)\,\text{Subst}\left(\int \frac{1}{x^2\left(1-c^2x^2\right)}\,dx, x, x^2\right)$$
$$= -\frac{bc}{4x^2} - \frac{a+b\tanh^{-1}\left(cx^2\right)}{4x^4} + \frac{1}{4}\left(bc^3\right)\text{Subst}\left(\int \frac{1}{1-c^2x^2}\,dx, x, x^2\right)$$
$$= -\frac{bc}{4x^2} + \frac{1}{4}bc^2\tanh^{-1}\left(cx^2\right) - \frac{a+b\tanh^{-1}\left(cx^2\right)}{4x^4}$$

Mathematica [A] time = 0.0113402, size = 65, normalized size = 1.59

$$-\frac{a}{4x^4} - \frac{1}{8}bc^2\log\left(1-cx^2\right) + \frac{1}{8}bc^2\log\left(cx^2+1\right) - \frac{bc}{4x^2} - \frac{b\tanh^{-1}\left(cx^2\right)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])/x^5,x]
```

```
[Out] -a/(4*x^4) - (b*c)/(4*x^2) - (b*ArcTanh[c*x^2])/(4*x^4) - (b*c^2*Log[1 - c*
x^2])/8 + (b*c^2*Log[1 + c*x^2])/8
```

Maple [A] time = 0.013, size = 55, normalized size = 1.3

$$-\frac{a}{4\,x^4} - \frac{b\text{Artanh}\left(cx^2\right)}{4\,x^4} + \frac{bc^2\ln\left(cx^2+1\right)}{8} - \frac{bc^2\ln\left(cx^2-1\right)}{8} - \frac{bc}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))/x^5,x)
```

```
[Out] -1/4*a/x^4-1/4*b/x^4*arctanh(c*x^2)+1/8*b*c^2*ln(c*x^2+1)-1/8*b*c^2*ln(c*x^
2-1)-1/4*b*c/x^2
```

Maxima [A] time = 0.958714, size = 69, normalized size = 1.68

$$\frac{1}{8}\left(\left(c\log\left(cx^2+1\right) - c\log\left(cx^2-1\right) - \frac{2}{x^2}\right)c - \frac{2\,\text{artanh}\left(cx^2\right)}{x^4}\right)b - \frac{a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/x^5,x, algorithm="maxima")
```

```
[Out] 1/8*((c*log(c*x^2 + 1) - c*log(c*x^2 - 1) - 2/x^2)*c - 2*arctanh(c*x^2)/x^4
)*b - 1/4*a/x^4
```

Fricas [A] time = 1.93919, size = 103, normalized size = 2.51

$$-\frac{2\,bcx^2 - \left(bc^2x^4 - b\right)\log\left(-\frac{cx^2+1}{cx^2-1}\right) + 2\,a}{8\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/x^5,x, algorithm="fricas")
```

```
[Out] -1/8*(2*b*c*x^2 - (b*c^2*x^4 - b)*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 2*a)/x^4
```

Sympy [A] time = 12.4636, size = 41, normalized size = 1.

$$-\frac{a}{4x^4} + \frac{bc^2\operatorname{atanh}\left(cx^2\right)}{4} - \frac{bc}{4x^2} - \frac{b\operatorname{atanh}\left(cx^2\right)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))/x**5,x)
```

```
[Out] -a/(4*x**4) + b*c**2*atanh(c*x**2)/4 - b*c/(4*x**2) - b*atanh(c*x**2)/(4*x*
*4)
```

Giac [A] time = 1.19515, size = 90, normalized size = 2.2

$$\frac{1}{8}\,bc^2\log\left(cx^2+1\right) - \frac{1}{8}\,bc^2\log\left(cx^2-1\right) - \frac{b\log\left(-\frac{cx^2+1}{cx^2-1}\right)}{8\,x^4} - \frac{bcx^2+a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/x^5,x, algorithm="giac")
```

```
[Out] 1/8*b*c^2*log(c*x^2 + 1) - 1/8*b*c^2*log(c*x^2 - 1) - 1/8*b*log(-(c*x^2 + 1
)/(c*x^2 - 1))/x^4 - 1/4*(b*c*x^2 + a)/x^4
```

3.57 $\int \frac{a+b\tanh^{-1}(cx^2)}{x^7}\,dx$

Optimal. Leaf size=56

$$-\frac{a+b\tanh^{-1}(cx^2)}{6x^6}-\frac{1}{12}bc^3\log\left(1-c^2x^4\right)+\frac{1}{3}bc^3\log(x)-\frac{bc}{12x^4}$$

```
[Out] -(b*c)/(12*x^4) - (a + b*ArcTanh[c*x^2])/(6*x^6) + (b*c^3*Log[x])/3 - (b*c^
3*Log[1 - c^2*x^4])/12
```

Rubi [A] time = 0.0340155, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {6097, 266, 44}

$$-\frac{a+b\tanh^{-1}(cx^2)}{6x^6}-\frac{1}{12}bc^3\log\left(1-c^2x^4\right)+\frac{1}{3}bc^3\log(x)-\frac{bc}{12x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^2])/x^7,x]
```

```
[Out] -(b*c)/(12*x^4) - (a + b*ArcTanh[c*x^2])/(6*x^6) + (b*c^3*Log[x])/3 - (b*c^
3*Log[1 - c^2*x^4])/12
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}(cx^2)}{x^7}\,dx = -\frac{a+b\tanh^{-1}(cx^2)}{6x^6} + \frac{1}{3}(bc)\int \frac{1}{x^5(1-c^2x^4)}\,dx$$
$$= -\frac{a+b\tanh^{-1}(cx^2)}{6x^6} + \frac{1}{12}(bc)\,\text{Subst}\left(\int \frac{1}{x^2(1-c^2x)}\,dx, x, x^4\right)$$
$$= -\frac{a+b\tanh^{-1}(cx^2)}{6x^6} + \frac{1}{12}(bc)\,\text{Subst}\left(\int \left(\frac{1}{x^2}+\frac{c^2}{x}-\frac{c^4}{-1+c^2x}\right)dx, x, x^4\right)$$
$$= -\frac{bc}{12x^4} - \frac{a+b\tanh^{-1}(cx^2)}{6x^6} + \frac{1}{3}bc^3\log(x) - \frac{1}{12}bc^3\log(1-c^2x^4)$$

Mathematica [A] time = 0.0112476, size = 61, normalized size = 1.09

$$-\frac{a}{6x^6} - \frac{1}{12}bc^3\log(1-c^2x^4) + \frac{1}{3}bc^3\log(x) - \frac{bc}{12x^4} - \frac{b\tanh^{-1}(cx^2)}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])/x^7,x]
```

```
[Out] -a/(6*x^6) - (b*c)/(12*x^4) - (b*ArcTanh[c*x^2])/(6*x^6) + (b*c^3*Log[x])/3
 - (b*c^3*Log[1 - c^2*x^4])/12
```

Maple [A] time = 0.015, size = 63, normalized size = 1.1

$$-\frac{a}{6x^6} - \frac{b\text{Artanh}(cx^2)}{6x^6} - \frac{bc^3\ln(cx^2+1)}{12} - \frac{bc}{12x^4} + \frac{bc^3\ln(x)}{3} - \frac{bc^3\ln(cx^2-1)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))/x^7,x)
```

```
[Out] -1/6*a/x^6-1/6*b/x^6*arctanh(c*x^2)-1/12*b*c^3*ln(c*x^2+1)-1/12*b*c/x^4+1/3
*b*c^3*ln(x)-1/12*b*c^3*ln(c*x^2-1)
```

Maxima [A] time = 0.975046, size = 69, normalized size = 1.23

$$-\frac{1}{12}\left(\left(c^2\log(c^2x^4-1) - c^2\log(x^4) + \frac{1}{x^4}\right)c + \frac{2\,\text{artanh}(cx^2)}{x^6}\right)b - \frac{a}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/x^7,x, algorithm="maxima")
```

```
[Out] -1/12*((c^2*log(c^2*x^4 - 1) - c^2*log(x^4) + 1/x^4)*c + 2*arctanh(c*x^2)/x
^6)*b - 1/6*a/x^6
```

Fricas [A] time = 2.11026, size = 150, normalized size = 2.68

$$-\frac{bc^3x^6\log\left(c^2x^4-1\right)-4\,bc^3x^6\log\left(x\right)+bcx^2+b\log\left(-\frac{cx^2+1}{cx^2-1}\right)+2\,a}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/x^7,x, algorithm="fricas")
```

```
[Out] -1/12*(b*c^3*x^6*log(c^2*x^4 - 1) - 4*b*c^3*x^6*log(x) + b*c*x^2 + b*log(-(
c*x^2 + 1)/(c*x^2 - 1)) + 2*a)/x^6
```

Sympy [A] time = 34.4314, size = 97, normalized size = 1.73

$$\begin{cases} -\frac{a}{6x^6}+\frac{bc^3\log\left(x\right)}{3}-\frac{bc^3\log\left(x-i\sqrt{\frac{1}{c}}\right)}{6}-\frac{bc^3\log\left(x+i\sqrt{\frac{1}{c}}\right)}{6}+\frac{bc^3\operatorname{atanh}\left(cx^2\right)}{6}-\frac{bc}{12x^4}-\frac{b\operatorname{atanh}\left(cx^2\right)}{6x^6} & \text{for}\ c\neq 0\\ -\frac{a}{6x^6} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))/x**7,x)
```

```
[Out] Piecewise((-a/(6*x**6) + b*c**3*log(x)/3 - b*c**3*log(x - I*sqrt(1/c))/6 -
b*c**3*log(x + I*sqrt(1/c))/6 + b*c**3*atanh(c*x**2)/6 - b*c/(12*x**4) - b*
atanh(c*x**2)/(6*x**6), Ne(c, 0)), (-a/(6*x**6), True))
```

Giac [A] time = 1.33286, size = 88, normalized size = 1.57

$$-\frac{1}{12}\,bc^3\log\left(c^2x^4-1\right)+\frac{1}{3}\,bc^3\log\left(x\right)-\frac{b\log\left(-\frac{cx^2+1}{cx^2-1}\right)}{12\,x^6}-\frac{bcx^2+2\,a}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/x^7,x, algorithm="giac")
```

```
[Out] -1/12*b*c^3*log(c^2*x^4 - 1) + 1/3*b*c^3*log(x) - 1/12*b*log(-(c*x^2 + 1)/(
c*x^2 - 1))/x^6 - 1/12*(b*c*x^2 + 2*a)/x^6
```

3.58 $\int x^4 \left(a+b\tanh^{-1}\left(cx^2\right)\right)\,dx$

Optimal. Leaf size=65

$$\frac{1}{5}x^5\left(a+b\tanh^{-1}\left(cx^2\right)\right)+\frac{b\tan^{-1}\left(\sqrt{c}x\right)}{5c^{5/2}}-\frac{b\tanh^{-1}\left(\sqrt{c}x\right)}{5c^{5/2}}+\frac{2bx^3}{15c}$$

```
[Out] (2*b*x^3)/(15*c) + (b*ArcTan[Sqrt[c]*x])/(5*c^(5/2)) - (b*ArcTanh[Sqrt[c]*x
])/(5*c^(5/2)) + (x^5*(a + b*ArcTanh[c*x^2]))/5
```

Rubi [A] time = 0.0352452, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6097, 321, 298, 203, 206}

$$\frac{1}{5}x^5\left(a+b\tanh^{-1}\left(cx^2\right)\right)+\frac{b\tan^{-1}\left(\sqrt{c}x\right)}{5c^{5/2}}-\frac{b\tanh^{-1}\left(\sqrt{c}x\right)}{5c^{5/2}}+\frac{2bx^3}{15c}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*ArcTanh[c*x^2]),x]
```

```
[Out] (2*b*x^3)/(15*c) + (b*ArcTan[Sqrt[c]*x])/(5*c^(5/2)) - (b*ArcTanh[Sqrt[c]*x
])/(5*c^(5/2)) + (x^5*(a + b*ArcTanh[c*x^2]))/5
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int x^4\left(a+b\tanh^{-1}\left(cx^2\right)\right)dx &= \frac{1}{5}x^5\left(a+b\tanh^{-1}\left(cx^2\right)\right)-\frac{1}{5}(2bc)\int\frac{x^6}{1-c^2x^4}\,dx\\
&= \frac{2bx^3}{15c}+\frac{1}{5}x^5\left(a+b\tanh^{-1}\left(cx^2\right)\right)-\frac{(2b)\int\frac{x^2}{1-c^2x^4}\,dx}{5c}\\
&= \frac{2bx^3}{15c}+\frac{1}{5}x^5\left(a+b\tanh^{-1}\left(cx^2\right)\right)-\frac{b\int\frac{1}{1-cx^2}\,dx}{5c^2}+\frac{b\int\frac{1}{1+cx^2}\,dx}{5c^2}\\
&= \frac{2bx^3}{15c}+\frac{b\tan^{-1}\left(\sqrt{c}x\right)}{5c^{5/2}}-\frac{b\tanh^{-1}\left(\sqrt{c}x\right)}{5c^{5/2}}+\frac{1}{5}x^5\left(a+b\tanh^{-1}\left(cx^2\right)\right)
\end{aligned}
$$

Mathematica [A] time = 0.0239303, size = 93, normalized size = 1.43

$$\frac{ax^5}{5}+\frac{b\log\left(1-\sqrt{c}x\right)}{10c^{5/2}}-\frac{b\log\left(\sqrt{c}x+1\right)}{10c^{5/2}}+\frac{b\tan^{-1}\left(\sqrt{c}x\right)}{5c^{5/2}}+\frac{2bx^3}{15c}+\frac{1}{5}bx^5\tanh^{-1}\left(cx^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*ArcTanh[c*x^2]),x]
```

```
[Out] (2*b*x^3)/(15*c) + (a*x^5)/5 + (b*ArcTan[Sqrt[c]*x])/(5*c^(5/2)) + (b*x^5*A
rcTanh[c*x^2])/5 + (b*Log[1 - Sqrt[c]*x])/(10*c^(5/2)) - (b*Log[1 + Sqrt[c]
*x])/(10*c^(5/2))
```

Maple [A] time = 0.011, size = 53, normalized size = 0.8

$$\frac{ax^5}{5}+\frac{x^5b\,\mathrm{Artanh}\left(cx^2\right)}{5}+\frac{2\,bx^3}{15\,c}+\frac{b}{5}\arctan\left(x\sqrt{c}\right)c^{-\frac{5}{2}}-\frac{b}{5}\mathrm{Artanh}\left(x\sqrt{c}\right)c^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arctanh(c*x^2)),x)
```

```
[Out] 1/5*a*x^5+1/5*x^5*b*arctanh(c*x^2)+2/15*b*x^3/c+1/5*b*arctan(x*c^(1/2))/c^(
5/2)-1/5*b*arctanh(x*c^(1/2))/c^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x^2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.00913, size = 473, normalized size = 7.28

$$\left[\frac{3\,bc^3x^5\log\left(-\frac{cx^2+1}{cx^2-1}\right)+6\,ac^3x^5+4\,bc^2x^3+6\,b\sqrt{c}\arctan\left(\sqrt{c}x\right)+3\,b\sqrt{c}\log\left(\frac{cx^2-2\sqrt{c}x+1}{cx^2-1}\right)}{30\,c^3},\frac{3\,bc^3x^5\log\left(-\frac{cx^2+1}{cx^2-1}\right)+6\,ac^3}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x^2)),x, algorithm="fricas")
```

```
[Out] [1/30*(3*b*c^3*x^5*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 6*a*c^3*x^5 + 4*b*c^2*x^
3 + 6*b*sqrt(c)*arctan(sqrt(c)*x) + 3*b*sqrt(c)*log((c*x^2 - 2*sqrt(c)*x +
1)/(c*x^2 - 1)))/c^3, 1/30*(3*b*c^3*x^5*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 6*a
*c^3*x^5 + 4*b*c^2*x^3 + 6*b*sqrt(-c)*arctan(sqrt(-c)*x) - 3*b*sqrt(-c)*log
((c*x^2 - 2*sqrt(-c)*x - 1)/(c*x^2 + 1)))/c^3]
```

Sympy [A] time = 19.3632, size = 185, normalized size = 2.85

$$\begin{cases}\frac{ax^5}{5}+\frac{bx^5\operatorname{atanh}\left(cx^2\right)}{5}+\frac{2bx^3}{15c}-\frac{b\log\left(x-i\sqrt{\frac{1}{c}}\right)}{10c^3\sqrt{\frac{1}{c}}}-\frac{ib\log\left(x-i\sqrt{\frac{1}{c}}\right)}{10c^3\sqrt{\frac{1}{c}}}-\frac{b\log\left(x+i\sqrt{\frac{1}{c}}\right)}{10c^3\sqrt{\frac{1}{c}}}+\frac{ib\log\left(x+i\sqrt{\frac{1}{c}}\right)}{10c^3\sqrt{\frac{1}{c}}}+\frac{b\log\left(x-\sqrt{\frac{1}{c}}\right)}{5c^3\sqrt{\frac{1}{c}}}+\frac{b\operatorname{atanh}\left(cx^2\right)}{5c^3\sqrt{\frac{1}{c}}} & \text{fo} \\ \frac{ax^5}{5} & \text{ot}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*atanh(c*x**2)),x)
```

```
[Out] Piecewise((a*x**5/5 + b*x**5*atanh(c*x**2)/5 + 2*b*x**3/(15*c) - b*log(x -
I*sqrt(1/c))/(10*c**3*sqrt(1/c)) - I*b*log(x - I*sqrt(1/c))/(10*c**3*sqrt(1
/c)) - b*log(x + I*sqrt(1/c))/(10*c**3*sqrt(1/c)) + I*b*log(x + I*sqrt(1/c)
)/(10*c**3*sqrt(1/c)) + b*log(x - sqrt(1/c))/(5*c**3*sqrt(1/c)) + b*atanh(c
*x**2)/(5*c**3*sqrt(1/c)), Ne(c, 0)), (a*x**5/5, True))
```

Giac [A] time = 1.27424, size = 103, normalized size = 1.58

$$\frac{1}{5}\,bc^9\left(\frac{\arctan\left(\sqrt{c}x\right)}{c^{\frac{23}{2}}}+\frac{\arctan\left(\frac{cx}{\sqrt{-c}}\right)}{\sqrt{-c}c^{11}}\right)+\frac{1}{10}\,bx^5\log\left(-\frac{cx^2+1}{cx^2-1}\right)+\frac{1}{5}\,ax^5+\frac{2\,bx^3}{15\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x^2)),x, algorithm="giac")
```

```
[Out] 1/5*b*c^9*(arctan(sqrt(c)*x)/c^(23/2) + arctan(c*x/sqrt(-c))/(sqrt(-c)*c^11
)) + 1/10*b*x^5*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 1/5*a*x^5 + 2/15*b*x^3/c
```

3.59 $\int x^2 \left(a+b\tanh^{-1}\left(cx^2\right)\right) dx$

Optimal. Leaf size=63

$$\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(cx^2\right)\right)-\frac{b\tan^{-1}\left(\sqrt{c}x\right)}{3c^{3/2}}-\frac{b\tanh^{-1}\left(\sqrt{c}x\right)}{3c^{3/2}}+\frac{2bx}{3c}$$

```
[Out] (2*b*x)/(3*c) - (b*ArcTan[Sqrt[c]*x])/(3*c^(3/2)) - (b*ArcTanh[Sqrt[c]*x])/
(3*c^(3/2)) + (x^3*(a + b*ArcTanh[c*x^2]))/3
```

Rubi [A] time = 0.0330533, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6097, 321, 212, 206, 203}

$$\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(cx^2\right)\right)-\frac{b\tan^{-1}\left(\sqrt{c}x\right)}{3c^{3/2}}-\frac{b\tanh^{-1}\left(\sqrt{c}x\right)}{3c^{3/2}}+\frac{2bx}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcTanh[c*x^2]),x]
```

```
[Out] (2*b*x)/(3*c) - (b*ArcTan[Sqrt[c]*x])/(3*c^(3/2)) - (b*ArcTanh[Sqrt[c]*x])/
(3*c^(3/2)) + (x^3*(a + b*ArcTanh[c*x^2]))/3
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+b\tanh^{-1}\left(cx^2\right)\right)dx &= \frac{1}{3}x^3\left(a+b\tanh^{-1}\left(cx^2\right)\right)-\frac{1}{3}(2bc)\int\frac{x^4}{1-c^2x^4}\,dx\\ &= \frac{2bx}{3c}+\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(cx^2\right)\right)-\frac{(2b)\int\frac{1}{1-c^2x^4}\,dx}{3c}\\ &= \frac{2bx}{3c}+\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(cx^2\right)\right)-\frac{b\int\frac{1}{1-cx^2}\,dx}{3c}-\frac{b\int\frac{1}{1+cx^2}\,dx}{3c}\\ &= \frac{2bx}{3c}-\frac{b\tan^{-1}\left(\sqrt{c}x\right)}{3c^{3/2}}-\frac{b\tanh^{-1}\left(\sqrt{c}x\right)}{3c^{3/2}}+\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(cx^2\right)\right)\end{aligned}$$

Mathematica [A] time = 0.0188546, size = 91, normalized size = 1.44

$$\frac{ax^3}{3}+\frac{b\log\left(1-\sqrt{c}x\right)}{6c^{3/2}}-\frac{b\log\left(\sqrt{c}x+1\right)}{6c^{3/2}}-\frac{b\tan^{-1}\left(\sqrt{c}x\right)}{3c^{3/2}}+\frac{1}{3}bx^3\tanh^{-1}\left(cx^2\right)+\frac{2bx}{3c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcTanh[c*x^2]),x]
```

```
[Out] (2*b*x)/(3*c) + (a*x^3)/3 - (b*ArcTan[Sqrt[c]*x])/(3*c^(3/2)) + (b*x^3*ArcT
anh[c*x^2])/3 + (b*Log[1 - Sqrt[c]*x])/(6*c^(3/2)) - (b*Log[1 + Sqrt[c]*x])
/(6*c^(3/2))
```

Maple [A] time = 0.01, size = 51, normalized size = 0.8

$$\frac{x^3a}{3}+\frac{x^3b\text{Artanh}\left(cx^2\right)}{3}+\frac{2\,bx}{3\,c}-\frac{b}{3}\arctan\left(x\sqrt{c}\right)c^{-\frac{3}{2}}-\frac{b}{3}\text{Artanh}\left(x\sqrt{c}\right)c^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c*x^2)),x)
```

```
[Out] 1/3*x^3*a+1/3*x^3*b*arctanh(c*x^2)+2/3*b*x/c-1/3*b*arctan(x*c^(1/2))/c^(3/2
)-1/3*b*arctanh(x*c^(1/2))/c^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.22894, size = 448, normalized size = 7.11

$$\left[\frac{bc^2x^3\log\left(-\frac{cx^2+1}{cx^2-1}\right)+2\,ac^2x^3+4\,bcx-2\,b\sqrt{c}\arctan\left(\sqrt{c}x\right)+b\sqrt{c}\log\left(\frac{cx^2-2\,\sqrt{c}x+1}{cx^2-1}\right)}{6\,c^2},\ \frac{bc^2x^3\log\left(-\frac{cx^2+1}{cx^2-1}\right)+2\,ac^2x^3+4\,bcx\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^2)),x, algorithm="fricas")
```

```
[Out] [1/6*(b*c^2*x^3*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 2*a*c^2*x^3 + 4*b*c*x - 2*b
*sqrt(c)*arctan(sqrt(c)*x) + b*sqrt(c)*log((c*x^2 - 2*sqrt(c)*x + 1)/(c*x^2
 - 1)))/c^2, 1/6*(b*c^2*x^3*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 2*a*c^2*x^3 + 4
*b*c*x + 2*b*sqrt(-c)*arctan(sqrt(-c)*x) - b*sqrt(-c)*log((c*x^2 + 2*sqrt(-
c)*x - 1)/(c*x^2 + 1)))/c^2]
```

Sympy [A] time = 12.9636, size = 581, normalized size = 9.22

$$\begin{cases} -\frac{4ac^2x^3\sqrt{\frac{1}{c}}}{-12c^2\sqrt{\frac{1}{c}}-12ic^2\sqrt{\frac{1}{c}}}-\frac{4iac^2x^3\sqrt{\frac{1}{c}}}{-12c^2\sqrt{\frac{1}{c}}-12ic^2\sqrt{\frac{1}{c}}}-\frac{4bc^2x^3\sqrt{\frac{1}{c}}\operatorname{atanh}\left(cx^2\right)}{-12c^2\sqrt{\frac{1}{c}}-12ic^2\sqrt{\frac{1}{c}}}-\frac{4ibc^2x^3\sqrt{\frac{1}{c}}\operatorname{atanh}\left(cx^2\right)}{-12c^2\sqrt{\frac{1}{c}}-12ic^2\sqrt{\frac{1}{c}}}-\frac{2ibc^2\log\left(x+i\sqrt{\frac{1}{c}}\right)}{-12c^4\sqrt{\frac{1}{c}}-12ic^4\sqrt{\frac{1}{c}}}-\frac{8bcx\sqrt{\frac{1}{c}}}{-12c^2\sqrt{\frac{1}{c}}-12ic^2\sqrt{\frac{1}{c}}}-\cdots \\ \frac{ax^3}{3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c*x**2)),x)
```

```
[Out] Piecewise((-4*a*c**2*x**3*sqrt(1/c)/(-12*c**2*sqrt(1/c) - 12*I*c**2*sqrt(1/
c)) - 4*I*a*c**2*x**3*sqrt(1/c)/(-12*c**2*sqrt(1/c) - 12*I*c**2*sqrt(1/c))
- 4*b*c**2*x**3*sqrt(1/c)*atanh(c*x**2)/(-12*c**2*sqrt(1/c) - 12*I*c**2*sqr
t(1/c)) - 4*I*b*c**2*x**3*sqrt(1/c)*atanh(c*x**2)/(-12*c**2*sqrt(1/c) - 12*
I*c**2*sqrt(1/c)) - 2*I*b*c**2*log(x + I*sqrt(1/c))/(-12*c**4*sqrt(1/c) - 1
2*I*c**4*sqrt(1/c)) - 8*b*c*x*sqrt(1/c)/(-12*c**2*sqrt(1/c) - 12*I*c**2*sqr
t(1/c)) - 8*I*b*c*x*sqrt(1/c)/(-12*c**2*sqrt(1/c) - 12*I*c**2*sqrt(1/c)) +
6*I*b*c*log(x + I*sqrt(1/c))/(-12*c**3*sqrt(1/c) - 12*I*c**3*sqrt(1/c)) - 4
*I*b*c*log(x - sqrt(1/c))/(-12*c**3*sqrt(1/c) - 12*I*c**3*sqrt(1/c)) - 4*I*
b*c*atanh(c*x**2)/(-12*c**3*sqrt(1/c) - 12*I*c**3*sqrt(1/c)) + 4*b*log(x -
I*sqrt(1/c))/(-12*c**2*sqrt(1/c) - 12*I*c**2*sqrt(1/c)) - 4*b*log(x - sqrt(
1/c))/(-12*c**2*sqrt(1/c) - 12*I*c**2*sqrt(1/c)) - 4*b*atanh(c*x**2)/(-12*c
**2*sqrt(1/c) - 12*I*c**2*sqrt(1/c)), Ne(c, 0)), (a*x**3/3, True))
```

Giac [A] time = 1.27659, size = 101, normalized size = 1.6

$$-\frac{1}{3}\,bc^5\left(\frac{\arctan\left(\sqrt{c}x\right)}{c^{\frac{13}{2}}}-\frac{\arctan\left(\frac{cx}{\sqrt{-c}}\right)}{\sqrt{-c}c^6}\right)+\frac{1}{6}\,bx^3\log\left(-\frac{cx^2+1}{cx^2-1}\right)+\frac{1}{3}\,ax^3+\frac{2\,bx}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^2)),x, algorithm="giac")
```

```
[Out] -1/3*b*c^5*(arctan(sqrt(c)*x)/c^(13/2) - arctan(c*x/sqrt(-c))/(sqrt(-c)*c^6
)) + 1/6*b*x^3*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 1/3*a*x^3 + 2/3*b*x/c
```

3.60 $\int \left(a+b\tanh^{-1}\left(cx^2\right)\right)dx$

Optimal. Leaf size=44

$$ax+bx\tanh^{-1}\left(cx^2\right)+\frac{b\tan^{-1}\left(\sqrt{c}x\right)}{\sqrt{c}}-\frac{b\tanh^{-1}\left(\sqrt{c}x\right)}{\sqrt{c}}$$

```
[Out] a*x + (b*ArcTan[Sqrt[c]*x])/Sqrt[c] - (b*ArcTanh[Sqrt[c]*x])/Sqrt[c] + b*x*
ArcTanh[c*x^2]
```

Rubi [A] time = 0.0244469, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {6091, 298, 203, 206}

$$ax+bx\tanh^{-1}\left(cx^2\right)+\frac{b\tan^{-1}\left(\sqrt{c}x\right)}{\sqrt{c}}-\frac{b\tanh^{-1}\left(\sqrt{c}x\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcTanh[c*x^2],x]
```

```
[Out] a*x + (b*ArcTan[Sqrt[c]*x])/Sqrt[c] - (b*ArcTanh[Sqrt[c]*x])/Sqrt[c] + b*x*
ArcTanh[c*x^2]
```

Rule 6091

```
Int[ArcTanh[(c_.)*(x_)^(n_)], x_Symbol] :> Simp[x*ArcTanh[c*x^n], x] - Dist
[c*n, Int[x^n/(1 - c^2*x^(2*n)), x], x] /; FreeQ[{c, n}, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(a+b\tanh^{-1}\left(cx^2\right)\right)dx = ax+b\int \tanh^{-1}\left(cx^2\right)dx$$
$$= ax+bx\tanh^{-1}\left(cx^2\right)-(2bc)\int \frac{x^2}{1-c^2x^4}\,dx$$
$$= ax+bx\tanh^{-1}\left(cx^2\right)-b\int \frac{1}{1-cx^2}\,dx+b\int \frac{1}{1+cx^2}\,dx$$
$$= ax+\frac{b\tan^{-1}\left(\sqrt{c}x\right)}{\sqrt{c}}-\frac{b\tanh^{-1}\left(\sqrt{c}x\right)}{\sqrt{c}}+bx\tanh^{-1}\left(cx^2\right)$$

Mathematica [A] time = 0.0196384, size = 57, normalized size = 1.3

$$ax+bx\tanh^{-1}\left(cx^2\right)+\frac{b\left(\log\left(1-\sqrt{c}x\right)-\log\left(\sqrt{c}x+1\right)+2\tan^{-1}\left(\sqrt{c}x\right)\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcTanh[c*x^2],x]
```

```
[Out] a*x + b*x*ArcTanh[c*x^2] + (b*(2*ArcTan[Sqrt[c]*x] + Log[1 - Sqrt[c]*x] - L
og[1 + Sqrt[c]*x]))/(2*Sqrt[c])
```

Maple [A] time = 0.007, size = 37, normalized size = 0.8

$$ax+bx\mathrm{Artanh}\left(cx^2\right)+b\arctan\left(x\sqrt{c}\right)\frac{1}{\sqrt{c}}-b\mathrm{Artanh}\left(x\sqrt{c}\right)\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arctanh(c*x^2),x)
```

```
[Out] a*x+b*x*arctanh(c*x^2)+b*arctan(x*c^(1/2))/c^(1/2)-b*arctanh(x*c^(1/2))/c^(
1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.07371, size = 394, normalized size = 8.95

$$\left[\frac{bcx\log\left(-\frac{cx^2+1}{cx^2-1}\right)+2\,acx+2\,b\sqrt{c}\arctan\left(\sqrt{c}x\right)+b\sqrt{c}\log\left(\frac{cx^2-2\,\sqrt{c}x+1}{cx^2-1}\right)}{2\,c}, \frac{bcx\log\left(-\frac{cx^2+1}{cx^2-1}\right)+2\,acx+2\,b\sqrt{-c}\arctan\left(\sqrt{-c}\right.}{2\,c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c*x^2),x, algorithm="fricas")
```

```
[Out] [1/2*(b*c*x*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 2*a*c*x + 2*b*sqrt(c)*arctan(sq
rt(c)*x) + b*sqrt(c)*log((c*x^2 - 2*sqrt(c)*x + 1)/(c*x^2 - 1)))/c, 1/2*(b*
c*x*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 2*a*c*x + 2*b*sqrt(-c)*arctan(sqrt(-c)*
x) - b*sqrt(-c)*log((c*x^2 - 2*sqrt(-c)*x - 1)/(c*x^2 + 1)))/c]
```

Sympy [A] time = 8.39401, size = 139, normalized size = 3.16

$$ax + b\left(\begin{cases} x\operatorname{atanh}\left(cx^2\right) - \frac{\log\left(x - i\sqrt{\frac{1}{c}}\right)}{2c\sqrt{\frac{1}{c}}} - \frac{i\log\left(x - i\sqrt{\frac{1}{c}}\right)}{2c\sqrt{\frac{1}{c}}} - \frac{\log\left(x + i\sqrt{\frac{1}{c}}\right)}{2c\sqrt{\frac{1}{c}}} + \frac{i\log\left(x + i\sqrt{\frac{1}{c}}\right)}{2c\sqrt{\frac{1}{c}}} + \frac{\log\left(x - \sqrt{\frac{1}{c}}\right)}{c\sqrt{\frac{1}{c}}} + \frac{\operatorname{atanh}\left(cx^2\right)}{c\sqrt{\frac{1}{c}}} & \text{for } c \neq 0 \\ 0 & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*atanh(c*x**2),x)
```

```
[Out] a*x + b*Piecewise((x*atanh(c*x**2) - log(x - I*sqrt(1/c))/(2*c*sqrt(1/c)) -
 I*log(x - I*sqrt(1/c))/(2*c*sqrt(1/c)) - log(x + I*sqrt(1/c))/(2*c*sqrt(1/
c)) + I*log(x + I*sqrt(1/c))/(2*c*sqrt(1/c)) + log(x - sqrt(1/c))/(c*sqrt(1
/c)) + atanh(c*x**2)/(c*sqrt(1/c)), Ne(c, 0)), (0, True))
```

Giac [B] time = 1.19516, size = 112, normalized size = 2.55

$$\frac{1}{2}\left(c\left(\frac{2\sqrt{|c|}\arctan\left(x\sqrt{|c|}\right)}{c^2} - \frac{\sqrt{|c|}\log\left(\left|x + \frac{1}{\sqrt{|c|}}\right|\right)}{c^2} + \frac{\sqrt{|c|}\log\left(\left|x - \frac{1}{\sqrt{|c|}}\right|\right)}{c^2}\right) + x\log\left(-\frac{cx^2 + 1}{cx^2 - 1}\right)\right)b + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c*x^2),x, algorithm="giac")
```

```
[Out] 1/2*(c*(2*sqrt(abs(c))*arctan(x*sqrt(abs(c)))/c^2 - sqrt(abs(c))*log(abs(x
+ 1/sqrt(abs(c))))/c^2 + sqrt(abs(c))*log(abs(x - 1/sqrt(abs(c))))/c^2) + x
*log(-(c*x^2 + 1)/(c*x^2 - 1)))*b + a*x
```

3.61 $\int \frac{a+b\tanh^{-1}(cx^2)}{x^2}\,dx$

Optimal. Leaf size=46

$$-\frac{a+b\tanh^{-1}\left(cx^2\right)}{x}+b\sqrt{c}\tan^{-1}\left(\sqrt{c}x\right)+b\sqrt{c}\tanh^{-1}\left(\sqrt{c}x\right)$$

```
[Out] b*Sqrt[c]*ArcTan[Sqrt[c]*x] + b*Sqrt[c]*ArcTanh[Sqrt[c]*x] - (a + b*ArcTanh
[c*x^2])/x
```

Rubi [A] time = 0.0256217, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6097, 212, 206, 203}

$$-\frac{a+b\tanh^{-1}\left(cx^2\right)}{x}+b\sqrt{c}\tan^{-1}\left(\sqrt{c}x\right)+b\sqrt{c}\tanh^{-1}\left(\sqrt{c}x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^2])/x^2,x]
```

```
[Out] b*Sqrt[c]*ArcTan[Sqrt[c]*x] + b*Sqrt[c]*ArcTanh[Sqrt[c]*x] - (a + b*ArcTanh
[c*x^2])/x
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}\left(cx^2\right)}{x^2}\,dx &= -\frac{a+b\tanh^{-1}\left(cx^2\right)}{x}+(2bc)\int \frac{1}{1-c^2x^4}\,dx\\ &= -\frac{a+b\tanh^{-1}\left(cx^2\right)}{x}+(bc)\int \frac{1}{1-cx^2}\,dx+(bc)\int \frac{1}{1+cx^2}\,dx\\ &= b\sqrt{c}\tan^{-1}\left(\sqrt{c}x\right)+b\sqrt{c}\tanh^{-1}\left(\sqrt{c}x\right)-\frac{a+b\tanh^{-1}\left(cx^2\right)}{x}\end{aligned}$$

Mathematica [A] time = 0.0170497, size = 75, normalized size = 1.63

$$-\frac{a}{x}-\frac{b\tanh^{-1}\left(cx^2\right)}{x}-\frac{1}{2}b\sqrt{c}\log\left(1-\sqrt{c}x\right)+\frac{1}{2}b\sqrt{c}\log\left(\sqrt{c}x+1\right)+b\sqrt{c}\tan^{-1}\left(\sqrt{c}x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])/x^2,x]
```

```
[Out] -(a/x) + b*Sqrt[c]*ArcTan[Sqrt[c]*x] - (b*ArcTanh[c*x^2])/x - (b*Sqrt[c]*Lo
g[1 - Sqrt[c]*x])/2 + (b*Sqrt[c]*Log[1 + Sqrt[c]*x])/2
```

Maple [A] time = 0.013, size = 42, normalized size = 0.9

$$-\frac{a}{x}-\frac{b\mathrm{Artanh}\left(cx^2\right)}{x}+b\arctan\left(x\sqrt{c}\right)\sqrt{c}+b\mathrm{Artanh}\left(x\sqrt{c}\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))/x^2,x)
```

```
[Out] -a/x-b/x*arctanh(c*x^2)+b*arctan(x*c^(1/2))*c^(1/2)+b*arctanh(x*c^(1/2))*c^
(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.12528, size = 385, normalized size = 8.37

$$\left[\frac{2\,b\sqrt{c}x\arctan\left(\sqrt{c}x\right)+b\sqrt{c}x\log\left(\frac{cx^2+2\,\sqrt{c}x+1}{cx^2-1}\right)-b\log\left(-\frac{cx^2+1}{cx^2-1}\right)-2\,a}{2\,x},-\frac{2\,b\sqrt{-c}x\arctan\left(\sqrt{-c}x\right)-b\sqrt{-c}x\log\left(\frac{cx^2+2\,\cdots}{cx^2\cdots}\right.\cdots}{2\,x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*b*sqrt(c)*x*arctan(sqrt(c)*x) + b*sqrt(c)*x*log((c*x^2 + 2*sqrt(c)*
x + 1)/(c*x^2 - 1)) - b*log(-(c*x^2 + 1)/(c*x^2 - 1)) - 2*a)/x, -1/2*(2*b*s
qrt(-c)*x*arctan(sqrt(-c)*x) - b*sqrt(-c)*x*log((c*x^2 + 2*sqrt(-c)*x - 1)/
(c*x^2 + 1)) + b*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 2*a)/x]
```

Sympy [A] time = 15.1448, size = 699, normalized size = 15.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))/x**2,x)
```

```
[Out] Piecewise((-(a - oo*b)/x, Eq(c, -1/x**2)), (-(a + oo*b)/x, Eq(c, x**(-2))),
 (-a/x, Eq(c, 0)), (-4*a*x**4/(4*x**5 - 4*x/c**2) + 4*a/(4*c**2*x**5 - 4*x)
 - b*c**2*x**5*(1/c)**(3/2)*log(x + I*sqrt(1/c))/(4*x**5 - 4*x/c**2) - I*b*
c**2*x**5*(1/c)**(3/2)*log(x + I*sqrt(1/c))/(4*x**5 - 4*x/c**2) + 2*b*c*x**
5*sqrt(1/c)*log(x - I*sqrt(1/c))/(4*x**5 - 4*x/c**2) - 2*I*b*c*x**5*sqrt(1/
c)*log(x - I*sqrt(1/c))/(4*x**5 - 4*x/c**2) + 3*b*c*x**5*sqrt(1/c)*log(x +
I*sqrt(1/c))/(4*x**5 - 4*x/c**2) + 3*I*b*c*x**5*sqrt(1/c)*log(x + I*sqrt(1/
c))/(4*x**5 - 4*x/c**2) - 4*b*c*x**5*sqrt(1/c)*log(x - sqrt(1/c))/(4*x**5 -
 4*x/c**2) - 4*b*c*x**5*sqrt(1/c)*atanh(c*x**2)/(4*x**5 - 4*x/c**2) - 4*b*x
**4*atanh(c*x**2)/(4*x**5 - 4*x/c**2) - 2*b*x*sqrt(1/c)*log(x - I*sqrt(1/c)
)/(4*c*x**5 - 4*x/c) + 2*I*b*x*sqrt(1/c)*log(x - I*sqrt(1/c))/(4*c*x**5 - 4
*x/c) - 3*b*x*sqrt(1/c)*log(x + I*sqrt(1/c))/(4*c*x**5 - 4*x/c) - 3*I*b*x*s
qrt(1/c)*log(x + I*sqrt(1/c))/(4*c*x**5 - 4*x/c) + 4*b*x*sqrt(1/c)*log(x -
sqrt(1/c))/(4*c*x**5 - 4*x/c) + 4*b*x*sqrt(1/c)*atanh(c*x**2)/(4*c*x**5 - 4
*x/c) + b*x*(1/c)**(3/2)*log(x + I*sqrt(1/c))/(4*x**5 - 4*x/c**2) + I*b*x*(
1/c)**(3/2)*log(x + I*sqrt(1/c))/(4*x**5 - 4*x/c**2) + 4*b*atanh(c*x**2)/(4
*c**2*x**5 - 4*x), True))
```

Giac [B] time = 1.22457, size = 107, normalized size = 2.33

$$\frac{1}{2}\,bc\left(\frac{2\,\arctan\left(x\sqrt{|c|}\right)}{\sqrt{|c|}}+\frac{\log\left(\left|x+\frac{1}{\sqrt{|c|}}\right|\right)}{\sqrt{|c|}}-\frac{\log\left(\left|x-\frac{1}{\sqrt{|c|}}\right|\right)}{\sqrt{|c|}}\right)-\frac{b\log\left(-\frac{cx^2+1}{cx^2-1}\right)}{2\,x}-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/x^2,x, algorithm="giac")
```

```
[Out] 1/2*b*c*(2*arctan(x*sqrt(abs(c)))/sqrt(abs(c)) + log(abs(x + 1/sqrt(abs(c))
))/sqrt(abs(c)) - log(abs(x - 1/sqrt(abs(c))))/sqrt(abs(c))) - 1/2*b*log(-(
c*x^2 + 1)/(c*x^2 - 1))/x - a/x
```

3.62 $\int \frac{a+b\tanh^{-1}(cx^2)}{x^4}\,dx$

Optimal. Leaf size=63

$$-\frac{a+b\tanh^{-1}\left(cx^2\right)}{3x^3}-\frac{1}{3}bc^{3/2}\tan^{-1}\left(\sqrt{c}x\right)+\frac{1}{3}bc^{3/2}\tanh^{-1}\left(\sqrt{c}x\right)-\frac{2bc}{3x}$$

```
[Out] (-2*b*c)/(3*x) - (b*c^(3/2)*ArcTan[Sqrt[c]*x])/3 + (b*c^(3/2)*ArcTanh[Sqrt[
c]*x])/3 - (a + b*ArcTanh[c*x^2])/(3*x^3)
```

Rubi [A] time = 0.0334463, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6097, 325, 298, 203, 206}

$$-\frac{a+b\tanh^{-1}\left(cx^2\right)}{3x^3}-\frac{1}{3}bc^{3/2}\tan^{-1}\left(\sqrt{c}x\right)+\frac{1}{3}bc^{3/2}\tanh^{-1}\left(\sqrt{c}x\right)-\frac{2bc}{3x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^2])/x^4,x]
```

```
[Out] (-2*b*c)/(3*x) - (b*c^(3/2)*ArcTan[Sqrt[c]*x])/3 + (b*c^(3/2)*ArcTanh[Sqrt[
c]*x])/3 - (a + b*ArcTanh[c*x^2])/(3*x^3)
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\tanh^{-1}\left(cx^2\right)}{x^4}\,dx &= -\frac{a+b\tanh^{-1}\left(cx^2\right)}{3x^3}+\frac{1}{3}(2bc)\int \frac{1}{x^2\left(1-c^2x^4\right)}\,dx\\
&= -\frac{2bc}{3x}-\frac{a+b\tanh^{-1}\left(cx^2\right)}{3x^3}+\frac{1}{3}\left(2bc^3\right)\int \frac{x^2}{1-c^2x^4}\,dx\\
&= -\frac{2bc}{3x}-\frac{a+b\tanh^{-1}\left(cx^2\right)}{3x^3}+\frac{1}{3}\left(bc^2\right)\int \frac{1}{1-cx^2}\,dx-\frac{1}{3}\left(bc^2\right)\int \frac{1}{1+cx^2}\,dx\\
&= -\frac{2bc}{3x}-\frac{1}{3}bc^{3/2}\tan^{-1}\left(\sqrt{c}x\right)+\frac{1}{3}bc^{3/2}\tanh^{-1}\left(\sqrt{c}x\right)-\frac{a+b\tanh^{-1}\left(cx^2\right)}{3x^3}
\end{aligned}
$$

Mathematica [A] time = 0.0273203, size = 91, normalized size = 1.44

$$
-\frac{a}{3x^3}-\frac{1}{6}bc^{3/2}\log\left(1-\sqrt{c}x\right)+\frac{1}{6}bc^{3/2}\log\left(\sqrt{c}x+1\right)-\frac{1}{3}bc^{3/2}\tan^{-1}\left(\sqrt{c}x\right)-\frac{b\tanh^{-1}\left(cx^2\right)}{3x^3}-\frac{2bc}{3x}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])/x^4,x]
```

```
[Out] -a/(3*x^3) - (2*b*c)/(3*x) - (b*c^(3/2)*ArcTan[Sqrt[c]*x])/3 - (b*ArcTanh[c
*x^2])/(3*x^3) - (b*c^(3/2)*Log[1 - Sqrt[c]*x])/6 + (b*c^(3/2)*Log[1 + Sqrt
[c]*x])/6
```

Maple [A] time = 0.014, size = 51, normalized size = 0.8

$$
-\frac{a}{3\,x^3}-\frac{b\mathrm{Artanh}\left(cx^2\right)}{3\,x^3}-\frac{b}{3}c^{\frac{3}{2}}\arctan\left(x\sqrt{c}\right)-\frac{2\,bc}{3\,x}+\frac{b}{3}c^{\frac{3}{2}}\mathrm{Artanh}\left(x\sqrt{c}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))/x^4,x)
```

```
[Out] -1/3*a/x^3-1/3*b/x^3*arctanh(c*x^2)-1/3*b*c^(3/2)*arctan(x*c^(1/2))-2/3*b*c
/x+1/3*b*c^(3/2)*arctanh(x*c^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.13394, size = 440, normalized size = 6.98

$$\left[-\frac{2\,bc^{\frac{3}{2}}x^3\arctan\left(\sqrt{c}x\right)-bc^{\frac{3}{2}}x^3\log\left(\frac{cx^2+2\,\sqrt{c}x+1}{cx^2-1}\right)+4\,bcx^2+b\log\left(-\frac{cx^2+1}{cx^2-1}\right)+2\,a}{6\,x^3},-\frac{2\,b\sqrt{-c}cx^3\arctan\left(\sqrt{-c}x\right)-b\sqrt{-}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/x^4,x, algorithm="fricas")
```

```
[Out] [-1/6*(2*b*c^(3/2)*x^3*arctan(sqrt(c)*x) - b*c^(3/2)*x^3*log((c*x^2 + 2*sqr
t(c)*x + 1)/(c*x^2 - 1)) + 4*b*c*x^2 + b*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 2*
a)/x^3, -1/6*(2*b*sqrt(-c)*c*x^3*arctan(sqrt(-c)*x) - b*sqrt(-c)*c*x^3*log(
(c*x^2 - 2*sqrt(-c)*x - 1)/(c*x^2 + 1)) + 4*b*c*x^2 + b*log(-(c*x^2 + 1)/(c
*x^2 - 1)) + 2*a)/x^3]
```

Sympy [A] time = 21.2908, size = 813, normalized size = 12.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))/x**4,x)
```

```
[Out] Piecewise((-a/(3*x**3), Eq(c, 0)), (-(a - oo*b)/(3*x**3), Eq(c, -1/x**2)),
(-(a + oo*b)/(3*x**3), Eq(c, x**(-2))), (-4*a*x**4/(12*x**7 - 12*x**3/c**2)
 + 4*a/(12*c**2*x**7 - 12*x**3) - b*c**3*x**7*(1/c)**(3/2)*log(x + I*sqrt(1
/c))/(12*x**7 - 12*x**3/c**2) + I*b*c**3*x**7*(1/c)**(3/2)*log(x + I*sqrt(1
/c))/(12*x**7 - 12*x**3/c**2) + 2*b*c**2*x**7*sqrt(1/c)*log(x - I*sqrt(1/c)
)/(12*x**7 - 12*x**3/c**2) + 2*I*b*c**2*x**7*sqrt(1/c)*log(x - I*sqrt(1/c))
/(12*x**7 - 12*x**3/c**2) + 3*b*c**2*x**7*sqrt(1/c)*log(x + I*sqrt(1/c))/(1
2*x**7 - 12*x**3/c**2) - 3*I*b*c**2*x**7*sqrt(1/c)*log(x + I*sqrt(1/c))/(12
*x**7 - 12*x**3/c**2) - 4*b*c**2*x**7*sqrt(1/c)*log(x - sqrt(1/c))/(12*x**7
 - 12*x**3/c**2) - 4*b*c**2*x**7*sqrt(1/c)*atanh(c*x**2)/(12*x**7 - 12*x**3
/c**2) - 8*b*c*x**6/(12*x**7 - 12*x**3/c**2) + b*c*x**3*(1/c)**(3/2)*log(x
+ I*sqrt(1/c))/(12*x**7 - 12*x**3/c**2) - I*b*c*x**3*(1/c)**(3/2)*log(x + I
*sqrt(1/c))/(12*x**7 - 12*x**3/c**2) - 4*b*x**4*atanh(c*x**2)/(12*x**7 - 12
*x**3/c**2) - 2*b*x**3*sqrt(1/c)*log(x - I*sqrt(1/c))/(12*x**7 - 12*x**3/c*
*2) - 2*I*b*x**3*sqrt(1/c)*log(x - I*sqrt(1/c))/(12*x**7 - 12*x**3/c**2) -
3*b*x**3*sqrt(1/c)*log(x + I*sqrt(1/c))/(12*x**7 - 12*x**3/c**2) + 3*I*b*x*
*3*sqrt(1/c)*log(x + I*sqrt(1/c))/(12*x**7 - 12*x**3/c**2) + 4*b*x**3*sqrt(
1/c)*log(x - sqrt(1/c))/(12*x**7 - 12*x**3/c**2) + 4*b*x**3*sqrt(1/c)*atanh
(c*x**2)/(12*x**7 - 12*x**3/c**2) + 8*b*x**2/(12*c*x**7 - 12*x**3/c) + 4*b*
atanh(c*x**2)/(12*c**2*x**7 - 12*x**3), True))
```

Giac [B] time = 1.35212, size = 132, normalized size = 2.1

$$-\frac{1}{6}\,bc^3\left(\frac{2\,\sqrt{|c|}\arctan\left(x\sqrt{|c|}\right)}{c^2}-\frac{\sqrt{|c|}\log\left(\left|x+\frac{1}{\sqrt{|c|}}\right|\right)}{c^2}+\frac{\sqrt{|c|}\log\left(\left|x-\frac{1}{\sqrt{|c|}}\right|\right)}{c^2}\right)-\frac{b\log\left(-\frac{cx^2+1}{cx^2-1}\right)}{6\,x^3}-\frac{2\,bcx^2+a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/x^4,x, algorithm="giac")
```

```
[Out] -1/6*b*c^3*(2*sqrt(abs(c))*arctan(x*sqrt(abs(c)))/c^2 - sqrt(abs(c))*log(ab
s(x + 1/sqrt(abs(c))))/c^2 + sqrt(abs(c))*log(abs(x - 1/sqrt(abs(c))))/c^2)
 - 1/6*b*log(-(c*x^2 + 1)/(c*x^2 - 1))/x^3 - 1/3*(2*b*c*x^2 + a)/x^3
```

3.63 $\int \frac{a+b\tanh^{-1}(cx^2)}{x^6}\,dx$

Optimal. Leaf size=63

$$-\frac{a+b\tanh^{-1}\left(cx^2\right)}{5x^5}+\frac{1}{5}bc^{5/2}\tan^{-1}\left(\sqrt{c}x\right)+\frac{1}{5}bc^{5/2}\tanh^{-1}\left(\sqrt{c}x\right)-\frac{2bc}{15x^3}$$

```
[Out] (-2*b*c)/(15*x^3) + (b*c^(5/2)*ArcTan[Sqrt[c]*x])/5 + (b*c^(5/2)*ArcTanh[Sq
rt[c]*x])/5 - (a + b*ArcTanh[c*x^2])/(5*x^5)
```

Rubi [A] time = 0.0319471, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6097, 325, 212, 206, 203}

$$-\frac{a+b\tanh^{-1}\left(cx^2\right)}{5x^5}+\frac{1}{5}bc^{5/2}\tan^{-1}\left(\sqrt{c}x\right)+\frac{1}{5}bc^{5/2}\tanh^{-1}\left(\sqrt{c}x\right)-\frac{2bc}{15x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^2])/x^6,x]
```

```
[Out] (-2*b*c)/(15*x^3) + (b*c^(5/2)*ArcTan[Sqrt[c]*x])/5 + (b*c^(5/2)*ArcTanh[Sq
rt[c]*x])/5 - (a + b*ArcTanh[c*x^2])/(5*x^5)
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}\left(cx^2\right)}{x^6}\,dx &= -\frac{a+b\tanh^{-1}\left(cx^2\right)}{5x^5}+\frac{1}{5}(2bc)\int \frac{1}{x^4\left(1-c^2x^4\right)}\,dx\\ &= -\frac{2bc}{15x^3}-\frac{a+b\tanh^{-1}\left(cx^2\right)}{5x^5}+\frac{1}{5}\left(2bc^3\right)\int \frac{1}{1-c^2x^4}\,dx\\ &= -\frac{2bc}{15x^3}-\frac{a+b\tanh^{-1}\left(cx^2\right)}{5x^5}+\frac{1}{5}\left(bc^3\right)\int \frac{1}{1-cx^2}\,dx+\frac{1}{5}\left(bc^3\right)\int \frac{1}{1+cx^2}\,dx\\ &= -\frac{2bc}{15x^3}+\frac{1}{5}bc^{5/2}\tan^{-1}\left(\sqrt{c}x\right)+\frac{1}{5}bc^{5/2}\tanh^{-1}\left(\sqrt{c}x\right)-\frac{a+b\tanh^{-1}\left(cx^2\right)}{5x^5}\end{aligned}$$

Mathematica [A] time = 0.0272086, size = 91, normalized size = 1.44

$$-\frac{a}{5x^5}-\frac{1}{10}bc^{5/2}\log\left(1-\sqrt{c}x\right)+\frac{1}{10}bc^{5/2}\log\left(\sqrt{c}x+1\right)+\frac{1}{5}bc^{5/2}\tan^{-1}\left(\sqrt{c}x\right)-\frac{2bc}{15x^3}-\frac{b\tanh^{-1}\left(cx^2\right)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])/x^6,x]
```

```
[Out] -a/(5*x^5) - (2*b*c)/(15*x^3) + (b*c^(5/2)*ArcTan[Sqrt[c]*x])/5 - (b*ArcTan
h[c*x^2])/(5*x^5) - (b*c^(5/2)*Log[1 - Sqrt[c]*x])/10 + (b*c^(5/2)*Log[1 +
Sqrt[c]*x])/10
```

Maple [A] time = 0.013, size = 51, normalized size = 0.8

$$-\frac{a}{5\,x^5}-\frac{b\mathrm{Artanh}\left(cx^2\right)}{5\,x^5}+\frac{b}{5}c^{\frac{5}{2}}\arctan\left(x\sqrt{c}\right)+\frac{b}{5}c^{\frac{5}{2}}\mathrm{Artanh}\left(x\sqrt{c}\right)-\frac{2\,bc}{15\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))/x^6,x)
```

```
[Out] -1/5*a/x^5-1/5*b/x^5*arctanh(c*x^2)+1/5*b*c^(5/2)*arctan(x*c^(1/2))+1/5*b*c
^(5/2)*arctanh(x*c^(1/2))-2/15*b*c/x^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.14109, size = 458, normalized size = 7.27

$$\left[\frac{6\,bc^{\frac{5}{2}}x^5\arctan\left(\sqrt{c}x\right)+3\,bc^{\frac{5}{2}}x^5\log\left(\frac{cx^2+2\,\sqrt{c}x+1}{cx^2-1}\right)-4\,bcx^2-3\,b\log\left(-\frac{cx^2+1}{cx^2-1}\right)-6\,a}{30\,x^5},-\frac{6\,b\sqrt{-c}c^2x^5\arctan\left(\sqrt{-c}x\right)-3\,b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/x^6,x, algorithm="fricas")
```

```
[Out] [1/30*(6*b*c^(5/2)*x^5*arctan(sqrt(c)*x) + 3*b*c^(5/2)*x^5*log((c*x^2 + 2*s
qrt(c)*x + 1)/(c*x^2 - 1)) - 4*b*c*x^2 - 3*b*log(-(c*x^2 + 1)/(c*x^2 - 1))
- 6*a)/x^5, -1/30*(6*b*sqrt(-c)*c^2*x^5*arctan(sqrt(-c)*x) - 3*b*sqrt(-c)*c
^2*x^5*log((c*x^2 + 2*sqrt(-c)*x - 1)/(c*x^2 + 1)) + 4*b*c*x^2 + 3*b*log(-(
c*x^2 + 1)/(c*x^2 - 1)) + 6*a)/x^5]
```

Sympy [A] time = 30.8841, size = 833, normalized size = 13.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))/x**6,x)
```

```
[Out] Piecewise((-a/(5*x**5), Eq(c, 0)), (-(a - oo*b)/(5*x**5), Eq(c, -1/x**2)),
(-(a + oo*b)/(5*x**5), Eq(c, x**(-2))), (-12*a*x**4/(60*x**9 - 60*x**5/c**2
) + 12*a/(60*c**2*x**9 - 60*x**5) - 3*b*c**4*x**9*(1/c)**(3/2)*log(x + I*sq
rt(1/c))/(60*x**9 - 60*x**5/c**2) - 3*I*b*c**4*x**9*(1/c)**(3/2)*log(x + I*
sqrt(1/c))/(60*x**9 - 60*x**5/c**2) + 6*b*c**3*x**9*sqrt(1/c)*log(x - I*sqr
t(1/c))/(60*x**9 - 60*x**5/c**2) - 6*I*b*c**3*x**9*sqrt(1/c)*log(x - I*sqrt
(1/c))/(60*x**9 - 60*x**5/c**2) + 9*b*c**3*x**9*sqrt(1/c)*log(x + I*sqrt(1/
c))/(60*x**9 - 60*x**5/c**2) + 9*I*b*c**3*x**9*sqrt(1/c)*log(x + I*sqrt(1/c
))/(60*x**9 - 60*x**5/c**2) - 12*b*c**3*x**9*sqrt(1/c)*log(x - sqrt(1/c))/(
60*x**9 - 60*x**5/c**2) - 12*b*c**3*x**9*sqrt(1/c)*atanh(c*x**2)/(60*x**9 -
 60*x**5/c**2) + 3*b*c**2*x**5*(1/c)**(3/2)*log(x + I*sqrt(1/c))/(60*x**9 -
 60*x**5/c**2) + 3*I*b*c**2*x**5*(1/c)**(3/2)*log(x + I*sqrt(1/c))/(60*x**9
 - 60*x**5/c**2) - 8*b*c*x**6/(60*x**9 - 60*x**5/c**2) - 6*b*c*x**5*sqrt(1/
c)*log(x - I*sqrt(1/c))/(60*x**9 - 60*x**5/c**2) + 6*I*b*c*x**5*sqrt(1/c)*l
og(x - I*sqrt(1/c))/(60*x**9 - 60*x**5/c**2) - 9*b*c*x**5*sqrt(1/c)*log(x +
 I*sqrt(1/c))/(60*x**9 - 60*x**5/c**2) - 9*I*b*c*x**5*sqrt(1/c)*log(x + I*s
qrt(1/c))/(60*x**9 - 60*x**5/c**2) + 12*b*c*x**5*sqrt(1/c)*log(x - sqrt(1/c
))/(60*x**9 - 60*x**5/c**2) + 12*b*c*x**5*sqrt(1/c)*atanh(c*x**2)/(60*x**9
- 60*x**5/c**2) - 12*b*x**4*atanh(c*x**2)/(60*x**9 - 60*x**5/c**2) + 8*b*x*
*2/(60*c*x**9 - 60*x**5/c) + 12*b*atanh(c*x**2)/(60*c**2*x**9 - 60*x**5), T
rue))
```

Giac [A] time = 1.53314, size = 123, normalized size = 1.95

$$\frac{1}{10}\,bc^3\left(\frac{2\,\arctan\left(x\sqrt{|c|}\right)}{\sqrt{|c|}}+\frac{\log\left(\left|x+\frac{1}{\sqrt{|c|}}\right|\right)}{\sqrt{|c|}}-\frac{\log\left(\left|x-\frac{1}{\sqrt{|c|}}\right|\right)}{\sqrt{|c|}}\right)-\frac{b\log\left(-\frac{cx^2+1}{cx^2-1}\right)}{10\,x^5}-\frac{2\,bcx^2+3\,a}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/x^6,x, algorithm="giac")
```

```
[Out] 1/10*b*c^3*(2*arctan(x*sqrt(abs(c)))/sqrt(abs(c)) + log(abs(x + 1/sqrt(abs(
c))))/sqrt(abs(c)) - log(abs(x - 1/sqrt(abs(c))))/sqrt(abs(c))) - 1/10*b*lo
g(-(c*x^2 + 1)/(c*x^2 - 1))/x^5 - 1/15*(2*b*c*x^2 + 3*a)/x^5
```

3.64 $\int x^7 \left(a+b\tanh^{-1}\left(cx^2\right)\right)^2 dx$

Optimal. Leaf size=125

$$\frac{abx^2}{4c^3}-\frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{8c^4}+\frac{1}{8}x^8\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2+\frac{bx^6\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{12c}+\frac{b^2x^4}{24c^2}+\frac{b^2\log\left(1-c^2x^4\right)}{6c^4}+\cdots$$

```
[Out] (a*b*x^2)/(4*c^3) + (b^2*x^4)/(24*c^2) + (b^2*x^2*ArcTanh[c*x^2])/(4*c^3) +
 (b*x^6*(a + b*ArcTanh[c*x^2]))/(12*c) - (a + b*ArcTanh[c*x^2])^2/(8*c^4) +
 (x^8*(a + b*ArcTanh[c*x^2])^2)/8 + (b^2*Log[1 - c^2*x^4])/(6*c^4)
```

Rubi [C] time = 1.54731, antiderivative size = 636, normalized size of antiderivative = 5.09, number of steps used = 62, number of rules used = 19, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.187, Rules used = {6099, 2454, 2398, 2411, 43, 2334, 12, 14, 2301, 2395, 2439, 2416, 2389, 2295, 2394, 2393, 2391, 2410, 2390}

$$\frac{b^2\text{PolyLog}\left(2,\frac{1}{2}\left(1-cx^2\right)\right)}{16c^4}+\frac{b^2\text{PolyLog}\left(2,\frac{1}{2}\left(cx^2+1\right)\right)}{16c^4}+\frac{abx^2}{8c^3}-\frac{bx^4\left(2a-b\log\left(1-cx^2\right)\right)}{32c^2}-\frac{1}{192}b\left(-\frac{3\left(1-cx^2\right)^4}{c^4}\cdots\right.$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^7*(a + b*ArcTanh[c*x^2])^2,x]
```

```
[Out] (a*b*x^2)/(8*c^3) + (23*b^2*x^2)/(192*c^3) + (b^2*x^4)/(128*c^2) - (7*b^2*x
^6)/(576*c) - (b^2*x^8)/256 + (3*b^2*(1 - c*x^2)^2)/(32*c^4) - (b^2*(1 - c*
x^2)^3)/(36*c^4) + (b^2*(1 - c*x^2)^4)/(256*c^4) - (5*b^2*Log[1 - c*x^2])/(
192*c^4) + (b^2*(1 - c*x^2)*Log[1 - c*x^2])/(16*c^4) + (b^2*Log[1 - c*x^2]^
2)/(32*c^4) - (b*x^4*(2*a - b*Log[1 - c*x^2]))/(32*c^2) + (b*x^6*(2*a - b*L
og[1 - c*x^2]))/(48*c) - (b*x^8*(2*a - b*Log[1 - c*x^2]))/64 + (x^8*(2*a -
b*Log[1 - c*x^2])^2)/32 - (b*(2*a - b*Log[1 - c*x^2])*((48*(1 - c*x^2))/c^4
 - (36*(1 - c*x^2)^2)/c^4 + (16*(1 - c*x^2)^3)/c^4 - (3*(1 - c*x^2)^4)/c^4
- (12*Log[1 - c*x^2])/c^4))/192 - (b*(2*a - b*Log[1 - c*x^2])*Log[(1 + c*x^
2)/2])/(16*c^4) + (b^2*Log[1 + c*x^2])/(24*c^4) + (b^2*x^6*Log[1 + c*x^2])/
(24*c) + (b^2*(1 + c*x^2)*Log[1 + c*x^2])/(8*c^4) + (b^2*Log[(1 - c*x^2)/2]
*Log[1 + c*x^2])/(16*c^4) + (b*x^8*(2*a - b*Log[1 - c*x^2])*Log[1 + c*x^2])
/16 - (b^2*Log[1 + c*x^2]^2)/(32*c^4) + (b^2*x^8*Log[1 + c*x^2]^2)/32 + (b^
2*PolyLog[2, (1 - c*x^2)/2])/(16*c^4) + (b^2*PolyLog[2, (1 + c*x^2)/2])/(16
*c^4)
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1
] && EqQ[m, -1])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[(x^
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
```

```
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
 + j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rubi steps

$$\int x^7 \left(a+b\tanh^{-1}\left(cx^2\right)\right)^2 dx = \int \left(\frac{1}{4}x^7\left(2a-b\log\left(1-cx^2\right)\right)^2 - \frac{1}{2}bx^7\left(-2a+b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right) + \frac{1}{4}b^2x^7\log$$

$$= \frac{1}{4}\int x^7\left(2a-b\log\left(1-cx^2\right)\right)^2 dx - \frac{1}{2}b\int x^7\left(-2a+b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right) dx +$$

$$= \frac{1}{8}\text{Subst}\left(\int x^3(2a-b\log(1-cx))^2 dx, x, x^2\right) - \frac{1}{4}b\,\text{Subst}\left(\int x^3(-2a+b\log(1-cx))\log$$

$$= \frac{1}{32}x^8\left(2a-b\log\left(1-cx^2\right)\right)^2 + \frac{1}{16}bx^8\left(2a-b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right) + \frac{1}{32}b^2x^8\log^2$$

$$= \frac{1}{32}x^8\left(2a-b\log\left(1-cx^2\right)\right)^2 + \frac{1}{16}bx^8\left(2a-b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right) + \frac{1}{32}b^2x^8\log^2$$

$$= \frac{1}{32}x^8\left(2a-b\log\left(1-cx^2\right)\right)^2 - \frac{1}{192}b\left(2a-b\log\left(1-cx^2\right)\right)\left(\frac{48\left(1-cx^2\right)}{c^4} - \frac{36\left(1-cx^2\right)^2}{c^4}\right.$$

$$= \frac{abx^2}{8c^3} - \frac{bx^4\left(2a-b\log\left(1-cx^2\right)\right)}{32c^2} + \frac{bx^6\left(2a-b\log\left(1-cx^2\right)\right)}{48c} - \frac{1}{64}bx^8\left(2a-b\log\left(1-\right.\right.$$

$$= \frac{abx^2}{8c^3} - \frac{bx^4\left(2a-b\log\left(1-cx^2\right)\right)}{32c^2} + \frac{bx^6\left(2a-b\log\left(1-cx^2\right)\right)}{48c} - \frac{1}{64}bx^8\left(2a-b\log\left(1-\right.\right.$$

$$= \frac{abx^2}{8c^3} + \frac{55b^2x^2}{192c^3} - \frac{5b^2x^4}{384c^2} + \frac{b^2x^6}{576c} - \frac{b^2x^8}{256} + \frac{3b^2\left(1-cx^2\right)^2}{32c^4} - \frac{b^2\left(1-cx^2\right)^3}{36c^4} + \frac{b^2\left(1-cx^2\right)}{256c^4}$$

$$= \frac{abx^2}{8c^3} + \frac{55b^2x^2}{192c^3} - \frac{5b^2x^4}{384c^2} + \frac{b^2x^6}{576c} - \frac{b^2x^8}{256} + \frac{3b^2\left(1-cx^2\right)^2}{32c^4} - \frac{b^2\left(1-cx^2\right)^3}{36c^4} + \frac{b^2\left(1-cx^2\right)}{256c^4}$$

Mathematica [A] time = 0.0734534, size = 146, normalized size = 1.17

$$\frac{3a^2c^4x^8 + 2abc^3x^6 + 2bcx^2\tanh^{-1}\left(cx^2\right)\left(3ac^3x^6 + b\left(c^2x^4+3\right)\right) + 6abcx^2 + b(3a+4b)\log\left(1-cx^2\right) - 3ab\log\left(cx^2+1\right)}{24c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*ArcTanh[c*x^2])^2,x]
```

```
[Out] (6*a*b*c*x^2 + b^2*c^2*x^4 + 2*a*b*c^3*x^6 + 3*a^2*c^4*x^8 + 2*b*c*x^2*(3*a
*c^3*x^6 + b*(3 + c^2*x^4))*ArcTanh[c*x^2] + 3*b^2*(-1 + c^4*x^8)*ArcTanh[c
*x^2]^2 + b*(3*a + 4*b)*Log[1 - c*x^2] - 3*a*b*Log[1 + c*x^2] + 4*b^2*Log[1
 + c*x^2])/(24*c^4)
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int x^7\left(a+b\text{Artanh}\left(cx^2\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a+b*arctanh(c*x^2))^2,x)
```

```
[Out] int(x^7*(a+b*arctanh(c*x^2))^2,x)
```

Maxima [A] time = 1.00696, size = 293, normalized size = 2.34

$$\frac{1}{8}b^2x^8\operatorname{artanh}\left(cx^2\right)^2+\frac{1}{8}a^2x^8+\frac{1}{24}\left(6x^8\operatorname{artanh}\left(cx^2\right)+c\left(\frac{2\left(c^2x^6+3x^2\right)}{c^4}-\frac{3\log\left(cx^2+1\right)}{c^5}+\frac{3\log\left(cx^2-1\right)}{c^5}\right)\right)ab$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arctanh(c*x^2))^2,x, algorithm="maxima")
```

```
[Out] 1/8*b^2*x^8*arctanh(c*x^2)^2 + 1/8*a^2*x^8 + 1/24*(6*x^8*arctanh(c*x^2) + c
*(2*(c^2*x^6 + 3*x^2)/c^4 - 3*log(c*x^2 + 1)/c^5 + 3*log(c*x^2 - 1)/c^5))*a
*b + 1/96*(4*c*(2*(c^2*x^6 + 3*x^2)/c^4 - 3*log(c*x^2 + 1)/c^5 + 3*log(c*x^
2 - 1)/c^5)*arctanh(c*x^2) + (4*c^2*x^4 - 2*(3*log(c*x^2 - 1) - 8)*log(c*x^
2 + 1) + 3*log(c*x^2 + 1)^2 + 3*log(c*x^2 - 1)^2 + 16*log(c*x^2 - 1))/c^4)*
b^2
```

Fricas [A] time = 2.0428, size = 375, normalized size = 3.

$$\frac{12a^2c^4x^8+8abc^3x^6+4b^2c^2x^4+24abcx^2+3\left(b^2c^4x^8-b^2\right)\log\left(-\frac{cx^2+1}{cx^2-1}\right)^2-4\left(3ab-4b^2\right)\log\left(cx^2+1\right)+4\left(3ab+4\right.}{96c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arctanh(c*x^2))^2,x, algorithm="fricas")
```

```
[Out] 1/96*(12*a^2*c^4*x^8 + 8*a*b*c^3*x^6 + 4*b^2*c^2*x^4 + 24*a*b*c*x^2 + 3*(b^
2*c^4*x^8 - b^2)*log(-(c*x^2 + 1)/(c*x^2 - 1))^2 - 4*(3*a*b - 4*b^2)*log(c*
x^2 + 1) + 4*(3*a*b + 4*b^2)*log(c*x^2 - 1) + 4*(3*a*b*c^4*x^8 + b^2*c^3*x^
6 + 3*b^2*c*x^2)*log(-(c*x^2 + 1)/(c*x^2 - 1)))/c^4
```

Sympy [A] time = 39.731, size = 206, normalized size = 1.65

$$\begin{cases}\frac{a^2x^8}{8}+\frac{abx^8\operatorname{atanh}\left(cx^2\right)}{4}+\frac{abx^6}{12c}+\frac{abx^2}{4c^3}-\frac{ab\operatorname{atanh}\left(cx^2\right)}{4c^4}+\frac{b^2x^8\operatorname{atanh}^2\left(cx^2\right)}{8}+\frac{b^2x^6\operatorname{atanh}\left(cx^2\right)}{12c}+\frac{b^2x^4}{24c^2}+\frac{b^2x^2\operatorname{atanh}\left(cx^2\right)}{4c^3}+\frac{b^2\log\left(x-i\right.}{3c^4} \\ \frac{a^2x^8}{8}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(a+b*atanh(c*x**2))**2,x)
```

```
[Out] Piecewise((a**2*x**8/8 + a*b*x**8*atanh(c*x**2)/4 + a*b*x**6/(12*c) + a*b*x
**2/(4*c**3) - a*b*atanh(c*x**2)/(4*c**4) + b**2*x**8*atanh(c*x**2)**2/8 +
b**2*x**6*atanh(c*x**2)/(12*c) + b**2*x**4/(24*c**2) + b**2*x**2*atanh(c*x*
*2)/(4*c**3) + b**2*log(x - I*sqrt(1/c))/(3*c**4) + b**2*log(x + I*sqrt(1/c
))/(3*c**4) - b**2*atanh(c*x**2)**2/(8*c**4) - b**2*atanh(c*x**2)/(3*c**4),
 Ne(c, 0)), (a**2*x**8/8, True))
```

Giac [A] time = 1.50282, size = 236, normalized size = 1.89

$$\frac{1}{8}\,a^2x^8 + \frac{abx^6}{12\,c} + \frac{b^2x^4}{24\,c^2} + \frac{1}{32}\left(b^2x^8 - \frac{b^2}{c^4}\right)\log\left(-\frac{cx^2+1}{cx^2-1}\right)^2 + \frac{1}{24}\left(3\,abx^8 + \frac{b^2x^6}{c} + \frac{3\,b^2x^2}{c^3}\right)\log\left(-\frac{cx^2+1}{cx^2-1}\right) + \frac{abx^2}{4\,c^3} - \frac{(3\,ab}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arctanh(c*x^2))^2,x, algorithm="giac")
```

```
[Out] 1/8*a^2*x^8 + 1/12*a*b*x^6/c + 1/24*b^2*x^4/c^2 + 1/32*(b^2*x^8 - b^2/c^4)*
log(-(c*x^2 + 1)/(c*x^2 - 1))^2 + 1/24*(3*a*b*x^8 + b^2*x^6/c + 3*b^2*x^2/c
^3)*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 1/4*a*b*x^2/c^3 - 1/24*(3*a*b - 4*b^2)*
log(c*x^2 + 1)/c^4 + 1/24*(3*a*b + 4*b^2)*log(c*x^2 - 1)/c^4
```

3.65 $\int x^5 \left(a+b\tanh^{-1}\left(cx^2\right)\right)^2 dx$

Optimal. Leaf size=146

$$-\frac{b^2\text{PolyLog}\left(2,1-\frac{2}{1-cx^2}\right)}{6c^3}+\frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{6c^3}-\frac{b\log\left(\frac{2}{1-cx^2}\right)\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{3c^3}+\frac{1}{6}x^6\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2$$

```
[Out] (b^2*x^2)/(6*c^2) - (b^2*ArcTanh[c*x^2])/(6*c^3) + (b*x^4*(a + b*ArcTanh[c*
x^2]))/(6*c) + (a + b*ArcTanh[c*x^2])^2/(6*c^3) + (x^6*(a + b*ArcTanh[c*x^2
])^2)/6 - (b*(a + b*ArcTanh[c*x^2])*Log[2/(1 - c*x^2)])/(3*c^3) - (b^2*Poly
Log[2, 1 - 2/(1 - c*x^2)])/(6*c^3)
```

Rubi [B] time = 1.29604, antiderivative size = 536, normalized size of antiderivative = 3.67, number of steps used = 53, number of rules used = 19, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.187, Rules used = {6099, 2454, 2398, 2411, 43, 2334, 12, 14, 2301, 2395, 2439, 2416, 2389, 2295, 2394, 2393, 2391, 2410, 2390}

$$-\frac{b^2\text{PolyLog}\left(2,\frac{1}{2}\left(1-cx^2\right)\right)}{12c^3}+\frac{b^2\text{PolyLog}\left(2,\frac{1}{2}\left(cx^2+1\right)\right)}{12c^3}-\frac{abx^2}{6c^2}-\frac{1}{72}b\left(\frac{2\left(1-cx^2\right)^3}{c^3}-\frac{9\left(1-cx^2\right)^2}{c^3}+\frac{18\left(1-cx^2\right)}{c^3}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^5*(a + b*ArcTanh[c*x^2])^2,x]
```

```
[Out] -(a*b*x^2)/(6*c^2) + (19*b^2*x^2)/(72*c^2) - (5*b^2*x^4)/(144*c) - (b^2*x^6
)/108 + (b^2*(1 - c*x^2)^2)/(16*c^3) - (b^2*(1 - c*x^2)^3)/(108*c^3) + (b^2
*Log[1 - c*x^2])/(72*c^3) - (b^2*(1 - c*x^2)*Log[1 - c*x^2])/(12*c^3) + (b^
2*Log[1 - c*x^2]^2)/(24*c^3) + (b*x^4*(2*a - b*Log[1 - c*x^2]))/(24*c) - (b
*x^6*(2*a - b*Log[1 - c*x^2]))/36 + (x^6*(2*a - b*Log[1 - c*x^2])^2)/24 - (
b*(2*a - b*Log[1 - c*x^2])*((18*(1 - c*x^2))/c^3 - (9*(1 - c*x^2)^2)/c^3 +
(2*(1 - c*x^2)^3)/c^3 - (6*Log[1 - c*x^2])/c^3))/72 + (b*(2*a - b*Log[1 - c
*x^2])*Log[(1 + c*x^2)/2])/(12*c^3) - (b^2*Log[1 + c*x^2])/(12*c^3) + (b^2*
x^4*Log[1 + c*x^2])/(12*c) + (b^2*Log[(1 - c*x^2)/2]*Log[1 + c*x^2])/(12*c^
3) + (b*x^6*(2*a - b*Log[1 - c*x^2])*Log[1 + c*x^2])/12 + (b^2*Log[1 + c*x^
2]^2)/(24*c^3) + (b^2*x^6*Log[1 + c*x^2]^2)/24 - (b^2*PolyLog[2, (1 - c*x^2
)/2])/(12*c^3) + (b^2*PolyLog[2, (1 + c*x^2)/2])/(12*c^3)
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1
] && EqQ[m, -1])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[(x^
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
```

```
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
 + j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rubi steps

$$\int x^5\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2dx=\int\left(\frac{1}{4}x^5\left(2a-b\log\left(1-cx^2\right)\right)^2-\frac{1}{2}bx^5\left(-2a+b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right)+\frac{1}{4}b^2x^5\log\right.$$

$$=\frac{1}{4}\int x^5\left(2a-b\log\left(1-cx^2\right)\right)^2dx-\frac{1}{2}b\int x^5\left(-2a+b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right)dx+$$

$$=\frac{1}{8}\text{Subst}\left(\int x^2(2a-b\log(1-cx))^2\,dx,x,x^2\right)-\frac{1}{4}b\,\text{Subst}\left(\int x^2(-2a+b\log(1-cx))\log\right.$$

$$=\frac{1}{24}x^6\left(2a-b\log\left(1-cx^2\right)\right)^2+\frac{1}{12}bx^6\left(2a-b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right)+\frac{1}{24}b^2x^6\log^2$$

$$=\frac{1}{24}x^6\left(2a-b\log\left(1-cx^2\right)\right)^2+\frac{1}{12}bx^6\left(2a-b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right)+\frac{1}{24}b^2x^6\log^2$$

$$=\frac{1}{24}x^6\left(2a-b\log\left(1-cx^2\right)\right)^2-\frac{1}{72}b\left(2a-b\log\left(1-cx^2\right)\right)\left(\frac{18\left(1-cx^2\right)}{c^3}-\frac{9\left(1-cx^2\right)^2}{c^3}+\right.$$

$$=-\frac{abx^2}{6c^2}+\frac{bx^4\left(2a-b\log\left(1-cx^2\right)\right)}{24c}-\frac{1}{36}bx^6\left(2a-b\log\left(1-cx^2\right)\right)+\frac{1}{24}x^6\left(2a-b\log\left(1\right.\right.$$

$$=-\frac{abx^2}{6c^2}+\frac{bx^4\left(2a-b\log\left(1-cx^2\right)\right)}{24c}-\frac{1}{36}bx^6\left(2a-b\log\left(1-cx^2\right)\right)+\frac{1}{24}x^6\left(2a-b\log\left(1\right.\right.$$

$$=-\frac{abx^2}{6c^2}+\frac{13b^2x^2}{72c^2}+\frac{b^2x^4}{144c}-\frac{b^2x^6}{108}+\frac{b^2\left(1-cx^2\right)^2}{16c^3}-\frac{b^2\left(1-cx^2\right)^3}{108c^3}+\frac{b^2\log\left(1-cx^2\right)}{72c^3}-\frac{b^2}{}$$

$$=-\frac{abx^2}{6c^2}+\frac{13b^2x^2}{72c^2}+\frac{b^2x^4}{144c}-\frac{b^2x^6}{108}+\frac{b^2\left(1-cx^2\right)^2}{16c^3}-\frac{b^2\left(1-cx^2\right)^3}{108c^3}+\frac{b^2\log\left(1-cx^2\right)}{72c^3}-\frac{b^2}{}$$

Mathematica [A] time = 0.277859, size = 132, normalized size = 0.9

$$\frac{b^2\text{PolyLog}\left(2,-e^{-2\tanh^{-1}\left(cx^2\right)}\right)+a^2c^3x^6+abc^2x^4+ab\log\left(c^2x^4-1\right)+b\tanh^{-1}\left(cx^2\right)\left(2ac^3x^6+bc^2x^4-2b\log\left(e^{-2\tanh^{-1}}\right.\right.}{6c^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^5*(a + b*ArcTanh[c*x^2])^2,x]
```

```
[Out] (b^2*c*x^2 + a*b*c^2*x^4 + a^2*c^3*x^6 + b^2*(-1 + c^3*x^6)*ArcTanh[c*x^2]^
2 + b*ArcTanh[c*x^2]*(-b + b*c^2*x^4 + 2*a*c^3*x^6 - 2*b*Log[1 + E^(-2*ArcT
anh[c*x^2])]) + a*b*Log[-1 + c^2*x^4] + b^2*PolyLog[2, -E^(-2*ArcTanh[c*x^2
])])/(6*c^3)
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int x^5\left(a+b\text{Artanh}\left(cx^2\right)\right)^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arctanh(c*x^2))^2,x)
```

```
[Out] int(x^5*(a+b*arctanh(c*x^2))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{6}a^2x^6 + \frac{1}{6}\left(2x^6 \operatorname{artanh}\left(cx^2\right) + \left(\frac{x^4}{c^2} + \frac{\log\left(c^2x^4 - 1\right)}{c^4}\right)c\right)ab + \frac{1}{432}\left(18x^6 \log\left(-cx^2 + 1\right)^2 - 2c^4\left(\frac{2\left(c^2x^6 + 3x^2\right)}{c^6} - \frac{3\log}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x^2))^2,x, algorithm="maxima")
```

```
[Out] 1/6*a^2*x^6 + 1/6*(2*x^6*arctanh(c*x^2) + (x^4/c^2 + log(c^2*x^4 - 1)/c^4)*
c)*a*b + 1/432*(18*x^6*log(-c*x^2 + 1)^2 - 2*c^4*(2*(c^2*x^6 + 3*x^2)/c^6 -
 3*log(c*x^2 + 1)/c^7 + 3*log(c*x^2 - 1)/c^7) + 3*c^3*(x^4/c^4 + log(c^2*x^
4 - 1)/c^6) + 1296*c^3*integrate(1/9*x^7*log(c*x^2 + 1)/(c^4*x^4 - c^2), x)
 - 9*c^2*(2*x^2/c^4 - log(c*x^2 + 1)/c^5 + log(c*x^2 - 1)/c^5) - 6*c*((2*c^
2*x^6 + 3*c*x^4 + 6*x^2)/c^3 + 6*log(c*x^2 - 1)/c^4)*log(-c*x^2 + 1) + 648*
c*integrate(1/9*x^3*log(c*x^2 + 1)/(c^4*x^4 - c^2), x) + 6*(3*c^3*x^6*log(c
*x^2 + 1)^2 + (2*c^3*x^6 - 3*c^2*x^4 + 6*c*x^2 - 6*(c^3*x^6 + 1)*log(c*x^2
+ 1))*log(-c*x^2 + 1))/c^3 + (4*c^3*x^6 + 15*c^2*x^4 + 66*c*x^2 + 18*log(c*
x^2 - 1)^2 + 66*log(c*x^2 - 1))/c^3 - 18*log(9*c^4*x^4 - 9*c^2)/c^3 + 648*i
ntegrate(1/9*x*log(c*x^2 + 1)/(c^4*x^4 - c^2), x))*b^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(b^2x^5 \operatorname{artanh}\left(cx^2\right)^2 + 2\,abx^5 \operatorname{artanh}\left(cx^2\right) + a^2x^5, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x^2))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^5*arctanh(c*x^2)^2 + 2*a*b*x^5*arctanh(c*x^2) + a^2*x^5, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^5 \left(a + b \operatorname{atanh}\left(cx^2\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*atanh(c*x**2))**2,x)
```

```
[Out] Integral(x**5*(a + b*atanh(c*x**2))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \operatorname{artanh}\left(cx^2\right) + a\right)^2 x^5 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x^2))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^2) + a)^2*x^5, x)
```

3.66 $\int x^3 \left(a+b\tanh^{-1}\left(cx^2\right)\right)^2 dx$

Optimal. Leaf size=91

$$-\frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{4c^2}+\frac{abx^2}{2c}+\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2+\frac{b^2\log\left(1-c^2x^4\right)}{4c^2}+\frac{b^2x^2\tanh^{-1}\left(cx^2\right)}{2c}$$

```
[Out] (a*b*x^2)/(2*c) + (b^2*x^2*ArcTanh[c*x^2])/(2*c) - (a + b*ArcTanh[c*x^2])^2
/(4*c^2) + (x^4*(a + b*ArcTanh[c*x^2])^2)/4 + (b^2*Log[1 - c^2*x^4])/(4*c^2
)
```

Rubi [C] time = 0.96549, antiderivative size = 524, normalized size of antiderivative = 5.76, number of steps used = 44, number of rules used = 16, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {6099, 2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304, 2395, 43, 2439, 2416, 2394, 2393, 2391}

$$\frac{b^2\text{PolyLog}\left(2,\frac{1}{2}\left(1-cx^2\right)\right)}{8c^2}+\frac{b^2\text{PolyLog}\left(2,\frac{1}{2}\left(cx^2+1\right)\right)}{8c^2}+\frac{\left(1-cx^2\right)^2\left(2a-b\log\left(1-cx^2\right)\right)^2}{16c^2}-\frac{\left(1-cx^2\right)\left(2a-b\log\right.}{8c^2}$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^3*(a + b*ArcTanh[c*x^2])^2,x]
```

```
[Out] (3*a*b*x^2)/(4*c) - (b^2*x^4)/16 + (b^2*(1 - c*x^2)^2)/(32*c^2) + (b^2*(1 +
 c*x^2)^2)/(32*c^2) - (b^2*Log[1 - c*x^2])/(16*c^2) + (3*b^2*(1 - c*x^2)*Lo
g[1 - c*x^2])/(8*c^2) - (b*x^4*(2*a - b*Log[1 - c*x^2]))/16 + (b*(1 - c*x^2
)^2*(2*a - b*Log[1 - c*x^2]))/(16*c^2) - ((1 - c*x^2)*(2*a - b*Log[1 - c*x^
2])^2)/(8*c^2) + ((1 - c*x^2)^2*(2*a - b*Log[1 - c*x^2])^2)/(16*c^2) - (b*(
2*a - b*Log[1 - c*x^2])*Log[(1 + c*x^2)/2])/(8*c^2) - (b^2*Log[1 + c*x^2])/
(16*c^2) + (b^2*x^4*Log[1 + c*x^2])/16 + (3*b^2*(1 + c*x^2)*Log[1 + c*x^2])
/(8*c^2) - (b^2*(1 + c*x^2)^2*Log[1 + c*x^2])/(16*c^2) + (b^2*Log[(1 - c*x^
2)/2]*Log[1 + c*x^2])/(8*c^2) + (b*x^4*(2*a - b*Log[1 - c*x^2])*Log[1 + c*x
^2])/8 - (b^2*(1 + c*x^2)*Log[1 + c*x^2]^2)/(8*c^2) + (b^2*(1 + c*x^2)^2*Lo
g[1 + c*x^2]^2)/(16*c^2) + (b^2*PolyLog[2, (1 - c*x^2)/2])/(8*c^2) + (b^2*P
olyLog[2, (1 + c*x^2)/2])/(8*c^2)
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
```

```
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[(x^
```

```
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
 + j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int x^3 \left(a+b\tanh^{-1}\left(cx^2\right)\right)^2 dx = \int \left(\frac{1}{4}x^3\left(2a-b\log\left(1-cx^2\right)\right)^2 - \frac{1}{2}bx^3\left(-2a+b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right) + \frac{1}{4}b^2x^3\log$$
$$= \frac{1}{4}\int x^3\left(2a-b\log\left(1-cx^2\right)\right)^2 dx - \frac{1}{2}b\int x^3\left(-2a+b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right)dx +$$
$$= \frac{1}{8}\text{Subst}\left(\int x(2a-b\log(1-cx))^2 dx, x, x^2\right) - \frac{1}{4}b\,\text{Subst}\left(\int x(-2a+b\log(1-cx))\log($$
$$= \frac{1}{8}bx^4\left(2a-b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right) + \frac{1}{8}\text{Subst}\left(\int\left(\frac{(2a-b\log(1-cx))^2}{c} - \frac{(1-cx}{}\right.\right.$$
$$= \frac{1}{8}bx^4\left(2a-b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right) + \frac{\text{Subst}\left(\int(2a-b\log(1-cx))^2 dx, x, x^2\right)}{8c} - \frac{\text{S}}{}$$
$$= \frac{1}{8}bx^4\left(2a-b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right) + \frac{1}{8}b\,\text{Subst}\left(\int x(-2a+b\log(1-cx))\,dx, x, x^2\right.$$
$$= \frac{abx^2}{4c} - \frac{1}{16}bx^4\left(2a-b\log\left(1-cx^2\right)\right) - \frac{\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^2}{8c^2} + \frac{\left(1-cx^2\right)^2\left(2a\right.}{}$$
$$= \frac{3abx^2}{4c} - \frac{3b^2x^2}{8c} + \frac{b^2\left(1-cx^2\right)^2}{32c^2} + \frac{b^2\left(1+cx^2\right)^2}{32c^2} - \frac{1}{16}bx^4\left(2a-b\log\left(1-cx^2\right)\right) + \frac{b\left(1-\right.}{}$$
$$= \frac{3abx^2}{4c} - \frac{b^2x^4}{16} + \frac{b^2\left(1-cx^2\right)^2}{32c^2} + \frac{b^2\left(1+cx^2\right)^2}{32c^2} - \frac{b^2\log\left(1-cx^2\right)}{16c^2} + \frac{3b^2\left(1-cx^2\right)\log\left(1\right.}{8c^2}$$

Mathematica [A] time = 0.0603613, size = 106, normalized size = 1.16

$$\frac{a^2c^2x^4 + 2abcx^2 + b(a+b)\log\left(1-cx^2\right) - ab\log\left(cx^2+1\right) + 2bcx^2\tanh^{-1}\left(cx^2\right)\left(acx^2+b\right) + b^2\left(c^2x^4-1\right)\tanh^{-1}\left(cx^2\right)^2}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcTanh[c*x^2])^2,x]
```

```
[Out] (2*a*b*c*x^2 + a^2*c^2*x^4 + 2*b*c*x^2*(b + a*c*x^2)*ArcTanh[c*x^2] + b^2*(
-1 + c^2*x^4)*ArcTanh[c*x^2]^2 + b*(a + b)*Log[1 - c*x^2] - a*b*Log[1 + c*x
^2] + b^2*Log[1 + c*x^2])/(4*c^2)
```

Maple [B] time = 0.163, size = 247, normalized size = 2.7

$$\frac{b^2\left(c^2x^4-1\right)\left(\ln\left(cx^2+1\right)\right)^2}{16\,c^2} + \frac{b\left(-x^4b\ln\left(-cx^2+1\right)c^2 + 2\,ac^2x^4 + 2\,bcx^2 + b\ln\left(-cx^2+1\right)\right)\ln\left(cx^2+1\right)}{8\,c^2} + \frac{b^2x^4\left(\ln\left(-cx\right.\right.}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c*x^2))^2,x)
```

```
[Out] 1/16*b^2*(c^2*x^4-1)/c^2*ln(c*x^2+1)^2+1/8*b*(-x^4*b*ln(-c*x^2+1)*c^2+2*a*c
^2*x^4+2*b*c*x^2+b*ln(-c*x^2+1))/c^2*ln(c*x^2+1)+1/16*b^2*x^4*ln(-c*x^2+1)^
2-1/4*a*b*x^4*ln(-c*x^2+1)+1/4*a^2*x^4-1/4/c*b^2*x^2*ln(-c*x^2+1)+1/2*a*b*x
^2/c-1/16/c^2*b^2*ln(-c*x^2+1)^2+1/4/c^2*b*ln(-c*x^2+1)*a+1/4/c^2*b^2*ln(-c
*x^2+1)-1/4/c^2*b*ln(-c*x^2-1)*a+1/4/c^2*b^2*ln(-c*x^2-1)
```

Maxima [B] time = 0.976063, size = 251, normalized size = 2.76

$$\frac{1}{4}b^2x^4\operatorname{artanh}\left(cx^2\right)^2+\frac{1}{4}a^2x^4+\frac{1}{4}\left(2x^4\operatorname{artanh}\left(cx^2\right)+c\left(\frac{2x^2}{c^2}-\frac{\log\left(cx^2+1\right)}{c^3}+\frac{\log\left(cx^2-1\right)}{c^3}\right)\right)ab+\frac{1}{16}\left(4c\left(\frac{2x^2}{c^2}-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^2))^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*x^4*arctanh(c*x^2)^2 + 1/4*a^2*x^4 + 1/4*(2*x^4*arctanh(c*x^2) + c*
(2*x^2/c^2 - log(c*x^2 + 1)/c^3 + log(c*x^2 - 1)/c^3))*a*b + 1/16*(4*c*(2*x
^2/c^2 - log(c*x^2 + 1)/c^3 + log(c*x^2 - 1)/c^3)*arctanh(c*x^2) - (2*(log(
c*x^2 - 1) - 2)*log(c*x^2 + 1) - log(c*x^2 + 1)^2 - log(c*x^2 - 1)^2 - 4*lo
g(c*x^2 - 1))/c^2)*b^2
```

Fricas [A] time = 2.11289, size = 292, normalized size = 3.21

$$\frac{4a^2c^2x^4+8abcx^2+\left(b^2c^2x^4-b^2\right)\log\left(-\frac{cx^2+1}{cx^2-1}\right)^2-4\left(ab-b^2\right)\log\left(cx^2+1\right)+4\left(ab+b^2\right)\log\left(cx^2-1\right)+4\left(abc^2x^4+b\right.}{16c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^2))^2,x, algorithm="fricas")
```

```
[Out] 1/16*(4*a^2*c^2*x^4 + 8*a*b*c*x^2 + (b^2*c^2*x^4 - b^2)*log(-(c*x^2 + 1)/(c
*x^2 - 1))^2 - 4*(a*b - b^2)*log(c*x^2 + 1) + 4*(a*b + b^2)*log(c*x^2 - 1)
+ 4*(a*b*c^2*x^4 + b^2*c*x^2)*log(-(c*x^2 + 1)/(c*x^2 - 1)))/c^2
```

Sympy [A] time = 19.1199, size = 163, normalized size = 1.79

$$\begin{cases}\frac{a^2x^4}{4}+\frac{abx^4\operatorname{atanh}\left(cx^2\right)}{2}+\frac{abx^2}{2c}-\frac{ab\operatorname{atanh}\left(cx^2\right)}{2c^2}+\frac{b^2x^4\operatorname{atanh}^2\left(cx^2\right)}{4}+\frac{b^2x^2\operatorname{atanh}\left(cx^2\right)}{2c}+\frac{b^2\log\left(x-i\sqrt{\frac{1}{c}}\right)}{2c^2}+\frac{b^2\log\left(x+i\sqrt{\frac{1}{c}}\right)}{2c^2}-\frac{b^2\operatorname{atanh}^2}{4c^2} \\ \frac{a^2x^4}{4}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c*x**2))**2,x)
```

```
[Out] Piecewise((a**2*x**4/4 + a*b*x**4*atanh(c*x**2)/2 + a*b*x**2/(2*c) - a*b*at
anh(c*x**2)/(2*c**2) + b**2*x**4*atanh(c*x**2)**2/4 + b**2*x**2*atanh(c*x**
2)/(2*c) + b**2*log(x - I*sqrt(1/c))/(2*c**2) + b**2*log(x + I*sqrt(1/c))/(
2*c**2) - b**2*atanh(c*x**2)**2/(4*c**2) - b**2*atanh(c*x**2)/(2*c**2), Ne(
c, 0)), (a**2*x**4/4, True))
```

Giac [A] time = 1.38177, size = 186, normalized size = 2.04

$$\frac{1}{4}a^2x^4+\frac{abx^2}{2c}+\frac{1}{16}\left(b^2x^4-\frac{b^2}{c^2}\right)\log\left(-\frac{cx^2+1}{cx^2-1}\right)^2+\frac{1}{4}\left(abx^4+\frac{b^2x^2}{c}\right)\log\left(-\frac{cx^2+1}{cx^2-1}\right)-\frac{\left(ab-b^2\right)\log\left(cx^2+1\right)}{4c^2}+\frac{(ab+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^2))^2,x, algorithm="giac")
```

```
[Out] 1/4*a^2*x^4 + 1/2*a*b*x^2/c + 1/16*(b^2*x^4 - b^2/c^2)*log(-(c*x^2 + 1)/(c*
x^2 - 1))^2 + 1/4*(a*b*x^4 + b^2*x^2/c)*log(-(c*x^2 + 1)/(c*x^2 - 1)) - 1/4
*(a*b - b^2)*log(c*x^2 + 1)/c^2 + 1/4*(a*b + b^2)*log(c*x^2 - 1)/c^2
```

3.67 $\int x\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2\,dx$

Optimal. Leaf size=94

$$-\frac{b^2\text{PolyLog}\left(2,1-\frac{2}{1-cx^2}\right)}{2c}+\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2+\frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{2c}-\frac{b\log\left(\frac{2}{1-cx^2}\right)\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{c}$$

```
[Out] (a + b*ArcTanh[c*x^2])^2/(2*c) + (x^2*(a + b*ArcTanh[c*x^2])^2)/2 - (b*(a +
 b*ArcTanh[c*x^2])*Log[2/(1 - c*x^2)])/c - (b^2*PolyLog[2, 1 - 2/(1 - c*x^2
)])/(2*c)
```

Rubi [B] time = 0.513899, antiderivative size = 207, normalized size of antiderivative = 2.2, number of steps used = 28, number of rules used = 12, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {6099, 2454, 2389, 2296, 2295, 6715, 2430, 43, 2416, 2394, 2393, 2391}

$$-\frac{b^2\text{PolyLog}\left(2,\frac{1}{2}\left(1-cx^2\right)\right)}{4c}+\frac{b^2\text{PolyLog}\left(2,\frac{1}{2}\left(cx^2+1\right)\right)}{4c}+\frac{b\log\left(\frac{1}{2}\left(cx^2+1\right)\right)\left(2a-b\log\left(1-cx^2\right)\right)}{4c}+\frac{1}{4}bx^2\log\left(c\right.$$

Warning: Unable to verify antiderivative.

```
[In] Int[x*(a + b*ArcTanh[c*x^2])^2,x]
```

```
[Out] -((1 - c*x^2)*(2*a - b*Log[1 - c*x^2])^2)/(8*c) + (b*(2*a - b*Log[1 - c*x^2
])*Log[(1 + c*x^2)/2])/(4*c) + (b^2*Log[(1 - c*x^2)/2]*Log[1 + c*x^2])/(4*c
) + (b*x^2*(2*a - b*Log[1 - c*x^2])*Log[1 + c*x^2])/4 + (b^2*(1 + c*x^2)*Lo
g[1 + c*x^2]^2)/(8*c) - (b^2*PolyLog[2, (1 - c*x^2)/2])/(4*c) + (b^2*PolyLo
g[2, (1 + c*x^2)/2])/(4*c)
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 2430

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.)), x_Symbol] :> Simp[x*(a + b*Log[c
*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]), x] + (-Dist[g*j*m, Int[(x*(a +
 b*Log[c*(d + e*x)^n])^p)/(i + j*x), x], x] - Dist[b*e*n*p, Int[(x*(a + b*L
og[c*(d + e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /
; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int x\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2\,dx=\int\left(\frac{1}{4}x\left(2a-b\log\left(1-cx^2\right)\right)^2-\frac{1}{2}bx\left(-2a+b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right)+\frac{1}{4}b^2x\log\right.$$
$$=\frac{1}{4}\int x\left(2a-b\log\left(1-cx^2\right)\right)^2\,dx-\frac{1}{2}b\int x\left(-2a+b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right)\,dx+$$
$$=\frac{1}{8}\text{Subst}\left(\int(2a-b\log(1-cx))^2\,dx,x,x^2\right)-\frac{1}{4}b\,\text{Subst}\left(\int(-2a+b\log(1-cx))\log(1\right.$$
$$=\frac{1}{4}bx^2\left(2a-b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right)-\frac{\text{Subst}\left(\int(2a-b\log(x))^2\,dx,x,1-cx^2\right)}{8c}+$$
$$=-\frac{\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^2}{8c}+\frac{1}{4}bx^2\left(2a-b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right)+\frac{b^2(1}{}$$
$$=\frac{1}{2}abx^2+\frac{b^2x^2}{4}-\frac{\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^2}{8c}-\frac{b^2\left(1+cx^2\right)\log\left(1+cx^2\right)}{4c}+\frac{1}{4}bx^2$$
$$=\frac{b^2x^2}{2}+\frac{b^2\left(1-cx^2\right)\log\left(1-cx^2\right)}{4c}-\frac{\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^2}{8c}+\frac{b\left(2a-b\log\right.}{}$$
$$=\frac{b^2x^2}{4}+\frac{b^2\left(1-cx^2\right)\log\left(1-cx^2\right)}{4c}-\frac{\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^2}{8c}+\frac{b\left(2a-b\log\right.}{}$$
$$=-\frac{\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^2}{8c}+\frac{b\left(2a-b\log\left(1-cx^2\right)\right)\log\left(\frac{1}{2}\left(1+cx^2\right)\right)}{4c}+\frac{b^2\log}{}$$

Mathematica [A] time = 0.0656213, size = 99, normalized size = 1.05

$$\frac{b^2\text{PolyLog}\left(2,-e^{-2\tanh^{-1}\left(cx^2\right)}\right)+a\left(acx^2+b\log\left(1-c^2x^4\right)\right)+2b\tanh^{-1}\left(cx^2\right)\left(acx^2-b\log\left(e^{-2\tanh^{-1}\left(cx^2\right)}+1\right)\right)+b^2}{2c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(a + b*ArcTanh[c*x^2])^2,x]
```

```
[Out] (b^2*(-1 + c*x^2)*ArcTanh[c*x^2]^2 + 2*b*ArcTanh[c*x^2]*(a*c*x^2 - b*Log[1
+ E^(-2*ArcTanh[c*x^2])]) + a*(a*c*x^2 + b*Log[1 - c^2*x^4]) + b^2*PolyLog[
2, -E^(-2*ArcTanh[c*x^2])])/(2*c)
```

Maple [A] time = 0.035, size = 144, normalized size = 1.5

$$\frac{\left(\text{Artanh}\left(cx^2\right)\right)^2x^2b^2}{2}+abx^2\text{Artanh}\left(cx^2\right)+\frac{a^2x^2}{2}+\frac{b^2\left(\text{Artanh}\left(cx^2\right)\right)^2}{2c}-\frac{\text{Artanh}\left(cx^2\right)b^2}{c}\ln\left(\frac{\left(cx^2+1\right)^2}{-c^2x^4+1}+1\right)-\frac{b}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c*x^2))^2,x)
```

```
[Out] 1/2*arctanh(c*x^2)^2*x^2*b^2+a*b*x^2*arctanh(c*x^2)+1/2*a^2*x^2+1/2/c*b^2*a
rctanh(c*x^2)^2-1/c*arctanh(c*x^2)*ln((c*x^2+1)^2/(-c^2*x^4+1)+1)*b^2-1/2/c
*polylog(2,-(c*x^2+1)^2/(-c^2*x^4+1))*b^2+1/2/c*a*b*ln(-c^2*x^4+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}a^2x^2 + \frac{1}{8}\left(x^2\log\left(-cx^2+1\right)^2 - c^2\left(\frac{2x^2}{c^2} - \frac{\log\left(cx^2+1\right)}{c^3} + \frac{\log\left(cx^2-1\right)}{c^3}\right) - 2c\left(\frac{x^2}{c} + \frac{\log\left(cx^2-1\right)}{c^2}\right)\log\left(-cx^2+1\right) + 12$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^2))^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*x^2 + 1/8*(x^2*log(-c*x^2 + 1)^2 - c^2*(2*x^2/c^2 - log(c*x^2 + 1)/
c^3 + log(c*x^2 - 1)/c^3) - 2*c*(x^2/c + log(c*x^2 - 1)/c^2)*log(-c*x^2 + 1
) + 12*c*integrate(x^3*log(c*x^2 + 1)/(c^2*x^4 - 1), x) + (c*x^2*log(c*x^2
+ 1)^2 + 2*(c*x^2 - (c*x^2 + 1)*log(c*x^2 + 1))*log(-c*x^2 + 1))/c + (2*c*x
^2 + log(c*x^2 - 1)^2 + 2*log(c*x^2 - 1))/c - log(c^2*x^4 - 1)/c + 4*integr
ate(x*log(c*x^2 + 1)/(c^2*x^4 - 1), x))*b^2 + 1/2*(2*c*x^2*arctanh(c*x^2) +
 log(-c^2*x^4 + 1))*a*b/c
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(b^2x\,\text{artanh}\left(cx^2\right)^2 + 2\,abx\,\text{artanh}\left(cx^2\right) + a^2x, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^2))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x*arctanh(c*x^2)^2 + 2*a*b*x*arctanh(c*x^2) + a^2*x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(a + b\,\text{atanh}\left(cx^2\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c*x**2))**2,x)
```

```
[Out] Integral(x*(a + b*atanh(c*x**2))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\,\text{artanh}\left(cx^2\right) + a\right)^2 x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^2))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^2) + a)^2*x, x)
```

3.68 $\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{x}\,dx$

Optimal. Leaf size=137

$$-\frac{1}{2}b\text{PolyLog}\left(2,1-\frac{2}{1-cx^2}\right)\left(a+b\tanh^{-1}\left(cx^2\right)\right)+\frac{1}{2}b\text{PolyLog}\left(2,\frac{2}{1-cx^2}-1\right)\left(a+b\tanh^{-1}\left(cx^2\right)\right)+\frac{1}{4}b^2\text{PolyLo}$$

```
[Out] (a + b*ArcTanh[c*x^2])^2*ArcTanh[1 - 2/(1 - c*x^2)] - (b*(a + b*ArcTanh[c*x
^2])*PolyLog[2, 1 - 2/(1 - c*x^2)])/2 + (b*(a + b*ArcTanh[c*x^2])*PolyLog[2
, -1 + 2/(1 - c*x^2)])/2 + (b^2*PolyLog[3, 1 - 2/(1 - c*x^2)])/4 - (b^2*Pol
yLog[3, -1 + 2/(1 - c*x^2)])/4
```

Rubi [A] time = 0.335539, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6095, 5914, 6052, 5948, 6058, 6610}

$$-\frac{1}{2}b\text{PolyLog}\left(2,1-\frac{2}{1-cx^2}\right)\left(a+b\tanh^{-1}\left(cx^2\right)\right)+\frac{1}{2}b\text{PolyLog}\left(2,\frac{2}{1-cx^2}-1\right)\left(a+b\tanh^{-1}\left(cx^2\right)\right)+\frac{1}{4}b^2\text{PolyLo}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^2])^2/x,x]
```


```
[Out] (a + b*ArcTanh[c*x^2])^2*ArcTanh[1 - 2/(1 - c*x^2)] - (b*(a + b*ArcTanh[c*x
^2])*PolyLog[2, 1 - 2/(1 - c*x^2)])/2 + (b*(a + b*ArcTanh[c*x^2])*PolyLog[2
, -1 + 2/(1 - c*x^2)])/2 + (b^2*PolyLog[3, 1 - 2/(1 - c*x^2)])/4 - (b^2*Pol
yLog[3, -1 + 2/(1 - c*x^2)])/4
```


Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[
1/n, Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
 n}, x] && IGtQ[p, 0]
```

Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
 e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{x}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x}\,dx, x, x^2\right)$$

$$= \left(a+b\tanh^{-1}\left(cx^2\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx^2}\right)-(2bc)\,\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(cx)\right)\tanh^{-1}\left(\right.}{1-c^2x^2}\right.$$

$$= \left(a+b\tanh^{-1}\left(cx^2\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx^2}\right)+(bc)\,\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1-cx}\right.}{1-c^2x^2}\right.$$

$$= \left(a+b\tanh^{-1}\left(cx^2\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx^2}\right)-\frac{1}{2}b\left(a+b\tanh^{-1}\left(cx^2\right)\right)\text{Li}_2\left(1-\frac{2}{1-cx^2}\right)+$$

$$= \left(a+b\tanh^{-1}\left(cx^2\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx^2}\right)-\frac{1}{2}b\left(a+b\tanh^{-1}\left(cx^2\right)\right)\text{Li}_2\left(1-\frac{2}{1-cx^2}\right)+$$

Mathematica [A] time = 0.100625, size = 141, normalized size = 1.03

$$\frac{1}{4}b\left(2\text{PolyLog}\left(2,\frac{cx^2+1}{1-cx^2}\right)\left(a+b\tanh^{-1}\left(cx^2\right)\right)-2\text{PolyLog}\left(2,\frac{cx^2+1}{cx^2-1}\right)\left(a+b\tanh^{-1}\left(cx^2\right)\right)+b\left(\text{PolyLog}\left(3,\frac{cx^2+1}{cx^2-1}\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])^2/x,x]
```

```
[Out] (a + b*ArcTanh[c*x^2])^2*ArcTanh[1 + 2/(-1 + c*x^2)] + (b*(2*(a + b*ArcTanh
[c*x^2])*PolyLog[2, (1 + c*x^2)/(1 - c*x^2)] - 2*(a + b*ArcTanh[c*x^2])*Pol
yLog[2, (1 + c*x^2)/(-1 + c*x^2)] + b*(-PolyLog[3, (1 + c*x^2)/(1 - c*x^2)]
 + PolyLog[3, (1 + c*x^2)/(-1 + c*x^2)])))/4
```

Maple [F] time = 0.133, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\text{Artanh}\left(cx^2\right)\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))^2/x,x)
```

```
[Out] int((a+b*arctanh(c*x^2))^2/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$a^2\log(x)+\int\frac{b^2\left(\log\left(cx^2+1\right)-\log\left(-cx^2+1\right)\right)^2}{4x}+\frac{ab\left(\log\left(cx^2+1\right)-\log\left(-cx^2+1\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/x,x, algorithm="maxima")
```

```
[Out] a^2*log(x) + integrate(1/4*b^2*(log(c*x^2 + 1) - log(-c*x^2 + 1))^2/x + a*b
*(log(c*x^2 + 1) - log(-c*x^2 + 1))/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^2\,\mathrm{artanh}\left(cx^2\right)^2+2\,ab\,\mathrm{artanh}\left(cx^2\right)+a^2}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x^2)^2 + 2*a*b*arctanh(c*x^2) + a^2)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+b\,\mathrm{atanh}\left(cx^2\right)\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))**2/x,x)
```

```
[Out] Integral((a + b*atanh(c*x**2))**2/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(b\,\mathrm{artanh}\left(cx^2\right)+a\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^2) + a)^2/x, x)
```

3.69 $\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{x^3}\,dx$

Optimal. Leaf size=87

$$-\frac{1}{2}b^2c\,\text{PolyLog}\left(2,\frac{2}{cx^2+1}-1\right)+\frac{1}{2}c\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2-\frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{2x^2}+bc\log\left(2-\frac{2}{cx^2+1}\right)\left(a+b\tanh^{-1}\right.$$

```
[Out] (c*(a + b*ArcTanh[c*x^2])^2)/2 - (a + b*ArcTanh[c*x^2])^2/(2*x^2) + b*c*(a
+ b*ArcTanh[c*x^2])*Log[2 - 2/(1 + c*x^2)] - (b^2*c*PolyLog[2, -1 + 2/(1 +
c*x^2)])/2
```

Rubi [B] time = 0.63159, antiderivative size = 237, normalized size of antiderivative = 2.72, number of steps used = 24, number of rules used = 13, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.812, Rules used = {6099, 2454, 2397, 2392, 2391, 2395, 36, 29, 31, 2439, 2416, 2394, 2393}

$$-\frac{1}{2}b^2c\,\text{PolyLog}\left(2,-cx^2\right)+\frac{1}{2}b^2c\,\text{PolyLog}\left(2,cx^2\right)+\frac{1}{4}b^2c\,\text{PolyLog}\left(2,\frac{1}{2}\left(1-cx^2\right)\right)-\frac{1}{4}b^2c\,\text{PolyLog}\left(2,\frac{1}{2}\left(cx^2+1\right)\right)-\frac{1}{4}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcTanh[c*x^2])^2/x^3,x]
```

```
[Out] 2*a*b*c*Log[x] - ((1 - c*x^2)*(2*a - b*Log[1 - c*x^2])^2)/(8*x^2) - (b*c*(2
*a - b*Log[1 - c*x^2])*Log[(1 + c*x^2)/2])/4 - (b^2*c*Log[(1 - c*x^2)/2]*Lo
g[1 + c*x^2])/4 - (b*(2*a - b*Log[1 - c*x^2])*Log[1 + c*x^2])/(4*x^2) - (b^
2*(1 + c*x^2)*Log[1 + c*x^2]^2)/(8*x^2) - (b^2*c*PolyLog[2, -(c*x^2)])/2 +
(b^2*c*PolyLog[2, c*x^2])/2 + (b^2*c*PolyLog[2, (1 - c*x^2)/2])/4 - (b^2*c*
PolyLog[2, (1 + c*x^2)/2])/4
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2397

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_))^2, x_Symbol] :> Simp[((d + e*x)*(a + b*Log[c*(d + e*x)^n])^p)/((e*f
 - d*g)*(f + g*x)), x] - Dist[(b*e*n*p)/(e*f - d*g), Int[(a + b*Log[c*(d +
e*x)^n])^(p - 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] &&
 NeQ[e*f - d*g, 0] && GtQ[p, 0]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/(x_), x_Symbol] :> Simp[
(a + b*Log[c*d])*Log[x], x] + Dist[b, Int[Log[1 + (e*x)/d]/x, x], x] /; Fre
eQ[{a, b, c, d, e}, x] && GtQ[c*d, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[(x^
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
 + j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{x^3}\,dx = \int \left(\frac{\left(2a-b\log\left(1-cx^2\right)\right)^2}{4x^3} - \frac{b\left(-2a+b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right)}{2x^3} + \frac{b^2\log^2\left(1+cx^2\right)}{4x^3}\right)$$

$$= \frac{1}{4}\int \frac{\left(2a-b\log\left(1-cx^2\right)\right)^2}{x^3}\,dx - \frac{1}{2}b\int \frac{\left(-2a+b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right)}{x^3}\,dx + \frac{1}{4}b^2\int$$

$$= \frac{1}{8}\operatorname{Subst}\left(\int \frac{(2a-b\log(1-cx))^2}{x^2}\,dx, x, x^2\right) - \frac{1}{4}b\operatorname{Subst}\left(\int \frac{(-2a+b\log(1-cx))\log(1+}{x^2}\right.$$

$$= -\frac{\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^2}{8x^2} - \frac{b\left(2a-b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right)}{4x^2} - \frac{b^2\left(1+cx^2\right)\mathrm{lo}}{8x}$$

$$= abc\log(x) - \frac{\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^2}{8x^2} - \frac{b\left(2a-b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right)}{4x^2} - \frac{b^2}{}$$

$$= abc\log(x) - \frac{\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^2}{8x^2} - \frac{b\left(2a-b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right)}{4x^2} - \frac{b^2}{}$$

$$= 2abc\log(x) - \frac{\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^2}{8x^2} - \frac{1}{4}bc\left(2a-b\log\left(1-cx^2\right)\right)\log\left(\frac{1}{2}\left(1+cx^2\right.\right.$$

$$= 2abc\log(x) - \frac{\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^2}{8x^2} - \frac{1}{4}bc\left(2a-b\log\left(1-cx^2\right)\right)\log\left(\frac{1}{2}\left(1+cx^2\right.\right.$$

$$= 2abc\log(x) - \frac{\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^2}{8x^2} - \frac{1}{4}bc\left(2a-b\log\left(1-cx^2\right)\right)\log\left(\frac{1}{2}\left(1+cx^2\right.\right.$$

Mathematica [A] time = 0.156982, size = 119, normalized size = 1.37

$$\frac{1}{2}b^2c\left(\tanh^{-1}\left(cx^2\right)\left(-\frac{\tanh^{-1}\left(cx^2\right)}{cx^2} + \tanh^{-1}\left(cx^2\right) + 2\log\left(1-e^{-2\tanh^{-1}\left(cx^2\right)}\right)\right) - \operatorname{PolyLog}\left(2, e^{-2\tanh^{-1}\left(cx^2\right)}\right)\right) - \frac{a^2}{2x^2} + a$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])^2/x^3,x]
```

```
[Out] -a^2/(2*x^2) + a*b*c*(-(ArcTanh[c*x^2]/(c*x^2)) + Log[c*x^2] - Log[1 - c^2*
x^4]/2) + (b^2*c*(ArcTanh[c*x^2]*(ArcTanh[c*x^2] - ArcTanh[c*x^2]/(c*x^2) +
 2*Log[1 - E^(-2*ArcTanh[c*x^2])]) - PolyLog[2, E^(-2*ArcTanh[c*x^2])]))/2
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \frac{\left(a+b\operatorname{Artanh}\left(cx^2\right)\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))^2/x^3,x)
```

```
[Out] int((a+b*arctanh(c*x^2))^2/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}\left(c\left(\log\left(c^2x^4-1\right)-\log\left(x^4\right)\right)+\frac{2\,\operatorname{artanh}\left(cx^2\right)}{x^2}\right)ab-\frac{1}{8}\,b^2\left(\frac{\log\left(-cx^2+1\right)^2}{x^2}+2\int-\frac{\left(cx^2-1\right)\log\left(cx^2+1\right)^2+2\left(c\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*(c*(log(c^2*x^4 - 1) - log(x^4)) + 2*arctanh(c*x^2)/x^2)*a*b - 1/8*b^2
*(log(-c*x^2 + 1)^2/x^2 + 2*integrate(-((c*x^2 - 1)*log(c*x^2 + 1)^2 + 2*(c
*x^2 - (c*x^2 - 1)*log(c*x^2 + 1))*log(-c*x^2 + 1))/(c*x^5 - x^3), x)) - 1/
2*a^2/x^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2\operatorname{artanh}\left(cx^2\right)^2+2\,ab\operatorname{artanh}\left(cx^2\right)+a^2}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/x^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x^2)^2 + 2*a*b*arctanh(c*x^2) + a^2)/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+b\operatorname{atanh}\left(cx^2\right)\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))**2/x**3,x)
```

```
[Out] Integral((a + b*atanh(c*x**2))**2/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(b\operatorname{artanh}\left(cx^2\right)+a\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^2) + a)^2/x^3, x)
```

3.70 $\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{x^5}\,dx$

Optimal. Leaf size=88

$$\frac{1}{4}c^2\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2-\frac{bc\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{2x^2}-\frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{4x^4}-\frac{1}{4}b^2c^2\log\left(1-c^2x^4\right)+b^2c^2\log(x)$$

```
[Out] -(b*c*(a + b*ArcTanh[c*x^2]))/(2*x^2) + (c^2*(a + b*ArcTanh[c*x^2])^2)/4 -
(a + b*ArcTanh[c*x^2])^2/(4*x^4) + b^2*c^2*Log[x] - (b^2*c^2*Log[1 - c^2*x^
4])/4
```

Rubi [C] time = 1.05672, antiderivative size = 360, normalized size of antiderivative = 4.09, number of steps used = 46, number of rules used = 23, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.438, Rules used = {6099, 2454, 2398, 2411, 2347, 2344, 2301, 2316, 2315, 2314, 31, 2395, 44, 2439, 2416, 36, 29, 2392, 2391, 2394, 2393, 2410, 2390}

$$-\frac{1}{8}b^2c^2\text{PolyLog}\left(2,\frac{1}{2}\left(1-cx^2\right)\right)-\frac{1}{8}b^2c^2\text{PolyLog}\left(2,\frac{1}{2}\left(cx^2+1\right)\right)+\frac{1}{8}bc^2\log\left(\frac{1}{2}\left(cx^2+1\right)\right)\left(2a-b\log\left(1-cx^2\right)\right)+$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcTanh[c*x^2])^2/x^5,x]
```

```
[Out] b^2*c^2*Log[x] - (b^2*c^2*Log[1 - c*x^2])/8 - (b*c*(2*a - b*Log[1 - c*x^2])
)/(8*x^2) - (b*c*(1 - c*x^2)*(2*a - b*Log[1 - c*x^2]))/(8*x^2) + (c^2*(2*a
- b*Log[1 - c*x^2])^2)/16 - (2*a - b*Log[1 - c*x^2])^2/(16*x^4) + (b*c^2*(2
*a - b*Log[1 - c*x^2])*Log[(1 + c*x^2)/2])/8 - (b^2*c^2*Log[1 + c*x^2])/4 -
 (b^2*c*Log[1 + c*x^2])/(4*x^2) - (b^2*c^2*Log[(1 - c*x^2)/2]*Log[1 + c*x^2
])/8 - (b*(2*a - b*Log[1 - c*x^2])*Log[1 + c*x^2])/(8*x^4) + (b^2*c^2*Log[1
 + c*x^2]^2)/16 - (b^2*Log[1 + c*x^2]^2)/(16*x^4) - (b^2*c^2*PolyLog[2, (1
- c*x^2)/2])/8 - (b^2*c^2*PolyLog[2, (1 + c*x^2)/2])/8
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
```

```
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2316

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[
((a + b*Log[-((c*d)/e)])*Log[d + e*x])/e, x] + Dist[b, Int[Log[-((e*x)/d)]/
(d + e*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[-((c*d)/e), 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
```

```
eQ[q, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[(x^
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
 + j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/(x_), x_Symbol] :> Simp[
(a + b*Log[c*d])*Log[x], x] + Dist[b, Int[Log[1 + (e*x)/d]/x, x], x] /; Fre
eQ[{a, b, c, d, e}, x] && GtQ[c*d, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
```

```
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{x^5}\,dx=\int \left(\frac{\left(2a-b\log\left(1-cx^2\right)\right)^2}{4x^5}-\frac{b\left(-2a+b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right)}{2x^5}+\frac{b^2\log^2\left(1+cx^2\right)}{4x^5}\right)$$

$$=\frac{1}{4}\int \frac{\left(2a-b\log\left(1-cx^2\right)\right)^2}{x^5}\,dx-\frac{1}{2}b\int \frac{\left(-2a+b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right)}{x^5}\,dx+\frac{1}{4}b^2\int$$

$$=\frac{1}{8}\text{Subst}\left(\int \frac{(2a-b\log(1-cx))^2}{x^3}\,dx,x,x^2\right)-\frac{1}{4}b\,\text{Subst}\left(\int \frac{(-2a+b\log(1-cx))\log(1+}{x^3}\right.$$

$$=-\frac{\left(2a-b\log\left(1-cx^2\right)\right)^2}{16x^4}-\frac{b\left(2a-b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right)}{8x^4}-\frac{b^2\log^2\left(1+cx^2\right)}{16x^4}+\frac{1}{8}(b$$

$$=-\frac{\left(2a-b\log\left(1-cx^2\right)\right)^2}{16x^4}-\frac{b\left(2a-b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right)}{8x^4}-\frac{b^2\log^2\left(1+cx^2\right)}{16x^4}-\frac{1}{8}b$$

$$=-\frac{\left(2a-b\log\left(1-cx^2\right)\right)^2}{16x^4}-\frac{b\left(2a-b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right)}{8x^4}-\frac{b^2\log^2\left(1+cx^2\right)}{16x^4}-\frac{1}{8}b$$

$$=-\frac{1}{2}abc^2\log(x)-\frac{bc\left(2a-b\log\left(1-cx^2\right)\right)}{8x^2}-\frac{bc\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)}{8x^2}-\frac{\left(2a-b\log\right.}{}$$

$$=\frac{1}{4}b^2c^2\log(x)-\frac{bc\left(2a-b\log\left(1-cx^2\right)\right)}{8x^2}-\frac{bc\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)}{8x^2}+\frac{1}{16}c^2\left(2a-\right.$$

$$=\frac{1}{2}b^2c^2\log(x)-\frac{1}{8}b^2c^2\log\left(1-cx^2\right)-\frac{bc\left(2a-b\log\left(1-cx^2\right)\right)}{8x^2}-\frac{bc\left(1-cx^2\right)\left(2a-b\log\left(1\right.\right.}{8x^2}$$

Mathematica [A] time = 0.083499, size = 111, normalized size = 1.26

$$\frac{1}{4}\left(-\frac{a^2}{x^4}-bc^2(a+b)\log\left(1-cx^2\right)+bc^2(a-b)\log\left(cx^2+1\right)-\frac{2abc}{x^2}-\frac{2b\tanh^{-1}\left(cx^2\right)\left(a+bcx^2\right)}{x^4}+\frac{b^2\left(c^2x^4-1\right)\tanh^{-1}\left(\right.}{x^4}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])^2/x^5,x]
```

```
[Out] (-(a^2/x^4) - (2*a*b*c)/x^2 - (2*b*(a + b*c*x^2)*ArcTanh[c*x^2])/x^4 + (b^2
*(-1 + c^2*x^4)*ArcTanh[c*x^2]^2)/x^4 + 4*b^2*c^2*Log[x] - b*(a + b)*c^2*Lo
g[1 - c*x^2] + (a - b)*b*c^2*Log[1 + c*x^2])/4
```

Maple [B] time = 0.188, size = 257, normalized size = 2.9

$$\frac{b^2\left(c^2x^4-1\right)\left(\ln\left(cx^2+1\right)\right)^2}{16\,x^4}-\frac{b\left(x^4b\ln\left(-cx^2+1\right)c^2+2\,bcx^2-b\ln\left(-cx^2+1\right)+2\,a\right)\ln\left(cx^2+1\right)}{8\,x^4}+\frac{b^2c^2x^4\left(\ln\left(-cx^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))^2/x^5,x)
```

```
[Out] 1/16*b^2*(c^2*x^4-1)/x^4*ln(c*x^2+1)^2-1/8*b*(x^4*b*ln(-c*x^2+1)*c^2+2*b*c*
x^2-b*ln(-c*x^2+1)+2*a)/x^4*ln(c*x^2+1)+1/16*(b^2*c^2*x^4*ln(-c*x^2+1)^2+4*
b*c^2*ln(c*x^2+1)*x^4*a-4*b^2*c^2*ln(c*x^2+1)*x^4-4*b*c^2*ln(c*x^2-1)*x^4*a
-4*b^2*c^2*ln(c*x^2-1)*x^4+16*b^2*c^2*ln(x)*x^4+4*b^2*c*x^2*ln(-c*x^2+1)-8*
a*b*c*x^2-b^2*ln(-c*x^2+1)^2+4*b*ln(-c*x^2+1)*a-4*a^2)/x^4
```

Maxima [B] time = 1.00288, size = 236, normalized size = 2.68

$$\frac{1}{4}\left(\left(c\log\left(cx^2+1\right)-c\log\left(cx^2-1\right)-\frac{2}{x^2}\right)c-\frac{2\,\operatorname{artanh}\left(cx^2\right)}{x^4}\right)ab+\frac{1}{16}\left(\left(2\left(\log\left(cx^2-1\right)-2\right)\log\left(cx^2+1\right)-\log\left(cx^2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/x^5,x, algorithm="maxima")
```

```
[Out] 1/4*((c*log(c*x^2 + 1) - c*log(c*x^2 - 1) - 2/x^2)*c - 2*arctanh(c*x^2)/x^4
)*a*b + 1/16*((2*(log(c*x^2 - 1) - 2)*log(c*x^2 + 1) - log(c*x^2 + 1)^2 - l
og(c*x^2 - 1)^2 - 4*log(c*x^2 - 1) + 16*log(x))*c^2 + 4*(c*log(c*x^2 + 1) -
 c*log(c*x^2 - 1) - 2/x^2)*c*arctanh(c*x^2))*b^2 - 1/4*b^2*arctanh(c*x^2)^2
/x^4 - 1/4*a^2/x^4
```

Fricas [A] time = 2.16351, size = 324, normalized size = 3.68

$$\frac{16\,b^2c^2x^4\log\left(x\right)+4\left(ab-b^2\right)c^2x^4\log\left(cx^2+1\right)-4\left(ab+b^2\right)c^2x^4\log\left(cx^2-1\right)-8\,abcx^2+\left(b^2c^2x^4-b^2\right)\log\left(-\frac{cx^2+1}{cx^2-1}\right)}{16\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/16*(16*b^2*c^2*x^4*log(x) + 4*(a*b - b^2)*c^2*x^4*log(c*x^2 + 1) - 4*(a*b
 + b^2)*c^2*x^4*log(c*x^2 - 1) - 8*a*b*c*x^2 + (b^2*c^2*x^4 - b^2)*log(-(c*
x^2 + 1)/(c*x^2 - 1))^2 - 4*a^2 - 4*(b^2*c*x^2 + a*b)*log(-(c*x^2 + 1)/(c*x
^2 - 1)))/x^4
```

Sympy [A] time = 26.0661, size = 175, normalized size = 1.99

$$\begin{cases} -\frac{a^2}{4x^4} + \frac{abc^2\,\mathrm{atanh}\left(cx^2\right)}{2} - \frac{abc}{2x^2} - \frac{ab\,\mathrm{atanh}\left(cx^2\right)}{2x^4} + b^2c^2\log\left(x\right) - \frac{b^2c^2\log\left(x-i\sqrt{\frac{1}{c}}\right)}{2} - \frac{b^2c^2\log\left(x+i\sqrt{\frac{1}{c}}\right)}{2} + \frac{b^2c^2\,\mathrm{atanh}^2\left(cx^2\right)}{4} + \frac{b^2c^2\,\mathrm{atanh}\left(cx\right.}{2} \\ -\frac{a^2}{4x^4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))**2/x**5,x)
```

```
[Out] Piecewise((-a**2/(4*x**4) + a*b*c**2*atanh(c*x**2)/2 - a*b*c/(2*x**2) - a*b
*atanh(c*x**2)/(2*x**4) + b**2*c**2*log(x) - b**2*c**2*log(x - I*sqrt(1/c))
/2 - b**2*c**2*log(x + I*sqrt(1/c))/2 + b**2*c**2*atanh(c*x**2)**2/4 + b**2
*c**2*atanh(c*x**2)/2 - b**2*c*atanh(c*x**2)/(2*x**2) - b**2*atanh(c*x**2)*
*2/(4*x**4), Ne(c, 0)), (-a**2/(4*x**4), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\,\mathrm{artanh}\left(cx^2\right) + a\right)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/x^5,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^2) + a)^2/x^5, x)
```

3.71 $\int x^4 \left(a+b\tanh^{-1}\left(cx^2\right)\right)^2 \, dx$

Optimal. Leaf size=1173

result too large to display

```
[Out] (8*b^2*x)/(15*c^2) + (2*a*b*x^3)/(15*c) - (2*a*b*x^5)/25 + (2*a*b*ArcTan[Sq
rt[c]*x])/(5*c^(5/2)) - (4*b^2*ArcTan[Sqrt[c]*x])/(15*c^(5/2)) + ((I/5)*b^2
*ArcTan[Sqrt[c]*x]^2)/c^(5/2) - (4*b^2*ArcTanh[Sqrt[c]*x])/(15*c^(5/2)) - (
b^2*ArcTanh[Sqrt[c]*x]^2)/(5*c^(5/2)) + (2*b^2*ArcTanh[Sqrt[c]*x]*Log[2/(1
- Sqrt[c]*x)])/(5*c^(5/2)) - (2*b^2*ArcTan[Sqrt[c]*x]*Log[2/(1 - I*Sqrt[c]*
x)])/(5*c^(5/2)) + (b^2*ArcTan[Sqrt[c]*x]*Log[((1 + I)*(1 - Sqrt[c]*x))/(1
- I*Sqrt[c]*x)])/(5*c^(5/2)) + (2*b^2*ArcTan[Sqrt[c]*x]*Log[2/(1 + I*Sqrt[c
]*x)])/(5*c^(5/2)) - (2*b^2*ArcTanh[Sqrt[c]*x]*Log[2/(1 + Sqrt[c]*x)])/(5*c
^(5/2)) + (b^2*ArcTanh[Sqrt[c]*x]*Log[(-2*Sqrt[c]*(1 - Sqrt[-c]*x))/((Sqrt[
-c] - Sqrt[c])*(1 + Sqrt[c]*x))])/(5*c^(5/2)) + (b^2*ArcTanh[Sqrt[c]*x]*Log
[(2*Sqrt[c]*(1 + Sqrt[-c]*x))/((Sqrt[-c] + Sqrt[c])*(1 + Sqrt[c]*x))])/(5*c
^(5/2)) + (b^2*ArcTan[Sqrt[c]*x]*Log[((1 - I)*(1 + Sqrt[c]*x))/(1 - I*Sqrt[
c]*x)])/(5*c^(5/2)) - (b^2*x^3*Log[1 - c*x^2])/(15*c) + (b^2*x^5*Log[1 - c*
x^2])/25 - (b^2*ArcTan[Sqrt[c]*x]*Log[1 - c*x^2])/(5*c^(5/2)) + (b*x^3*(2*a
 - b*Log[1 - c*x^2]))/(15*c) + (b*x^5*(2*a - b*Log[1 - c*x^2]))/25 - (b*Arc
Tanh[Sqrt[c]*x]*(2*a - b*Log[1 - c*x^2]))/(5*c^(5/2)) + (x^5*(2*a - b*Log[1
 - c*x^2])^2)/20 + (2*b^2*x^3*Log[1 + c*x^2])/(15*c) + (a*b*x^5*Log[1 + c*x
^2])/5 + (b^2*ArcTan[Sqrt[c]*x]*Log[1 + c*x^2])/(5*c^(5/2)) - (b^2*ArcTanh[
Sqrt[c]*x]*Log[1 + c*x^2])/(5*c^(5/2)) - (b^2*x^5*Log[1 - c*x^2]*Log[1 + c*
x^2])/10 + (b^2*x^5*Log[1 + c*x^2]^2)/20 + (b^2*PolyLog[2, 1 - 2/(1 - Sqrt[
c]*x)])/(5*c^(5/2)) + ((I/5)*b^2*PolyLog[2, 1 - 2/(1 - I*Sqrt[c]*x)])/c^(5/
2) - ((I/10)*b^2*PolyLog[2, 1 - ((1 + I)*(1 - Sqrt[c]*x))/(1 - I*Sqrt[c]*x)
])/c^(5/2) + ((I/5)*b^2*PolyLog[2, 1 - 2/(1 + I*Sqrt[c]*x)])/c^(5/2) + (b^2
*PolyLog[2, 1 - 2/(1 + Sqrt[c]*x)])/(5*c^(5/2)) - (b^2*PolyLog[2, 1 + (2*Sq
rt[c]*(1 - Sqrt[-c]*x))/((Sqrt[-c] - Sqrt[c])*(1 + Sqrt[c]*x))])/(10*c^(5/2
)) - (b^2*PolyLog[2, 1 - (2*Sqrt[c]*(1 + Sqrt[-c]*x))/((Sqrt[-c] + Sqrt[c])
*(1 + Sqrt[c]*x))])/(10*c^(5/2)) - ((I/10)*b^2*PolyLog[2, 1 - ((1 - I)*(1 +
 Sqrt[c]*x))/(1 - I*Sqrt[c]*x)])/c^(5/2)
```

Rubi [A] time = 2.33394, antiderivative size = 1173, normalized size of antiderivative = 1., number of steps used = 102, number of rules used = 26, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.625, Rules used = {6099, 2457, 2476, 2448, 321, 206, 2455, 302, 2470, 12, 5984, 5918, 2402, 2315, 6742, 203, 30, 2557, 207, 5992, 5920, 2447, 4928, 4856, 4920, 4854}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*ArcTanh[c*x^2])^2,x]
```


```
[Out] (8*b^2*x)/(15*c^2) + (2*a*b*x^3)/(15*c) - (2*a*b*x^5)/25 + (2*a*b*ArcTan[Sq
rt[c]*x])/(5*c^(5/2)) - (4*b^2*ArcTan[Sqrt[c]*x])/(15*c^(5/2)) + ((I/5)*b^2
*ArcTan[Sqrt[c]*x]^2)/c^(5/2) - (4*b^2*ArcTanh[Sqrt[c]*x])/(15*c^(5/2)) - (
b^2*ArcTanh[Sqrt[c]*x]^2)/(5*c^(5/2)) + (2*b^2*ArcTanh[Sqrt[c]*x]*Log[2/(1
- Sqrt[c]*x)])/(5*c^(5/2)) - (2*b^2*ArcTan[Sqrt[c]*x]*Log[2/(1 - I*Sqrt[c]*
x)])/(5*c^(5/2)) + (b^2*ArcTan[Sqrt[c]*x]*Log[((1 + I)*(1 - Sqrt[c]*x))/(1
- I*Sqrt[c]*x)])/(5*c^(5/2)) + (2*b^2*ArcTan[Sqrt[c]*x]*Log[2/(1 + I*Sqrt[c
]*x)])/(5*c^(5/2)) - (2*b^2*ArcTanh[Sqrt[c]*x]*Log[2/(1 + Sqrt[c]*x)])/(5*c
^(5/2)) + (b^2*ArcTanh[Sqrt[c]*x]*Log[(-2*Sqrt[c]*(1 - Sqrt[-c]*x))/((Sqrt[
-c] - Sqrt[c])*(1 + Sqrt[c]*x))])/(5*c^(5/2)) + (b^2*ArcTanh[Sqrt[c]*x]*Log
```

```
[(2*Sqrt[c]*(1 + Sqrt[-c]*x))/((Sqrt[-c] + Sqrt[c])*(1 + Sqrt[c]*x))])/(5*c
^(5/2)) + (b^2*ArcTan[Sqrt[c]*x]*Log[((1 - I)*(1 + Sqrt[c]*x))/(1 - I*Sqrt[
c]*x)])/(5*c^(5/2)) - (b^2*x^3*Log[1 - c*x^2])/(15*c) + (b^2*x^5*Log[1 - c*
x^2])/25 - (b^2*ArcTan[Sqrt[c]*x]*Log[1 - c*x^2])/(5*c^(5/2)) + (b*x^3*(2*a
 - b*Log[1 - c*x^2]))/(15*c) + (b*x^5*(2*a - b*Log[1 - c*x^2]))/25 - (b*Arc
Tanh[Sqrt[c]*x]*(2*a - b*Log[1 - c*x^2]))/(5*c^(5/2)) + (x^5*(2*a - b*Log[1
 - c*x^2])^2)/20 + (2*b^2*x^3*Log[1 + c*x^2])/(15*c) + (a*b*x^5*Log[1 + c*x
^2])/5 + (b^2*ArcTan[Sqrt[c]*x]*Log[1 + c*x^2])/(5*c^(5/2)) - (b^2*ArcTanh[
Sqrt[c]*x]*Log[1 + c*x^2])/(5*c^(5/2)) - (b^2*x^5*Log[1 - c*x^2]*Log[1 + c*
x^2])/10 + (b^2*x^5*Log[1 + c*x^2]^2)/20 + (b^2*PolyLog[2, 1 - 2/(1 - Sqrt[
c]*x)])/(5*c^(5/2)) + ((I/5)*b^2*PolyLog[2, 1 - 2/(1 - I*Sqrt[c]*x)])/c^(5/
2) - ((I/10)*b^2*PolyLog[2, 1 - ((1 + I)*(1 - Sqrt[c]*x))/(1 - I*Sqrt[c]*x)
])/c^(5/2) + ((I/5)*b^2*PolyLog[2, 1 - 2/(1 + I*Sqrt[c]*x)])/c^(5/2) + (b^2
*PolyLog[2, 1 - 2/(1 + Sqrt[c]*x)])/(5*c^(5/2)) - (b^2*PolyLog[2, 1 + (2*Sq
rt[c]*(1 - Sqrt[-c]*x))/((Sqrt[-c] - Sqrt[c])*(1 + Sqrt[c]*x))])/(10*c^(5/2
)) - (b^2*PolyLog[2, 1 - (2*Sqrt[c]*(1 + Sqrt[-c]*x))/((Sqrt[-c] + Sqrt[c])
*(1 + Sqrt[c]*x))])/(10*c^(5/2)) - ((I/10)*b^2*PolyLog[2, 1 - ((1 - I)*(1 +
 Sqrt[c]*x))/(1 - I*Sqrt[c]*x)])/c^(5/2)
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q
)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a +
 b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
```

```
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2557

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] :> With[{z = IntHide[u, x]}, Dist[Log[v
]*Log[w], z, x] + (-Int[SimplifyIntegrand[(z*Log[w]*D[v, x])/v, x], x] - In
t[SimplifyIntegrand[(z*Log[v]*D[w, x])/w, x], x]) /; InverseFunctionFreeQ[z
, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 5992

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
```

```
c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rubi steps

$$\int x^4 \left(a+b\tanh^{-1}\left(cx^2\right)\right)^2 dx = \int \left(\frac{1}{4}x^4\left(2a-b\log\left(1-cx^2\right)\right)^2 - \frac{1}{2}bx^4\left(-2a+b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right)+\frac{1}{4}b^2x^4\log\right.$$

$$= \frac{1}{4}\int x^4\left(2a-b\log\left(1-cx^2\right)\right)^2 dx - \frac{1}{2}b\int x^4\left(-2a+b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right)dx +$$

$$= \frac{1}{20}x^5\left(2a-b\log\left(1-cx^2\right)\right)^2 + \frac{1}{20}b^2x^5\log^2\left(1+cx^2\right) - \frac{1}{2}b\int\left(-2ax^4\log\left(1+cx^2\right)+bx\right.$$

$$= \frac{1}{20}x^5\left(2a-b\log\left(1-cx^2\right)\right)^2 + \frac{1}{20}b^2x^5\log^2\left(1+cx^2\right) + (ab)\int x^4\log\left(1+cx^2\right)dx - \frac{1}{2}b$$

$$= \frac{1}{20}x^5\left(2a-b\log\left(1-cx^2\right)\right)^2 + \frac{1}{5}abx^5\log\left(1+cx^2\right) - \frac{1}{10}b^2x^5\log\left(1-cx^2\right)\log\left(1+cx^2\right)$$

$$= \frac{2abx}{5c^2} + \frac{bx^3\left(2a-b\log\left(1-cx^2\right)\right)}{15c} + \frac{1}{25}bx^5\left(2a-b\log\left(1-cx^2\right)\right) - \frac{b\tanh^{-1}\left(\sqrt{c}x\right)\left(2a\right.}{5c^5}$$

$$= \frac{2b^2x}{5c^2} + \frac{2abx^3}{15c} - \frac{2}{25}abx^5 - \frac{b^2x\log\left(1-cx^2\right)}{5c^2} + \frac{bx^3\left(2a-b\log\left(1-cx^2\right)\right)}{15c} + \frac{1}{25}bx^5\left(2a-\right.$$

$$= \frac{92b^2x}{75c^2} + \frac{2abx^3}{15c} - \frac{2}{25}abx^5 + \frac{4b^2x^5}{125} + \frac{2ab\tan^{-1}\left(\sqrt{c}x\right)}{5c^{5/2}} - \frac{2b^2\tan^{-1}\left(\sqrt{c}x\right)}{5c^{5/2}} + \frac{ib^2\tan^{-1}\left(\right.}{5c^{5/2}}$$

$$= \frac{92b^2x}{75c^2} + \frac{2abx^3}{15c} - \frac{2}{25}abx^5 + \frac{4b^2x^5}{125} + \frac{2ab\tan^{-1}\left(\sqrt{c}x\right)}{5c^{5/2}} - \frac{46b^2\tan^{-1}\left(\sqrt{c}x\right)}{75c^{5/2}} + \frac{ib^2\tan^{-1}}{5c^5}$$

$$= \frac{32b^2x}{75c^2} + \frac{2abx^3}{15c} - \frac{2}{25}abx^5 + \frac{4b^2x^5}{125} + \frac{2ab\tan^{-1}\left(\sqrt{c}x\right)}{5c^{5/2}} - \frac{46b^2\tan^{-1}\left(\sqrt{c}x\right)}{75c^{5/2}} + \frac{ib^2\tan^{-1}}{5c^5}$$

$$= \frac{8b^2x}{15c^2} + \frac{2abx^3}{15c} - \frac{2}{25}abx^5 + \frac{2ab\tan^{-1}\left(\sqrt{c}x\right)}{5c^{5/2}} - \frac{16b^2\tan^{-1}\left(\sqrt{c}x\right)}{75c^{5/2}} + \frac{ib^2\tan^{-1}\left(\sqrt{c}x\right)^2}{5c^{5/2}} -$$

$$= \frac{8b^2x}{15c^2} + \frac{2abx^3}{15c} - \frac{2}{25}abx^5 + \frac{2ab\tan^{-1}\left(\sqrt{c}x\right)}{5c^{5/2}} - \frac{4b^2\tan^{-1}\left(\sqrt{c}x\right)}{15c^{5/2}} + \frac{ib^2\tan^{-1}\left(\sqrt{c}x\right)^2}{5c^{5/2}} -$$

$$= \frac{8b^2x}{15c^2} + \frac{2abx^3}{15c} - \frac{2}{25}abx^5 + \frac{2ab\tan^{-1}\left(\sqrt{c}x\right)}{5c^{5/2}} - \frac{4b^2\tan^{-1}\left(\sqrt{c}x\right)}{15c^{5/2}} + \frac{ib^2\tan^{-1}\left(\sqrt{c}x\right)^2}{5c^{5/2}} -$$

$$= \frac{8b^2x}{15c^2} + \frac{2abx^3}{15c} - \frac{2}{25}abx^5 + \frac{2ab\tan^{-1}\left(\sqrt{c}x\right)}{5c^{5/2}} - \frac{4b^2\tan^{-1}\left(\sqrt{c}x\right)}{15c^{5/2}} + \frac{ib^2\tan^{-1}\left(\sqrt{c}x\right)^2}{5c^{5/2}} -$$

$$= \frac{8b^2x}{15c^2} + \frac{2abx^3}{15c} - \frac{2}{25}abx^5 + \frac{2ab\tan^{-1}\left(\sqrt{c}x\right)}{5c^{5/2}} - \frac{4b^2\tan^{-1}\left(\sqrt{c}x\right)}{15c^{5/2}} + \frac{ib^2\tan^{-1}\left(\sqrt{c}x\right)^2}{5c^{5/2}} -$$

Mathematica [F] time = 9.30766, size = 0, normalized size = 0.

$$\int x^4\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2 dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^4*(a + b*ArcTanh[c*x^2])^2,x]
```

```
[Out] Integrate[x^4*(a + b*ArcTanh[c*x^2])^2, x]
```

Maple [F] time = 0.216, size = 0, normalized size = 0.

$$\int x^4 \left(a + b\mathrm{Artanh}\left(c x^2\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arctanh(c*x^2))^2,x)
```

```
[Out] int(x^4*(a+b*arctanh(c*x^2))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x^2))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(b^2 x^4 \operatorname{artanh}\left(c x^2\right)^2 + 2\, a b x^4 \operatorname{artanh}\left(c x^2\right) + a^2 x^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x^2))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^4*arctanh(c*x^2)^2 + 2*a*b*x^4*arctanh(c*x^2) + a^2*x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \left(a + b \operatorname{atanh}\left(c x^2\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*atanh(c*x**2))**2,x)
```

```
[Out] Integral(x**4*(a + b*atanh(c*x**2))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \operatorname{artanh}\left(c x^2\right) + a\right)^2 x^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x^2))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^2) + a)^2*x^4, x)
```

3.72 $\int x^2 \left(a+b \tanh^{-1}\left(cx^2\right)\right)^2 dx$

Optimal. Leaf size=1129

result too large to display

```
[Out] (4*a*b*x)/(3*c) - (2*a*b*x^3)/9 - (2*a*b*ArcTan[Sqrt[c]*x])/(3*c^(3/2)) + (
4*b^2*ArcTan[Sqrt[c]*x])/(3*c^(3/2)) - ((I/3)*b^2*ArcTan[Sqrt[c]*x]^2)/c^(3
/2) - (4*b^2*ArcTanh[Sqrt[c]*x])/(3*c^(3/2)) - (b^2*ArcTanh[Sqrt[c]*x]^2)/(
3*c^(3/2)) + (2*b^2*ArcTanh[Sqrt[c]*x]*Log[2/(1 - Sqrt[c]*x)])/(3*c^(3/2))
+ (2*b^2*ArcTan[Sqrt[c]*x]*Log[2/(1 - I*Sqrt[c]*x)])/(3*c^(3/2)) - (b^2*Arc
Tan[Sqrt[c]*x]*Log[((1 + I)*(1 - Sqrt[c]*x))/(1 - I*Sqrt[c]*x)])/(3*c^(3/2)
) - (2*b^2*ArcTan[Sqrt[c]*x]*Log[2/(1 + I*Sqrt[c]*x)])/(3*c^(3/2)) - (2*b^2
*ArcTanh[Sqrt[c]*x]*Log[2/(1 + Sqrt[c]*x)])/(3*c^(3/2)) + (b^2*ArcTanh[Sqrt
[c]*x]*Log[(-2*Sqrt[c]*(1 - Sqrt[-c]*x))/((Sqrt[-c] - Sqrt[c])*(1 + Sqrt[c]
*x))])/(3*c^(3/2)) + (b^2*ArcTanh[Sqrt[c]*x]*Log[(2*Sqrt[c]*(1 + Sqrt[-c]*x
))/((Sqrt[-c] + Sqrt[c])*(1 + Sqrt[c]*x))])/(3*c^(3/2)) - (b^2*ArcTan[Sqrt[
c]*x]*Log[((1 - I)*(1 + Sqrt[c]*x))/(1 - I*Sqrt[c]*x)])/(3*c^(3/2)) - (2*b^
2*x*Log[1 - c*x^2])/(3*c) + (b^2*x^3*Log[1 - c*x^2])/9 + (b^2*ArcTan[Sqrt[c
]*x]*Log[1 - c*x^2])/(3*c^(3/2)) + (b*x^3*(2*a - b*Log[1 - c*x^2]))/9 - (b*
ArcTanh[Sqrt[c]*x]*(2*a - b*Log[1 - c*x^2]))/(3*c^(3/2)) + (x^3*(2*a - b*Lo
g[1 - c*x^2])^2)/12 + (2*b^2*x*Log[1 + c*x^2])/(3*c) + (a*b*x^3*Log[1 + c*x
^2])/3 - (b^2*ArcTan[Sqrt[c]*x]*Log[1 + c*x^2])/(3*c^(3/2)) - (b^2*ArcTanh[
Sqrt[c]*x]*Log[1 + c*x^2])/(3*c^(3/2)) - (b^2*x^3*Log[1 - c*x^2]*Log[1 + c*
x^2])/6 + (b^2*x^3*Log[1 + c*x^2]^2)/12 + (b^2*PolyLog[2, 1 - 2/(1 - Sqrt[c
]*x)])/(3*c^(3/2)) - ((I/3)*b^2*PolyLog[2, 1 - 2/(1 - I*Sqrt[c]*x)])/c^(3/2
) + ((I/6)*b^2*PolyLog[2, 1 - ((1 + I)*(1 - Sqrt[c]*x))/(1 - I*Sqrt[c]*x)])
/c^(3/2) - ((I/3)*b^2*PolyLog[2, 1 - 2/(1 + I*Sqrt[c]*x)])/c^(3/2) + (b^2*P
olyLog[2, 1 - 2/(1 + Sqrt[c]*x)])/(3*c^(3/2)) - (b^2*PolyLog[2, 1 + (2*Sqrt
[c]*(1 - Sqrt[-c]*x))/((Sqrt[-c] - Sqrt[c])*(1 + Sqrt[c]*x))])/(6*c^(3/2))
- (b^2*PolyLog[2, 1 - (2*Sqrt[c]*(1 + Sqrt[-c]*x))/((Sqrt[-c] + Sqrt[c])*(1
 + Sqrt[c]*x))])/(6*c^(3/2)) + ((I/6)*b^2*PolyLog[2, 1 - ((1 - I)*(1 + Sqrt
[c]*x))/(1 - I*Sqrt[c]*x)])/c^(3/2)
```

Rubi [A] time = 2.06081, antiderivative size = 1129, normalized size of antiderivative = 1., number of steps used = 86, number of rules used = 26, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.625, Rules used = {6099, 2457, 2476, 2448, 321, 206, 2455, 302, 2470, 12, 5984, 5918, 2402, 2315, 6742, 203, 30, 2557, 207, 5992, 5920, 2447, 4928, 4856, 4920, 4854}

$$\frac{1}{12}\left(2a-b\log\left(1-cx^2\right)\right)^2 x^3+\frac{1}{12}b^2\log^2\left(cx^2+1\right)x^3-\frac{2}{9}abx^3+\frac{1}{9}b^2\log\left(1-cx^2\right)x^3+\frac{1}{9}b\left(2a-b\log\left(1-cx^2\right)\right)x^3+\frac{1}{3}a$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcTanh[c*x^2])^2,x]
```


```
[Out] (4*a*b*x)/(3*c) - (2*a*b*x^3)/9 - (2*a*b*ArcTan[Sqrt[c]*x])/(3*c^(3/2)) + (
4*b^2*ArcTan[Sqrt[c]*x])/(3*c^(3/2)) - ((I/3)*b^2*ArcTan[Sqrt[c]*x]^2)/c^(3
/2) - (4*b^2*ArcTanh[Sqrt[c]*x])/(3*c^(3/2)) - (b^2*ArcTanh[Sqrt[c]*x]^2)/(
3*c^(3/2)) + (2*b^2*ArcTanh[Sqrt[c]*x]*Log[2/(1 - Sqrt[c]*x)])/(3*c^(3/2))
+ (2*b^2*ArcTan[Sqrt[c]*x]*Log[2/(1 - I*Sqrt[c]*x)])/(3*c^(3/2)) - (b^2*Arc
Tan[Sqrt[c]*x]*Log[((1 + I)*(1 - Sqrt[c]*x))/(1 - I*Sqrt[c]*x)])/(3*c^(3/2)
) - (2*b^2*ArcTan[Sqrt[c]*x]*Log[2/(1 + I*Sqrt[c]*x)])/(3*c^(3/2)) - (2*b^2
*ArcTanh[Sqrt[c]*x]*Log[2/(1 + Sqrt[c]*x)])/(3*c^(3/2)) + (b^2*ArcTanh[Sqrt
[c]*x]*Log[(-2*Sqrt[c]*(1 - Sqrt[-c]*x))/((Sqrt[-c] - Sqrt[c])*(1 + Sqrt[c]
```

```
*x))])/(3*c^(3/2)) + (b^2*ArcTanh[Sqrt[c]*x]*Log[(2*Sqrt[c]*(1 + Sqrt[-c]*x
))/((Sqrt[-c] + Sqrt[c])*(1 + Sqrt[c]*x))])/(3*c^(3/2)) - (b^2*ArcTan[Sqrt[
c]*x]*Log[((1 - I)*(1 + Sqrt[c]*x))/(1 - I*Sqrt[c]*x)])/(3*c^(3/2)) - (2*b^
2*x*Log[1 - c*x^2])/(3*c) + (b^2*x^3*Log[1 - c*x^2])/9 + (b^2*ArcTan[Sqrt[c
]*x]*Log[1 - c*x^2])/(3*c^(3/2)) + (b*x^3*(2*a - b*Log[1 - c*x^2]))/9 - (b*
ArcTanh[Sqrt[c]*x]*(2*a - b*Log[1 - c*x^2]))/(3*c^(3/2)) + (x^3*(2*a - b*Lo
g[1 - c*x^2])^2)/12 + (2*b^2*x*Log[1 + c*x^2])/(3*c) + (a*b*x^3*Log[1 + c*x
^2])/3 - (b^2*ArcTan[Sqrt[c]*x]*Log[1 + c*x^2])/(3*c^(3/2)) - (b^2*ArcTanh[
Sqrt[c]*x]*Log[1 + c*x^2])/(3*c^(3/2)) - (b^2*x^3*Log[1 - c*x^2]*Log[1 + c*
x^2])/6 + (b^2*x^3*Log[1 + c*x^2]^2)/12 + (b^2*PolyLog[2, 1 - 2/(1 - Sqrt[c
]*x)])/(3*c^(3/2)) - ((I/3)*b^2*PolyLog[2, 1 - 2/(1 - I*Sqrt[c]*x)])/c^(3/2
) + ((I/6)*b^2*PolyLog[2, 1 - ((1 + I)*(1 - Sqrt[c]*x))/(1 - I*Sqrt[c]*x)])
/c^(3/2) - ((I/3)*b^2*PolyLog[2, 1 - 2/(1 + I*Sqrt[c]*x)])/c^(3/2) + (b^2*P
olyLog[2, 1 - 2/(1 + Sqrt[c]*x)])/(3*c^(3/2)) - (b^2*PolyLog[2, 1 + (2*Sqrt
[c]*(1 - Sqrt[-c]*x))/((Sqrt[-c] - Sqrt[c])*(1 + Sqrt[c]*x))])/(6*c^(3/2))
- (b^2*PolyLog[2, 1 - (2*Sqrt[c]*(1 + Sqrt[-c]*x))/((Sqrt[-c] + Sqrt[c])*(1
 + Sqrt[c]*x))])/(6*c^(3/2)) + ((I/6)*b^2*PolyLog[2, 1 - ((1 - I)*(1 + Sqrt
[c]*x))/(1 - I*Sqrt[c]*x)])/c^(3/2)
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q
)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a +
 b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
```

```
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2557

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] :> With[{z = IntHide[u, x]}, Dist[Log[v
]*Log[w], z, x] + (-Int[SimplifyIntegrand[(z*Log[w]*D[v, x])/v, x], x] - In
t[SimplifyIntegrand[(z*Log[v]*D[w, x])/w, x], x]) /; InverseFunctionFreeQ[z
, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 5992

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
```

```
c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rubi steps

$$\int x^2\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2 dx = \int\left(\frac{1}{4}x^2\left(2a-b\log\left(1-cx^2\right)\right)^2-\frac{1}{2}bx^2\left(-2a+b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right)+\frac{1}{4}b^2x^2\right.$$

$$=\frac{1}{4}\int x^2\left(2a-b\log\left(1-cx^2\right)\right)^2 dx-\frac{1}{2}b\int x^2\left(-2a+b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right) dx$$

$$=\frac{1}{12}x^3\left(2a-b\log\left(1-cx^2\right)\right)^2+\frac{1}{12}b^2x^3\log^2\left(1+cx^2\right)-\frac{1}{2}b\int\left(-2ax^2\log\left(1+cx^2\right)+\right.$$

$$=\frac{1}{12}x^3\left(2a-b\log\left(1-cx^2\right)\right)^2+\frac{1}{12}b^2x^3\log^2\left(1+cx^2\right)+(ab)\int x^2\log\left(1+cx^2\right) dx-$$

$$=\frac{1}{12}x^3\left(2a-b\log\left(1-cx^2\right)\right)^2+\frac{1}{3}abx^3\log\left(1+cx^2\right)-\frac{1}{6}b^2x^3\log\left(1-cx^2\right)\log\left(1+cx^2\right.$$

$$=\frac{2abx}{3c}+\frac{1}{9}bx^3\left(2a-b\log\left(1-cx^2\right)\right)-\frac{b\tanh^{-1}\left(\sqrt{c}x\right)\left(2a-b\log\left(1-cx^2\right)\right)}{3c^{3/2}}+\frac{1}{12}x^3$$

$$=\frac{4abx}{3c}-\frac{2b^2x}{3c}-\frac{2}{9}abx^3-\frac{b^2x\log\left(1-cx^2\right)}{3c}+\frac{1}{9}bx^3\left(2a-b\log\left(1-cx^2\right)\right)-\frac{b\tanh^{-1}}{}$$

$$=\frac{4abx}{3c}-\frac{2}{9}abx^3+\frac{4b^2x^3}{27}-\frac{2ab\tan^{-1}\left(\sqrt{c}x\right)}{3c^{3/2}}+\frac{2b^2\tan^{-1}\left(\sqrt{c}x\right)}{3c^{3/2}}-\frac{ib^2\tan^{-1}\left(\sqrt{c}x\right)^2}{3c^{3/2}}-$$

$$=\frac{4abx}{3c}-\frac{2}{9}abx^3+\frac{4b^2x^3}{27}-\frac{2ab\tan^{-1}\left(\sqrt{c}x\right)}{3c^{3/2}}+\frac{8b^2\tan^{-1}\left(\sqrt{c}x\right)}{9c^{3/2}}-\frac{ib^2\tan^{-1}\left(\sqrt{c}x\right)^2}{3c^{3/2}}-$$

$$=\frac{4abx}{3c}-\frac{2}{9}abx^3+\frac{4b^2x^3}{27}-\frac{2ab\tan^{-1}\left(\sqrt{c}x\right)}{3c^{3/2}}+\frac{8b^2\tan^{-1}\left(\sqrt{c}x\right)}{9c^{3/2}}-\frac{ib^2\tan^{-1}\left(\sqrt{c}x\right)^2}{3c^{3/2}}-$$

$$=\frac{4abx}{3c}-\frac{2}{9}abx^3-\frac{2ab\tan^{-1}\left(\sqrt{c}x\right)}{3c^{3/2}}+\frac{14b^2\tan^{-1}\left(\sqrt{c}x\right)}{9c^{3/2}}-\frac{ib^2\tan^{-1}\left(\sqrt{c}x\right)^2}{3c^{3/2}}-\frac{14b^2\tan}{}$$

$$=\frac{4abx}{3c}-\frac{2}{9}abx^3-\frac{2ab\tan^{-1}\left(\sqrt{c}x\right)}{3c^{3/2}}+\frac{4b^2\tan^{-1}\left(\sqrt{c}x\right)}{3c^{3/2}}-\frac{ib^2\tan^{-1}\left(\sqrt{c}x\right)^2}{3c^{3/2}}-\frac{4b^2\tan}{3}$$

$$=\frac{4abx}{3c}-\frac{2}{9}abx^3-\frac{2ab\tan^{-1}\left(\sqrt{c}x\right)}{3c^{3/2}}+\frac{4b^2\tan^{-1}\left(\sqrt{c}x\right)}{3c^{3/2}}-\frac{ib^2\tan^{-1}\left(\sqrt{c}x\right)^2}{3c^{3/2}}-\frac{4b^2\tan}{3}$$

$$=\frac{4abx}{3c}-\frac{2}{9}abx^3-\frac{2ab\tan^{-1}\left(\sqrt{c}x\right)}{3c^{3/2}}+\frac{4b^2\tan^{-1}\left(\sqrt{c}x\right)}{3c^{3/2}}-\frac{ib^2\tan^{-1}\left(\sqrt{c}x\right)^2}{3c^{3/2}}-\frac{4b^2\tan}{3}$$

$$=\frac{4abx}{3c}-\frac{2}{9}abx^3-\frac{2ab\tan^{-1}\left(\sqrt{c}x\right)}{3c^{3/2}}+\frac{4b^2\tan^{-1}\left(\sqrt{c}x\right)}{3c^{3/2}}-\frac{ib^2\tan^{-1}\left(\sqrt{c}x\right)^2}{3c^{3/2}}-\frac{4b^2\tan}{3}$$

Mathematica [F] time = 9.0584, size = 0, normalized size = 0.

$$\int x^2\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2 dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2*(a + b*ArcTanh[c*x^2])^2,x]
```

```
[Out] Integrate[x^2*(a + b*ArcTanh[c*x^2])^2, x]
```

Maple [F] time = 0.184, size = 0, normalized size = 0.

$$\int x^2 \left(a + b\text{Artanh}\left(cx^2\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c*x^2))^2,x)
```

```
[Out] int(x^2*(a+b*arctanh(c*x^2))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^2))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(b^2x^2\,\text{artanh}\left(cx^2\right)^2 + 2\,abx^2\,\text{artanh}\left(cx^2\right) + a^2x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^2))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^2*arctanh(c*x^2)^2 + 2*a*b*x^2*arctanh(c*x^2) + a^2*x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \left(a + b\,\text{atanh}\left(cx^2\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c*x**2))**2,x)
```

```
[Out] Integral(x**2*(a + b*atanh(c*x**2))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\,\text{artanh}\left(cx^2\right) + a\right)^2 x^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^2))^2,x, algorithm="giac")

[Out] integrate((b*arctanh(c*x^2) + a)^2*x^2, x)
```

3.73 $\int \left(a+b\tanh^{-1}\left(cx^2\right)\right)^2 dx$

Optimal. Leaf size=958

result too large to display

```
[Out] a^2*x + (2*a*b*ArcTan[Sqrt[c]*x])/Sqrt[c] + (I*b^2*ArcTan[Sqrt[c]*x]^2)/Sqr
t[c] - (2*a*b*ArcTanh[Sqrt[c]*x])/Sqrt[c] - (b^2*ArcTanh[Sqrt[c]*x]^2)/Sqrt
[c] + (2*b^2*ArcTanh[Sqrt[c]*x]*Log[2/(1 - Sqrt[c]*x)])/Sqrt[c] - (2*b^2*Ar
cTan[Sqrt[c]*x]*Log[2/(1 - I*Sqrt[c]*x)])/Sqrt[c] + (b^2*ArcTan[Sqrt[c]*x]*
Log[((1 + I)*(1 - Sqrt[c]*x))/(1 - I*Sqrt[c]*x)])/Sqrt[c] + (2*b^2*ArcTan[S
qrt[c]*x]*Log[2/(1 + I*Sqrt[c]*x)])/Sqrt[c] - (2*b^2*ArcTanh[Sqrt[c]*x]*Log
[2/(1 + Sqrt[c]*x)])/Sqrt[c] + (b^2*ArcTanh[Sqrt[c]*x]*Log[(-2*Sqrt[c]*(1 -
 Sqrt[-c]*x))/((Sqrt[-c] - Sqrt[c])*(1 + Sqrt[c]*x))])/Sqrt[c] + (b^2*ArcTa
nh[Sqrt[c]*x]*Log[(2*Sqrt[c]*(1 + Sqrt[-c]*x))/((Sqrt[-c] + Sqrt[c])*(1 + S
qrt[c]*x))])/Sqrt[c] + (b^2*ArcTan[Sqrt[c]*x]*Log[((1 - I)*(1 + Sqrt[c]*x))
/(1 - I*Sqrt[c]*x)])/Sqrt[c] - a*b*x*Log[1 - c*x^2] - (b^2*ArcTan[Sqrt[c]*x
]*Log[1 - c*x^2])/Sqrt[c] + (b^2*ArcTanh[Sqrt[c]*x]*Log[1 - c*x^2])/Sqrt[c]
 + (b^2*x*Log[1 - c*x^2]^2)/4 + a*b*x*Log[1 + c*x^2] + (b^2*ArcTan[Sqrt[c]*
x]*Log[1 + c*x^2])/Sqrt[c] - (b^2*ArcTanh[Sqrt[c]*x]*Log[1 + c*x^2])/Sqrt[c
] - (b^2*x*Log[1 - c*x^2]*Log[1 + c*x^2])/2 + (b^2*x*Log[1 + c*x^2]^2)/4 +
(b^2*PolyLog[2, 1 - 2/(1 - Sqrt[c]*x)])/Sqrt[c] + (I*b^2*PolyLog[2, 1 - 2/(
1 - I*Sqrt[c]*x)])/Sqrt[c] - ((I/2)*b^2*PolyLog[2, 1 - ((1 + I)*(1 - Sqrt[c
]*x))/(1 - I*Sqrt[c]*x)])/Sqrt[c] + (I*b^2*PolyLog[2, 1 - 2/(1 + I*Sqrt[c]*
x)])/Sqrt[c] + (b^2*PolyLog[2, 1 - 2/(1 + Sqrt[c]*x)])/Sqrt[c] - (b^2*PolyL
og[2, 1 + (2*Sqrt[c]*(1 - Sqrt[-c]*x))/((Sqrt[-c] - Sqrt[c])*(1 + Sqrt[c]*x
))])/(2*Sqrt[c]) - (b^2*PolyLog[2, 1 - (2*Sqrt[c]*(1 + Sqrt[-c]*x))/((Sqrt[
-c] + Sqrt[c])*(1 + Sqrt[c]*x))])/(2*Sqrt[c]) - ((I/2)*b^2*PolyLog[2, 1 - (
(1 - I)*(1 + Sqrt[c]*x))/(1 - I*Sqrt[c]*x)])/Sqrt[c]
```

Rubi [A] time = 1.46813, antiderivative size = 958, normalized size of antiderivative = 1., number of steps used = 69, number of rules used = 21, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.75, Rules used = {6093, 2448, 321, 206, 2450, 2476, 2470, 12, 5984, 5918, 2402, 2315, 203, 2556, 5992, 5920, 2447, 4928, 4856, 4920, 4854}

$$xa^2+\frac{2b\tan^{-1}\left(\sqrt{c}x\right)a}{\sqrt{c}}-\frac{2b\tanh^{-1}\left(\sqrt{c}x\right)a}{\sqrt{c}}-bx\log\left(1-cx^2\right)a+bx\log\left(cx^2+1\right)a+\frac{ib^2\tan^{-1}\left(\sqrt{c}x\right)^2}{\sqrt{c}}-\frac{b^2\tanh^{-1}(\sqrt{}}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^2])^2,x]
```

```
[Out] a^2*x + (2*a*b*ArcTan[Sqrt[c]*x])/Sqrt[c] + (I*b^2*ArcTan[Sqrt[c]*x]^2)/Sqr
t[c] - (2*a*b*ArcTanh[Sqrt[c]*x])/Sqrt[c] - (b^2*ArcTanh[Sqrt[c]*x]^2)/Sqrt
[c] + (2*b^2*ArcTanh[Sqrt[c]*x]*Log[2/(1 - Sqrt[c]*x)])/Sqrt[c] - (2*b^2*Ar
cTan[Sqrt[c]*x]*Log[2/(1 - I*Sqrt[c]*x)])/Sqrt[c] + (b^2*ArcTan[Sqrt[c]*x]*
Log[((1 + I)*(1 - Sqrt[c]*x))/(1 - I*Sqrt[c]*x)])/Sqrt[c] + (2*b^2*ArcTan[S
qrt[c]*x]*Log[2/(1 + I*Sqrt[c]*x)])/Sqrt[c] - (2*b^2*ArcTanh[Sqrt[c]*x]*Log
[2/(1 + Sqrt[c]*x)])/Sqrt[c] + (b^2*ArcTanh[Sqrt[c]*x]*Log[(-2*Sqrt[c]*(1 -
 Sqrt[-c]*x))/((Sqrt[-c] - Sqrt[c])*(1 + Sqrt[c]*x))])/Sqrt[c] + (b^2*ArcTa
nh[Sqrt[c]*x]*Log[(2*Sqrt[c]*(1 + Sqrt[-c]*x))/((Sqrt[-c] + Sqrt[c])*(1 + S
qrt[c]*x))])/Sqrt[c] + (b^2*ArcTan[Sqrt[c]*x]*Log[((1 - I)*(1 + Sqrt[c]*x))
/(1 - I*Sqrt[c]*x)])/Sqrt[c] - a*b*x*Log[1 - c*x^2] - (b^2*ArcTan[Sqrt[c]*x
]*Log[1 - c*x^2])/Sqrt[c] + (b^2*ArcTanh[Sqrt[c]*x]*Log[1 - c*x^2])/Sqrt[c]
 + (b^2*x*Log[1 - c*x^2]^2)/4 + a*b*x*Log[1 + c*x^2] + (b^2*ArcTan[Sqrt[c]*
```

```
x]*Log[1 + c*x^2])/Sqrt[c] - (b^2*ArcTanh[Sqrt[c]*x]*Log[1 + c*x^2])/Sqrt[c
] - (b^2*x*Log[1 - c*x^2]*Log[1 + c*x^2])/2 + (b^2*x*Log[1 + c*x^2]^2)/4 +
(b^2*PolyLog[2, 1 - 2/(1 - Sqrt[c]*x)])/Sqrt[c] + (I*b^2*PolyLog[2, 1 - 2/(
1 - I*Sqrt[c]*x)])/Sqrt[c] - ((I/2)*b^2*PolyLog[2, 1 - ((1 + I)*(1 - Sqrt[c
]*x))/(1 - I*Sqrt[c]*x)])/Sqrt[c] + (I*b^2*PolyLog[2, 1 - 2/(1 + I*Sqrt[c]*
x)])/Sqrt[c] + (b^2*PolyLog[2, 1 - 2/(1 + Sqrt[c]*x)])/Sqrt[c] - (b^2*PolyL
og[2, 1 + (2*Sqrt[c]*(1 - Sqrt[-c]*x))/((Sqrt[-c] - Sqrt[c])*(1 + Sqrt[c]*x
))])/(2*Sqrt[c]) - (b^2*PolyLog[2, 1 - (2*Sqrt[c]*(1 + Sqrt[-c]*x))/((Sqrt[
-c] + Sqrt[c])*(1 + Sqrt[c]*x))])/(2*Sqrt[c]) - ((I/2)*b^2*PolyLog[2, 1 - (
(1 - I)*(1 + Sqrt[c]*x))/(1 - I*Sqrt[c]*x)])/Sqrt[c]
```

Rule 6093

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.), x_Symbol] :> Int[Expand
Integrand[(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 - c*x^n])/2)^p, x], x] /; Fr
eeQ[{a, b, c, n}, x] && IGtQ[p, 0] && IntegerQ[n]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2450

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> Simp[x*(a + b*Log[c*(d + e*x^n)^p])^q, x] - Dist[b*e*n*p*q, Int[(x^n*
(a + b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c
, d, e, n, p}, x] && IGtQ[q, 0] && (EqQ[q, 1] || IntegerQ[n])
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2556

```
Int[Log[v_]*Log[w_], x_Symbol] :> Simp[x*Log[v]*Log[w], x] + (-Int[Simplify
Integrand[(x*Log[w]*D[v, x])/v, x], x] - Int[SimplifyIntegrand[(x*Log[v]*D[
w, x])/w, x], x]) /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w,
x]
```

Rule 5992

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rubi steps

$$\int \left(a+b\tanh^{-1}\left(cx^2\right)\right)^2 dx = \int \left(a^2 - ab\log\left(1-cx^2\right) + \frac{1}{4}b^2\log^2\left(1-cx^2\right) + ab\log\left(1+cx^2\right) - \frac{1}{2}b^2\log\left(1-cx^2\right)\log\left(1\right.\right.$$

$$= a^2x - (ab)\int \log\left(1-cx^2\right) dx + (ab)\int \log\left(1+cx^2\right) dx + \frac{1}{4}b^2\int \log^2\left(1-cx^2\right) dx + \frac{1}{4}b^2$$

$$= a^2x - abx\log\left(1-cx^2\right) + \frac{1}{4}b^2x\log^2\left(1-cx^2\right) + abx\log\left(1+cx^2\right) - \frac{1}{2}b^2x\log\left(1-cx^2\right)\log$$

$$= a^2x - abx\log\left(1-cx^2\right) + \frac{1}{4}b^2x\log^2\left(1-cx^2\right) + abx\log\left(1+cx^2\right) - \frac{1}{2}b^2x\log\left(1-cx^2\right)\log$$

$$= a^2x + \frac{2ab\tan^{-1}\left(\sqrt{c}x\right)}{\sqrt{c}} - \frac{2ab\tanh^{-1}\left(\sqrt{c}x\right)}{\sqrt{c}} - abx\log\left(1-cx^2\right) + \frac{1}{4}b^2x\log^2\left(1-cx^2\right) + a$$

$$= a^2x + \frac{2ab\tan^{-1}\left(\sqrt{c}x\right)}{\sqrt{c}} - \frac{2ab\tanh^{-1}\left(\sqrt{c}x\right)}{\sqrt{c}} - abx\log\left(1-cx^2\right) - b^2x\log\left(1-cx^2\right) + \frac{b^2\,\mathrm{t}}{}$$

$$= a^2x + 4b^2x + \frac{2ab\tan^{-1}\left(\sqrt{c}x\right)}{\sqrt{c}} - \frac{2ab\tanh^{-1}\left(\sqrt{c}x\right)}{\sqrt{c}} - abx\log\left(1-cx^2\right) - \frac{b^2\tan^{-1}\left(\sqrt{c}x\right)\mathrm{l}}{\sqrt{c}}$$

$$= a^2x + \frac{2ab\tan^{-1}\left(\sqrt{c}x\right)}{\sqrt{c}} - \frac{2b^2\tan^{-1}\left(\sqrt{c}x\right)}{\sqrt{c}} + \frac{ib^2\tan^{-1}\left(\sqrt{c}x\right)^2}{\sqrt{c}} - \frac{2ab\tanh^{-1}\left(\sqrt{c}x\right)}{\sqrt{c}} - 2b^2$$

$$= a^2x + \frac{2ab\tan^{-1}\left(\sqrt{c}x\right)}{\sqrt{c}} + \frac{ib^2\tan^{-1}\left(\sqrt{c}x\right)^2}{\sqrt{c}} - \frac{2ab\tanh^{-1}\left(\sqrt{c}x\right)}{\sqrt{c}} - \frac{b^2\tanh^{-1}\left(\sqrt{c}x\right)^2}{\sqrt{c}} + \frac{2b}{}$$

$$= a^2x + \frac{2ab\tan^{-1}\left(\sqrt{c}x\right)}{\sqrt{c}} + \frac{ib^2\tan^{-1}\left(\sqrt{c}x\right)^2}{\sqrt{c}} - \frac{2ab\tanh^{-1}\left(\sqrt{c}x\right)}{\sqrt{c}} - \frac{b^2\tanh^{-1}\left(\sqrt{c}x\right)^2}{\sqrt{c}} + \frac{2b}{}$$

$$= a^2x + \frac{2ab\tan^{-1}\left(\sqrt{c}x\right)}{\sqrt{c}} + \frac{ib^2\tan^{-1}\left(\sqrt{c}x\right)^2}{\sqrt{c}} - \frac{2ab\tanh^{-1}\left(\sqrt{c}x\right)}{\sqrt{c}} - \frac{b^2\tanh^{-1}\left(\sqrt{c}x\right)^2}{\sqrt{c}} + \frac{2b}{}$$

$$= a^2x + \frac{2ab\tan^{-1}\left(\sqrt{c}x\right)}{\sqrt{c}} + \frac{ib^2\tan^{-1}\left(\sqrt{c}x\right)^2}{\sqrt{c}} - \frac{2ab\tanh^{-1}\left(\sqrt{c}x\right)}{\sqrt{c}} - \frac{b^2\tanh^{-1}\left(\sqrt{c}x\right)^2}{\sqrt{c}} + \frac{2b}{}$$

$$= a^2x + \frac{2ab\tan^{-1}\left(\sqrt{c}x\right)}{\sqrt{c}} + \frac{ib^2\tan^{-1}\left(\sqrt{c}x\right)^2}{\sqrt{c}} - \frac{2ab\tanh^{-1}\left(\sqrt{c}x\right)}{\sqrt{c}} - \frac{b^2\tanh^{-1}\left(\sqrt{c}x\right)^2}{\sqrt{c}} + \frac{2b}{}$$

<u>Mathematica</u> [A] time = 2.56231, size = 566, normalized size = 0.59

$$\frac{1}{2}x\left(\frac{b^2\left(\mathrm{PolyLog}\left(2,\frac{1}{2}\left(1-\sqrt{cx^2}\right)\right) - \mathrm{PolyLog}\left(2,\left(-\frac{1}{2}-\frac{i}{2}\right)\left(\sqrt{cx^2}-1\right)\right) - \mathrm{PolyLog}\left(2,\left(-\frac{1}{2}+\frac{i}{2}\right)\left(\sqrt{cx^2}-1\right)\right) - \mathrm{PolyLo}\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])^2,x]
```

```
[Out] (x*(2*a^2 + 4*a*b*ArcTanh[c*x^2] + (4*a*b*(ArcTan[Sqrt[c*x^2]] - ArcTanh[Sq
rt[c*x^2]]))/Sqrt[c*x^2] + (b^2*((-2*I)*ArcTan[Sqrt[c*x^2]]^2 + 4*ArcTan[Sq
rt[c*x^2]]*ArcTanh[c*x^2] + 2*Sqrt[c*x^2]*ArcTanh[c*x^2]^2 + 2*ArcTan[Sqrt[
c*x^2]]*Log[1 + E^((4*I)*ArcTan[Sqrt[c*x^2]])] + 2*ArcTanh[c*x^2]*Log[1 - S
qrt[c*x^2]] - Log[2]*Log[1 - Sqrt[c*x^2]] + Log[1 - Sqrt[c*x^2]]^2/2 - Log[
1 - Sqrt[c*x^2]]*Log[(1/2 + I/2)*(-I + Sqrt[c*x^2])] - 2*ArcTanh[c*x^2]*Log
[1 + Sqrt[c*x^2]] + Log[2]*Log[1 + Sqrt[c*x^2]] + Log[((1 + I) - (1 - I)*Sq
```

```
rt[c*x^2])/2]*Log[1 + Sqrt[c*x^2]] + Log[(-1/2 - I/2)*(I + Sqrt[c*x^2])]*Lo
g[1 + Sqrt[c*x^2]] - Log[1 + Sqrt[c*x^2]]^2/2 - Log[1 - Sqrt[c*x^2]]*Log[((
1 + I) + (1 - I)*Sqrt[c*x^2])/2] - (I/2)*PolyLog[2, -E^((4*I)*ArcTan[Sqrt[c
*x^2]])] + PolyLog[2, (1 - Sqrt[c*x^2])/2] - PolyLog[2, (-1/2 - I/2)*(-1 +
Sqrt[c*x^2])] - PolyLog[2, (-1/2 + I/2)*(-1 + Sqrt[c*x^2])] - PolyLog[2, (1
 + Sqrt[c*x^2])/2] + PolyLog[2, (1/2 - I/2)*(1 + Sqrt[c*x^2])] + PolyLog[2,
 (1/2 + I/2)*(1 + Sqrt[c*x^2])]))/Sqrt[c*x^2]))/2
```

Maple [F] time = 0.125, size = 0, normalized size = 0.

$$\int \left(a+b\mathrm{Artanh}\left(c x^{2}\right)\right)^{2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))^2,x)
```

```
[Out] int((a+b*arctanh(c*x^2))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(b^{2}\,\mathrm{artanh}\left(c x^{2}\right)^{2}+2\,ab\,\mathrm{artanh}\left(c x^{2}\right)+a^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*arctanh(c*x^2)^2 + 2*a*b*arctanh(c*x^2) + a^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+b\,\mathrm{atanh}\left(c x^{2}\right)\right)^{2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))**2,x)
```

```
[Out] Integral((a + b*atanh(c*x**2))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\,\text{artanh}\left(cx^2\right)+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^2) + a)^2, x)
```

3.74 $\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{x^2}\,dx$

Optimal. Leaf size=942

result too large to display

```
[Out] 2*a*b*Sqrt[c]*ArcTan[Sqrt[c]*x] + I*b^2*Sqrt[c]*ArcTan[Sqrt[c]*x]^2 + b^2*S
qrt[c]*ArcTanh[Sqrt[c]*x]^2 - 2*b^2*Sqrt[c]*ArcTanh[Sqrt[c]*x]*Log[2/(1 - S
qrt[c]*x)] - 2*b^2*Sqrt[c]*ArcTan[Sqrt[c]*x]*Log[2/(1 - I*Sqrt[c]*x)] + b^2
*Sqrt[c]*ArcTan[Sqrt[c]*x]*Log[((1 + I)*(1 - Sqrt[c]*x))/(1 - I*Sqrt[c]*x)]
 + 2*b^2*Sqrt[c]*ArcTan[Sqrt[c]*x]*Log[2/(1 + I*Sqrt[c]*x)] + 2*b^2*Sqrt[c]
*ArcTanh[Sqrt[c]*x]*Log[2/(1 + Sqrt[c]*x)] - b^2*Sqrt[c]*ArcTanh[Sqrt[c]*x]
*Log[(-2*Sqrt[c]*(1 - Sqrt[-c]*x))/((Sqrt[-c] - Sqrt[c])*(1 + Sqrt[c]*x))]
- b^2*Sqrt[c]*ArcTanh[Sqrt[c]*x]*Log[(2*Sqrt[c]*(1 + Sqrt[-c]*x))/((Sqrt[-c
] + Sqrt[c])*(1 + Sqrt[c]*x))] + b^2*Sqrt[c]*ArcTan[Sqrt[c]*x]*Log[((1 - I)
*(1 + Sqrt[c]*x))/(1 - I*Sqrt[c]*x)] - b^2*Sqrt[c]*ArcTan[Sqrt[c]*x]*Log[1
- c*x^2] + b*Sqrt[c]*ArcTanh[Sqrt[c]*x]*(2*a - b*Log[1 - c*x^2]) - (2*a - b
*Log[1 - c*x^2])^2/(4*x) - (a*b*Log[1 + c*x^2])/x + b^2*Sqrt[c]*ArcTan[Sqrt
[c]*x]*Log[1 + c*x^2] + b^2*Sqrt[c]*ArcTanh[Sqrt[c]*x]*Log[1 + c*x^2] + (b^
2*Log[1 - c*x^2]*Log[1 + c*x^2])/(2*x) - (b^2*Log[1 + c*x^2]^2)/(4*x) - b^2
*Sqrt[c]*PolyLog[2, 1 - 2/(1 - Sqrt[c]*x)] + I*b^2*Sqrt[c]*PolyLog[2, 1 - 2
/(1 - I*Sqrt[c]*x)] - (I/2)*b^2*Sqrt[c]*PolyLog[2, 1 - ((1 + I)*(1 - Sqrt[c
]*x))/(1 - I*Sqrt[c]*x)] + I*b^2*Sqrt[c]*PolyLog[2, 1 - 2/(1 + I*Sqrt[c]*x)
] - b^2*Sqrt[c]*PolyLog[2, 1 - 2/(1 + Sqrt[c]*x)] + (b^2*Sqrt[c]*PolyLog[2,
 1 + (2*Sqrt[c]*(1 - Sqrt[-c]*x))/((Sqrt[-c] - Sqrt[c])*(1 + Sqrt[c]*x))])/
2 + (b^2*Sqrt[c]*PolyLog[2, 1 - (2*Sqrt[c]*(1 + Sqrt[-c]*x))/((Sqrt[-c] + S
qrt[c])*(1 + Sqrt[c]*x))])/2 - (I/2)*b^2*Sqrt[c]*PolyLog[2, 1 - ((1 - I)*(1
 + Sqrt[c]*x))/(1 - I*Sqrt[c]*x)]
```

Rubi [A] time = 1.33509, antiderivative size = 942, normalized size of antiderivative = 1., number of steps used = 47, number of rules used = 21, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.313, Rules used = {6099, 2457, 206, 2470, 12, 5984, 5918, 2402, 2315, 2455, 6742, 203, 30, 2557, 5992, 5920, 2447, 4928, 4856, 4920, 4854}

$$i\sqrt{c}\tan^{-1}\left(\sqrt{c}x\right)^2 b^2+\sqrt{c}\tanh^{-1}\left(\sqrt{c}x\right)^2 b^2-\frac{\log^2\left(cx^2+1\right)b^2}{4x}-2\sqrt{c}\tanh^{-1}\left(\sqrt{c}x\right)\log\left(\frac{2}{1-\sqrt{c}x}\right)b^2-2\sqrt{c}\tan^{-1}\Big($$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^2])^2/x^2,x]
```

```
[Out] 2*a*b*Sqrt[c]*ArcTan[Sqrt[c]*x] + I*b^2*Sqrt[c]*ArcTan[Sqrt[c]*x]^2 + b^2*S
qrt[c]*ArcTanh[Sqrt[c]*x]^2 - 2*b^2*Sqrt[c]*ArcTanh[Sqrt[c]*x]*Log[2/(1 - S
qrt[c]*x)] - 2*b^2*Sqrt[c]*ArcTan[Sqrt[c]*x]*Log[2/(1 - I*Sqrt[c]*x)] + b^2
*Sqrt[c]*ArcTan[Sqrt[c]*x]*Log[((1 + I)*(1 - Sqrt[c]*x))/(1 - I*Sqrt[c]*x)]
 + 2*b^2*Sqrt[c]*ArcTan[Sqrt[c]*x]*Log[2/(1 + I*Sqrt[c]*x)] + 2*b^2*Sqrt[c]
*ArcTanh[Sqrt[c]*x]*Log[2/(1 + Sqrt[c]*x)] - b^2*Sqrt[c]*ArcTanh[Sqrt[c]*x]
*Log[(-2*Sqrt[c]*(1 - Sqrt[-c]*x))/((Sqrt[-c] - Sqrt[c])*(1 + Sqrt[c]*x))]
- b^2*Sqrt[c]*ArcTanh[Sqrt[c]*x]*Log[(2*Sqrt[c]*(1 + Sqrt[-c]*x))/((Sqrt[-c
] + Sqrt[c])*(1 + Sqrt[c]*x))] + b^2*Sqrt[c]*ArcTan[Sqrt[c]*x]*Log[((1 - I)
*(1 + Sqrt[c]*x))/(1 - I*Sqrt[c]*x)] - b^2*Sqrt[c]*ArcTan[Sqrt[c]*x]*Log[1
- c*x^2] + b*Sqrt[c]*ArcTanh[Sqrt[c]*x]*(2*a - b*Log[1 - c*x^2]) - (2*a - b
*Log[1 - c*x^2])^2/(4*x) - (a*b*Log[1 + c*x^2])/x + b^2*Sqrt[c]*ArcTan[Sqrt
[c]*x]*Log[1 + c*x^2] + b^2*Sqrt[c]*ArcTanh[Sqrt[c]*x]*Log[1 + c*x^2] + (b^
```

```
2*Log[1 - c*x^2]*Log[1 + c*x^2])/(2*x) - (b^2*Log[1 + c*x^2]^2)/(4*x) - b^2
*Sqrt[c]*PolyLog[2, 1 - 2/(1 - Sqrt[c]*x)] + I*b^2*Sqrt[c]*PolyLog[2, 1 - 2
/(1 - I*Sqrt[c]*x)] - (I/2)*b^2*Sqrt[c]*PolyLog[2, 1 - ((1 + I)*(1 - Sqrt[c
]*x))/(1 - I*Sqrt[c]*x)] + I*b^2*Sqrt[c]*PolyLog[2, 1 - 2/(1 + I*Sqrt[c]*x)
] - b^2*Sqrt[c]*PolyLog[2, 1 - 2/(1 + Sqrt[c]*x)] + (b^2*Sqrt[c]*PolyLog[2,
 1 + (2*Sqrt[c]*(1 - Sqrt[-c]*x))/((Sqrt[-c] - Sqrt[c])*(1 + Sqrt[c]*x))])/
2 + (b^2*Sqrt[c]*PolyLog[2, 1 - (2*Sqrt[c]*(1 + Sqrt[-c]*x))/((Sqrt[-c] + S
qrt[c])*(1 + Sqrt[c]*x))])/2 - (I/2)*b^2*Sqrt[c]*PolyLog[2, 1 - ((1 - I)*(1
 + Sqrt[c]*x))/(1 - I*Sqrt[c]*x)]
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q
)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a +
 b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2557

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] :> With[{z = IntHide[u, x]}, Dist[Log[v
]*Log[w], z, x] + (-Int[SimplifyIntegrand[(z*Log[w]*D[v, x])/v, x], x] - In
t[SimplifyIntegrand[(z*Log[v]*D[w, x])/w, x], x]) /; InverseFunctionFreeQ[z
, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 5992

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
```

```
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{x^2}dx = \int \left(\frac{\left(2a-b\log\left(1-cx^2\right)\right)^2}{4x^2} - \frac{b\left(-2a+b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right)}{2x^2} + \frac{b^2\log^2\left(1+cx^2\right)}{4x^2}$$

$$= \frac{1}{4}\int \frac{\left(2a-b\log\left(1-cx^2\right)\right)^2}{x^2}dx - \frac{1}{2}b\int \frac{\left(-2a+b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right)}{x^2}dx + \frac{1}{4}b^2$$

$$= -\frac{\left(2a-b\log\left(1-cx^2\right)\right)^2}{4x} - \frac{b^2\log^2\left(1+cx^2\right)}{4x} - \frac{1}{2}b\int \left(-\frac{2a\log\left(1+cx^2\right)}{x^2} + \frac{b\log\left(1-c\right.}{}\right.$$

$$= b\sqrt{c}\tanh^{-1}\left(\sqrt{c}x\right)\left(2a-b\log\left(1-cx^2\right)\right) - \frac{\left(2a-b\log\left(1-cx^2\right)\right)^2}{4x} + b^2\sqrt{c}\tan^{-1}\left(\sqrt{c}x\right)$$

$$= b\sqrt{c}\tanh^{-1}\left(\sqrt{c}x\right)\left(2a-b\log\left(1-cx^2\right)\right) - \frac{\left(2a-b\log\left(1-cx^2\right)\right)^2}{4x} - \frac{ab\log\left(1+cx^2\right)}{x} +$$

$$= 2ab\sqrt{c}\tan^{-1}\left(\sqrt{c}x\right) + ib^2\sqrt{c}\tan^{-1}\left(\sqrt{c}x\right)^2 + b^2\sqrt{c}\tanh^{-1}\left(\sqrt{c}x\right)^2 + b\sqrt{c}\tanh^{-1}\left(\sqrt{c}x\right)$$

$$= 2ab\sqrt{c}\tan^{-1}\left(\sqrt{c}x\right) + ib^2\sqrt{c}\tan^{-1}\left(\sqrt{c}x\right)^2 + b^2\sqrt{c}\tanh^{-1}\left(\sqrt{c}x\right)^2 - 2b^2\sqrt{c}\tanh^{-1}\left(\sqrt{c}x\right.$$

$$= 2ab\sqrt{c}\tan^{-1}\left(\sqrt{c}x\right) + ib^2\sqrt{c}\tan^{-1}\left(\sqrt{c}x\right)^2 + b^2\sqrt{c}\tanh^{-1}\left(\sqrt{c}x\right)^2 - 2b^2\sqrt{c}\tanh^{-1}\left(\sqrt{c}x\right.$$

$$= 2ab\sqrt{c}\tan^{-1}\left(\sqrt{c}x\right) + ib^2\sqrt{c}\tan^{-1}\left(\sqrt{c}x\right)^2 + b^2\sqrt{c}\tanh^{-1}\left(\sqrt{c}x\right)^2 - 2b^2\sqrt{c}\tanh^{-1}\left(\sqrt{c}x\right.$$

$$= 2ab\sqrt{c}\tan^{-1}\left(\sqrt{c}x\right) + ib^2\sqrt{c}\tan^{-1}\left(\sqrt{c}x\right)^2 + b^2\sqrt{c}\tanh^{-1}\left(\sqrt{c}x\right)^2 - 2b^2\sqrt{c}\tanh^{-1}\left(\sqrt{c}x\right.$$

$$= 2ab\sqrt{c}\tan^{-1}\left(\sqrt{c}x\right) + ib^2\sqrt{c}\tan^{-1}\left(\sqrt{c}x\right)^2 + b^2\sqrt{c}\tanh^{-1}\left(\sqrt{c}x\right)^2 - 2b^2\sqrt{c}\tanh^{-1}\left(\sqrt{c}x\right.$$

$$= 2ab\sqrt{c}\tan^{-1}\left(\sqrt{c}x\right) + ib^2\sqrt{c}\tan^{-1}\left(\sqrt{c}x\right)^2 + b^2\sqrt{c}\tanh^{-1}\left(\sqrt{c}x\right)^2 - 2b^2\sqrt{c}\tanh^{-1}\left(\sqrt{c}x\right.$$

$$= 2ab\sqrt{c}\tan^{-1}\left(\sqrt{c}x\right) + ib^2\sqrt{c}\tan^{-1}\left(\sqrt{c}x\right)^2 + b^2\sqrt{c}\tanh^{-1}\left(\sqrt{c}x\right)^2 - 2b^2\sqrt{c}\tanh^{-1}\left(\sqrt{c}x\right.$$

Mathematica [A] time = 3.36907, size = 566, normalized size = 0.6

$$\frac{b^2\sqrt{cx^2}\left(-\text{PolyLog}\left(2,\frac{1}{2}\left(1-\sqrt{cx^2}\right)\right)+\text{PolyLog}\left(2,\left(-\frac{1}{2}-\frac{i}{2}\right)\left(\sqrt{cx^2}-1\right)\right)+\text{PolyLog}\left(2,\left(-\frac{1}{2}+\frac{i}{2}\right)\left(\sqrt{cx^2}-1\right)\right)+\text{Po}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])^2/x^2,x]
```

```
[Out] (-2*a^2 - 4*a*b*ArcTanh[c*x^2] + 4*a*b*Sqrt[c*x^2]*(ArcTan[Sqrt[c*x^2]] + A
rcTanh[Sqrt[c*x^2]]) + b^2*Sqrt[c*x^2]*((-2*I)*ArcTan[Sqrt[c*x^2]]^2 + 4*Ar
cTan[Sqrt[c*x^2]]*ArcTanh[c*x^2] - (2*ArcTanh[c*x^2]^2)/Sqrt[c*x^2] + 2*Arc
Tan[Sqrt[c*x^2]]*Log[1 + E^((4*I)*ArcTan[Sqrt[c*x^2]])] - 2*ArcTanh[c*x^2]*
Log[1 - Sqrt[c*x^2]] + Log[2]*Log[1 - Sqrt[c*x^2]] - Log[1 - Sqrt[c*x^2]]^2
/2 + Log[1 - Sqrt[c*x^2]]*Log[(1/2 + I/2)*(-I + Sqrt[c*x^2])] + 2*ArcTanh[c
```

```
*x^2]*Log[1 + Sqrt[c*x^2]] - Log[2]*Log[1 + Sqrt[c*x^2]] - Log[((1 + I) - (
1 - I)*Sqrt[c*x^2])/2]*Log[1 + Sqrt[c*x^2]] - Log[(-1/2 - I/2)*(I + Sqrt[c*
x^2])]*Log[1 + Sqrt[c*x^2]] + Log[1 + Sqrt[c*x^2]]^2/2 + Log[1 - Sqrt[c*x^2
]]*Log[((1 + I) + (1 - I)*Sqrt[c*x^2])/2] - (I/2)*PolyLog[2, -E^((4*I)*ArcT
an[Sqrt[c*x^2]])] - PolyLog[2, (1 - Sqrt[c*x^2])/2] + PolyLog[2, (-1/2 - I/
2)*(-1 + Sqrt[c*x^2])] + PolyLog[2, (-1/2 + I/2)*(-1 + Sqrt[c*x^2])] + Poly
Log[2, (1 + Sqrt[c*x^2])/2] - PolyLog[2, (1/2 - I/2)*(1 + Sqrt[c*x^2])] - P
olyLog[2, (1/2 + I/2)*(1 + Sqrt[c*x^2])]))/(2*x)
```

Maple [F] time = 0.175, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\,\mathrm{Artanh}\left(c x^2\right)\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))^2/x^2,x)
```

```
[Out] int((a+b*arctanh(c*x^2))^2/x^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^2\,\mathrm{artanh}\left(c x^2\right)^2+2\,ab\,\mathrm{artanh}\left(c x^2\right)+a^2}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x^2)^2 + 2*a*b*arctanh(c*x^2) + a^2)/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+b\,\mathrm{atanh}\left(c x^2\right)\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))**2/x**2,x)
```

```
[Out] Integral((a + b*atanh(c*x**2))**2/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\,\mathrm{artanh}\left(cx^2\right)+a\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^2) + a)^2/x^2, x)
```

3.75 $\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{x^4}\,dx$

Optimal. Leaf size=1102

result too large to display

```
[Out] (-2*a*b*c)/(3*x) - (2*a*b*c^(3/2)*ArcTan[Sqrt[c]*x])/3 + (4*b^2*c^(3/2)*Arc
Tan[Sqrt[c]*x])/3 - (I/3)*b^2*c^(3/2)*ArcTan[Sqrt[c]*x]^2 + (4*b^2*c^(3/2)*
ArcTanh[Sqrt[c]*x])/3 + (b^2*c^(3/2)*ArcTanh[Sqrt[c]*x]^2)/3 - (2*b^2*c^(3/
2)*ArcTanh[Sqrt[c]*x]*Log[2/(1 - Sqrt[c]*x)])/3 + (2*b^2*c^(3/2)*ArcTan[Sqr
t[c]*x]*Log[2/(1 - I*Sqrt[c]*x)])/3 - (b^2*c^(3/2)*ArcTan[Sqrt[c]*x]*Log[((
1 + I)*(1 - Sqrt[c]*x))/(1 - I*Sqrt[c]*x)])/3 - (2*b^2*c^(3/2)*ArcTan[Sqrt[
c]*x]*Log[2/(1 + I*Sqrt[c]*x)])/3 + (2*b^2*c^(3/2)*ArcTanh[Sqrt[c]*x]*Log[2
/(1 + Sqrt[c]*x)])/3 - (b^2*c^(3/2)*ArcTanh[Sqrt[c]*x]*Log[(-2*Sqrt[c]*(1 -
 Sqrt[-c]*x))/((Sqrt[-c] - Sqrt[c])*(1 + Sqrt[c]*x))])/3 - (b^2*c^(3/2)*Arc
Tanh[Sqrt[c]*x]*Log[(2*Sqrt[c]*(1 + Sqrt[-c]*x))/((Sqrt[-c] + Sqrt[c])*(1 +
 Sqrt[c]*x))])/3 - (b^2*c^(3/2)*ArcTan[Sqrt[c]*x]*Log[((1 - I)*(1 + Sqrt[c]
*x))/(1 - I*Sqrt[c]*x)])/3 + (b^2*c*Log[1 - c*x^2])/(3*x) + (b^2*c^(3/2)*Ar
cTan[Sqrt[c]*x]*Log[1 - c*x^2])/3 - (b*c*(2*a - b*Log[1 - c*x^2]))/(3*x) +
(b*c^(3/2)*ArcTanh[Sqrt[c]*x]*(2*a - b*Log[1 - c*x^2]))/3 - (2*a - b*Log[1
- c*x^2])^2/(12*x^3) - (a*b*Log[1 + c*x^2])/(3*x^3) - (2*b^2*c*Log[1 + c*x^
2])/(3*x) - (b^2*c^(3/2)*ArcTan[Sqrt[c]*x]*Log[1 + c*x^2])/3 + (b^2*c^(3/2)
*ArcTanh[Sqrt[c]*x]*Log[1 + c*x^2])/3 + (b^2*Log[1 - c*x^2]*Log[1 + c*x^2])
/(6*x^3) - (b^2*Log[1 + c*x^2]^2)/(12*x^3) - (b^2*c^(3/2)*PolyLog[2, 1 - 2/
(1 - Sqrt[c]*x)])/3 - (I/3)*b^2*c^(3/2)*PolyLog[2, 1 - 2/(1 - I*Sqrt[c]*x)]
 + (I/6)*b^2*c^(3/2)*PolyLog[2, 1 - ((1 + I)*(1 - Sqrt[c]*x))/(1 - I*Sqrt[c
]*x)] - (I/3)*b^2*c^(3/2)*PolyLog[2, 1 - 2/(1 + I*Sqrt[c]*x)] - (b^2*c^(3/2
)*PolyLog[2, 1 - 2/(1 + Sqrt[c]*x)])/3 + (b^2*c^(3/2)*PolyLog[2, 1 + (2*Sqr
t[c]*(1 - Sqrt[-c]*x))/((Sqrt[-c] - Sqrt[c])*(1 + Sqrt[c]*x))])/6 + (b^2*c^
(3/2)*PolyLog[2, 1 - (2*Sqrt[c]*(1 + Sqrt[-c]*x))/((Sqrt[-c] + Sqrt[c])*(1
+ Sqrt[c]*x))])/6 + (I/6)*b^2*c^(3/2)*PolyLog[2, 1 - ((1 - I)*(1 + Sqrt[c]*
x))/(1 - I*Sqrt[c]*x)]
```

Rubi [A] time = 1.84194, antiderivative size = 1102, normalized size of antiderivative = 1., number of steps used = 64, number of rules used = 24, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.5, Rules used = {6099, 2457, 2476, 2455, 206, 207, 2470, 12, 5984, 5918, 2402, 2315, 325, 6742, 203, 30, 2557, 5992, 5920, 2447, 4928, 4856, 4920, 4854}

$$-\frac{1}{3}ic^{3/2}\tan^{-1}\left(\sqrt{c}x\right)^2b^2+\frac{1}{3}c^{3/2}\tanh^{-1}\left(\sqrt{c}x\right)^2b^2-\frac{\log^2\left(cx^2+1\right)b^2}{12x^3}+\frac{4}{3}c^{3/2}\tan^{-1}\left(\sqrt{c}x\right)b^2+\frac{4}{3}c^{3/2}\tanh^{-1}\left(\sqrt{c}x\right)b^2-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^2])^2/x^4,x]
```

```
[Out] (-2*a*b*c)/(3*x) - (2*a*b*c^(3/2)*ArcTan[Sqrt[c]*x])/3 + (4*b^2*c^(3/2)*Arc
Tan[Sqrt[c]*x])/3 - (I/3)*b^2*c^(3/2)*ArcTan[Sqrt[c]*x]^2 + (4*b^2*c^(3/2)*
ArcTanh[Sqrt[c]*x])/3 + (b^2*c^(3/2)*ArcTanh[Sqrt[c]*x]^2)/3 - (2*b^2*c^(3/
2)*ArcTanh[Sqrt[c]*x]*Log[2/(1 - Sqrt[c]*x)])/3 + (2*b^2*c^(3/2)*ArcTan[Sqr
t[c]*x]*Log[2/(1 - I*Sqrt[c]*x)])/3 - (b^2*c^(3/2)*ArcTan[Sqrt[c]*x]*Log[((
1 + I)*(1 - Sqrt[c]*x))/(1 - I*Sqrt[c]*x)])/3 - (2*b^2*c^(3/2)*ArcTan[Sqrt[
c]*x]*Log[2/(1 + I*Sqrt[c]*x)])/3 + (2*b^2*c^(3/2)*ArcTanh[Sqrt[c]*x]*Log[2
/(1 + Sqrt[c]*x)])/3 - (b^2*c^(3/2)*ArcTanh[Sqrt[c]*x]*Log[(-2*Sqrt[c]*(1 -
 Sqrt[-c]*x))/((Sqrt[-c] - Sqrt[c])*(1 + Sqrt[c]*x))])/3 - (b^2*c^(3/2)*Arc
```

```
Tanh[Sqrt[c]*x]*Log[(2*Sqrt[c]*(1 + Sqrt[-c]*x))/((Sqrt[-c] + Sqrt[c])*(1 +
 Sqrt[c]*x))]])/3 - (b^2*c^(3/2)*ArcTan[Sqrt[c]*x]*Log[((1 - I)*(1 + Sqrt[c]
*x))/(1 - I*Sqrt[c]*x)]])/3 + (b^2*c*Log[1 - c*x^2])/(3*x) + (b^2*c^(3/2)*Ar
cTan[Sqrt[c]*x]*Log[1 - c*x^2])/3 - (b*c*(2*a - b*Log[1 - c*x^2]))/(3*x) +
(b*c^(3/2)*ArcTanh[Sqrt[c]*x]*(2*a - b*Log[1 - c*x^2]))/3 - (2*a - b*Log[1
- c*x^2])^2/(12*x^3) - (a*b*Log[1 + c*x^2])/(3*x^3) - (2*b^2*c*Log[1 + c*x^
2])/(3*x) - (b^2*c^(3/2)*ArcTan[Sqrt[c]*x]*Log[1 + c*x^2])/3 + (b^2*c^(3/2)
*ArcTanh[Sqrt[c]*x]*Log[1 + c*x^2])/3 + (b^2*Log[1 - c*x^2]*Log[1 + c*x^2])
/(6*x^3) - (b^2*Log[1 + c*x^2]^2)/(12*x^3) - (b^2*c^(3/2)*PolyLog[2, 1 - 2/
(1 - Sqrt[c]*x)])/3 - (I/3)*b^2*c^(3/2)*PolyLog[2, 1 - 2/(1 - I*Sqrt[c]*x)]
 + (I/6)*b^2*c^(3/2)*PolyLog[2, 1 - ((1 + I)*(1 - Sqrt[c]*x))/(1 - I*Sqrt[c
]*x)] - (I/3)*b^2*c^(3/2)*PolyLog[2, 1 - 2/(1 + I*Sqrt[c]*x)] - (b^2*c^(3/2
)*PolyLog[2, 1 - 2/(1 + Sqrt[c]*x)])/3 + (b^2*c^(3/2)*PolyLog[2, 1 + (2*Sqr
t[c]*(1 - Sqrt[-c]*x))/((Sqrt[-c] - Sqrt[c])*(1 + Sqrt[c]*x))])/6 + (b^2*c^
(3/2)*PolyLog[2, 1 - (2*Sqrt[c]*(1 + Sqrt[-c]*x))/((Sqrt[-c] + Sqrt[c])*(1
+ Sqrt[c]*x))])/6 + (I/6)*b^2*c^(3/2)*PolyLog[2, 1 - ((1 - I)*(1 + Sqrt[c]*
x))/(1 - I*Sqrt[c]*x)]
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q
)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a +
 b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2557

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] :> With[{z = IntHide[u, x]}, Dist[Log[v
]*Log[w], z, x] + (-Int[SimplifyIntegrand[(z*Log[w]*D[v, x])/v, x], x] - In
t[SimplifyIntegrand[(z*Log[v]*D[w, x])/w, x], x]) /; InverseFunctionFreeQ[z
, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 5992

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
```

```
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{x^4}\,dx = \int \left(\frac{\left(2a-b\log\left(1-cx^2\right)\right)^2}{4x^4} - \frac{b\left(-2a+b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right)}{2x^4} + \frac{b^2\log^2\left(1+cx^2\right)}{4x^4}\right)$$

$$= \frac{1}{4}\int \frac{\left(2a-b\log\left(1-cx^2\right)\right)^2}{x^4}\,dx - \frac{1}{2}b\int \frac{\left(-2a+b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right)}{x^4}\,dx + \frac{1}{4}b^2\int$$

$$= -\frac{\left(2a-b\log\left(1-cx^2\right)\right)^2}{12x^3} - \frac{b^2\log^2\left(1+cx^2\right)}{12x^3} - \frac{1}{2}b\int\left(-\frac{2a\log\left(1+cx^2\right)}{x^4} + \frac{b\log\left(1-cx^2\right)}{x^4}\right.$$

$$= -\frac{\left(2a-b\log\left(1-cx^2\right)\right)^2}{12x^3} - \frac{b^2\log^2\left(1+cx^2\right)}{12x^3} + (ab)\int \frac{\log\left(1+cx^2\right)}{x^4}\,dx - \frac{1}{2}b^2\int \frac{\log\left(1-c\right.}{}$$

$$= -\frac{\left(2a-b\log\left(1-cx^2\right)\right)^2}{12x^3} - \frac{ab\log\left(1+cx^2\right)}{3x^3} + \frac{b^2\log\left(1-cx^2\right)\log\left(1+cx^2\right)}{6x^3} - \frac{b^2\log^2\left(1+\right.}{12x^3}$$

$$= -\frac{2abc}{3x} - \frac{bc\left(2a-b\log\left(1-cx^2\right)\right)}{3x} + \frac{1}{3}bc^{3/2}\tanh^{-1}\left(\sqrt{c}x\right)\left(2a-b\log\left(1-cx^2\right)\right) - \frac{\left(2a-b\right.}{}$$

$$= -\frac{2abc}{3x} - \frac{2}{3}abc^{3/2}\tan^{-1}\left(\sqrt{c}x\right) + \frac{2}{3}b^2c^{3/2}\tan^{-1}\left(\sqrt{c}x\right) + \frac{2}{3}b^2c^{3/2}\tanh^{-1}\left(\sqrt{c}x\right) - \frac{bc\left(2a-b\right.}{}$$

$$= -\frac{2abc}{3x} - \frac{2}{3}abc^{3/2}\tan^{-1}\left(\sqrt{c}x\right) + \frac{2}{3}b^2c^{3/2}\tan^{-1}\left(\sqrt{c}x\right) - \frac{1}{3}ib^2c^{3/2}\tan^{-1}\left(\sqrt{c}x\right)^2 + \frac{2}{3}b^2c^{3/2}\mathrm{ta}$$

$$= -\frac{2abc}{3x} - \frac{2}{3}abc^{3/2}\tan^{-1}\left(\sqrt{c}x\right) + \frac{2}{3}b^2c^{3/2}\tan^{-1}\left(\sqrt{c}x\right) - \frac{1}{3}ib^2c^{3/2}\tan^{-1}\left(\sqrt{c}x\right)^2 + \frac{2}{3}b^2c^{3/2}\mathrm{ta}$$

$$= -\frac{2abc}{3x} - \frac{2}{3}abc^{3/2}\tan^{-1}\left(\sqrt{c}x\right) + \frac{4}{3}b^2c^{3/2}\tan^{-1}\left(\sqrt{c}x\right) - \frac{1}{3}ib^2c^{3/2}\tan^{-1}\left(\sqrt{c}x\right)^2 + \frac{4}{3}b^2c^{3/2}\mathrm{ta}$$

$$= -\frac{2abc}{3x} - \frac{2}{3}abc^{3/2}\tan^{-1}\left(\sqrt{c}x\right) + \frac{4}{3}b^2c^{3/2}\tan^{-1}\left(\sqrt{c}x\right) - \frac{1}{3}ib^2c^{3/2}\tan^{-1}\left(\sqrt{c}x\right)^2 + \frac{4}{3}b^2c^{3/2}\mathrm{ta}$$

$$= -\frac{2abc}{3x} - \frac{2}{3}abc^{3/2}\tan^{-1}\left(\sqrt{c}x\right) + \frac{4}{3}b^2c^{3/2}\tan^{-1}\left(\sqrt{c}x\right) - \frac{1}{3}ib^2c^{3/2}\tan^{-1}\left(\sqrt{c}x\right)^2 + \frac{4}{3}b^2c^{3/2}\mathrm{ta}$$

$$= -\frac{2abc}{3x} - \frac{2}{3}abc^{3/2}\tan^{-1}\left(\sqrt{c}x\right) + \frac{4}{3}b^2c^{3/2}\tan^{-1}\left(\sqrt{c}x\right) - \frac{1}{3}ib^2c^{3/2}\tan^{-1}\left(\sqrt{c}x\right)^2 + \frac{4}{3}b^2c^{3/2}\mathrm{ta}$$

$$= -\frac{2abc}{3x} - \frac{2}{3}abc^{3/2}\tan^{-1}\left(\sqrt{c}x\right) + \frac{4}{3}b^2c^{3/2}\tan^{-1}\left(\sqrt{c}x\right) - \frac{1}{3}ib^2c^{3/2}\tan^{-1}\left(\sqrt{c}x\right)^2 + \frac{4}{3}b^2c^{3/2}\mathrm{ta}$$

$$= -\frac{2abc}{3x} - \frac{2}{3}abc^{3/2}\tan^{-1}\left(\sqrt{c}x\right) + \frac{4}{3}b^2c^{3/2}\tan^{-1}\left(\sqrt{c}x\right) - \frac{1}{3}ib^2c^{3/2}\tan^{-1}\left(\sqrt{c}x\right)^2 + \frac{4}{3}b^2c^{3/2}\mathrm{ta}$$

Mathematica [F] time = 2.5538, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{x^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])^2/x^4,x]

[Out] Integrate[(a + b*ArcTanh[c*x^2])^2/x^4, x]
```

Maple [F] time = 0.175, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\operatorname{Artanh}\left(c x^{2}\right)\right)^{2}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))^2/x^4,x)

[Out] int((a+b*arctanh(c*x^2))^2/x^4,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/x^4,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{b^{2}\operatorname{artanh}\left(c x^{2}\right)^{2}+2\,ab\operatorname{artanh}\left(c x^{2}\right)+a^{2}}{x^{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/x^4,x, algorithm="fricas")

[Out] integral((b^2*arctanh(c*x^2)^2 + 2*a*b*arctanh(c*x^2) + a^2)/x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+b\operatorname{atanh}\left(c x^{2}\right)\right)^{2}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))**2/x**4,x)

[Out] Integral((a + b*atanh(c*x**2))**2/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\operatorname{artanh}\left(cx^2\right)+a\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^2) + a)^2/x^4, x)
```

3.76 $\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{x^6}\,dx$

Optimal. Leaf size=1176

result too large to display

```
[Out] (-2*a*b*c)/(15*x^3) + (2*a*b*c^2)/(5*x) - (8*b^2*c^2)/(15*x) + (2*a*b*c^(5/
2)*ArcTan[Sqrt[c]*x])/5 - (4*b^2*c^(5/2)*ArcTan[Sqrt[c]*x])/15 + (I/5)*b^2*
c^(5/2)*ArcTan[Sqrt[c]*x]^2 + (4*b^2*c^(5/2)*ArcTanh[Sqrt[c]*x])/15 + (b^2*
c^(5/2)*ArcTanh[Sqrt[c]*x]^2)/5 - (2*b^2*c^(5/2)*ArcTanh[Sqrt[c]*x]*Log[2/(
1 - Sqrt[c]*x)])/5 - (2*b^2*c^(5/2)*ArcTan[Sqrt[c]*x]*Log[2/(1 - I*Sqrt[c]*
x)])/5 + (b^2*c^(5/2)*ArcTan[Sqrt[c]*x]*Log[((1 + I)*(1 - Sqrt[c]*x))/(1 -
I*Sqrt[c]*x)])/5 + (2*b^2*c^(5/2)*ArcTan[Sqrt[c]*x]*Log[2/(1 + I*Sqrt[c]*x)
])/5 + (2*b^2*c^(5/2)*ArcTanh[Sqrt[c]*x]*Log[2/(1 + Sqrt[c]*x)])/5 - (b^2*c
^(5/2)*ArcTanh[Sqrt[c]*x]*Log[(-2*Sqrt[c]*(1 - Sqrt[-c]*x))/((Sqrt[-c] - Sq
rt[c])*(1 + Sqrt[c]*x))])/5 - (b^2*c^(5/2)*ArcTanh[Sqrt[c]*x]*Log[(2*Sqrt[c
]*(1 + Sqrt[-c]*x))/((Sqrt[-c] + Sqrt[c])*(1 + Sqrt[c]*x))])/5 + (b^2*c^(5/
2)*ArcTan[Sqrt[c]*x]*Log[((1 - I)*(1 + Sqrt[c]*x))/(1 - I*Sqrt[c]*x)])/5 +
(b^2*c*Log[1 - c*x^2])/(15*x^3) - (b^2*c^2*Log[1 - c*x^2])/(5*x) - (b^2*c^(
5/2)*ArcTan[Sqrt[c]*x]*Log[1 - c*x^2])/5 - (b*c*(2*a - b*Log[1 - c*x^2]))/(
15*x^3) - (b*c^2*(2*a - b*Log[1 - c*x^2]))/(5*x) + (b*c^(5/2)*ArcTanh[Sqrt[
c]*x]*(2*a - b*Log[1 - c*x^2]))/5 - (2*a - b*Log[1 - c*x^2])^2/(20*x^5) - (
a*b*Log[1 + c*x^2])/(5*x^5) - (2*b^2*c*Log[1 + c*x^2])/(15*x^3) + (b^2*c^(5
/2)*ArcTan[Sqrt[c]*x]*Log[1 + c*x^2])/5 + (b^2*c^(5/2)*ArcTanh[Sqrt[c]*x]*L
og[1 + c*x^2])/5 + (b^2*Log[1 - c*x^2]*Log[1 + c*x^2])/(10*x^5) - (b^2*Log[
1 + c*x^2]^2)/(20*x^5) - (b^2*c^(5/2)*PolyLog[2, 1 - 2/(1 - Sqrt[c]*x)])/5
+ (I/5)*b^2*c^(5/2)*PolyLog[2, 1 - 2/(1 - I*Sqrt[c]*x)] - (I/10)*b^2*c^(5/2
)*PolyLog[2, 1 - ((1 + I)*(1 - Sqrt[c]*x))/(1 - I*Sqrt[c]*x)] + (I/5)*b^2*c
^(5/2)*PolyLog[2, 1 - 2/(1 + I*Sqrt[c]*x)] - (b^2*c^(5/2)*PolyLog[2, 1 - 2/
(1 + Sqrt[c]*x)])/5 + (b^2*c^(5/2)*PolyLog[2, 1 + (2*Sqrt[c]*(1 - Sqrt[-c]*
x))/((Sqrt[-c] - Sqrt[c])*(1 + Sqrt[c]*x))])/10 + (b^2*c^(5/2)*PolyLog[2, 1
 - (2*Sqrt[c]*(1 + Sqrt[-c]*x))/((Sqrt[-c] + Sqrt[c])*(1 + Sqrt[c]*x))])/10
 - (I/10)*b^2*c^(5/2)*PolyLog[2, 1 - ((1 - I)*(1 + Sqrt[c]*x))/(1 - I*Sqrt[
c]*x)]
```

Rubi [A] time = 1.98966, antiderivative size = 1176, normalized size of antiderivative = 1., number of steps used = 77, number of rules used = 24, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.5, Rules used = {6099, 2457, 2476, 2455, 325, 206, 207, 2470, 12, 5984, 5918, 2402, 2315, 6742, 203, 30, 2557, 5992, 5920, 2447, 4928, 4856, 4920, 4854}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^2])^2/x^6,x]
```


```
[Out] (-2*a*b*c)/(15*x^3) + (2*a*b*c^2)/(5*x) - (8*b^2*c^2)/(15*x) + (2*a*b*c^(5/
2)*ArcTan[Sqrt[c]*x])/5 - (4*b^2*c^(5/2)*ArcTan[Sqrt[c]*x])/15 + (I/5)*b^2*
c^(5/2)*ArcTan[Sqrt[c]*x]^2 + (4*b^2*c^(5/2)*ArcTanh[Sqrt[c]*x])/15 + (b^2*
c^(5/2)*ArcTanh[Sqrt[c]*x]^2)/5 - (2*b^2*c^(5/2)*ArcTanh[Sqrt[c]*x]*Log[2/(
1 - Sqrt[c]*x)])/5 - (2*b^2*c^(5/2)*ArcTan[Sqrt[c]*x]*Log[2/(1 - I*Sqrt[c]*
x)])/5 + (b^2*c^(5/2)*ArcTan[Sqrt[c]*x]*Log[((1 + I)*(1 - Sqrt[c]*x))/(1 -
I*Sqrt[c]*x)])/5 + (2*b^2*c^(5/2)*ArcTan[Sqrt[c]*x]*Log[2/(1 + I*Sqrt[c]*x)
])/5 + (2*b^2*c^(5/2)*ArcTanh[Sqrt[c]*x]*Log[2/(1 + Sqrt[c]*x)])/5 - (b^2*c
^(5/2)*ArcTanh[Sqrt[c]*x]*Log[(-2*Sqrt[c]*(1 - Sqrt[-c]*x))/((Sqrt[-c] - Sq
```

```
rt[c])*(1 + Sqrt[c]*x))])/5 - (b^2*c^(5/2)*ArcTanh[Sqrt[c]*x]*Log[(2*Sqrt[c
]*(1 + Sqrt[-c]*x))/((Sqrt[-c] + Sqrt[c])*(1 + Sqrt[c]*x))])/5 + (b^2*c^(5/
2)*ArcTan[Sqrt[c]*x]*Log[((1 - I)*(1 + Sqrt[c]*x))/(1 - I*Sqrt[c]*x)])/5 +
(b^2*c*Log[1 - c*x^2])/(15*x^3) - (b^2*c^2*Log[1 - c*x^2])/(5*x) - (b^2*c^(
5/2)*ArcTan[Sqrt[c]*x]*Log[1 - c*x^2])/5 - (b*c*(2*a - b*Log[1 - c*x^2]))/(
15*x^3) - (b*c^2*(2*a - b*Log[1 - c*x^2]))/(5*x) + (b*c^(5/2)*ArcTanh[Sqrt[
c]*x]*(2*a - b*Log[1 - c*x^2]))/5 - (2*a - b*Log[1 - c*x^2])^2/(20*x^5) - (
a*b*Log[1 + c*x^2])/(5*x^5) - (2*b^2*c*Log[1 + c*x^2])/(15*x^3) + (b^2*c^(5
/2)*ArcTan[Sqrt[c]*x]*Log[1 + c*x^2])/5 + (b^2*c^(5/2)*ArcTanh[Sqrt[c]*x]*L
og[1 + c*x^2])/5 + (b^2*Log[1 - c*x^2]*Log[1 + c*x^2])/(10*x^5) - (b^2*Log[
1 + c*x^2]^2)/(20*x^5) - (b^2*c^(5/2)*PolyLog[2, 1 - 2/(1 - Sqrt[c]*x)])/5
+ (I/5)*b^2*c^(5/2)*PolyLog[2, 1 - 2/(1 - I*Sqrt[c]*x)] - (I/10)*b^2*c^(5/2
)*PolyLog[2, 1 - ((1 + I)*(1 - Sqrt[c]*x))/(1 - I*Sqrt[c]*x)] + (I/5)*b^2*c
^(5/2)*PolyLog[2, 1 - 2/(1 + I*Sqrt[c]*x)] - (b^2*c^(5/2)*PolyLog[2, 1 - 2/
(1 + Sqrt[c]*x)])/5 + (b^2*c^(5/2)*PolyLog[2, 1 + (2*Sqrt[c]*(1 - Sqrt[-c]*
x))/((Sqrt[-c] - Sqrt[c])*(1 + Sqrt[c]*x))])/10 + (b^2*c^(5/2)*PolyLog[2, 1
 - (2*Sqrt[c]*(1 + Sqrt[-c]*x))/((Sqrt[-c] + Sqrt[c])*(1 + Sqrt[c]*x))])/10
 - (I/10)*b^2*c^(5/2)*PolyLog[2, 1 - ((1 - I)*(1 + Sqrt[c]*x))/(1 - I*Sqrt[
c]*x)]
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q
)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a +
 b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2557

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] :> With[{z = IntHide[u, x]}, Dist[Log[v
]*Log[w], z, x] + (-Int[SimplifyIntegrand[(z*Log[w]*D[v, x])/v, x], x] - In
t[SimplifyIntegrand[(z*Log[v]*D[w, x])/w, x], x]) /; InverseFunctionFreeQ[z
, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 5992

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{x^6}\,dx = \int \left(\frac{\left(2a-b\log\left(1-cx^2\right)\right)^2}{4x^6} - \frac{b\left(-2a+b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right)}{2x^6} + \frac{b^2\log^2\left(1+cx^2\right)}{4x^6}\right.$$

$$= \frac{1}{4}\int \frac{\left(2a-b\log\left(1-cx^2\right)\right)^2}{x^6}\,dx - \frac{1}{2}b\int \frac{\left(-2a+b\log\left(1-cx^2\right)\right)\log\left(1+cx^2\right)}{x^6}\,dx + \frac{1}{4}b^2$$

$$= -\frac{\left(2a-b\log\left(1-cx^2\right)\right)^2}{20x^5} - \frac{b^2\log^2\left(1+cx^2\right)}{20x^5} - \frac{1}{2}b\int\left(-\frac{2a\log\left(1+cx^2\right)}{x^6} + \frac{b\log\left(1-cx\right.}{}\right.$$

$$= -\frac{\left(2a-b\log\left(1-cx^2\right)\right)^2}{20x^5} - \frac{b^2\log^2\left(1+cx^2\right)}{20x^5} + (ab)\int \frac{\log\left(1+cx^2\right)}{x^6}\,dx - \frac{1}{2}b^2\int \frac{\log\left(1\right.}{}$$

$$= -\frac{\left(2a-b\log\left(1-cx^2\right)\right)^2}{20x^5} - \frac{ab\log\left(1+cx^2\right)}{5x^5} + \frac{b^2\log\left(1-cx^2\right)\log\left(1+cx^2\right)}{10x^5} - \frac{b^2\log^2\left(\right.}{20}$$

$$= -\frac{2abc}{15x^3} - \frac{bc\left(2a-b\log\left(1-cx^2\right)\right)}{15x^3} - \frac{bc^2\left(2a-b\log\left(1-cx^2\right)\right)}{5x} + \frac{1}{5}bc^{5/2}\tanh^{-1}\left(\sqrt{c}x\right)\left(\right.$$

$$= -\frac{2abc}{15x^3} + \frac{2abc^2}{5x} - \frac{4b^2c^2}{15x} - \frac{2}{5}b^2c^{5/2}\tan^{-1}\left(\sqrt{c}x\right) + \frac{2}{5}b^2c^{5/2}\tanh^{-1}\left(\sqrt{c}x\right) - \frac{bc\left(2a-b\log\right.}{15}$$

$$= -\frac{2abc}{15x^3} + \frac{2abc^2}{5x} - \frac{4b^2c^2}{15x} + \frac{2}{5}abc^{5/2}\tan^{-1}\left(\sqrt{c}x\right) - \frac{8}{15}b^2c^{5/2}\tan^{-1}\left(\sqrt{c}x\right) + \frac{1}{5}ib^2c^{5/2}\tan$$

$$= -\frac{2abc}{15x^3} + \frac{2abc^2}{5x} - \frac{4b^2c^2}{15x} + \frac{2}{5}abc^{5/2}\tan^{-1}\left(\sqrt{c}x\right) - \frac{8}{15}b^2c^{5/2}\tan^{-1}\left(\sqrt{c}x\right) + \frac{1}{5}ib^2c^{5/2}\tan$$

$$= -\frac{2abc}{15x^3} + \frac{2abc^2}{5x} - \frac{8b^2c^2}{15x} + \frac{2}{5}abc^{5/2}\tan^{-1}\left(\sqrt{c}x\right) - \frac{2}{15}b^2c^{5/2}\tan^{-1}\left(\sqrt{c}x\right) + \frac{1}{5}ib^2c^{5/2}\tan$$

$$= -\frac{2abc}{15x^3} + \frac{2abc^2}{5x} - \frac{8b^2c^2}{15x} + \frac{2}{5}abc^{5/2}\tan^{-1}\left(\sqrt{c}x\right) - \frac{4}{15}b^2c^{5/2}\tan^{-1}\left(\sqrt{c}x\right) + \frac{1}{5}ib^2c^{5/2}\tan$$

$$= -\frac{2abc}{15x^3} + \frac{2abc^2}{5x} - \frac{8b^2c^2}{15x} + \frac{2}{5}abc^{5/2}\tan^{-1}\left(\sqrt{c}x\right) - \frac{4}{15}b^2c^{5/2}\tan^{-1}\left(\sqrt{c}x\right) + \frac{1}{5}ib^2c^{5/2}\tan$$

$$= -\frac{2abc}{15x^3} + \frac{2abc^2}{5x} - \frac{8b^2c^2}{15x} + \frac{2}{5}abc^{5/2}\tan^{-1}\left(\sqrt{c}x\right) - \frac{4}{15}b^2c^{5/2}\tan^{-1}\left(\sqrt{c}x\right) + \frac{1}{5}ib^2c^{5/2}\tan$$

$$= -\frac{2abc}{15x^3} + \frac{2abc^2}{5x} - \frac{8b^2c^2}{15x} + \frac{2}{5}abc^{5/2}\tan^{-1}\left(\sqrt{c}x\right) - \frac{4}{15}b^2c^{5/2}\tan^{-1}\left(\sqrt{c}x\right) + \frac{1}{5}ib^2c^{5/2}\tan$$

$$= -\frac{2abc}{15x^3} + \frac{2abc^2}{5x} - \frac{8b^2c^2}{15x} + \frac{2}{5}abc^{5/2}\tan^{-1}\left(\sqrt{c}x\right) - \frac{4}{15}b^2c^{5/2}\tan^{-1}\left(\sqrt{c}x\right) + \frac{1}{5}ib^2c^{5/2}\tan$$

Mathematica [F] time = 3.01188, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{x^6}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])^2/x^6,x]
```

```
[Out] Integrate[(a + b*ArcTanh[c*x^2])^2/x^6, x]
```

Maple [F] time = 0.167, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\operatorname{Artanh}\left(c x^{2}\right)\right)^{2}}{x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))^2/x^6,x)
```

```
[Out] int((a+b*arctanh(c*x^2))^2/x^6,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^{2}\operatorname{artanh}\left(c x^{2}\right)^{2}+2\,ab\operatorname{artanh}\left(c x^{2}\right)+a^{2}}{x^{6}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/x^6,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x^2)^2 + 2*a*b*arctanh(c*x^2) + a^2)/x^6, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+b\operatorname{atanh}\left(c x^{2}\right)\right)^{2}}{x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))**2/x**6,x)
```

```
[Out] Integral((a + b*atanh(c*x**2))**2/x**6, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \operatorname{artanh}\left(cx^2\right) + a\right)^2}{x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/x^6,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^2) + a)^2/x^6, x)
```

3.77 $\int x^3 \left(a+b\tanh^{-1}\left(cx^2\right)\right)^3 dx$

Optimal. Leaf size=141

$$-\frac{3b^3\text{PolyLog}\left(2,1-\frac{2}{1-cx^2}\right)}{4c^2}-\frac{3b^2\log\left(\frac{2}{1-cx^2}\right)\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{2c^2}-\frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^3}{4c^2}+\frac{3b\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{4c^2}$$

```
[Out] (3*b*(a + b*ArcTanh[c*x^2])^2)/(4*c^2) + (3*b*x^2*(a + b*ArcTanh[c*x^2])^2)
/(4*c) - (a + b*ArcTanh[c*x^2])^3/(4*c^2) + (x^4*(a + b*ArcTanh[c*x^2])^3)/
4 - (3*b^2*(a + b*ArcTanh[c*x^2])*Log[2/(1 - c*x^2)])/(2*c^2) - (3*b^3*Poly
Log[2, 1 - 2/(1 - c*x^2)])/(4*c^2)
```

Rubi [B] time = 4.21552, antiderivative size = 479, normalized size of antiderivative = 3.4, number of steps used = 155, number of rules used = 30, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.875, Rules used = {6099, 2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304, 2439, 2416, 2396, 2433, 2374, 6589, 2411, 43, 2334, 12, 14, 2301, 6742, 2430, 2394, 2393, 2391, 2395, 2375, 2317, 2425}

$$-\frac{3b^3\text{PolyLog}\left(2,\frac{1}{2}\left(1-cx^2\right)\right)}{8c^2}+\frac{3b^3\text{PolyLog}\left(2,\frac{1}{2}\left(cx^2+1\right)\right)}{8c^2}-\frac{3b^2\log^2\left(cx^2+1\right)\left(2a-b\log\left(1-cx^2\right)\right)}{32c^2}+\frac{3b^2\log\left(\frac{1}{2}\left(c\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^3*(a + b*ArcTanh[c*x^2])^3,x]
```

```
[Out] (-3*b*(1 - c*x^2)*(2*a - b*Log[1 - c*x^2])^2)/(16*c^2) - ((1 - c*x^2)*(2*a
- b*Log[1 - c*x^2])^3)/(16*c^2) + ((1 - c*x^2)^2*(2*a - b*Log[1 - c*x^2])^3
)/(32*c^2) + (3*b^2*(2*a - b*Log[1 - c*x^2])*Log[(1 + c*x^2)/2])/(8*c^2) +
(3*b^3*Log[(1 - c*x^2)/2]*Log[1 + c*x^2])/(8*c^2) + (3*b^2*x^2*(2*a - b*Log
[1 - c*x^2])*Log[1 + c*x^2])/(8*c) - (3*b*(2*a - b*Log[1 - c*x^2])^2*Log[1
+ c*x^2])/(32*c^2) + (3*b*x^4*(2*a - b*Log[1 - c*x^2])^2*Log[1 + c*x^2])/32
 + (3*b^3*(1 + c*x^2)*Log[1 + c*x^2]^2)/(16*c^2) - (3*b^2*(2*a - b*Log[1 -
c*x^2])*Log[1 + c*x^2]^2)/(32*c^2) + (3*b^2*x^4*(2*a - b*Log[1 - c*x^2])*Lo
g[1 + c*x^2]^2)/32 - (b^3*(1 + c*x^2)*Log[1 + c*x^2]^3)/(16*c^2) + (b^3*(1
+ c*x^2)^2*Log[1 + c*x^2]^3)/(32*c^2) - (3*b^3*PolyLog[2, (1 - c*x^2)/2])/(
8*c^2) + (3*b^3*PolyLog[2, (1 + c*x^2)/2])/(8*c^2)
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
```

```
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[(x^
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
 + j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1
] && EqQ[m, -1])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
```

```
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2430

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.)), x_Symbol] :> Simp[x*(a + b*Log[c
*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]), x] + (-Dist[g*j*m, Int[(x*(a +
 b*Log[c*(d + e*x)^n])^p)/(i + j*x), x], x] - Dist[b*e*n*p, Int[(x*(a + b*L
og[c*(d + e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /
; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2375

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[(Log[d*(e + f*x^m)^r]*(a + b*Log[
c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[(f*m*r)/(b*n*(p + 1)), Int[(x^(m
- 1)*(a + b*Log[c*x^n])^(p + 1))/(e + f*x^m), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2425

```
Int[(Log[(f_.)*(x_)^(m_.)]*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b
_.)))/(x_), x_Symbol] :> Simp[(Log[f*x^m]^2*(a + b*Log[c*(d + e*x)^n]))/(2*
m), x] - Dist[(b*e*n)/(2*m), Int[Log[f*x^m]^2/(d + e*x), x], x] /; FreeQ[{a
, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int x^3\left(a+b\tanh^{-1}\left(cx^2\right)\right)^3dx = \int\left(\frac{1}{8}x^3\left(2a-b\log\left(1-cx^2\right)\right)^3+\frac{3}{8}bx^3\left(-2a+b\log\left(1-cx^2\right)\right)^2\log\left(1+cx^2\right)-\frac{3}{8}b^2x^3\left(\right.\right.$$

$$=\frac{1}{8}\int x^3\left(2a-b\log\left(1-cx^2\right)\right)^3dx+\frac{1}{8}(3b)\int x^3\left(-2a+b\log\left(1-cx^2\right)\right)^2\log\left(1+cx^2\right)d$$

$$=\frac{1}{16}\operatorname{Subst}\left(\int x(2a-b\log(1-cx))^3dx,x,x^2\right)+\frac{1}{16}(3b)\operatorname{Subst}\left(\int x(-2a+b\log(1-cx)\right.$$

$$=\frac{3}{32}bx^4\left(2a-b\log\left(1-cx^2\right)\right)^2\log\left(1+cx^2\right)+\frac{3}{32}b^2x^4\left(2a-b\log\left(1-cx^2\right)\right)\log^2\left(1+cx\right.$$

$$=\frac{3}{32}bx^4\left(2a-b\log\left(1-cx^2\right)\right)^2\log\left(1+cx^2\right)+\frac{3}{32}b^2x^4\left(2a-b\log\left(1-cx^2\right)\right)\log^2\left(1+cx\right.$$

$$=\frac{3}{32}bx^4\left(2a-b\log\left(1-cx^2\right)\right)^2\log\left(1+cx^2\right)+\frac{3}{32}b^2x^4\left(2a-b\log\left(1-cx^2\right)\right)\log^2\left(1+cx\right.$$

$$=-\frac{\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^3}{16c^2}+\frac{\left(1-cx^2\right)^2\left(2a-b\log\left(1-cx^2\right)\right)^3}{32c^2}-\frac{3b\left(2a-b\log\right.}{}$$

$$=-\frac{9b\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^2}{32c^2}+\frac{3b\left(1-cx^2\right)^2\left(2a-b\log\left(1-cx^2\right)\right)^2}{64c^2}-\frac{\left(1-cx^2\right)}{}$$

$$=\frac{9ab^2x^2}{8c}+\frac{9b^3x^2}{16c}+\frac{3b^3\left(1-cx^2\right)^2}{128c^2}-\frac{3b^3\left(1+cx^2\right)^2}{128c^2}+\frac{3b^2\left(1-cx^2\right)^2\left(2a-b\log\left(1-cx^2\right.\right.}{64c^2}$$

$$=\frac{9ab^2x^2}{8c}+\frac{9b^3x^2}{8c}+\frac{3b^3\left(1-cx^2\right)^2}{128c^2}-\frac{3b^3\left(1+cx^2\right)^2}{128c^2}+\frac{9b^3\left(1-cx^2\right)\log\left(1-cx^2\right)}{16c^2}+\frac{3b^2\left(\right.}{}$$

$$=\frac{3ab^2x^2}{4c}+\frac{15b^3x^2}{16c}+\frac{9b^3\left(1-cx^2\right)\log\left(1-cx^2\right)}{16c^2}-\frac{3b\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^2}{16c^2}-$$

$$=\frac{3ab^2x^2}{4c}+\frac{3b^3x^2}{4c}+\frac{3b^3\left(1-cx^2\right)\log\left(1-cx^2\right)}{8c^2}-\frac{3b\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^2}{16c^2}-\frac{\left(\right.}{}$$

Mathematica [A] time = 0.464082, size = 185, normalized size = 1.31

$$\frac{6b^3\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx^2)}\right)+a\left(2a^2c^2x^4+6abcx^2+3ab\log\left(1-cx^2\right)-3ab\log\left(cx^2+1\right)+6b^2\log\left(1-c^2x^4\right)\right)+\cdots}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*(a + b*ArcTanh[c*x^2])^3,x]
```

```
[Out] (6*b^2*(-1 + c*x^2)*(a + b + a*c*x^2)*ArcTanh[c*x^2]^2 + 2*b^3*(-1 + c^2*x^
4)*ArcTanh[c*x^2]^3 + 6*b*ArcTanh[c*x^2]*(a*c*x^2*(2*b + a*c*x^2) - 2*b^2*L
og[1 + E^(-2*ArcTanh[c*x^2])]) + a*(6*a*b*c*x^2 + 2*a^2*c^2*x^4 + 3*a*b*Log
[1 - c*x^2] - 3*a*b*Log[1 + c*x^2] + 6*b^2*Log[1 - c^2*x^4]) + 6*b^3*PolyLo
g[2, -E^(-2*ArcTanh[c*x^2])])/(8*c^2)
```

Maple [C] time = 0.269, size = 751, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c*x^2))^3,x)
```

```
[Out] 1/32*b^3*(c^2*x^4-1)/c^2*ln(c*x^2+1)^3+3/32*b^2*(-x^4*b*ln(-c*x^2+1)*c^2+2*
a*c^2*x^4+2*b*c*x^2+b*ln(-c*x^2+1)-2*a+2*b)/c^2*ln(c*x^2+1)^2+(3/32*b^3*(c^
2*x^4-1)/c^2*ln(-c*x^2+1)^2-3/8*b^2*x^2*(a*c*x^2+b)/c*ln(-c*x^2+1)-3/8*b*(-
a^2*c^2*x^4-2*a*b*c*x^2-b*ln(-c*x^2+1)*a-b^2*ln(-c*x^2+1))/c^2)*ln(c*x^2+1)
+3/16*b^3/c*x^2*ln(-c*x^2+1)^2-3/8*a^2*b*x^4*ln(-c*x^2+1)+3/8*a^2*b/c^2*ln(
c*x^2-1)-3/16*a*b^2/c^2*ln(-c*x^2+1)^2+3/8/c^2*b^3*ln(-c*x^2+1)-3/16*b^3/c^
2-3/8/c^2*b^3*ln(c*x^2-1)-1/32*b^3*x^4*ln(-c*x^2+1)^3-3/16*b^3/c^2*ln(-c*x^
2+1)^2+1/32*b^3/c^2*ln(-c*x^2+1)^3+3/4/c*a^2*b*x^2+3/16*b^2/c^2*a*ln(c*x^2-
1)-3/8*a^2*b/c^2*ln(c*x^2+1)+3/4*a*b^2/c^2*ln(c*x^2+1)+3/4*b^2/c*Sum(-(ln(x
-_alpha)*ln(-c*x^2+1)+2*c*(-1/2*ln(x-_alpha)*(ln((RootOf(_Z^2*c+2*_Z*_alpha
*c-2,index=1)-x+_alpha)/RootOf(_Z^2*c+2*_Z*_alpha*c-2,index=1))+ln((RootOf(
_Z^2*c+2*_Z*_alpha*c-2,index=2)-x+_alpha)/RootOf(_Z^2*c+2*_Z*_alpha*c-2,ind
ex=2)))/c-1/2*(dilog((RootOf(_Z^2*c+2*_Z*_alpha*c-2,index=1)-x+_alpha)/Root
Of(_Z^2*c+2*_Z*_alpha*c-2,index=1))+dilog((RootOf(_Z^2*c+2*_Z*_alpha*c-2,in
dex=2)-x+_alpha)/RootOf(_Z^2*c+2*_Z*_alpha*c-2,index=2)))/c))*b/c,_alpha=Ro
otOf(_Z^2*c+1))+1/4*x^4*a^3+3/16*a*b^2*x^4*ln(-c*x^2+1)^2+9/16*a*b^2/c^2*ln
(-c*x^2+1)-3/4*a*b^2/c*x^2*ln(-c*x^2+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^2))^3,x, algorithm="maxima")
```

```
[Out] 3/4*a*b^2*x^4*arctanh(c*x^2)^2 + 1/4*a^3*x^4 + 3/8*(2*x^4*arctanh(c*x^2) +
c*(2*x^2/c^2 - log(c*x^2 + 1)/c^3 + log(c*x^2 - 1)/c^3))*a^2*b + 3/16*(4*c*
(2*x^2/c^2 - log(c*x^2 + 1)/c^3 + log(c*x^2 - 1)/c^3)*arctanh(c*x^2) - (2*(
log(c*x^2 - 1) - 2)*log(c*x^2 + 1) - log(c*x^2 + 1)^2 - log(c*x^2 - 1)^2 -
```

```
4*log(c*x^2 - 1))/c^2)*a*b^2 - 1/128*(4*x^4*log(-c*x^2 + 1)^3 + 3*c^3*(x^4/
c^3 + log(c^2*x^4 - 1)/c^5) - 6*c*((c*x^4 + 2*x^2)/c^2 + 2*log(c*x^2 - 1)/c
^3)*log(-c*x^2 + 1)^2 + 21*c^2*(2*x^2/c^3 - log(c*x^2 + 1)/c^4 + log(c*x^2
- 1)/c^4) + c*(6*(c^2*x^4 + 6*c*x^2 + 2*log(c*x^2 - 1)^2 + 6*log(c*x^2 - 1)
)*log(-c*x^2 + 1)/c^3 - (3*c^2*x^4 + 42*c*x^2 + 4*log(c*x^2 - 1)^3 + 18*log
(c*x^2 - 1)^2 + 42*log(c*x^2 - 1))/c^3) - 1152*c*integrate(1/4*x^3*log(c*x^
2 + 1)/(c^3*x^4 - c), x) - 2*(12*c*x^2*log(c*x^2 + 1)^2 + 2*(c^2*x^4 - 1)*l
og(c*x^2 + 1)^3 - 3*(c^2*x^4 - 2*c*x^2 - 2*(c^2*x^4 - 1)*log(c*x^2 + 1) + 1
)*log(-c*x^2 + 1)^2 + 3*(c^2*x^4 + 6*c*x^2 - 2*(c^2*x^4 - 1)*log(c*x^2 + 1)
^2 - 8*(c*x^2 + 1)*log(c*x^2 + 1))*log(-c*x^2 + 1))/c^2 + 18*log(4*c^3*x^4
- 4*c)/c^2 - 384*integrate(1/4*x*log(c*x^2 + 1)/(c^3*x^4 - c), x))*b^3
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(b^3x^3\,\mathrm{artanh}\left(cx^2\right)^3+3\,ab^2x^3\,\mathrm{artanh}\left(cx^2\right)^2+3\,a^2bx^3\,\mathrm{artanh}\left(cx^2\right)+a^3x^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^2))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^3*arctanh(c*x^2)^3 + 3*a*b^2*x^3*arctanh(c*x^2)^2 + 3*a^2*b*
x^3*arctanh(c*x^2) + a^3*x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\left(a+b\,\mathrm{atanh}\left(cx^2\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c*x**2))**3,x)
```

```
[Out] Integral(x**3*(a + b*atanh(c*x**2))**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\,\mathrm{artanh}\left(cx^2\right)+a\right)^3x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^2))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^2) + a)^3*x^3, x)
```

3.78 $\int x \left(a + b \tanh^{-1}\left(cx^2\right)\right)^3 dx$

Optimal. Leaf size=134

$$-\frac{3b^2\text{PolyLog}\left(2, 1 - \frac{2}{1-cx^2}\right)\left(a + b\tanh^{-1}\left(cx^2\right)\right)}{2c} + \frac{3b^3\text{PolyLog}\left(3, 1 - \frac{2}{1-cx^2}\right)}{4c} + \frac{1}{2}x^2\left(a + b\tanh^{-1}\left(cx^2\right)\right)^3 + \frac{\left(a + b\right.}{}$$

```
[Out] (a + b*ArcTanh[c*x^2])^3/(2*c) + (x^2*(a + b*ArcTanh[c*x^2])^3)/2 - (3*b*(a
 + b*ArcTanh[c*x^2])^2*Log[2/(1 - c*x^2)])/(2*c) - (3*b^2*(a + b*ArcTanh[c*
x^2])*PolyLog[2, 1 - 2/(1 - c*x^2)])/(2*c) + (3*b^3*PolyLog[3, 1 - 2/(1 - c
*x^2)])/(4*c)
```

Rubi [B] time = 2.32313, antiderivative size = 390, normalized size of antiderivative = 2.91, number of steps used = 82, number of rules used = 23, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.643, Rules used = {6099, 2454, 2389, 2296, 2295, 6715, 2430, 2416, 2396, 2433, 2374, 6589, 2411, 2346, 2301, 6742, 43, 2394, 2393, 2391, 2375, 2317, 2425}

$$-\frac{3b^2\text{PolyLog}\left(2, \frac{1}{2}\left(1 - cx^2\right)\right)\left(2a - b\log\left(1 - cx^2\right)\right)}{4c} - \frac{3b^3\text{PolyLog}\left(3, \frac{1}{2}\left(1 - cx^2\right)\right)}{4c} - \frac{3b^3\text{PolyLog}\left(3, \frac{1}{2}\left(cx^2 + 1\right)\right)}{4c} +$$

Warning: Unable to verify antiderivative.

```
[In] Int[x*(a + b*ArcTanh[c*x^2])^3,x]
```

```
[Out] -((1 - c*x^2)*(2*a - b*Log[1 - c*x^2])^3)/(16*c) + (3*b*(2*a - b*Log[1 - c*
x^2])^2*Log[(1 + c*x^2)/2])/(8*c) - (3*b*(2*a - b*Log[1 - c*x^2])^2*Log[1 +
 c*x^2])/(16*c) + (3*b*x^2*(2*a - b*Log[1 - c*x^2])^2*Log[1 + c*x^2])/16 +
(3*b^3*Log[(1 - c*x^2)/2]*Log[1 + c*x^2]^2)/(8*c) + (3*b^2*(2*a - b*Log[1 -
 c*x^2])*Log[1 + c*x^2]^2)/(16*c) + (3*b^2*x^2*(2*a - b*Log[1 - c*x^2])*Log
[1 + c*x^2]^2)/16 + (b^3*(1 + c*x^2)*Log[1 + c*x^2]^3)/(16*c) - (3*b^2*(2*a
 - b*Log[1 - c*x^2])*PolyLog[2, (1 - c*x^2)/2])/(4*c) + (3*b^3*Log[1 + c*x^
2]*PolyLog[2, (1 + c*x^2)/2])/(4*c) - (3*b^3*PolyLog[3, (1 - c*x^2)/2])/(4*
c) - (3*b^3*PolyLog[3, (1 + c*x^2)/2])/(4*c)
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 2430

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.)), x_Symbol] :> Simp[x*(a + b*Log[c
*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]), x] + (-Dist[g*j*m, Int[(x*(a +
 b*Log[c*(d + e*x)^n])^p)/(i + j*x), x], x] - Dist[b*e*n*p, Int[(x*(a + b*L
og[c*(d + e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /
; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2346

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Dist[d, Int[((d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p)/x,
 x], x] + Dist[e, Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; Fre
eQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2375

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[(Log[d*(e + f*x^m)^r]*(a + b*Log[
```

```
c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[(f*m*r)/(b*n*(p + 1)), Int[(x^(m
- 1)*(a + b*Log[c*x^n])^(p + 1))/(e + f*x^m), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2425

```
Int[(Log[(f_.)*(x_)^(m_.)]*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b
_.)))/(x_), x_Symbol] :> Simp[(Log[f*x^m]^2*(a + b*Log[c*(d + e*x)^n]))/(2*
m), x] - Dist[(b*e*n)/(2*m), Int[Log[f*x^m]^2/(d + e*x), x], x] /; FreeQ[{a
, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int x\left(a+b\tanh^{-1}\left(cx^2\right)\right)^3\,dx=\int\left(\frac{1}{8}x\left(2a-b\log\left(1-cx^2\right)\right)^3+\frac{3}{8}bx\left(-2a+b\log\left(1-cx^2\right)\right)^2\log\left(1+cx^2\right)-\frac{3}{8}b^2x\left(-2a\right.\right.$$
$$=\frac{1}{8}\int x\left(2a-b\log\left(1-cx^2\right)\right)^3\,dx+\frac{1}{8}(3b)\int x\left(-2a+b\log\left(1-cx^2\right)\right)^2\log\left(1+cx^2\right)\,dx-$$
$$=\frac{1}{16}\,\mathrm{Subst}\left(\int(2a-b\log(1-cx))^3\,dx,x,x^2\right)+\frac{1}{16}(3b)\,\mathrm{Subst}\left(\int(-2a+b\log(1-cx))^2\log\right.$$
$$=\frac{3}{16}bx^2\left(2a-b\log\left(1-cx^2\right)\right)^2\log\left(1+cx^2\right)+\frac{3}{16}b^2x^2\left(2a-b\log\left(1-cx^2\right)\right)\log^2\left(1+cx^2\right)$$
$$=-\frac{\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^3}{16c}+\frac{3}{16}bx^2\left(2a-b\log\left(1-cx^2\right)\right)^2\log\left(1+cx^2\right)+\frac{3}{16}b^2x^2$$
$$=-\frac{3b\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^2}{16c}-\frac{\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^3}{16c}+\frac{3}{16}bx^2\left(2a-b\right.$$
$$=\frac{3}{4}ab^2x^2-\frac{3b^3x^2}{8}-\frac{3b\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^2}{16c}-\frac{\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^3}{16c}$$
$$=\frac{3}{4}ab^2x^2+\frac{3b^3\left(1-cx^2\right)\log\left(1-cx^2\right)}{8c}-\frac{\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^3}{16c}+\frac{3b\left(2a-b\log\right.}{}$$
$$=\frac{3b^3x^2}{8}+\frac{3b^3\left(1-cx^2\right)\log\left(1-cx^2\right)}{8c}-\frac{\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^3}{16c}+\frac{3b\left(2a-b\log\right.}{}$$
$$=-\frac{\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^3}{16c}+\frac{3b\left(2a-b\log\left(1-cx^2\right)\right)^2\log\left(\frac{1}{2}\left(1+cx^2\right)\right)}{8c}-\frac{3b\left(2a\right.}{}$$
$$=-\frac{\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^3}{16c}+\frac{3b\left(2a-b\log\left(1-cx^2\right)\right)^2\log\left(\frac{1}{2}\left(1+cx^2\right)\right)}{8c}-\frac{3b\left(2a\right.}{}$$

Mathematica [A] time = 0.222373, size = 213, normalized size = 1.59

$$\frac{3ab^2\left(\mathrm{PolyLog}\left(2,-e^{-2\tanh^{-1}\left(cx^2\right)}\right)+\tanh^{-1}\left(cx^2\right)\left(cx^2\tanh^{-1}\left(cx^2\right)-\tanh^{-1}\left(cx^2\right)-2\log\left(e^{-2\tanh^{-1}\left(cx^2\right)}+1\right)\right)\right)}{2c}+\frac{b^3\left(\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(a + b*ArcTanh[c*x^2])^3,x]
```

```
[Out] (a^3*x^2)/2 + (3*a^2*b*x^2*ArcTanh[c*x^2])/2 + (3*a^2*b*Log[1 - c^2*x^4])/(
4*c) + (3*a*b^2*(ArcTanh[c*x^2]*(-ArcTanh[c*x^2] + c*x^2*ArcTanh[c*x^2] - 2
*Log[1 + E^(-2*ArcTanh[c*x^2])]) + PolyLog[2, -E^(-2*ArcTanh[c*x^2])]))/(2*
c) + (b^3*(ArcTanh[c*x^2]^2*(-ArcTanh[c*x^2] + c*x^2*ArcTanh[c*x^2] - 3*Log
[1 + E^(-2*ArcTanh[c*x^2])]) + 3*ArcTanh[c*x^2]*PolyLog[2, -E^(-2*ArcTanh[c
*x^2])] + (3*PolyLog[3, -E^(-2*ArcTanh[c*x^2])])/2))/(2*c)
```

Maple [B] time = 0.007, size = 298, normalized size = 2.2

$$\frac{x^2a^3}{2}+\frac{b^3x^2\left(\mathrm{Artanh}\left(cx^2\right)\right)^3}{2}+\frac{b^3\left(\mathrm{Artanh}\left(cx^2\right)\right)^3}{2c}-\frac{3\,b^3\left(\mathrm{Artanh}\left(cx^2\right)\right)^2}{2c}\ln\left(\frac{\left(cx^2+1\right)^2}{-c^2x^4+1}+1\right)-\frac{3\,b^3\mathrm{Artanh}\left(cx^2\right)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c*x^2))^3,x)
```

```
[Out] 1/2*x^2*a^3+1/2*b^3*x^2*arctanh(c*x^2)^3+1/2/c*b^3*arctanh(c*x^2)^3-3/2/c*b
^3*arctanh(c*x^2)^2*ln((c*x^2+1)^2/(-c^2*x^4+1)+1)-3/2/c*b^3*arctanh(c*x^2)
*polylog(2,-(c*x^2+1)^2/(-c^2*x^4+1))+3/4/c*b^3*polylog(3,-(c*x^2+1)^2/(-c^
2*x^4+1))+3/2*arctanh(c*x^2)^2*x^2*a*b^2-3/c*ln((c*x^2+1)^2/(-c^2*x^4+1)+1)
*arctanh(c*x^2)*a*b^2+3/2/c*a*b^2*arctanh(c*x^2)^2-3/2/c*polylog(2,-(c*x^2+
1)^2/(-c^2*x^4+1))*a*b^2+3/2*x^2*a^2*b*arctanh(c*x^2)+3/4/c*a^2*b*ln(-c^2*x
^4+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\,a^3x^2+\frac{3\left(2\,cx^2\,\mathrm{artanh}\left(cx^2\right)+\log\left(-c^2x^4+1\right)\right)a^2b}{4\,c}-\frac{\left(b^3cx^2-b^3\right)\log\left(-cx^2+1\right)^3-3\left(2\,ab^2cx^2+\left(b^3cx^2+b^3\right)\log\right.}{16\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^2))^3,x, algorithm="maxima")
```

```
[Out] 1/2*a^3*x^2 + 3/4*(2*c*x^2*arctanh(c*x^2) + log(-c^2*x^4 + 1))*a^2*b/c - 1/
16*((b^3*c*x^2 - b^3)*log(-c*x^2 + 1)^3 - 3*(2*a*b^2*c*x^2 + (b^3*c*x^2 + b
^3)*log(c*x^2 + 1))*log(-c*x^2 + 1)^2)/c - integrate(-1/8*((b^3*c*x^3 - b^3
*x)*log(c*x^2 + 1)^3 + 6*(a*b^2*c*x^3 - a*b^2*x)*log(c*x^2 + 1)^2 - 3*(4*a*
b^2*c*x^3 + (b^3*c*x^3 - b^3*x)*log(c*x^2 + 1)^2 + 2*((2*a*b^2*c + b^3*c)*x
^3 - (2*a*b^2 - b^3)*x)*log(c*x^2 + 1))*log(-c*x^2 + 1))/(c*x^2 - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(b^3x\,\mathrm{artanh}\left(cx^2\right)^3+3\,ab^2x\,\mathrm{artanh}\left(cx^2\right)^2+3\,a^2bx\,\mathrm{artanh}\left(cx^2\right)+a^3x,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^2))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x*arctanh(c*x^2)^3 + 3*a*b^2*x*arctanh(c*x^2)^2 + 3*a^2*b*x*ar
ctanh(c*x^2) + a^3*x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(a+b\operatorname{atanh}\left(cx^2\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c*x**2))**3,x)
```

```
[Out] Integral(x*(a + b*atanh(c*x**2))**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\operatorname{artanh}\left(cx^2\right)+a\right)^3x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^2))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^2) + a)^3*x, x)
```

3.79 $\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^3}{x}\,dx$

Optimal. Leaf size=207

$$\frac{3}{4}b^2\text{PolyLog}\left(3,1-\frac{2}{1-cx^2}\right)\left(a+b\tanh^{-1}\left(cx^2\right)\right)-\frac{3}{4}b^2\text{PolyLog}\left(3,\frac{2}{1-cx^2}-1\right)\left(a+b\tanh^{-1}\left(cx^2\right)\right)-\frac{3}{4}b\text{PolyLog}$$

```
[Out] (a + b*ArcTanh[c*x^2])^3*ArcTanh[1 - 2/(1 - c*x^2)] - (3*b*(a + b*ArcTanh[c
*x^2])^2*PolyLog[2, 1 - 2/(1 - c*x^2)])/4 + (3*b*(a + b*ArcTanh[c*x^2])^2*P
olyLog[2, -1 + 2/(1 - c*x^2)])/4 + (3*b^2*(a + b*ArcTanh[c*x^2])*PolyLog[3,
 1 - 2/(1 - c*x^2)])/4 - (3*b^2*(a + b*ArcTanh[c*x^2])*PolyLog[3, -1 + 2/(1
 - c*x^2)])/4 - (3*b^3*PolyLog[4, 1 - 2/(1 - c*x^2)])/8 + (3*b^3*PolyLog[4,
 -1 + 2/(1 - c*x^2)])/8
```

Rubi [A] time = 0.559731, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {6095, 5914, 6052, 5948, 6058, 6062, 6610}

$$\frac{3}{4}b^2\text{PolyLog}\left(3,1-\frac{2}{1-cx^2}\right)\left(a+b\tanh^{-1}\left(cx^2\right)\right)-\frac{3}{4}b^2\text{PolyLog}\left(3,\frac{2}{1-cx^2}-1\right)\left(a+b\tanh^{-1}\left(cx^2\right)\right)-\frac{3}{4}b\text{PolyLog}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^2])^3/x,x]
```

```
[Out] (a + b*ArcTanh[c*x^2])^3*ArcTanh[1 - 2/(1 - c*x^2)] - (3*b*(a + b*ArcTanh[c
*x^2])^2*PolyLog[2, 1 - 2/(1 - c*x^2)])/4 + (3*b*(a + b*ArcTanh[c*x^2])^2*P
olyLog[2, -1 + 2/(1 - c*x^2)])/4 + (3*b^2*(a + b*ArcTanh[c*x^2])*PolyLog[3,
 1 - 2/(1 - c*x^2)])/4 - (3*b^2*(a + b*ArcTanh[c*x^2])*PolyLog[3, -1 + 2/(1
 - c*x^2)])/4 - (3*b^3*PolyLog[4, 1 - 2/(1 - c*x^2)])/8 + (3*b^3*PolyLog[4,
 -1 + 2/(1 - c*x^2)])/8
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[
1/n, Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
 n}, x] && IGtQ[p, 0]
```

Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
 e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6062

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[k + 1, u])/(2
*c*d), x] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[k + 1,
u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && Eq
Q[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^3}{x}\,dx=\frac{1}{2}\,\mathrm{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{x}\,dx,x,x^2\right)$$

$$=\left(a+b\tanh^{-1}\left(cx^2\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1-cx^2}\right)-(3bc)\,\mathrm{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2\tanh^{-1}}{1-c^2x^2}\right.$$

$$=\left(a+b\tanh^{-1}\left(cx^2\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1-cx^2}\right)+\frac{1}{2}(3bc)\,\mathrm{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\right.}{1-c^2x^2}\right.$$

$$=\left(a+b\tanh^{-1}\left(cx^2\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1-cx^2}\right)-\frac{3}{4}b\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2\mathrm{Li}_2\left(1-\frac{2}{1-cx^2}\right)+$$

$$=\left(a+b\tanh^{-1}\left(cx^2\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1-cx^2}\right)-\frac{3}{4}b\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2\mathrm{Li}_2\left(1-\frac{2}{1-cx^2}\right)+$$

$$=\left(a+b\tanh^{-1}\left(cx^2\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1-cx^2}\right)-\frac{3}{4}b\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2\mathrm{Li}_2\left(1-\frac{2}{1-cx^2}\right)+$$

Mathematica [A] time = 0.191135, size = 211, normalized size = 1.02

$$\frac{3}{8}b\left(2\mathrm{PolyLog}\left(2,\frac{cx^2+1}{1-cx^2}\right)\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2-2\mathrm{PolyLog}\left(2,\frac{cx^2+1}{cx^2-1}\right)\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2+b\left(-2\mathrm{PolyLog}\left(3,\frac{cx}{1}\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])^3/x,x]
```

```
[Out] (a + b*ArcTanh[c*x^2])^3*ArcTanh[1 + 2/(-1 + c*x^2)] + (3*b*(2*(a + b*ArcTa
nh[c*x^2])^2*PolyLog[2, (1 + c*x^2)/(1 - c*x^2)] - 2*(a + b*ArcTanh[c*x^2])
^2*PolyLog[2, (1 + c*x^2)/(-1 + c*x^2)] + b*(-2*(a + b*ArcTanh[c*x^2])*Poly
Log[3, (1 + c*x^2)/(1 - c*x^2)] + 2*(a + b*ArcTanh[c*x^2])*PolyLog[3, (1 +
c*x^2)/(-1 + c*x^2)] + b*(PolyLog[4, (1 + c*x^2)/(1 - c*x^2)] - PolyLog[4,
(1 + c*x^2)/(-1 + c*x^2)]))))/8
```

Maple [F] time = 0.138, size = 0, normalized size = 0.

$$\int \frac{\left(a + b\text{Artanh}\left(cx^2\right)\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))^3/x,x)
```

```
[Out] int((a+b*arctanh(c*x^2))^3/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$a^3\log(x) + \int \frac{b^3\left(\log\left(cx^2+1\right) - \log\left(-cx^2+1\right)\right)^3}{8x} + \frac{3\,ab^2\left(\log\left(cx^2+1\right) - \log\left(-cx^2+1\right)\right)^2}{4x} + \frac{3\,a^2b\left(\log\left(cx^2+1\right) - \log\right.}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^3/x,x, algorithm="maxima")
```

```
[Out] a^3*log(x) + integrate(1/8*b^3*(log(c*x^2 + 1) - log(-c*x^2 + 1))^3/x + 3/4
*a*b^2*(log(c*x^2 + 1) - log(-c*x^2 + 1))^2/x + 3/2*a^2*b*(log(c*x^2 + 1) -
 log(-c*x^2 + 1))/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3\,\text{artanh}\left(cx^2\right)^3 + 3\,ab^2\,\text{artanh}\left(cx^2\right)^2 + 3\,a^2b\,\text{artanh}\left(cx^2\right) + a^3}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^3/x,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctanh(c*x^2)^3 + 3*a*b^2*arctanh(c*x^2)^2 + 3*a^2*b*arctanh
(c*x^2) + a^3)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + b\,\text{atanh}\left(cx^2\right)\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))**3/x,x)
```

```
[Out] Integral((a + b*atanh(c*x**2))**3/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\,\mathrm{artanh}\left(cx^2\right)+a\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^3/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^2) + a)^3/x, x)
```

3.80 $\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^3}{x^3} dx$

Optimal. Leaf size=125

$$-\frac{3}{2}b^2c\text{PolyLog}\left(2,\frac{2}{cx^2+1}-1\right)\left(a+b\tanh^{-1}\left(cx^2\right)\right)-\frac{3}{4}b^3c\text{PolyLog}\left(3,\frac{2}{cx^2+1}-1\right)+\frac{1}{2}c\left(a+b\tanh^{-1}\left(cx^2\right)\right)^3-$$

```
[Out] (c*(a + b*ArcTanh[c*x^2])^3)/2 - (a + b*ArcTanh[c*x^2])^3/(2*x^2) + (3*b*c*
(a + b*ArcTanh[c*x^2])^2*Log[2 - 2/(1 + c*x^2)])/2 - (3*b^2*c*(a + b*ArcTan
h[c*x^2])*PolyLog[2, -1 + 2/(1 + c*x^2)])/2 - (3*b^3*c*PolyLog[3, -1 + 2/(1
 + c*x^2)])/4
```

Rubi [F] time = 0.780071, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^3}{x^3} dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcTanh[c*x^2])^3/x^3,x]
```

```
[Out] (3*b*c*Log[c*x^2]*(2*a - b*Log[1 - c*x^2])^2)/16 - ((1 - c*x^2)*(2*a - b*Lo
g[1 - c*x^2])^3)/(16*x^2) + (3*b^3*c*Log[-(c*x^2)]*Log[1 + c*x^2]^2)/16 - (
b^3*(1 + c*x^2)*Log[1 + c*x^2]^3)/(16*x^2) - (3*b^2*c*(2*a - b*Log[1 - c*x^
2])*PolyLog[2, 1 - c*x^2])/8 + (3*b^3*c*Log[1 + c*x^2]*PolyLog[2, 1 + c*x^2
])/8 - (3*b^3*c*PolyLog[3, 1 - c*x^2])/8 - (3*b^3*c*PolyLog[3, 1 + c*x^2])/
8 + (3*b*Defer[Subst][Defer[Int][((-2*a + b*Log[1 - c*x])^2*Log[1 + c*x])/x
^2, x], x, x^2])/16 - (3*b^2*Defer[Subst][Defer[Int][((-2*a + b*Log[1 - c*x
])*Log[1 + c*x]^2)/x^2, x], x, x^2])/16
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^3}{x^3}\,dx = \int \left(\frac{\left(2a-b\log\left(1-cx^2\right)\right)^3}{8x^3} + \frac{3b\left(-2a+b\log\left(1-cx^2\right)\right)^2\log\left(1+cx^2\right)}{8x^3} - \frac{3b^2\left(-2a+b\log\right.}{}\right.$$

$$= \frac{1}{8}\int \frac{\left(2a-b\log\left(1-cx^2\right)\right)^3}{x^3}\,dx + \frac{1}{8}(3b)\int \frac{\left(-2a+b\log\left(1-cx^2\right)\right)^2\log\left(1+cx^2\right)}{x^3}\,dx - \frac{1}{8}($$

$$= \frac{1}{16}\,\text{Subst}\left(\int \frac{(2a-b\log(1-cx))^3}{x^2}\,dx, x, x^2\right) + \frac{1}{16}(3b)\,\text{Subst}\left(\int \frac{(-2a+b\log(1-cx))^2\,\text{lo}}{x^2}\right.$$

$$= -\frac{\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^3}{16x^2} - \frac{b^3\left(1+cx^2\right)\log^3\left(1+cx^2\right)}{16x^2} + \frac{1}{16}(3b)\,\text{Subst}\left(\int \frac{(-2a}{}\right.$$

$$= \frac{3}{16}bc\log\left(cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^2 - \frac{\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^3}{16x^2} + \frac{3}{16}b^3c\log\left(-cx\right.$$

$$= \frac{3}{16}bc\log\left(cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^2 - \frac{\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^3}{16x^2} + \frac{3}{16}b^3c\log\left(-cx\right.$$

$$= \frac{3}{16}bc\log\left(cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^2 - \frac{\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^3}{16x^2} + \frac{3}{16}b^3c\log\left(-cx\right.$$

$$= \frac{3}{16}bc\log\left(cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^2 - \frac{\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^3}{16x^2} + \frac{3}{16}b^3c\log\left(-cx\right.$$

Mathematica [C] time = 0.414398, size = 222, normalized size = 1.78

$$\frac{1}{4}\left(6ab^2c\left(\tanh^{-1}\left(cx^2\right)\left(\left(1-\frac{1}{cx^2}\right)\tanh^{-1}\left(cx^2\right)+2\log\left(1-e^{-2\tanh^{-1}\left(cx^2\right)}\right)\right)-\text{PolyLog}\left(2,e^{-2\tanh^{-1}\left(cx^2\right)}\right)\right)+2b^3c\left(3\tan\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])^3/x^3,x]
```

```
[Out] ((-2*a^3)/x^2 - (6*a^2*b*ArcTanh[c*x^2])/x^2 + 12*a^2*b*c*Log[x] - 3*a^2*b*
c*Log[1 - c^2*x^4] + 6*a*b^2*c*(ArcTanh[c*x^2]*((1 - 1/(c*x^2))*ArcTanh[c*x
^2] + 2*Log[1 - E^(-2*ArcTanh[c*x^2])]) - PolyLog[2, E^(-2*ArcTanh[c*x^2])]
) + 2*b^3*c*((I/8)*Pi^3 - ArcTanh[c*x^2]^3 - ArcTanh[c*x^2]^3/(c*x^2) + 3*A
rcTanh[c*x^2]^2*Log[1 - E^(2*ArcTanh[c*x^2])] + 3*ArcTanh[c*x^2]*PolyLog[2,
 E^(2*ArcTanh[c*x^2])] - (3*PolyLog[3, E^(2*ArcTanh[c*x^2])])/2))/4
```

Maple [F] time = 0.194, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\text{Artanh}\left(cx^2\right)\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))^3/x^3,x)
```

```
[Out] int((a+b*arctanh(c*x^2))^3/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{3}{4}\left(c\left(\log\left(c^2x^4-1\right)-\log\left(x^4\right)\right)+\frac{2\,\operatorname{artanh}\left(cx^2\right)}{x^2}\right)a^2b-\frac{a^3}{2\,x^2}-\frac{\left(b^3cx^2-b^3\right)\log\left(-cx^2+1\right)^3+3\left(2\,ab^2+\left(b^3cx^2+b^3\right.\right.}{16\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^3/x^3,x, algorithm="maxima")
```

```
[Out] -3/4*(c*(log(c^2*x^4 - 1) - log(x^4)) + 2*arctanh(c*x^2)/x^2)*a^2*b - 1/2*a
^3/x^2 - 1/16*((b^3*c*x^2 - b^3)*log(-c*x^2 + 1)^3 + 3*(2*a*b^2 + (b^3*c*x^
2 + b^3)*log(c*x^2 + 1))*log(-c*x^2 + 1)^2)/x^2 - integrate(-1/8*((b^3*c*x^
2 - b^3)*log(c*x^2 + 1)^3 + 6*(a*b^2*c*x^2 - a*b^2)*log(c*x^2 + 1)^2 + 3*(4
*a*b^2*c*x^2 - (b^3*c*x^2 - b^3)*log(c*x^2 + 1)^2 + 2*(b^3*c^2*x^4 + 2*a*b^
2 - (2*a*b^2*c - b^3*c)*x^2)*log(c*x^2 + 1))*log(-c*x^2 + 1))/(c*x^5 - x^3)
, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{b^3\operatorname{artanh}\left(cx^2\right)^3+3\,ab^2\operatorname{artanh}\left(cx^2\right)^2+3\,a^2b\operatorname{artanh}\left(cx^2\right)+a^3}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^3/x^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctanh(c*x^2)^3 + 3*a*b^2*arctanh(c*x^2)^2 + 3*a^2*b*arctanh
(c*x^2) + a^3)/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+b\operatorname{atanh}\left(cx^2\right)\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))**3/x**3,x)
```

```
[Out] Integral((a + b*atanh(c*x**2))**3/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(b\operatorname{artanh}\left(cx^2\right)+a\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^3/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^2) + a)^3/x^3, x)
```

3.81 $\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^3}{x^5}\,dx$

Optimal. Leaf size=139

$$-\frac{3}{4}b^3c^2\text{PolyLog}\left(2,\frac{2}{cx^2+1}-1\right)+\frac{3}{2}b^2c^2\log\left(2-\frac{2}{cx^2+1}\right)\left(a+b\tanh^{-1}\left(cx^2\right)\right)+\frac{3}{4}bc^2\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2+\frac{1}{4}c^2\left(a-\right.$$

```
[Out] (3*b*c^2*(a + b*ArcTanh[c*x^2])^2)/4 - (3*b*c*(a + b*ArcTanh[c*x^2])^2)/(4*
x^2) + (c^2*(a + b*ArcTanh[c*x^2])^3)/4 - (a + b*ArcTanh[c*x^2])^3/(4*x^4)
+ (3*b^2*c^2*(a + b*ArcTanh[c*x^2])*Log[2 - 2/(1 + c*x^2)])/2 - (3*b^3*c^2*
PolyLog[2, -1 + 2/(1 + c*x^2)])/4
```

Rubi [F] time = 1.55794, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^3}{x^5}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcTanh[c*x^2])^3/x^5,x]
```

```
[Out] (3*a*b^2*c^2*Log[x])/4 - (3*b*c*(1 - c*x^2)*(2*a - b*Log[1 - c*x^2])^2)/(32
*x^2) + (3*b*c^2*Log[c*x^2]*(2*a - b*Log[1 - c*x^2])^2)/32 + (c^2*(2*a - b*
Log[1 - c*x^2])^3)/32 - (2*a - b*Log[1 - c*x^2])^3/(32*x^4) - (3*b^3*c*(1 +
 c*x^2)*Log[1 + c*x^2]^2)/(32*x^2) - (3*b^3*c^2*Log[-(c*x^2)]*Log[1 + c*x^2
]^2)/32 + (b^3*c^2*Log[1 + c*x^2]^3)/32 - (b^3*Log[1 + c*x^2]^3)/(32*x^4) -
 (3*b^3*c^2*PolyLog[2, -(c*x^2)])/16 + (3*b^3*c^2*PolyLog[2, c*x^2])/16 - (
3*b^2*c^2*(2*a - b*Log[1 - c*x^2])*PolyLog[2, 1 - c*x^2])/16 - (3*b^3*c^2*L
og[1 + c*x^2]*PolyLog[2, 1 + c*x^2])/16 - (3*b^3*c^2*PolyLog[3, 1 - c*x^2])
/16 + (3*b^3*c^2*PolyLog[3, 1 + c*x^2])/16 + (3*b*Defer[Subst][Defer[Int][(
(-2*a + b*Log[1 - c*x])^2*Log[1 + c*x])/x^3, x], x, x^2])/16 - (3*b^2*Defer
[Subst][Defer[Int][((-2*a + b*Log[1 - c*x])*Log[1 + c*x]^2)/x^3, x], x, x^2
])/16
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^3}{x^5}\,dx = \int \left(\frac{\left(2a-b\log\left(1-cx^2\right)\right)^3}{8x^5} + \frac{3b\left(-2a+b\log\left(1-cx^2\right)\right)^2\log\left(1+cx^2\right)}{8x^5} - \frac{3b^2\left(-2a+b\right.}{}\right.$$

$$= \frac{1}{8}\int \frac{\left(2a-b\log\left(1-cx^2\right)\right)^3}{x^5}\,dx + \frac{1}{8}(3b)\int \frac{\left(-2a+b\log\left(1-cx^2\right)\right)^2\log\left(1+cx^2\right)}{x^5}\,dx -$$

$$= \frac{1}{16}\,\text{Subst}\left(\int \frac{(2a-b\log(1-cx))^3}{x^3}\,dx, x, x^2\right) + \frac{1}{16}(3b)\,\text{Subst}\left(\int \frac{(-2a+b\log(1-cx))}{x^3}\right.$$

$$= -\frac{\left(2a-b\log\left(1-cx^2\right)\right)^3}{32x^4} - \frac{b^3\log^3\left(1+cx^2\right)}{32x^4} + \frac{1}{16}(3b)\,\text{Subst}\left(\int \frac{(-2a+b\log(1-cx))^2}{x^3}\right.$$

$$= -\frac{\left(2a-b\log\left(1-cx^2\right)\right)^3}{32x^4} - \frac{b^3\log^3\left(1+cx^2\right)}{32x^4} - \frac{1}{32}(3b)\,\text{Subst}\left(\int \frac{(2a-b\log(x))^2}{x\left(\frac{1}{c}-\frac{x}{c}\right)^2}\,dx, x,\right.$$

$$= -\frac{\left(2a-b\log\left(1-cx^2\right)\right)^3}{32x^4} - \frac{b^3\log^3\left(1+cx^2\right)}{32x^4} - \frac{1}{32}(3b)\,\text{Subst}\left(\int \frac{(2a-b\log(x))^2}{\left(\frac{1}{c}-\frac{x}{c}\right)^2}\,dx, x,\right.$$

$$= -\frac{3bc\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^2}{32x^2} - \frac{\left(2a-b\log\left(1-cx^2\right)\right)^3}{32x^4} - \frac{3b^3c\left(1+cx^2\right)\log^2\left(\right.}{32x^2}$$

$$= \frac{3}{4}ab^2c^2\log(x) - \frac{3bc\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^2}{32x^2} + \frac{3}{32}bc^2\log\left(cx^2\right)\left(2a-b\log\left(1-\right.\right.$$

$$= \frac{3}{4}ab^2c^2\log(x) - \frac{3bc\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^2}{32x^2} + \frac{3}{32}bc^2\log\left(cx^2\right)\left(2a-b\log\left(1-\right.\right.$$

$$= \frac{3}{4}ab^2c^2\log(x) - \frac{3bc\left(1-cx^2\right)\left(2a-b\log\left(1-cx^2\right)\right)^2}{32x^2} + \frac{3}{32}bc^2\log\left(cx^2\right)\left(2a-b\log\left(1-\right.\right.$$

Mathematica [A] time = 0.290196, size = 218, normalized size = 1.57

$$\frac{-6b^3c^2x^4\text{PolyLog}\left(2, e^{-2\tanh^{-1}\left(cx^2\right)}\right) + a\left(-2a^2 - 3abc^2x^4\log\left(1-cx^2\right) + 3abc^2x^4\log\left(cx^2+1\right) - 6abcx^2 + 12b^2c^2x^4\,\text{lo}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])^3/x^5,x]
```

```
[Out] (6*b^2*(-1 + c*x^2)*(a + a*c*x^2 + b*c*x^2)*ArcTanh[c*x^2]^2 + 2*b^3*(-1 +
c^2*x^4)*ArcTanh[c*x^2]^3 - 6*b*ArcTanh[c*x^2]*(a^2 + 2*a*b*c*x^2 - 2*b^2*c
^2*x^4*Log[1 - E^(-2*ArcTanh[c*x^2])]) + a*(-2*a^2 - 6*a*b*c*x^2 - 3*a*b*c^
2*x^4*Log[1 - c*x^2] + 3*a*b*c^2*x^4*Log[1 + c*x^2] + 12*b^2*c^2*x^4*Log[(c
*x^2)/Sqrt[1 - c^2*x^4]]) - 6*b^3*c^2*x^4*PolyLog[2, E^(-2*ArcTanh[c*x^2])]
)/(8*x^4)
```

Maple [F] time = 0.231, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\text{Artanh}\left(cx^2\right)\right)^3}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))^3/x^5,x)
```

```
[Out] int((a+b*arctanh(c*x^2))^3/x^5,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{3}{8}\left(\left(c\log\left(cx^2+1\right)-c\log\left(cx^2-1\right)-\frac{2}{x^2}\right)c-\frac{2\,\mathrm{artanh}\left(cx^2\right)}{x^4}\right)a^2b+\frac{3}{16}\left(\left(2\left(\log\left(cx^2-1\right)-2\right)\log\left(cx^2+1\right)-\log\left(cx^2+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^3/x^5,x, algorithm="maxima")
```

```
[Out] 3/8*((c*log(c*x^2 + 1) - c*log(c*x^2 - 1) - 2/x^2)*c - 2*arctanh(c*x^2)/x^4
)*a^2*b + 3/16*((2*(log(c*x^2 - 1) - 2)*log(c*x^2 + 1) - log(c*x^2 + 1)^2 -
 log(c*x^2 - 1)^2 - 4*log(c*x^2 - 1) + 16*log(x))*c^2 + 4*(c*log(c*x^2 + 1)
 - c*log(c*x^2 - 1) - 2/x^2)*c*arctanh(c*x^2))*a*b^2 - 1/32*b^3*(((c^2*x^4
- 1)*log(-c*x^2 + 1)^3 + 3*(2*c*x^2 - (c^2*x^4 - 1)*log(c*x^2 + 1))*log(-c*
x^2 + 1)^2)/x^4 + 4*integrate(-((c*x^2 - 1)*log(c*x^2 + 1)^3 + 3*(2*c^2*x^4
 - (c*x^2 - 1)*log(c*x^2 + 1)^2 - (c^3*x^6 - c*x^2)*log(c*x^2 + 1))*log(-c*
x^2 + 1))/(c*x^7 - x^5), x)) - 3/4*a*b^2*arctanh(c*x^2)^2/x^4 - 1/4*a^3/x^4
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^3\,\mathrm{artanh}\left(cx^2\right)^3+3\,ab^2\,\mathrm{artanh}\left(cx^2\right)^2+3\,a^2b\,\mathrm{artanh}\left(cx^2\right)+a^3}{x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^3/x^5,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctanh(c*x^2)^3 + 3*a*b^2*arctanh(c*x^2)^2 + 3*a^2*b*arctanh
(c*x^2) + a^3)/x^5, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+b\,\mathrm{atanh}\left(cx^2\right)\right)^3}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))**3/x**5,x)
```

```
[Out] Integral((a + b*atanh(c*x**2))**3/x**5, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \operatorname{artanh}\left(c x^2\right)+a\right)^3}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^3/x^5,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^2) + a)^3/x^5, x)
```

3.82 $\int (dx)^{5/2} \left(a + b \tanh^{-1}\left(cx^2\right)\right) dx$

Optimal. Leaf size=317

$$\frac{2(dx)^{7/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{7d}-\frac{bd^{5/2}\log\left(\sqrt{c}\sqrt{d}x-\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{7\sqrt{2}c^{7/4}}+\frac{bd^{5/2}\log\left(\sqrt{c}\sqrt{d}x+\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{7\sqrt{2}c^{7/4}}+\frac{2bd^{5/2}\mathrm{t}}{}$$

```
[Out] (8*b*d*(d*x)^(3/2))/(21*c) + (2*b*d^(5/2)*ArcTan[(c^(1/4)*Sqrt[d*x])/Sqrt[d
]])/(7*c^(7/4)) + (Sqrt[2]*b*d^(5/2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[d*x])
/Sqrt[d]])/(7*c^(7/4)) - (Sqrt[2]*b*d^(5/2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqr
t[d*x])/Sqrt[d]])/(7*c^(7/4)) + (2*(d*x)^(7/2)*(a + b*ArcTanh[c*x^2]))/(7*d
) - (2*b*d^(5/2)*ArcTanh[(c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(7*c^(7/4)) - (b*d^(
5/2)*Log[Sqrt[d] + Sqrt[c]*Sqrt[d]*x - Sqrt[2]*c^(1/4)*Sqrt[d*x]])/(7*Sqrt[
2]*c^(7/4)) + (b*d^(5/2)*Log[Sqrt[d] + Sqrt[c]*Sqrt[d]*x + Sqrt[2]*c^(1/4)*
Sqrt[d*x]])/(7*Sqrt[2]*c^(7/4))
```

Rubi [A] time = 0.334361, antiderivative size = 317, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 14, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {6097, 16, 321, 329, 300, 297, 1162, 617, 204, 1165, 628, 298, 205, 208}

$$\frac{2(dx)^{7/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{7d}-\frac{bd^{5/2}\log\left(\sqrt{c}\sqrt{d}x-\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{7\sqrt{2}c^{7/4}}+\frac{bd^{5/2}\log\left(\sqrt{c}\sqrt{d}x+\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{7\sqrt{2}c^{7/4}}+\frac{2bd^{5/2}\mathrm{t}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(5/2)*(a + b*ArcTanh[c*x^2]),x]
```


```
[Out] (8*b*d*(d*x)^(3/2))/(21*c) + (2*b*d^(5/2)*ArcTan[(c^(1/4)*Sqrt[d*x])/Sqrt[d
]])/(7*c^(7/4)) + (Sqrt[2]*b*d^(5/2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[d*x])
/Sqrt[d]])/(7*c^(7/4)) - (Sqrt[2]*b*d^(5/2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqr
t[d*x])/Sqrt[d]])/(7*c^(7/4)) + (2*(d*x)^(7/2)*(a + b*ArcTanh[c*x^2]))/(7*d
) - (2*b*d^(5/2)*ArcTanh[(c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(7*c^(7/4)) - (b*d^(
5/2)*Log[Sqrt[d] + Sqrt[c]*Sqrt[d]*x - Sqrt[2]*c^(1/4)*Sqrt[d*x]])/(7*Sqrt[
2]*c^(7/4)) + (b*d^(5/2)*Log[Sqrt[d] + Sqrt[c]*Sqrt[d]*x + Sqrt[2]*c^(1/4)*
Sqrt[d*x]])/(7*Sqrt[2]*c^(7/4))
```


Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 300

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[x^m/(r + s
*x^(n/2)), x], x] + Dist[r/(2*a), Int[x^m/(r - s*x^(n/2)), x], x]] /; FreeQ
[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n/2] &&  !GtQ[a/b, 0]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int (dx)^{5/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)dx &= \frac{2(dx)^{7/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{7d}-\frac{(4bc)\int \frac{x(dx)^{7/2}}{1-c^2x^4}\,dx}{7d}\\
&= \frac{2(dx)^{7/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{7d}-\frac{(4bc)\int \frac{(dx)^{9/2}}{1-c^2x^4}\,dx}{7d^2}\\
&= \frac{8bd(dx)^{3/2}}{21c}+\frac{2(dx)^{7/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{7d}-\frac{\left(4bd^2\right)\int \frac{\sqrt{dx}}{1-c^2x^4}\,dx}{7c}\\
&= \frac{8bd(dx)^{3/2}}{21c}+\frac{2(dx)^{7/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{7d}-\frac{(8bd)\,\text{Subst}\left(\int \frac{x^2}{1-\frac{c^2x^8}{d^4}}\,dx,x,\sqrt{dx}\right)}{7c}\\
&= \frac{8bd(dx)^{3/2}}{21c}+\frac{2(dx)^{7/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{7d}-\frac{\left(4bd^3\right)\text{Subst}\left(\int \frac{x^2}{d^2-cx^4}\,dx,x,\sqrt{dx}\right)}{7c}-
\\
&= \frac{8bd(dx)^{3/2}}{21c}+\frac{2(dx)^{7/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{7d}-\frac{\left(2bd^3\right)\text{Subst}\left(\int \frac{1}{d-\sqrt{c}x^2}\,dx,x,\sqrt{dx}\right)}{7c^{3/2}}+
\\
&= \frac{8bd(dx)^{3/2}}{21c}+\frac{2bd^{5/2}\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{7c^{7/4}}+\frac{2(dx)^{7/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{7d}-\frac{2bd^{5/2}\tanh^{-1}}{7c^{7/4}}\\
&= \frac{8bd(dx)^{3/2}}{21c}+\frac{2bd^{5/2}\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{7c^{7/4}}+\frac{2(dx)^{7/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{7d}-\frac{2bd^{5/2}\tanh^{-1}}{7c^{7/4}}\\
&= \frac{8bd(dx)^{3/2}}{21c}+\frac{2bd^{5/2}\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{7c^{7/4}}+\frac{\sqrt{2}bd^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{7c^{7/4}}-\frac{\sqrt{2}bd^{5/2}\tan^{-1}}{7c}
\end{aligned}$$

Mathematica [A] time = 0.125296, size = 241, normalized size = 0.76

$$\frac{(dx)^{5/2}\left(12ac^{7/4}x^{7/2}+16bc^{3/4}x^{3/2}+12bc^{7/4}x^{7/2}\tanh^{-1}\left(cx^2\right)+6b\log\left(1-\sqrt[4]{c}\sqrt{x}\right)-6b\log\left(\sqrt[4]{c}\sqrt{x}+1\right)-3\sqrt{2}b\log\left(\sqrt{c}x-\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(5/2)*(a + b*ArcTanh[c*x^2]),x]
```

```
[Out] ((d*x)^(5/2)*(16*b*c^(3/4)*x^(3/2) + 12*a*c^(7/4)*x^(7/2) + 6*Sqrt[2]*b*Arc
Tan[1 - Sqrt[2]*c^(1/4)*Sqrt[x]] - 6*Sqrt[2]*b*ArcTan[1 + Sqrt[2]*c^(1/4)*S
```

```
qrt[x]] + 12*b*ArcTan[c^(1/4)*Sqrt[x]] + 12*b*c^(7/4)*x^(7/2)*ArcTanh[c*x^2
] + 6*b*Log[1 - c^(1/4)*Sqrt[x]] - 6*b*Log[1 + c^(1/4)*Sqrt[x]] - 3*Sqrt[2]
*b*Log[1 - Sqrt[2]*c^(1/4)*Sqrt[x] + Sqrt[c]*x] + 3*Sqrt[2]*b*Log[1 + Sqrt[
2]*c^(1/4)*Sqrt[x] + Sqrt[c]*x]))/(42*c^(7/4)*x^(5/2))
```

Maple [A] time = 0.022, size = 302, normalized size = 1.

$$\frac{2a}{7d}(dx)^{\frac{7}{2}}+\frac{2b\mathrm{Artanh}\left(cx^{2}\right)}{7d}(dx)^{\frac{7}{2}}+\frac{8bd}{21c}(dx)^{\frac{3}{2}}-\frac{d^{3}b\sqrt{2}}{14c^{2}}\ln\left(\left(dx-\sqrt[4]{\frac{d^{2}}{c}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{d^{2}}{c}}\right)\left(dx+\sqrt[4]{\frac{d^{2}}{c}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{d^{2}}{c}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)*(a+b*arctanh(c*x^2)),x)
```

```
[Out] 2/7/d*(d*x)^(7/2)*a+2/7/d*b*(d*x)^(7/2)*arctanh(c*x^2)+8/21*b*d*(d*x)^(3/2)
/c-1/14*d^3*b/c^2/(d^2/c)^(1/4)*2^(1/2)*ln((d*x-(d^2/c)^(1/4)*(d*x)^(1/2)*2
^(1/2)+(d^2/c)^(1/2))/(d*x+(d^2/c)^(1/4)*(d*x)^(1/2)*2^(1/2)+(d^2/c)^(1/2))
)-1/7*d^3*b/c^2/(d^2/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(d^2/c)^(1/4)*(d*x)^(1
/2)+1)-1/7*d^3*b/c^2/(d^2/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(d^2/c)^(1/4)*(d*
x)^(1/2)-1)+2/7*d^3*b/c^2/(d^2/c)^(1/4)*arctan((d*x)^(1/2)/(d^2/c)^(1/4))-1
/7*d^3*b/c^2/(d^2/c)^(1/4)*ln(((d*x)^(1/2)+(d^2/c)^(1/4))/((d*x)^(1/2)-(d^2
/c)^(1/4)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(a+b*arctanh(c*x^2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.2931, size = 124, normalized size = 0.39

$$\frac{\left(3\,bcd^{2}x^{3}\log\left(-\frac{cx^{2}+1}{cx^{2}-1}\right)+6\,acd^{2}x^{3}+8\,bd^{2}x\right)\sqrt{dx}}{21\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(a+b*arctanh(c*x^2)),x, algorithm="fricas")
```

```
[Out] 1/21*(3*b*c*d^2*x^3*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 6*a*c*d^2*x^3 + 8*b*d^2
*x)*sqrt(d*x)/c
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(5/2)*(a+b*atanh(c*x**2)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^{\frac{5}{2}} \left(b \operatorname{artanh}\left(cx^2\right) + a\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(a+b*arctanh(c*x^2)),x, algorithm="giac")
```

```
[Out] integrate((d*x)^(5/2)*(b*arctanh(c*x^2) + a), x)
```

3.83 $\int (dx)^{3/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)dx$

Optimal. Leaf size=317

$$\frac{2(dx)^{5/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{5d}+\frac{bd^{3/2}\log\left(\sqrt{c}\sqrt{d}x-\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{5\sqrt{2}c^{5/4}}-\frac{bd^{3/2}\log\left(\sqrt{c}\sqrt{d}x+\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{5\sqrt{2}c^{5/4}}-\frac{2bd^{3}}{}$$

```
[Out] (8*b*d*Sqrt[d*x])/(5*c) - (2*b*d^(3/2)*ArcTan[(c^(1/4)*Sqrt[d*x])/Sqrt[d]])
/(5*c^(5/4)) + (Sqrt[2]*b*d^(3/2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[d*x])/Sq
rt[d]])/(5*c^(5/4)) - (Sqrt[2]*b*d^(3/2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[d
*x])/Sqrt[d]])/(5*c^(5/4)) + (2*(d*x)^(5/2)*(a + b*ArcTanh[c*x^2]))/(5*d) -
 (2*b*d^(3/2)*ArcTanh[(c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(5*c^(5/4)) + (b*d^(3/2
)*Log[Sqrt[d] + Sqrt[c]*Sqrt[d]*x - Sqrt[2]*c^(1/4)*Sqrt[d*x]])/(5*Sqrt[2]*
c^(5/4)) - (b*d^(3/2)*Log[Sqrt[d] + Sqrt[c]*Sqrt[d]*x + Sqrt[2]*c^(1/4)*Sqr
t[d*x]])/(5*Sqrt[2]*c^(5/4))
```

Rubi [A] time = 0.299853, antiderivative size = 317, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 14, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {6097, 16, 321, 329, 214, 212, 208, 205, 211, 1165, 628, 1162, 617, 204}

$$\frac{2(dx)^{5/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{5d}+\frac{bd^{3/2}\log\left(\sqrt{c}\sqrt{d}x-\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{5\sqrt{2}c^{5/4}}-\frac{bd^{3/2}\log\left(\sqrt{c}\sqrt{d}x+\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{5\sqrt{2}c^{5/4}}-\frac{2bd^{3}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)*(a + b*ArcTanh[c*x^2]),x]
```

```
[Out] (8*b*d*Sqrt[d*x])/(5*c) - (2*b*d^(3/2)*ArcTan[(c^(1/4)*Sqrt[d*x])/Sqrt[d]])
/(5*c^(5/4)) + (Sqrt[2]*b*d^(3/2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[d*x])/Sq
rt[d]])/(5*c^(5/4)) - (Sqrt[2]*b*d^(3/2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[d
*x])/Sqrt[d]])/(5*c^(5/4)) + (2*(d*x)^(5/2)*(a + b*ArcTanh[c*x^2]))/(5*d) -
 (2*b*d^(3/2)*ArcTanh[(c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(5*c^(5/4)) + (b*d^(3/2
)*Log[Sqrt[d] + Sqrt[c]*Sqrt[d]*x - Sqrt[2]*c^(1/4)*Sqrt[d*x]])/(5*Sqrt[2]*
c^(5/4)) - (b*d^(3/2)*Log[Sqrt[d] + Sqrt[c]*Sqrt[d]*x + Sqrt[2]*c^(1/4)*Sqr
t[d*x]])/(5*Sqrt[2]*c^(5/4))
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/
b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^(n/2
)), x], x] + Dist[r/(2*a), Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b},
x] && IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (dx)^{3/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)dx = \frac{2(dx)^{5/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{5d} - \frac{(4bc)\int \frac{x(dx)^{5/2}}{1-c^2x^4}\,dx}{5d}$$

$$= \frac{2(dx)^{5/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{5d} - \frac{(4bc)\int \frac{(dx)^{7/2}}{1-c^2x^4}\,dx}{5d^2}$$

$$= \frac{8bd\sqrt{dx}}{5c} + \frac{2(dx)^{5/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{5d} - \frac{\left(4bd^2\right)\int \frac{1}{\sqrt{dx}\left(1-c^2x^4\right)}\,dx}{5c}$$

$$= \frac{8bd\sqrt{dx}}{5c} + \frac{2(dx)^{5/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{5d} - \frac{(8bd)\,\text{Subst}\left(\int \frac{1}{1-\frac{c^2x^8}{d^4}}\,dx, x, \sqrt{dx}\right)}{5c}$$

$$= \frac{8bd\sqrt{dx}}{5c} + \frac{2(dx)^{5/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{5d} - \frac{\left(4bd^3\right)\text{Subst}\left(\int \frac{1}{d^2-cx^4}\,dx, x, \sqrt{dx}\right)}{5c} -$$

$$= \frac{8bd\sqrt{dx}}{5c} + \frac{2(dx)^{5/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{5d} - \frac{\left(2bd^2\right)\text{Subst}\left(\int \frac{1}{d-\sqrt{c}x^2}\,dx, x, \sqrt{dx}\right)}{5c} -$$

$$= \frac{8bd\sqrt{dx}}{5c} - \frac{2bd^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{5c^{5/4}} + \frac{2(dx)^{5/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{5d} - \frac{2bd^{3/2}\tanh^{-1}}{5c^{5/4}}$$

$$= \frac{8bd\sqrt{dx}}{5c} - \frac{2bd^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{5c^{5/4}} + \frac{2(dx)^{5/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{5d} - \frac{2bd^{3/2}\tanh^{-1}}{5c^{5/4}}$$

$$= \frac{8bd\sqrt{dx}}{5c} - \frac{2bd^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{5c^{5/4}} + \frac{\sqrt{2}bd^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{5c^{5/4}} - \frac{\sqrt{2}bd^{3/2}\tan^{-}}{5}$$

Mathematica [A] time = 0.0933528, size = 240, normalized size = 0.76

$$\frac{(dx)^{3/2}\left(4ac^{5/4}x^{5/2}+4bc^{5/4}x^{5/2}\tanh^{-1}\left(cx^2\right)+16b\sqrt[4]{c}\sqrt{x}+2b\log\left(1-\sqrt[4]{c}\sqrt{x}\right)-2b\log\left(\sqrt[4]{c}\sqrt{x}+1\right)+\sqrt{2}b\log\left(\sqrt{c}x-\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)*(a + b*ArcTanh[c*x^2]),x]
```

```
[Out] ((d*x)^(3/2)*(16*b*c^(1/4)*Sqrt[x] + 4*a*c^(5/4)*x^(5/2) + 2*Sqrt[2]*b*ArcT
an[1 - Sqrt[2]*c^(1/4)*Sqrt[x]] - 2*Sqrt[2]*b*ArcTan[1 + Sqrt[2]*c^(1/4)*Sq
```

```
rt[x]] - 4*b*ArcTan[c^(1/4)*Sqrt[x]] + 4*b*c^(5/4)*x^(5/2)*ArcTanh[c*x^2] +
 2*b*Log[1 - c^(1/4)*Sqrt[x]] - 2*b*Log[1 + c^(1/4)*Sqrt[x]] + Sqrt[2]*b*Lo
g[1 - Sqrt[2]*c^(1/4)*Sqrt[x] + Sqrt[c]*x] - Sqrt[2]*b*Log[1 + Sqrt[2]*c^(1
/4)*Sqrt[x] + Sqrt[c]*x]))/(10*c^(5/4)*x^(3/2))
```

Maple [A] time = 0.016, size = 292, normalized size = 0.9

$$\frac{2a}{5d}(dx)^{\frac{5}{2}}+\frac{2b\mathrm{Artanh}\left(cx^2\right)}{5d}(dx)^{\frac{5}{2}}+\frac{8bd}{5c}\sqrt{dx}-\frac{bd\sqrt{2}}{10c}\sqrt[4]{\frac{d^2}{c}}\ln\left(\left(dx+\sqrt[4]{\frac{d^2}{c}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{d^2}{c}}\right)\left(dx-\sqrt[4]{\frac{d^2}{c}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{d^2}{c}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*(a+b*arctanh(c*x^2)),x)
```

```
[Out] 2/5/d*(d*x)^(5/2)*a+2/5/d*b*(d*x)^(5/2)*arctanh(c*x^2)+8/5*b*d*(d*x)^(1/2)/
c-1/10*d*b/c*(d^2/c)^(1/4)*2^(1/2)*ln((d*x+(d^2/c)^(1/4)*(d*x)^(1/2)*2^(1/2
)+(d^2/c)^(1/2))/(d*x-(d^2/c)^(1/4)*(d*x)^(1/2)*2^(1/2)+(d^2/c)^(1/2)))-1/5
*d*b/c*(d^2/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(d^2/c)^(1/4)*(d*x)^(1/2)+1)-1/
5*d*b/c*(d^2/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(d^2/c)^(1/4)*(d*x)^(1/2)-1)-1
/5*d*b/c*(d^2/c)^(1/4)*ln(((d*x)^(1/2)+(d^2/c)^(1/4))/((d*x)^(1/2)-(d^2/c)^
(1/4)))-2/5*d*b/c*(d^2/c)^(1/4)*arctan((d*x)^(1/2)/(d^2/c)^(1/4))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(a+b*arctanh(c*x^2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.27177, size = 109, normalized size = 0.34

$$\frac{\left(bcdx^2\log\left(-\frac{cx^2+1}{cx^2-1}\right)+2\,acdx^2+8\,bd\right)\sqrt{dx}}{5\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(a+b*arctanh(c*x^2)),x, algorithm="fricas")
```

```
[Out] 1/5*(b*c*d*x^2*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 2*a*c*d*x^2 + 8*b*d)*sqrt(d*
x)/c
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)*(a+b*atanh(c*x**2)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^{\frac{3}{2}} \left(b \operatorname{artanh}\left(cx^2\right) + a\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(a+b*arctanh(c*x^2)),x, algorithm="giac")
```

```
[Out] integrate((d*x)^(3/2)*(b*arctanh(c*x^2) + a), x)
```

3.84 $\int \sqrt{dx}\left(a+b\tanh^{-1}\left(cx^2\right)\right)dx$

Optimal. Leaf size=301

$$\frac{2(dx)^{3/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{3d}+\frac{b\sqrt{d}\log\left(\sqrt{c}\sqrt{dx}-\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{3\sqrt{2}c^{3/4}}-\frac{b\sqrt{d}\log\left(\sqrt{c}\sqrt{dx}+\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{3\sqrt{2}c^{3/4}}+\frac{2b\sqrt{d}\,\text{ta}}{3}$$

```
[Out] (2*b*Sqrt[d]*ArcTan[(c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(3*c^(3/4)) - (Sqrt[2]*b*
Sqrt[d]*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(3*c^(3/4)) + (Sqr
t[2]*b*Sqrt[d]*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(3*c^(3/4))
 + (2*(d*x)^(3/2)*(a + b*ArcTanh[c*x^2]))/(3*d) - (2*b*Sqrt[d]*ArcTanh[(c^(
1/4)*Sqrt[d*x])/Sqrt[d]])/(3*c^(3/4)) + (b*Sqrt[d]*Log[Sqrt[d] + Sqrt[c]*Sq
rt[d]*x - Sqrt[2]*c^(1/4)*Sqrt[d*x]])/(3*Sqrt[2]*c^(3/4)) - (b*Sqrt[d]*Log[
Sqrt[d] + Sqrt[c]*Sqrt[d]*x + Sqrt[2]*c^(1/4)*Sqrt[d*x]])/(3*Sqrt[2]*c^(3/4
))
```

Rubi [A] time = 0.250824, antiderivative size = 301, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.722, Rules used = {6097, 16, 329, 301, 297, 1162, 617, 204, 1165, 628, 298, 205, 208}

$$\frac{2(dx)^{3/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{3d}+\frac{b\sqrt{d}\log\left(\sqrt{c}\sqrt{dx}-\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{3\sqrt{2}c^{3/4}}-\frac{b\sqrt{d}\log\left(\sqrt{c}\sqrt{dx}+\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{3\sqrt{2}c^{3/4}}+\frac{2b\sqrt{d}\,\text{ta}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]*(a + b*ArcTanh[c*x^2]),x]
```


```
[Out] (2*b*Sqrt[d]*ArcTan[(c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(3*c^(3/4)) - (Sqrt[2]*b*
Sqrt[d]*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(3*c^(3/4)) + (Sqr
t[2]*b*Sqrt[d]*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(3*c^(3/4))
 + (2*(d*x)^(3/2)*(a + b*ArcTanh[c*x^2]))/(3*d) - (2*b*Sqrt[d]*ArcTanh[(c^(
1/4)*Sqrt[d*x])/Sqrt[d]])/(3*c^(3/4)) + (b*Sqrt[d]*Log[Sqrt[d] + Sqrt[c]*Sq
rt[d]*x - Sqrt[2]*c^(1/4)*Sqrt[d*x]])/(3*Sqrt[2]*c^(3/4)) - (b*Sqrt[d]*Log[
Sqrt[d] + Sqrt[c]*Sqrt[d]*x + Sqrt[2]*c^(1/4)*Sqrt[d*x]])/(3*Sqrt[2]*c^(3/4
))
```


Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 301

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[x^(m - n/2
)/(r + s*x^(n/2)), x], x] - Dist[s/(2*b), Int[x^(m - n/2)/(r - s*x^(n/2)),
x], x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LeQ[n/2, m] &&
 LtQ[m, n] &&  !GtQ[a/b, 0]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \sqrt{dx}\left(a+b\tanh^{-1}\left(cx^2\right)\right)dx = \frac{2(dx)^{3/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{3d} - \frac{(4bc)\int \frac{x(dx)^{3/2}}{1-c^2x^4}\,dx}{3d}$$

$$= \frac{2(dx)^{3/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{3d} - \frac{(4bc)\int \frac{(dx)^{5/2}}{1-c^2x^4}\,dx}{3d^2}$$

$$= \frac{2(dx)^{3/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{3d} - \frac{(8bc)\,\mathrm{Subst}\left(\int \frac{x^6}{1-\frac{c^2x^8}{d^4}}\,dx, x, \sqrt{dx}\right)}{3d^3}$$

$$= \frac{2(dx)^{3/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{3d} - \frac{1}{3}(4bd)\,\mathrm{Subst}\left(\int \frac{x^2}{d^2-cx^4}\,dx, x, \sqrt{dx}\right) + \frac{1}{3}(4bd)\,\mathrm{Subs}$$

$$= \frac{2(dx)^{3/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{3d} - \frac{(2bd)\,\mathrm{Subst}\left(\int \frac{1}{d-\sqrt{c}x^2}\,dx, x, \sqrt{dx}\right)}{3\sqrt{c}} + \frac{(2bd)\,\mathrm{Subst}\left(\int \frac{}{d}\right.}{3}$$

$$= \frac{2b\sqrt{d}\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{3c^{3/4}} + \frac{2(dx)^{3/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{3d} - \frac{2b\sqrt{d}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{3c^{3/4}} + \frac{(b\sqrt{d}}{}$$

$$= \frac{2b\sqrt{d}\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{3c^{3/4}} + \frac{2(dx)^{3/2}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{3d} - \frac{2b\sqrt{d}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{3c^{3/4}} + \frac{b\sqrt{d}}{}$$

$$= \frac{2b\sqrt{d}\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{3c^{3/4}} - \frac{\sqrt{2}b\sqrt{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{3c^{3/4}} + \frac{\sqrt{2}b\sqrt{d}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{3c^{3/4}} +$$

Mathematica [A] time = 0.071632, size = 227, normalized size = 0.75

$$\frac{\sqrt{dx}\left(4ac^{3/4}x^{3/2}+4bc^{3/4}x^{3/2}\tanh^{-1}\left(cx^2\right)+2b\log\left(1-\sqrt[4]{c}\sqrt{x}\right)-2b\log\left(\sqrt[4]{c}\sqrt{x}+1\right)+\sqrt{2}b\log\left(\sqrt{c}x-\sqrt{2}\sqrt[4]{c}\sqrt{x}+1\right)-\right.}{6c^{3/4}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]*(a + b*ArcTanh[c*x^2]),x]
```

```
[Out] (Sqrt[d*x]*(4*a*c^(3/4)*x^(3/2) - 2*Sqrt[2]*b*ArcTan[1 - Sqrt[2]*c^(1/4)*Sq
rt[x]] + 2*Sqrt[2]*b*ArcTan[1 + Sqrt[2]*c^(1/4)*Sqrt[x]] + 4*b*ArcTan[c^(1/
4)*Sqrt[x]] + 4*b*c^(3/4)*x^(3/2)*ArcTanh[c*x^2] + 2*b*Log[1 - c^(1/4)*Sqrt
[x]] - 2*b*Log[1 + c^(1/4)*Sqrt[x]] + Sqrt[2]*b*Log[1 - Sqrt[2]*c^(1/4)*Sqr
t[x] + Sqrt[c]*x] - Sqrt[2]*b*Log[1 + Sqrt[2]*c^(1/4)*Sqrt[x] + Sqrt[c]*x])
)/(6*c^(3/4)*Sqrt[x])
```

Maple [A] time = 0.011, size = 280, normalized size = 0.9

$$\frac{2a}{3d}(dx)^{\frac{3}{2}} + \frac{2b\mathrm{Artanh}\left(cx^2\right)}{3d}(dx)^{\frac{3}{2}} + \frac{db\sqrt{2}}{6c}\ln\left(\left(dx-\sqrt[4]{\frac{d^2}{c}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{d^2}{c}}\right)\left(dx+\sqrt[4]{\frac{d^2}{c}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{d^2}{c}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{d^2}{c}}} + \frac{db\sqrt{}}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)*(a+b*arctanh(c*x^2)),x)
```

```
[Out] 2/3/d*(d*x)^(3/2)*a+2/3/d*b*(d*x)^(3/2)*arctanh(c*x^2)+1/6*d*b/c/(d^2/c)^(1
/4)*2^(1/2)*ln((d*x-(d^2/c)^(1/4)*(d*x)^(1/2)*2^(1/2)+(d^2/c)^(1/2))/(d*x+(
d^2/c)^(1/4)*(d*x)^(1/2)*2^(1/2)+(d^2/c)^(1/2)))+1/3*d*b/c/(d^2/c)^(1/4)*2^
(1/2)*arctan(2^(1/2)/(d^2/c)^(1/4)*(d*x)^(1/2)+1)+1/3*d*b/c/(d^2/c)^(1/4)*2
^(1/2)*arctan(2^(1/2)/(d^2/c)^(1/4)*(d*x)^(1/2)-1)+2/3*d*b/c/(d^2/c)^(1/4)*
arctan((d*x)^(1/2)/(d^2/c)^(1/4))-1/3*d*b/c/(d^2/c)^(1/4)*ln(((d*x)^(1/2)+(
d^2/c)^(1/4))/((d*x)^(1/2)-(d^2/c)^(1/4)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*(a+b*arctanh(c*x^2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.15992, size = 80, normalized size = 0.27

$$\frac{1}{3}\left(bx\log\left(-\frac{cx^2+1}{cx^2-1}\right)+2\,ax\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*(a+b*arctanh(c*x^2)),x, algorithm="fricas")
```

```
[Out] 1/3*(b*x*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 2*a*x)*sqrt(d*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)*(a+b*atanh(c*x**2)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dx}\left(b\,\operatorname{artanh}\left(cx^2\right)+a\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*(a+b*arctanh(c*x^2)),x, algorithm="giac")

[Out] integrate(sqrt(d*x)*(b*arctanh(c*x^2) + a), x)
```

3.85 $\int \frac{a+b\tanh^{-1}(cx^2)}{\sqrt{dx}}\,dx$

Optimal. Leaf size=285

$$\frac{2\sqrt{dx}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{d}-\frac{b\log\left(\sqrt{c}\sqrt{dx}-\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}+\frac{b\log\left(\sqrt{c}\sqrt{dx}+\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}-\frac{2b\tan^{-1}\left(\frac{\sqrt[4]{c}\cdots}{\sqrt{\cdots}}\right.}{\sqrt[4]{c}\sqrt{d}}$$

```
[Out] (-2*b*ArcTan[(c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(c^(1/4)*Sqrt[d]) - (Sqrt[2]*b*A
rcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(c^(1/4)*Sqrt[d]) + (Sqrt[2
]*b*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(c^(1/4)*Sqrt[d]) + (2
*Sqrt[d*x]*(a + b*ArcTanh[c*x^2]))/d - (2*b*ArcTanh[(c^(1/4)*Sqrt[d*x])/Sqr
t[d]])/(c^(1/4)*Sqrt[d]) - (b*Log[Sqrt[d] + Sqrt[c]*Sqrt[d]*x - Sqrt[2]*c^(
1/4)*Sqrt[d*x]])/(Sqrt[2]*c^(1/4)*Sqrt[d]) + (b*Log[Sqrt[d] + Sqrt[c]*Sqrt[
d]*x + Sqrt[2]*c^(1/4)*Sqrt[d*x]])/(Sqrt[2]*c^(1/4)*Sqrt[d])
```

Rubi [A] time = 0.24214, antiderivative size = 285, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.722, Rules used = {6097, 16, 329, 301, 211, 1165, 628, 1162, 617, 204, 212, 208, 205}

$$\frac{2\sqrt{dx}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{d}-\frac{b\log\left(\sqrt{c}\sqrt{dx}-\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}+\frac{b\log\left(\sqrt{c}\sqrt{dx}+\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}-\frac{2b\tan^{-1}\left(\frac{\sqrt[4]{c}\cdots}{\sqrt{\cdots}}\right.}{\sqrt[4]{c}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^2])/Sqrt[d*x],x]
```

```
[Out] (-2*b*ArcTan[(c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(c^(1/4)*Sqrt[d]) - (Sqrt[2]*b*A
rcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(c^(1/4)*Sqrt[d]) + (Sqrt[2
]*b*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(c^(1/4)*Sqrt[d]) + (2
*Sqrt[d*x]*(a + b*ArcTanh[c*x^2]))/d - (2*b*ArcTanh[(c^(1/4)*Sqrt[d*x])/Sqr
t[d]])/(c^(1/4)*Sqrt[d]) - (b*Log[Sqrt[d] + Sqrt[c]*Sqrt[d]*x - Sqrt[2]*c^(
1/4)*Sqrt[d*x]])/(Sqrt[2]*c^(1/4)*Sqrt[d]) + (b*Log[Sqrt[d] + Sqrt[c]*Sqrt[
d]*x + Sqrt[2]*c^(1/4)*Sqrt[d*x]])/(Sqrt[2]*c^(1/4)*Sqrt[d])
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 301

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[x^(m - n/2
)/(r + s*x^(n/2)), x], x] - Dist[s/(2*b), Int[x^(m - n/2)/(r - s*x^(n/2)),
x], x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LeQ[n/2, m] &&
 LtQ[m, n] &&  !GtQ[a/b, 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\tanh^{-1}\left(cx^2\right)}{\sqrt{dx}}\,dx &= \frac{2\sqrt{dx}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{d} - \frac{(4bc)\int \frac{x\sqrt{dx}}{1-c^2x^4}\,dx}{d} \\
&= \frac{2\sqrt{dx}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{d} - \frac{(4bc)\int \frac{(dx)^{3/2}}{1-c^2x^4}\,dx}{d^2} \\
&= \frac{2\sqrt{dx}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{d} - \frac{(8bc)\,\text{Subst}\left(\int \frac{x^4}{1-\frac{c^2x^8}{d^4}}\,dx, x, \sqrt{dx}\right)}{d^3} \\
&= \frac{2\sqrt{dx}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{d} - (4bd)\,\text{Subst}\left(\int \frac{1}{d^2-cx^4}\,dx, x, \sqrt{dx}\right) + (4bd)\,\text{Subst}\left(\int \frac{}{d^2 -}\right. \\
&= \frac{2\sqrt{dx}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{d} - (2b)\,\text{Subst}\left(\int \frac{1}{d-\sqrt{c}x^2}\,dx, x, \sqrt{dx}\right) - (2b)\,\text{Subst}\left(\int \frac{1}{d+\sqrt{}}\right. \\
&= -\frac{2b\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{c}\sqrt{d}} + \frac{2\sqrt{dx}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{d} - \frac{2b\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{c}\sqrt{d}} + \frac{b\,\text{Subst}\left(\int \frac{}{\frac{d}{\sqrt{c}}-}\right.}{} \\
&= -\frac{2b\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{c}\sqrt{d}} + \frac{2\sqrt{dx}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{d} - \frac{2b\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{c}\sqrt{d}} - \frac{b\log\left(\sqrt{d}+\sqrt{c}\right.}{\sqrt{2}} \\
&= -\frac{2b\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{c}\sqrt{d}} - \frac{\sqrt{2}b\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{c}\sqrt{d}} + \frac{\sqrt{2}b\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{c}\sqrt{d}} + \frac{2\sqrt{dx}\left(a+\right.}{}
\end{aligned}
$$

Mathematica [A] time = 0.0586972, size = 227, normalized size = 0.8

$$
\frac{\sqrt{x}\left(4a\sqrt[4]{c}\sqrt{x}+4b\sqrt[4]{c}\sqrt{x}\tanh^{-1}\left(cx^2\right)+2b\log\left(1-\sqrt[4]{c}\sqrt{x}\right)-2b\log\left(\sqrt[4]{c}\sqrt{x}+1\right)-\sqrt{2}b\log\left(\sqrt{c}x-\sqrt{2}\sqrt[4]{c}\sqrt{x}+1\right)+\right.}{2\sqrt[4]{c}\sqrt{dx}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])/Sqrt[d*x],x]
```

```
[Out] (Sqrt[x]*(4*a*c^(1/4)*Sqrt[x] - 2*Sqrt[2]*b*ArcTan[1 - Sqrt[2]*c^(1/4)*Sqrt
[x]] + 2*Sqrt[2]*b*ArcTan[1 + Sqrt[2]*c^(1/4)*Sqrt[x]] - 4*b*ArcTan[c^(1/4)
*Sqrt[x]] + 4*b*c^(1/4)*Sqrt[x]*ArcTanh[c*x^2] + 2*b*Log[1 - c^(1/4)*Sqrt[x
]] - 2*b*Log[1 + c^(1/4)*Sqrt[x]] - Sqrt[2]*b*Log[1 - Sqrt[2]*c^(1/4)*Sqrt[
x] + Sqrt[c]*x] + Sqrt[2]*b*Log[1 + Sqrt[2]*c^(1/4)*Sqrt[x] + Sqrt[c]*x]))/
(2*c^(1/4)*Sqrt[d*x])
```

Maple [A] time = 0.013, size = 273, normalized size = 1.

$$
2\frac{a\sqrt{dx}}{d}+2\frac{b\sqrt{dx}\,\text{Artanh}\left(cx^2\right)}{d}+\frac{b\sqrt{2}}{2d}\sqrt[4]{\frac{d^2}{c}}\ln\left(\left(dx+\sqrt[4]{\frac{d^2}{c}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{d^2}{c}}\right)\left(dx-\sqrt[4]{\frac{d^2}{c}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{d^2}{c}}\right)^{-1}\right)+\frac{b\sqrt{2}}{d}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))/(d*x)^(1/2),x)
```

```
[Out] 2/d*a*(d*x)^(1/2)+2/d*b*(d*x)^(1/2)*arctanh(c*x^2)+1/2/d*b*(d^2/c)^(1/4)*2^
(1/2)*ln((d*x+(d^2/c)^(1/4)*(d*x)^(1/2)*2^(1/2)+(d^2/c)^(1/2))/(d*x-(d^2/c)
^(1/4)*(d*x)^(1/2)*2^(1/2)+(d^2/c)^(1/2)))+1/d*b*(d^2/c)^(1/4)*2^(1/2)*arct
an(2^(1/2)/(d^2/c)^(1/4)*(d*x)^(1/2)+1)+1/d*b*(d^2/c)^(1/4)*2^(1/2)*arctan(
2^(1/2)/(d^2/c)^(1/4)*(d*x)^(1/2)-1)-1/d*b*(d^2/c)^(1/4)*ln(((d*x)^(1/2)+(d
^2/c)^(1/4))/((d*x)^(1/2)-(d^2/c)^(1/4)))-2/d*b*(d^2/c)^(1/4)*arctan((d*x)^
(1/2)/(d^2/c)^(1/4))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.22864, size = 72, normalized size = 0.25

$$\frac{\sqrt{dx}\left(b\log\left(-\frac{cx^2+1}{cx^2-1}\right)+2\,a\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(d*x)*(b*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 2*a)/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+b\,\mathrm{atanh}\left(cx^2\right)}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))/(d*x)**(1/2),x)
```

```
[Out] Integral((a + b*atanh(c*x**2))/sqrt(d*x), x)
```

Giac [B] time = 1.24348, size = 666, normalized size = 2.34

$$\left(cd^2\left(\frac{2\sqrt{2}\left(c^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{d^2}{c}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)}{c^2d^2}+\frac{2\sqrt{2}\left(c^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{d^2}{c}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)}{c^2d^2}-\frac{2\sqrt{2}\left(-c^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{d^2}{c}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(-\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)}{c^2d^2}-\frac{2\sqrt{2}\cdots}{\cdots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*((c*d^2*(2*sqrt(2)*(c^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(d^2/c)^
(1/4) + 2*sqrt(d*x))/(d^2/c)^(1/4))/(c^2*d^2) + 2*sqrt(2)*(c^3*d^2)^(1/4)*a
rctan(-1/2*sqrt(2)*(sqrt(2)*(d^2/c)^(1/4) - 2*sqrt(d*x))/(d^2/c)^(1/4))/(c^
2*d^2) - 2*sqrt(2)*(-c^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-d^2/c)^(1
/4) + 2*sqrt(d*x))/(-d^2/c)^(1/4))/(c^2*d^2) - 2*sqrt(2)*(-c^3*d^2)^(1/4)*a
rctan(-1/2*sqrt(2)*(sqrt(2)*(-d^2/c)^(1/4) - 2*sqrt(d*x))/(-d^2/c)^(1/4))/(
c^2*d^2) + sqrt(2)*(c^3*d^2)^(1/4)*log(d*x + sqrt(2)*sqrt(d*x)*(d^2/c)^(1/4
) + sqrt(d^2/c))/(c^2*d^2) - sqrt(2)*(c^3*d^2)^(1/4)*log(d*x - sqrt(2)*sqrt
(d*x)*(d^2/c)^(1/4) + sqrt(d^2/c))/(c^2*d^2) - sqrt(2)*(-c^3*d^2)^(1/4)*log
(d*x + sqrt(2)*sqrt(d*x)*(-d^2/c)^(1/4) + sqrt(-d^2/c))/(c^2*d^2) + sqrt(2)
*(-c^3*d^2)^(1/4)*log(d*x - sqrt(2)*sqrt(d*x)*(-d^2/c)^(1/4) + sqrt(-d^2/c)
)/(c^2*d^2)) + 2*sqrt(d*x)*log(-(c*x^2 + 1)/(c*x^2 - 1)))*b + 4*sqrt(d*x)*a
)/d
```

3.86 $\int \frac{a+b\tanh^{-1}(cx^2)}{(dx)^{3/2}}\,dx$

Optimal. Leaf size=285

$$-\frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{d\sqrt{dx}}+\frac{b\sqrt[4]{c}\log\left(\sqrt{c}\sqrt{d}x-\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{\sqrt{2}d^{3/2}}-\frac{b\sqrt[4]{c}\log\left(\sqrt{c}\sqrt{d}x+\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{\sqrt{2}d^{3/2}}-\frac{2b\sqrt[4]{c}\tan^{-1}\left(\right.}{d^{3/2}}$$

```
[Out] (-2*b*c^(1/4)*ArcTan[(c^(1/4)*Sqrt[d*x])/Sqrt[d]])/d^(3/2) - (Sqrt[2]*b*c^(
1/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[d*x])/Sqrt[d]])/d^(3/2) + (Sqrt[2]*b*
c^(1/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[d*x])/Sqrt[d]])/d^(3/2) - (2*(a +
b*ArcTanh[c*x^2]))/(d*Sqrt[d*x]) + (2*b*c^(1/4)*ArcTanh[(c^(1/4)*Sqrt[d*x])
/Sqrt[d]])/d^(3/2) + (b*c^(1/4)*Log[Sqrt[d] + Sqrt[c]*Sqrt[d]*x - Sqrt[2]*c
^(1/4)*Sqrt[d*x]])/(Sqrt[2]*d^(3/2)) - (b*c^(1/4)*Log[Sqrt[d] + Sqrt[c]*Sqr
t[d]*x + Sqrt[2]*c^(1/4)*Sqrt[d*x]])/(Sqrt[2]*d^(3/2))
```

Rubi [A] time = 0.248436, antiderivative size = 285, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.722, Rules used = {6097, 16, 329, 300, 297, 1162, 617, 204, 1165, 628, 298, 205, 208}

$$-\frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{d\sqrt{dx}}+\frac{b\sqrt[4]{c}\log\left(\sqrt{c}\sqrt{d}x-\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{\sqrt{2}d^{3/2}}-\frac{b\sqrt[4]{c}\log\left(\sqrt{c}\sqrt{d}x+\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{\sqrt{2}d^{3/2}}-\frac{2b\sqrt[4]{c}\tan^{-1}\left(\right.}{d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^2])/(d*x)^(3/2),x]
```


```
[Out] (-2*b*c^(1/4)*ArcTan[(c^(1/4)*Sqrt[d*x])/Sqrt[d]])/d^(3/2) - (Sqrt[2]*b*c^(
1/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[d*x])/Sqrt[d]])/d^(3/2) + (Sqrt[2]*b*
c^(1/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[d*x])/Sqrt[d]])/d^(3/2) - (2*(a +
b*ArcTanh[c*x^2]))/(d*Sqrt[d*x]) + (2*b*c^(1/4)*ArcTanh[(c^(1/4)*Sqrt[d*x])
/Sqrt[d]])/d^(3/2) + (b*c^(1/4)*Log[Sqrt[d] + Sqrt[c]*Sqrt[d]*x - Sqrt[2]*c
^(1/4)*Sqrt[d*x]])/(Sqrt[2]*d^(3/2)) - (b*c^(1/4)*Log[Sqrt[d] + Sqrt[c]*Sqr
t[d]*x + Sqrt[2]*c^(1/4)*Sqrt[d*x]])/(Sqrt[2]*d^(3/2))
```


Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 300

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[x^m/(r + s
*x^(n/2)), x], x] + Dist[r/(2*a), Int[x^m/(r - s*x^(n/2)), x], x]] /; FreeQ
[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n/2] &&  !GtQ[a/b, 0]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}\left(cx^2\right)}{(dx)^{3/2}}\,dx = -\frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{d\sqrt{dx}}+\frac{(4bc)\int \frac{x}{\sqrt{dx}\left(1-c^2x^4\right)}\,dx}{d}$$

$$= -\frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{d\sqrt{dx}}+\frac{(4bc)\int \frac{\sqrt{dx}}{1-c^2x^4}\,dx}{d^2}$$

$$= -\frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{d\sqrt{dx}}+\frac{(8bc)\,\text{Subst}\left(\int \frac{x^2}{1-\frac{c^2x^8}{d^4}}\,dx,x,\sqrt{dx}\right)}{d^3}$$

$$= -\frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{d\sqrt{dx}}+\frac{(4bc)\,\text{Subst}\left(\int \frac{x^2}{d^2-cx^4}\,dx,x,\sqrt{dx}\right)}{d}+\frac{(4bc)\,\text{Subst}\left(\int \frac{x^2}{d^2+cx^4}\,dx,x,\sqrt{\right.}$$

$$= -\frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{d\sqrt{dx}}+\frac{\left(2b\sqrt{c}\right)\text{Subst}\left(\int \frac{1}{d-\sqrt{c}x^2}\,dx,x,\sqrt{dx}\right)}{d}-\frac{\left(2b\sqrt{c}\right)\text{Subst}\left(\int \frac{1}{d+\sqrt{c}x^2}\,dx\right.}{d}$$

$$= -\frac{2b\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{d\sqrt{dx}}+\frac{2b\sqrt[4]{c}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{\left(b\sqrt[4]{c}\right)\text{Subst}\left(\int\right.}{}$$

$$= -\frac{2b\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{d\sqrt{dx}}+\frac{2b\sqrt[4]{c}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{b\sqrt[4]{c}\log\left(\sqrt{d}+\sqrt{\right.}}{\sqrt{}}$$

$$= -\frac{2b\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{\sqrt{2}b\sqrt[4]{c}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{\sqrt{2}b\sqrt[4]{c}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{2\left(a\right.}{}$$

Mathematica [A] time = 0.102945, size = 268, normalized size = 0.94

$$-\frac{x\left(4a+4b\tanh^{-1}\left(cx^2\right)+2b\sqrt[4]{c}\sqrt{x}\log\left(1-\sqrt[4]{c}\sqrt{x}\right)-2b\sqrt[4]{c}\sqrt{x}\log\left(\sqrt[4]{c}\sqrt{x}+1\right)-\sqrt{2}b\sqrt[4]{c}\sqrt{x}\log\left(\sqrt{c}x-\sqrt{2}\sqrt[4]{c}\sqrt{x}+1\right)+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])/(d*x)^(3/2),x]
```

```
[Out] -(x*(4*a + 2*Sqrt[2]*b*c^(1/4)*Sqrt[x]*ArcTan[1 - Sqrt[2]*c^(1/4)*Sqrt[x]]
- 2*Sqrt[2]*b*c^(1/4)*Sqrt[x]*ArcTan[1 + Sqrt[2]*c^(1/4)*Sqrt[x]] + 4*b*c^(
1/4)*Sqrt[x]*ArcTan[c^(1/4)*Sqrt[x]] + 4*b*ArcTanh[c*x^2] + 2*b*c^(1/4)*Sqr
t[x]*Log[1 - c^(1/4)*Sqrt[x]] - 2*b*c^(1/4)*Sqrt[x]*Log[1 + c^(1/4)*Sqrt[x]
] - Sqrt[2]*b*c^(1/4)*Sqrt[x]*Log[1 - Sqrt[2]*c^(1/4)*Sqrt[x] + Sqrt[c]*x]
+ Sqrt[2]*b*c^(1/4)*Sqrt[x]*Log[1 + Sqrt[2]*c^(1/4)*Sqrt[x] + Sqrt[c]*x]))/
(2*(d*x)^(3/2))
```

Maple [A] time = 0.013, size = 272, normalized size = 1.

$$-2\,\frac{a}{d\sqrt{dx}}-2\,\frac{b\text{Artanh}\left(cx^2\right)}{d\sqrt{dx}}+\frac{b\sqrt{2}}{2\,d}\ln\left(\left(dx-\sqrt[4]{\frac{d^2}{c}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{d^2}{c}}\right)\left(dx+\sqrt[4]{\frac{d^2}{c}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{d^2}{c}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{d^2}{c}}}+\frac{b\sqrt{2}}{d}\arctan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))/(d*x)^(3/2),x)
```

```
[Out] -2/d*a/(d*x)^(1/2)-2/d*b/(d*x)^(1/2)*arctanh(c*x^2)+1/2/d*b/(d^2/c)^(1/4)*2
^(1/2)*ln((d*x-(d^2/c)^(1/4)*(d*x)^(1/2)*2^(1/2)+(d^2/c)^(1/2))/(d*x+(d^2/c
)^(1/4)*(d*x)^(1/2)*2^(1/2)+(d^2/c)^(1/2)))+1/d*b/(d^2/c)^(1/4)*2^(1/2)*arc
tan(2^(1/2)/(d^2/c)^(1/4)*(d*x)^(1/2)+1)+1/d*b/(d^2/c)^(1/4)*2^(1/2)*arctan
(2^(1/2)/(d^2/c)^(1/4)*(d*x)^(1/2)-1)-2/d*b/(d^2/c)^(1/4)*arctan((d*x)^(1/2
)/(d^2/c)^(1/4))+1/d*b/(d^2/c)^(1/4)*ln(((d*x)^(1/2)+(d^2/c)^(1/4))/((d*x)^
(1/2)-(d^2/c)^(1/4)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/(d*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.2664, size = 81, normalized size = 0.28

$$-\frac{\sqrt{dx}\left(b\log\left(-\frac{cx^2+1}{cx^2-1}\right)+2\,a\right)}{d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] -sqrt(d*x)*(b*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 2*a)/(d^2*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))/(d*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.90645, size = 682, normalized size = 2.39

$$\frac{1}{2}\,bcd^2\left(\frac{2\,\sqrt{2}\,\left(c^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{d^2}{c}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)}{c^3d^5}+\frac{2\,\sqrt{2}\,\left(c^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{d^2}{c}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)}{c^3d^5}+\frac{2\,\sqrt{2}\,\left(-c^3d^2\right)^{\frac{3}{4}}\arctan\left(\right.}{c^3d^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*b*c*d^2*(2*sqrt(2)*(c^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(d^2/c)^
(1/4) + 2*sqrt(d*x))/(d^2/c)^(1/4))/(c^3*d^5) + 2*sqrt(2)*(c^3*d^2)^(3/4)*a
rctan(-1/2*sqrt(2)*(sqrt(2)*(d^2/c)^(1/4) - 2*sqrt(d*x))/(d^2/c)^(1/4))/(c^
3*d^5) + 2*sqrt(2)*(-c^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-d^2/c)^(1
/4) + 2*sqrt(d*x))/(-d^2/c)^(1/4))/(c^3*d^5) + 2*sqrt(2)*(-c^3*d^2)^(3/4)*a
rctan(-1/2*sqrt(2)*(sqrt(2)*(-d^2/c)^(1/4) - 2*sqrt(d*x))/(-d^2/c)^(1/4))/(
c^3*d^5) - sqrt(2)*(c^3*d^2)^(3/4)*log(d*x + sqrt(2)*sqrt(d*x)*(d^2/c)^(1/4
) + sqrt(d^2/c))/(c^3*d^5) + sqrt(2)*(c^3*d^2)^(3/4)*log(d*x - sqrt(2)*sqrt
(d*x)*(d^2/c)^(1/4) + sqrt(d^2/c))/(c^3*d^5) - sqrt(2)*(-c^3*d^2)^(3/4)*log
(d*x + sqrt(2)*sqrt(d*x)*(-d^2/c)^(1/4) + sqrt(-d^2/c))/(c^3*d^5) + sqrt(2)
*(-c^3*d^2)^(3/4)*log(d*x - sqrt(2)*sqrt(d*x)*(-d^2/c)^(1/4) + sqrt(-d^2/c)
)/(c^3*d^5)) - (b*log(-(c*d^2*x^2 + d^2)/(c*d^2*x^2 - d^2))/sqrt(d*x) + 2*a
/sqrt(d*x))/d
```

3.87 $\int \frac{a+b\tanh^{-1}(cx^2)}{(dx)^{5/2}}\,dx$

Optimal. Leaf size=301

$$-\frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{3d(dx)^{3/2}}-\frac{bc^{3/4}\log\left(\sqrt{c}\sqrt{d}x-\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{3\sqrt{2}d^{5/2}}+\frac{bc^{3/4}\log\left(\sqrt{c}\sqrt{d}x+\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{3\sqrt{2}d^{5/2}}+\frac{2bc^{3/4}\tan}{3}$$

```
[Out] (2*b*c^(3/4)*ArcTan[(c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(3*d^(5/2)) - (Sqrt[2]*b*
c^(3/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(3*d^(5/2)) + (Sqr
t[2]*b*c^(3/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(3*d^(5/2))
 - (2*(a + b*ArcTanh[c*x^2]))/(3*d*(d*x)^(3/2)) + (2*b*c^(3/4)*ArcTanh[(c^(
1/4)*Sqrt[d*x])/Sqrt[d]])/(3*d^(5/2)) - (b*c^(3/4)*Log[Sqrt[d] + Sqrt[c]*Sq
rt[d]*x - Sqrt[2]*c^(1/4)*Sqrt[d*x]])/(3*Sqrt[2]*d^(5/2)) + (b*c^(3/4)*Log[
Sqrt[d] + Sqrt[c]*Sqrt[d]*x + Sqrt[2]*c^(1/4)*Sqrt[d*x]])/(3*Sqrt[2]*d^(5/2
))
```

Rubi [A] time = 0.249511, antiderivative size = 301, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.722, Rules used = {6097, 16, 329, 214, 212, 208, 205, 211, 1165, 628, 1162, 617, 204}

$$-\frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{3d(dx)^{3/2}}-\frac{bc^{3/4}\log\left(\sqrt{c}\sqrt{d}x-\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{3\sqrt{2}d^{5/2}}+\frac{bc^{3/4}\log\left(\sqrt{c}\sqrt{d}x+\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{3\sqrt{2}d^{5/2}}+\frac{2bc^{3/4}\tan}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^2])/(d*x)^(5/2),x]
```


```
[Out] (2*b*c^(3/4)*ArcTan[(c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(3*d^(5/2)) - (Sqrt[2]*b*
c^(3/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(3*d^(5/2)) + (Sqr
t[2]*b*c^(3/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(3*d^(5/2))
 - (2*(a + b*ArcTanh[c*x^2]))/(3*d*(d*x)^(3/2)) + (2*b*c^(3/4)*ArcTanh[(c^(
1/4)*Sqrt[d*x])/Sqrt[d]])/(3*d^(5/2)) - (b*c^(3/4)*Log[Sqrt[d] + Sqrt[c]*Sq
rt[d]*x - Sqrt[2]*c^(1/4)*Sqrt[d*x]])/(3*Sqrt[2]*d^(5/2)) + (b*c^(3/4)*Log[
Sqrt[d] + Sqrt[c]*Sqrt[d]*x + Sqrt[2]*c^(1/4)*Sqrt[d*x]])/(3*Sqrt[2]*d^(5/2
))
```


Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/
b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^(n/2
)), x], x] + Dist[r/(2*a), Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b},
x] && IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}\left(cx^2\right)}{(dx)^{5/2}}\,dx = -\frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{3d(dx)^{3/2}}+\frac{(4bc)\int \frac{x}{(dx)^{3/2}\left(1-c^2x^4\right)}\,dx}{3d}$$
$$= -\frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{3d(dx)^{3/2}}+\frac{(4bc)\int \frac{1}{\sqrt{dx}\left(1-c^2x^4\right)}\,dx}{3d^2}$$
$$= -\frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{3d(dx)^{3/2}}+\frac{(8bc)\,\text{Subst}\left(\int \frac{1}{1-\frac{c^2x^8}{d^4}}\,dx,x,\sqrt{dx}\right)}{3d^3}$$
$$= -\frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{3d(dx)^{3/2}}+\frac{(4bc)\,\text{Subst}\left(\int \frac{1}{d^2-cx^4}\,dx,x,\sqrt{dx}\right)}{3d}+\frac{(4bc)\,\text{Subst}\left(\int \frac{1}{d^2+cx^4}\,dx,x\right.}{3d}$$
$$= -\frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{3d(dx)^{3/2}}+\frac{(2bc)\,\text{Subst}\left(\int \frac{1}{d-\sqrt{c}x^2}\,dx,x,\sqrt{dx}\right)}{3d^2}+\frac{(2bc)\,\text{Subst}\left(\int \frac{1}{d+\sqrt{c}x^2}\,dx,\right.}{3d^2}$$
$$= \frac{2bc^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}}-\frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{3d(dx)^{3/2}}+\frac{2bc^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}}-\frac{\left(bc^{3/4}\right)\text{Subst}}{}$$
$$= \frac{2bc^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}}-\frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{3d(dx)^{3/2}}+\frac{2bc^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}}-\frac{bc^{3/4}\log\left(\sqrt{d}+\right.}{}$$
$$= \frac{2bc^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}}-\frac{\sqrt{2}bc^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}}+\frac{\sqrt{2}bc^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}}-\frac{2}{}$$

Mathematica [A] time = 0.105879, size = 268, normalized size = 0.89

$$-\frac{x\left(4a+2bc^{3/4}x^{3/2}\log\left(1-\sqrt[4]{c}\sqrt{x}\right)-2bc^{3/4}x^{3/2}\log\left(\sqrt[4]{c}\sqrt{x}+1\right)+\sqrt{2}bc^{3/4}x^{3/2}\log\left(\sqrt{c}x-\sqrt{2}\sqrt[4]{c}\sqrt{x}+1\right)-\sqrt{2}bc^{3/4}x^{3/2}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])/(d*x)^(5/2),x]
```

```
[Out] -(x*(4*a + 2*Sqrt[2]*b*c^(3/4)*x^(3/2)*ArcTan[1 - Sqrt[2]*c^(1/4)*Sqrt[x]]
- 2*Sqrt[2]*b*c^(3/4)*x^(3/2)*ArcTan[1 + Sqrt[2]*c^(1/4)*Sqrt[x]] - 4*b*c^(
3/4)*x^(3/2)*ArcTan[c^(1/4)*Sqrt[x]] + 4*b*ArcTanh[c*x^2] + 2*b*c^(3/4)*x^(
3/2)*Log[1 - c^(1/4)*Sqrt[x]] - 2*b*c^(3/4)*x^(3/2)*Log[1 + c^(1/4)*Sqrt[x]
] + Sqrt[2]*b*c^(3/4)*x^(3/2)*Log[1 - Sqrt[2]*c^(1/4)*Sqrt[x] + Sqrt[c]*x]
- Sqrt[2]*b*c^(3/4)*x^(3/2)*Log[1 + Sqrt[2]*c^(1/4)*Sqrt[x] + Sqrt[c]*x]))/
(6*(d*x)^(5/2))
```

Maple [A] time = 0.01, size = 280, normalized size = 0.9

$$-\frac{2a}{3d}(dx)^{-\frac{3}{2}}-\frac{2b\mathrm{Artanh}\left(cx^2\right)}{3d}(dx)^{-\frac{3}{2}}+\frac{bc\sqrt{2}}{6d^3}\sqrt[4]{\frac{d^2}{c}}\ln\left(\left(dx+\sqrt[4]{\frac{d^2}{c}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{d^2}{c}}\right)\left(dx-\sqrt[4]{\frac{d^2}{c}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{d^2}{c}}\right)^{-1}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))/(d*x)^(5/2),x)
```

```
[Out] -2/3/d*a/(d*x)^(3/2)-2/3/d*b/(d*x)^(3/2)*arctanh(c*x^2)+1/6/d^3*b*c*(d^2/c)
^(1/4)*2^(1/2)*ln((d*x+(d^2/c)^(1/4)*(d*x)^(1/2)*2^(1/2)+(d^2/c)^(1/2))/(d*
x-(d^2/c)^(1/4)*(d*x)^(1/2)*2^(1/2)+(d^2/c)^(1/2)))+1/3/d^3*b*c*(d^2/c)^(1/
4)*2^(1/2)*arctan(2^(1/2)/(d^2/c)^(1/4)*(d*x)^(1/2)+1)+1/3/d^3*b*c*(d^2/c)^
(1/4)*2^(1/2)*arctan(2^(1/2)/(d^2/c)^(1/4)*(d*x)^(1/2)-1)+1/3/d^3*b*c*(d^2/
c)^(1/4)*ln(((d*x)^(1/2)+(d^2/c)^(1/4))/((d*x)^(1/2)-(d^2/c)^(1/4)))+2/3/d^
3*b*c*(d^2/c)^(1/4)*arctan((d*x)^(1/2)/(d^2/c)^(1/4))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/(d*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.26762, size = 89, normalized size = 0.3

$$-\frac{\sqrt{dx}\left(b\log\left(-\frac{cx^2+1}{cx^2-1}\right)+2a\right)}{3d^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/(d*x)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(d*x)*(b*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 2*a)/(d^3*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))/(d*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 3.18579, size = 698, normalized size = 2.32

$$\frac{1}{6}\,bcd^2\left(\frac{2\sqrt{2}\left(c^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{d^2}{c}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)}{cd^5}+\frac{2\sqrt{2}\left(c^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{d^2}{c}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)}{cd^5}+\frac{2\sqrt{2}\left(-c^3d^2\right)^{\frac{1}{4}}\operatorname{arcta}}{c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/(d*x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/6*b*c*d^2*(2*sqrt(2)*(c^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(d^2/c)^
(1/4) + 2*sqrt(d*x))/(d^2/c)^(1/4))/(c*d^5) + 2*sqrt(2)*(c^3*d^2)^(1/4)*arc
tan(-1/2*sqrt(2)*(sqrt(2)*(d^2/c)^(1/4) - 2*sqrt(d*x))/(d^2/c)^(1/4))/(c*d^
5) + 2*sqrt(2)*(-c^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-d^2/c)^(1/4)
+ 2*sqrt(d*x))/(-d^2/c)^(1/4))/(c*d^5) + 2*sqrt(2)*(-c^3*d^2)^(1/4)*arctan(
-1/2*sqrt(2)*(sqrt(2)*(-d^2/c)^(1/4) - 2*sqrt(d*x))/(-d^2/c)^(1/4))/(c*d^5)
 + sqrt(2)*(c^3*d^2)^(1/4)*log(d*x + sqrt(2)*sqrt(d*x)*(d^2/c)^(1/4) + sqrt
(d^2/c))/(c*d^5) - sqrt(2)*(c^3*d^2)^(1/4)*log(d*x - sqrt(2)*sqrt(d*x)*(d^2
/c)^(1/4) + sqrt(d^2/c))/(c*d^5) + sqrt(2)*(-c^3*d^2)^(1/4)*log(d*x + sqrt(
2)*sqrt(d*x)*(-d^2/c)^(1/4) + sqrt(-d^2/c))/(c*d^5) - sqrt(2)*(-c^3*d^2)^(1
/4)*log(d*x - sqrt(2)*sqrt(d*x)*(-d^2/c)^(1/4) + sqrt(-d^2/c))/(c*d^5)) - 1
/3*(b*log(-(c*d^2*x^2 + d^2)/(c*d^2*x^2 - d^2))/(sqrt(d*x)*d*x) + 2*a/(sqrt
(d*x)*d*x))/d
```

3.88 $\int \frac{a+b\tanh^{-1}\left(cx^2\right)}{(dx)^{7/2}}\,dx$

Optimal. Leaf size=317

$$-\frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{5d(dx)^{5/2}}-\frac{bc^{5/4}\log\left(\sqrt{c}\sqrt{d}x-\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{5\sqrt{2}d^{7/2}}+\frac{bc^{5/4}\log\left(\sqrt{c}\sqrt{d}x+\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{5\sqrt{2}d^{7/2}}-\frac{2bc^{5/4}\tan^{-1}}{5d^{7/2}}$$

```
[Out] (-8*b*c)/(5*d^3*Sqrt[d*x]) - (2*b*c^(5/4)*ArcTan[(c^(1/4)*Sqrt[d*x])/Sqrt[d
]])/(5*d^(7/2)) + (Sqrt[2]*b*c^(5/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[d*x])
/Sqrt[d]])/(5*d^(7/2)) - (Sqrt[2]*b*c^(5/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqr
t[d*x])/Sqrt[d]])/(5*d^(7/2)) - (2*(a + b*ArcTanh[c*x^2]))/(5*d*(d*x)^(5/2)
) + (2*b*c^(5/4)*ArcTanh[(c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(5*d^(7/2)) - (b*c^(
5/4)*Log[Sqrt[d] + Sqrt[c]*Sqrt[d]*x - Sqrt[2]*c^(1/4)*Sqrt[d*x]])/(5*Sqrt[
2]*d^(7/2)) + (b*c^(5/4)*Log[Sqrt[d] + Sqrt[c]*Sqrt[d]*x + Sqrt[2]*c^(1/4)*
Sqrt[d*x]])/(5*Sqrt[2]*d^(7/2))
```

Rubi [A] time = 0.277762, antiderivative size = 317, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 14, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {6097, 16, 325, 329, 301, 297, 1162, 617, 204, 1165, 628, 298, 205, 208}

$$-\frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{5d(dx)^{5/2}}-\frac{bc^{5/4}\log\left(\sqrt{c}\sqrt{d}x-\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{5\sqrt{2}d^{7/2}}+\frac{bc^{5/4}\log\left(\sqrt{c}\sqrt{d}x+\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{5\sqrt{2}d^{7/2}}-\frac{2bc^{5/4}\tan^{-1}}{5d^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^2])/(d*x)^(7/2),x]
```


```
[Out] (-8*b*c)/(5*d^3*Sqrt[d*x]) - (2*b*c^(5/4)*ArcTan[(c^(1/4)*Sqrt[d*x])/Sqrt[d
]])/(5*d^(7/2)) + (Sqrt[2]*b*c^(5/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[d*x])
/Sqrt[d]])/(5*d^(7/2)) - (Sqrt[2]*b*c^(5/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqr
t[d*x])/Sqrt[d]])/(5*d^(7/2)) - (2*(a + b*ArcTanh[c*x^2]))/(5*d*(d*x)^(5/2)
) + (2*b*c^(5/4)*ArcTanh[(c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(5*d^(7/2)) - (b*c^(
5/4)*Log[Sqrt[d] + Sqrt[c]*Sqrt[d]*x - Sqrt[2]*c^(1/4)*Sqrt[d*x]])/(5*Sqrt[
2]*d^(7/2)) + (b*c^(5/4)*Log[Sqrt[d] + Sqrt[c]*Sqrt[d]*x + Sqrt[2]*c^(1/4)*
Sqrt[d*x]])/(5*Sqrt[2]*d^(7/2))
```


Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
```

```
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 301

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[x^(m - n/2
)/(r + s*x^(n/2)), x], x] - Dist[s/(2*b), Int[x^(m - n/2)/(r - s*x^(n/2)),
x], x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LeQ[n/2, m] &&
 LtQ[m, n] &&  !GtQ[a/b, 0]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\tanh^{-1}\left(cx^2\right)}{(dx)^{7/2}}\,dx &= -\frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{5d(dx)^{5/2}}+\frac{(4bc)\int \frac{x}{(dx)^{5/2}\left(1-c^2x^4\right)}\,dx}{5d}\\
&= -\frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{5d(dx)^{5/2}}+\frac{(4bc)\int \frac{1}{(dx)^{3/2}\left(1-c^2x^4\right)}\,dx}{5d^2}\\
&= -\frac{8bc}{5d^3\sqrt{dx}}-\frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{5d(dx)^{5/2}}+\frac{\left(4bc^3\right)\int \frac{(dx)^{5/2}}{1-c^2x^4}\,dx}{5d^6}\\
&= -\frac{8bc}{5d^3\sqrt{dx}}-\frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{5d(dx)^{5/2}}+\frac{\left(8bc^3\right)\operatorname{Subst}\left(\int \frac{x^6}{1-\frac{c^2x^8}{d^4}}\,dx,x,\sqrt{dx}\right)}{5d^7}\\
&= -\frac{8bc}{5d^3\sqrt{dx}}-\frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{5d(dx)^{5/2}}+\frac{\left(4bc^2\right)\operatorname{Subst}\left(\int \frac{x^2}{d^2-cx^4}\,dx,x,\sqrt{dx}\right)}{5d^3}-\frac{\left(4bc^2\right)\operatorname{Subst}\left(\int\right.}{5\ldots}\\
&= -\frac{8bc}{5d^3\sqrt{dx}}-\frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{5d(dx)^{5/2}}+\frac{\left(2bc^{3/2}\right)\operatorname{Subst}\left(\int \frac{1}{d-\sqrt{c}x^2}\,dx,x,\sqrt{dx}\right)}{5d^3}-\frac{\left(2bc^{3/2}\right)\operatorname{Subst}}{\ldots}\\
&= -\frac{8bc}{5d^3\sqrt{dx}}-\frac{2bc^{5/4}\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{5d^{7/2}}-\frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{5d(dx)^{5/2}}+\frac{2bc^{5/4}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{5d^{7/2}}-\frac{\left(bc^{5/4}\ldots\right.}{\ldots}\\
&= -\frac{8bc}{5d^3\sqrt{dx}}-\frac{2bc^{5/4}\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{5d^{7/2}}-\frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{5d(dx)^{5/2}}+\frac{2bc^{5/4}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{5d^{7/2}}-\frac{bc^{5/4}\ldots}{\ldots}\\
&= -\frac{8bc}{5d^3\sqrt{dx}}-\frac{2bc^{5/4}\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{5d^{7/2}}+\frac{\sqrt{2}bc^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{5d^{7/2}}-\frac{\sqrt{2}bc^{5/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\ldots}{\sqrt{\ldots}}\right)}{5d^{7/2}}
\end{aligned}$$

Mathematica [A] time = 0.0857637, size = 275, normalized size = 0.87

$$\frac{x\left(-4a-2bc^{5/4}x^{5/2}\log\left(1-\sqrt[4]{c}\sqrt{x}\right)+2bc^{5/4}x^{5/2}\log\left(\sqrt[4]{c}\sqrt{x}+1\right)-\sqrt{2}bc^{5/4}x^{5/2}\log\left(\sqrt{c}x-\sqrt{2}\sqrt[4]{c}\sqrt{x}+1\right)+\sqrt{2}bc^{5/4}x^{5/2}\log\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])/(d*x)^(7/2),x]
```

```
[Out] (x*(-4*a - 16*b*c*x^2 + 2*Sqrt[2]*b*c^(5/4)*x^(5/2)*ArcTan[1 - Sqrt[2]*c^(1
/4)*Sqrt[x]] - 2*Sqrt[2]*b*c^(5/4)*x^(5/2)*ArcTan[1 + Sqrt[2]*c^(1/4)*Sqrt[
x]] - 4*b*c^(5/4)*x^(5/2)*ArcTan[c^(1/4)*Sqrt[x]] - 4*b*ArcTanh[c*x^2] - 2*
b*c^(5/4)*x^(5/2)*Log[1 - c^(1/4)*Sqrt[x]] + 2*b*c^(5/4)*x^(5/2)*Log[1 + c^
(1/4)*Sqrt[x]] - Sqrt[2]*b*c^(5/4)*x^(5/2)*Log[1 - Sqrt[2]*c^(1/4)*Sqrt[x]
+ Sqrt[c]*x] + Sqrt[2]*b*c^(5/4)*x^(5/2)*Log[1 + Sqrt[2]*c^(1/4)*Sqrt[x] +
Sqrt[c]*x]))/(10*(d*x)^(7/2))
```

Maple [A] time = 0.016, size = 292, normalized size = 0.9

$$-\frac{2a}{5d}(dx)^{-\frac{5}{2}}-\frac{2b\mathrm{Artanh}\left(cx^2\right)}{5d}(dx)^{-\frac{5}{2}}-\frac{bc\sqrt{2}}{10d^3}\ln\left(\left(dx-\sqrt[4]{\frac{d^2}{c}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{d^2}{c}}\right)\left(dx+\sqrt[4]{\frac{d^2}{c}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{d^2}{c}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{d^2}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))/(d*x)^(7/2),x)
```

```
[Out] -2/5/d*a/(d*x)^(5/2)-2/5/d*b/(d*x)^(5/2)*arctanh(c*x^2)-1/10/d^3*b*c/(d^2/c
)^(1/4)*2^(1/2)*ln((d*x-(d^2/c)^(1/4)*(d*x)^(1/2)*2^(1/2)+(d^2/c)^(1/2))/(d
*x+(d^2/c)^(1/4)*(d*x)^(1/2)*2^(1/2)+(d^2/c)^(1/2)))-1/5/d^3*b*c/(d^2/c)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(d^2/c)^(1/4)*(d*x)^(1/2)+1)-1/5/d^3*b*c/(d^2/c)
^(1/4)*2^(1/2)*arctan(2^(1/2)/(d^2/c)^(1/4)*(d*x)^(1/2)-1)-8/5*b*c/d^3/(d*x
)^(1/2)-2/5/d^3*b*c/(d^2/c)^(1/4)*arctan((d*x)^(1/2)/(d^2/c)^(1/4))+1/5/d^3
*b*c/(d^2/c)^(1/4)*ln(((d*x)^(1/2)+(d^2/c)^(1/4))/((d*x)^(1/2)-(d^2/c)^(1/4
)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/(d*x)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.23702, size = 105, normalized size = 0.33

$$-\frac{\left(8\,bcx^2+b\log\left(-\frac{cx^2+1}{cx^2-1}\right)+2\,a\right)\sqrt{dx}}{5\,d^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/(d*x)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/5*(8*b*c*x^2 + b*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 2*a)*sqrt(d*x)/(d^4*x^3
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))/(d*x)**(7/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 9.77957, size = 717, normalized size = 2.26

$$-\frac{1}{10}bc^3\left(\frac{2\sqrt{2}\left(c^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{d^2}{c}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)}{c^4d^5}+\frac{2\sqrt{2}\left(c^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{d^2}{c}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)}{c^4d^5}-\frac{2\sqrt{2}\left(-c^3d^2\right)^{\frac{3}{4}}\arctan}{c^4d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/(d*x)^(7/2),x, algorithm="giac")
```

```
[Out] -1/10*b*c^3*(2*sqrt(2)*(c^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(d^2/c)^
(1/4) + 2*sqrt(d*x))/(d^2/c)^(1/4))/(c^4*d^5) + 2*sqrt(2)*(c^3*d^2)^(3/4)*a
rctan(-1/2*sqrt(2)*(sqrt(2)*(d^2/c)^(1/4) - 2*sqrt(d*x))/(d^2/c)^(1/4))/(c^
4*d^5) - 2*sqrt(2)*(-c^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-d^2/c)^(1
/4) + 2*sqrt(d*x))/(-d^2/c)^(1/4))/(c^4*d^5) - 2*sqrt(2)*(-c^3*d^2)^(3/4)*a
rctan(-1/2*sqrt(2)*(sqrt(2)*(-d^2/c)^(1/4) - 2*sqrt(d*x))/(-d^2/c)^(1/4))/(
c^4*d^5) - sqrt(2)*(c^3*d^2)^(3/4)*log(d*x + sqrt(2)*sqrt(d*x)*(d^2/c)^(1/4
) + sqrt(d^2/c))/(c^4*d^5) + sqrt(2)*(c^3*d^2)^(3/4)*log(d*x - sqrt(2)*sqrt
(d*x)*(d^2/c)^(1/4) + sqrt(d^2/c))/(c^4*d^5) + sqrt(2)*(-c^3*d^2)^(3/4)*log
(d*x + sqrt(2)*sqrt(d*x)*(-d^2/c)^(1/4) + sqrt(-d^2/c))/(c^4*d^5) - sqrt(2)
*(-c^3*d^2)^(3/4)*log(d*x - sqrt(2)*sqrt(d*x)*(-d^2/c)^(1/4) + sqrt(-d^2/c)
)/(c^4*d^5)) - 1/5*(b*log(-(c*d^2*x^2 + d^2)/(c*d^2*x^2 - d^2))/(sqrt(d*x)*
d^2*x^2) + 2*(4*b*c*d^2*x^2 + a*d^2)/(sqrt(d*x)*d^4*x^2))/d
```

3.89 $\int \frac{a+b\tanh^{-1}(cx^2)}{(dx)^{9/2}}\,dx$

Optimal. Leaf size=317

$$-\frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{7d(dx)^{7/2}}+\frac{bc^{7/4}\log\left(\sqrt{c}\sqrt{d}x-\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{7\sqrt{2}d^{9/2}}-\frac{bc^{7/4}\log\left(\sqrt{c}\sqrt{d}x+\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{7\sqrt{2}d^{9/2}}+\frac{2bc^{7/4}\tan}{7}$$

```
[Out] (-8*b*c)/(21*d^3*(d*x)^(3/2)) + (2*b*c^(7/4)*ArcTan[(c^(1/4)*Sqrt[d*x])/Sqr
t[d]])/(7*d^(9/2)) + (Sqrt[2]*b*c^(7/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[d*
x])/Sqrt[d]])/(7*d^(9/2)) - (Sqrt[2]*b*c^(7/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*
Sqrt[d*x])/Sqrt[d]])/(7*d^(9/2)) - (2*(a + b*ArcTanh[c*x^2]))/(7*d*(d*x)^(7
/2)) + (2*b*c^(7/4)*ArcTanh[(c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(7*d^(9/2)) + (b*
c^(7/4)*Log[Sqrt[d] + Sqrt[c]*Sqrt[d]*x - Sqrt[2]*c^(1/4)*Sqrt[d*x]])/(7*Sq
rt[2]*d^(9/2)) - (b*c^(7/4)*Log[Sqrt[d] + Sqrt[c]*Sqrt[d]*x + Sqrt[2]*c^(1/
4)*Sqrt[d*x]])/(7*Sqrt[2]*d^(9/2))
```

Rubi [A] time = 0.286934, antiderivative size = 317, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 14, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {6097, 16, 325, 329, 301, 211, 1165, 628, 1162, 617, 204, 212, 208, 205}

$$-\frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{7d(dx)^{7/2}}+\frac{bc^{7/4}\log\left(\sqrt{c}\sqrt{d}x-\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{7\sqrt{2}d^{9/2}}-\frac{bc^{7/4}\log\left(\sqrt{c}\sqrt{d}x+\sqrt{2}\sqrt[4]{c}\sqrt{dx}+\sqrt{d}\right)}{7\sqrt{2}d^{9/2}}+\frac{2bc^{7/4}\tan}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^2])/(d*x)^(9/2),x]
```


```
[Out] (-8*b*c)/(21*d^3*(d*x)^(3/2)) + (2*b*c^(7/4)*ArcTan[(c^(1/4)*Sqrt[d*x])/Sqr
t[d]])/(7*d^(9/2)) + (Sqrt[2]*b*c^(7/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[d*
x])/Sqrt[d]])/(7*d^(9/2)) - (Sqrt[2]*b*c^(7/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*
Sqrt[d*x])/Sqrt[d]])/(7*d^(9/2)) - (2*(a + b*ArcTanh[c*x^2]))/(7*d*(d*x)^(7
/2)) + (2*b*c^(7/4)*ArcTanh[(c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(7*d^(9/2)) + (b*
c^(7/4)*Log[Sqrt[d] + Sqrt[c]*Sqrt[d]*x - Sqrt[2]*c^(1/4)*Sqrt[d*x]])/(7*Sq
rt[2]*d^(9/2)) - (b*c^(7/4)*Log[Sqrt[d] + Sqrt[c]*Sqrt[d]*x + Sqrt[2]*c^(1/
4)*Sqrt[d*x]])/(7*Sqrt[2]*d^(9/2))
```


Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
```

```
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 301

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[x^(m - n/2
)/(r + s*x^(n/2)), x], x] - Dist[s/(2*b), Int[x^(m - n/2)/(r - s*x^(n/2)),
x], x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LeQ[n/2, m] &&
 LtQ[m, n] &&  !GtQ[a/b, 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}\left(cx^2\right)}{(dx)^{9/2}}\,dx = -\frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{7d(dx)^{7/2}}+\frac{(4bc)\int \frac{x}{(dx)^{7/2}\left(1-c^2x^4\right)}\,dx}{7d}$$

$$= -\frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{7d(dx)^{7/2}}+\frac{(4bc)\int \frac{1}{(dx)^{5/2}\left(1-c^2x^4\right)}\,dx}{7d^2}$$

$$= -\frac{8bc}{21d^3(dx)^{3/2}}-\frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{7d(dx)^{7/2}}+\frac{\left(4bc^3\right)\int \frac{(dx)^{3/2}}{1-c^2x^4}\,dx}{7d^6}$$

$$= -\frac{8bc}{21d^3(dx)^{3/2}}-\frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{7d(dx)^{7/2}}+\frac{\left(8bc^3\right)\text{Subst}\left(\int \frac{x^4}{1-\frac{c^2x^8}{d^4}}\,dx,x,\sqrt{dx}\right)}{7d^7}$$

$$= -\frac{8bc}{21d^3(dx)^{3/2}}-\frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{7d(dx)^{7/2}}+\frac{\left(4bc^2\right)\text{Subst}\left(\int \frac{1}{d^2-cx^4}\,dx,x,\sqrt{dx}\right)}{7d^3}-\frac{\left(4bc^2\right)\text{Sul}}{}$$

$$= -\frac{8bc}{21d^3(dx)^{3/2}}-\frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{7d(dx)^{7/2}}+\frac{\left(2bc^2\right)\text{Subst}\left(\int \frac{1}{d-\sqrt{c}x^2}\,dx,x,\sqrt{dx}\right)}{7d^4}+\frac{\left(2bc^2\right)\text{Su}}{}$$

$$= -\frac{8bc}{21d^3(dx)^{3/2}}+\frac{2bc^{7/4}\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{7d^{9/2}}-\frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{7d(dx)^{7/2}}+\frac{2bc^{7/4}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{7d^{9/2}}-$$

$$= -\frac{8bc}{21d^3(dx)^{3/2}}+\frac{2bc^{7/4}\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{7d^{9/2}}-\frac{2\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{7d(dx)^{7/2}}+\frac{2bc^{7/4}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{7d^{9/2}}-$$

$$= -\frac{8bc}{21d^3(dx)^{3/2}}+\frac{2bc^{7/4}\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{7d^{9/2}}+\frac{\sqrt{2}bc^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{7d^{9/2}}-\frac{\sqrt{2}bc^{7/4}\tan^{-1}\left(1\right.}{7d^{9/2}}$$

Mathematica [A] time = 0.0976679, size = 281, normalized size = 0.89

$$\sqrt{dx}\left(-12a-6bc^{7/4}x^{7/2}\log\left(1-\sqrt[4]{c}\sqrt{x}\right)+6bc^{7/4}x^{7/2}\log\left(\sqrt[4]{c}\sqrt{x}+1\right)+3\sqrt{2}bc^{7/4}x^{7/2}\log\left(\sqrt{c}x-\sqrt{2}\sqrt[4]{c}\sqrt{x}+1\right)-3\sqrt{2}b\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])/(d*x)^(9/2),x]
```

```
[Out] (Sqrt[d*x]*(-12*a - 16*b*c*x^2 + 6*Sqrt[2]*b*c^(7/4)*x^(7/2)*ArcTan[1 - Sqr
t[2]*c^(1/4)*Sqrt[x]] - 6*Sqrt[2]*b*c^(7/4)*x^(7/2)*ArcTan[1 + Sqrt[2]*c^(1
/4)*Sqrt[x]] + 12*b*c^(7/4)*x^(7/2)*ArcTan[c^(1/4)*Sqrt[x]] - 12*b*ArcTanh[
c*x^2] - 6*b*c^(7/4)*x^(7/2)*Log[1 - c^(1/4)*Sqrt[x]] + 6*b*c^(7/4)*x^(7/2)
*Log[1 + c^(1/4)*Sqrt[x]] + 3*Sqrt[2]*b*c^(7/4)*x^(7/2)*Log[1 - Sqrt[2]*c^(
1/4)*Sqrt[x] + Sqrt[c]*x] - 3*Sqrt[2]*b*c^(7/4)*x^(7/2)*Log[1 + Sqrt[2]*c^(
1/4)*Sqrt[x] + Sqrt[c]*x]))/(42*d^5*x^4)
```

Maple [A] time = 0.014, size = 302, normalized size = 1.

$$-\frac{2a}{7d}(dx)^{-\frac{7}{2}}-\frac{2b\mathrm{Artanh}\left(cx^2\right)}{7d}(dx)^{-\frac{7}{2}}-\frac{bc^2\sqrt{2}}{14d^5}\sqrt[4]{\frac{d^2}{c}}\ln\left(\left(dx+\sqrt[4]{\frac{d^2}{c}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{d^2}{c}}\right)\left(dx-\sqrt[4]{\frac{d^2}{c}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{d^2}{c}}\right)^{-1}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))/(d*x)^(9/2),x)
```

```
[Out] -2/7/d*a/(d*x)^(7/2)-2/7/d*b/(d*x)^(7/2)*arctanh(c*x^2)-1/14/d^5*b*c^2*(d^2
/c)^(1/4)*2^(1/2)*ln((d*x+(d^2/c)^(1/4)*(d*x)^(1/2)*2^(1/2)+(d^2/c)^(1/2))/
(d*x-(d^2/c)^(1/4)*(d*x)^(1/2)*2^(1/2)+(d^2/c)^(1/2)))-1/7/d^5*b*c^2*(d^2/c
)^(1/4)*2^(1/2)*arctan(2^(1/2)/(d^2/c)^(1/4)*(d*x)^(1/2)+1)-1/7/d^5*b*c^2*(
d^2/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(d^2/c)^(1/4)*(d*x)^(1/2)-1)+1/7/d^5*b*
c^2*(d^2/c)^(1/4)*ln(((d*x)^(1/2)+(d^2/c)^(1/4))/((d*x)^(1/2)-(d^2/c)^(1/4)
))+2/7/d^5*b*c^2*(d^2/c)^(1/4)*arctan((d*x)^(1/2)/(d^2/c)^(1/4))-8/21*b*c/d
^3/(d*x)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/(d*x)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.23562, size = 109, normalized size = 0.34

$$-\frac{\left(8\,bcx^2+3\,b\log\left(-\frac{cx^2+1}{cx^2-1}\right)+6\,a\right)\sqrt{dx}}{21\,d^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/(d*x)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/21*(8*b*c*x^2 + 3*b*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 6*a)*sqrt(d*x)/(d^5*
x^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))/(d*x)**(9/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 25.291, size = 720, normalized size = 2.27

$$-\frac{1}{14}bc^3\left(\frac{2\sqrt{2}\left(c^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{d^2}{c}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)}{c^2d^5}+\frac{2\sqrt{2}\left(c^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{d^2}{c}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)}{c^2d^5}-\frac{2\sqrt{2}\left(-c^3d^2\right)^{\frac{1}{4}}\operatorname{arct}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/(d*x)^(9/2),x, algorithm="giac")
```

```
[Out] -1/14*b*c^3*(2*sqrt(2)*(c^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(d^2/c)^
(1/4) + 2*sqrt(d*x))/(d^2/c)^(1/4))/(c^2*d^5) + 2*sqrt(2)*(c^3*d^2)^(1/4)*a
rctan(-1/2*sqrt(2)*(sqrt(2)*(d^2/c)^(1/4) - 2*sqrt(d*x))/(d^2/c)^(1/4))/(c^
2*d^5) - 2*sqrt(2)*(-c^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-d^2/c)^(1
/4) + 2*sqrt(d*x))/(-d^2/c)^(1/4))/(c^2*d^5) - 2*sqrt(2)*(-c^3*d^2)^(1/4)*a
rctan(-1/2*sqrt(2)*(sqrt(2)*(-d^2/c)^(1/4) - 2*sqrt(d*x))/(-d^2/c)^(1/4))/(
c^2*d^5) + sqrt(2)*(c^3*d^2)^(1/4)*log(d*x + sqrt(2)*sqrt(d*x)*(d^2/c)^(1/4
) + sqrt(d^2/c))/(c^2*d^5) - sqrt(2)*(c^3*d^2)^(1/4)*log(d*x - sqrt(2)*sqrt
(d*x)*(d^2/c)^(1/4) + sqrt(d^2/c))/(c^2*d^5) - sqrt(2)*(-c^3*d^2)^(1/4)*log
(d*x + sqrt(2)*sqrt(d*x)*(-d^2/c)^(1/4) + sqrt(-d^2/c))/(c^2*d^5) + sqrt(2)
*(-c^3*d^2)^(1/4)*log(d*x - sqrt(2)*sqrt(d*x)*(-d^2/c)^(1/4) + sqrt(-d^2/c)
)/(c^2*d^5)) - 1/21*(3*b*log(-(c*d^2*x^2 + d^2)/(c*d^2*x^2 - d^2))/(sqrt(d*
x)*d^3*x^3) + 2*(4*b*c*d^2*x^2 + 3*a*d^2)/(sqrt(d*x)*d^5*x^3))/d
```

3.90 $\int \sqrt{dx}\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2\,dx$

Optimal. Leaf size=6327

result too large to display

```
[Out] (-8*a*b*x*Sqrt[d*x])/9 - (2*Sqrt[2]*a*b*Sqrt[d*x]*ArcTan[1 - Sqrt[2]*c^(1/4
)*Sqrt[x]])/(3*c^(3/4)*Sqrt[x]) + (2*Sqrt[2]*a*b*Sqrt[d*x]*ArcTan[1 + Sqrt[
2]*c^(1/4)*Sqrt[x]])/(3*c^(3/4)*Sqrt[x]) - (((2*I)/3)*b^2*Sqrt[d*x]*ArcTan[
(-c)^(1/4)*Sqrt[x]]^2)/((-c)^(3/4)*Sqrt[x]) - (((2*I)/3)*b^2*Sqrt[d*x]*ArcT
an[c^(1/4)*Sqrt[x]]^2)/(c^(3/4)*Sqrt[x]) - (2*b^2*Sqrt[d*x]*ArcTanh[(-c)^(1
/4)*Sqrt[x]]^2)/(3*(-c)^(3/4)*Sqrt[x]) - (2*b^2*Sqrt[d*x]*ArcTanh[c^(1/4)*S
qrt[x]]^2)/(3*c^(3/4)*Sqrt[x]) + (4*b^2*Sqrt[d*x]*ArcTanh[(-c)^(1/4)*Sqrt[x
]]*Log[2/(1 - (-c)^(1/4)*Sqrt[x])])/(3*(-c)^(3/4)*Sqrt[x]) + (4*b^2*Sqrt[d*
x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[2/(1 - I*(-c)^(1/4)*Sqrt[x])])/(3*(-c)^(3
/4)*Sqrt[x]) - (2*b^2*Sqrt[d*x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[(-2*(-c)^(1/
4)*(1 - Sqrt[-Sqrt[c]]*Sqrt[x]))/((I*Sqrt[-Sqrt[c]] - (-c)^(1/4))*(1 - I*(-
c)^(1/4)*Sqrt[x]))])/(3*(-c)^(3/4)*Sqrt[x]) - (2*b^2*Sqrt[d*x]*ArcTan[(-c)^
(1/4)*Sqrt[x]]*Log[(2*(-c)^(1/4)*(1 + Sqrt[-Sqrt[c]]*Sqrt[x]))/((I*Sqrt[-Sq
rt[c]] + (-c)^(1/4))*(1 - I*(-c)^(1/4)*Sqrt[x]))])/(3*(-c)^(3/4)*Sqrt[x]) +
 (2*b^2*Sqrt[d*x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[((1 + I)*(1 - (-c)^(1/4)*S
qrt[x]))/(1 - I*(-c)^(1/4)*Sqrt[x])])/(3*(-c)^(3/4)*Sqrt[x]) - (4*b^2*Sqrt[
d*x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[2/(1 + I*(-c)^(1/4)*Sqrt[x])])/(3*(-c)^
(3/4)*Sqrt[x]) - (4*b^2*Sqrt[d*x]*ArcTanh[(-c)^(1/4)*Sqrt[x]]*Log[2/(1 + (-
c)^(1/4)*Sqrt[x])])/(3*(-c)^(3/4)*Sqrt[x]) - (2*b^2*Sqrt[d*x]*ArcTanh[(-c)^
(1/4)*Sqrt[x]]*Log[(-2*(-c)^(1/4)*(1 - Sqrt[-Sqrt[-c]]*Sqrt[x]))/((Sqrt[-Sq
rt[-c]] - (-c)^(1/4))*(1 + (-c)^(1/4)*Sqrt[x]))])/(3*(-c)^(3/4)*Sqrt[x]) -
(2*b^2*Sqrt[d*x]*ArcTanh[(-c)^(1/4)*Sqrt[x]]*Log[(2*(-c)^(1/4)*(1 + Sqrt[-S
qrt[-c]]*Sqrt[x]))/((Sqrt[-Sqrt[-c]] + (-c)^(1/4))*(1 + (-c)^(1/4)*Sqrt[x])
)])/(3*(-c)^(3/4)*Sqrt[x]) + (2*b^2*Sqrt[d*x]*ArcTanh[(-c)^(1/4)*Sqrt[x]]*L
og[(-2*(-c)^(1/4)*(1 - Sqrt[-Sqrt[c]]*Sqrt[x]))/((Sqrt[-Sqrt[c]] - (-c)^(1/
4))*(1 + (-c)^(1/4)*Sqrt[x]))])/(3*(-c)^(3/4)*Sqrt[x]) + (2*b^2*Sqrt[d*x]*A
rcTanh[(-c)^(1/4)*Sqrt[x]]*Log[(2*(-c)^(1/4)*(1 + Sqrt[-Sqrt[c]]*Sqrt[x]))/
((Sqrt[-Sqrt[c]] + (-c)^(1/4))*(1 + (-c)^(1/4)*Sqrt[x]))])/(3*(-c)^(3/4)*Sq
rt[x]) + (2*b^2*Sqrt[d*x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[((1 - I)*(1 + (-c)
^(1/4)*Sqrt[x]))/(1 - I*(-c)^(1/4)*Sqrt[x])])/(3*(-c)^(3/4)*Sqrt[x]) + (4*b
^2*Sqrt[d*x]*ArcTanh[c^(1/4)*Sqrt[x]]*Log[2/(1 - c^(1/4)*Sqrt[x])])/(3*c^(3
/4)*Sqrt[x]) - (2*b^2*Sqrt[d*x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[(2*(-c)^(1/4
)*(1 - c^(1/4)*Sqrt[x]))/((((-c)^(1/4) - I*c^(1/4))*(1 - I*(-c)^(1/4)*Sqrt[x
]))])/(3*(-c)^(3/4)*Sqrt[x]) + (2*b^2*Sqrt[d*x]*ArcTanh[(-c)^(1/4)*Sqrt[x]]
*Log[(2*(-c)^(1/4)*(1 - c^(1/4)*Sqrt[x]))/((((-c)^(1/4) - c^(1/4))*(1 + (-c)
^(1/4)*Sqrt[x]))])/(3*(-c)^(3/4)*Sqrt[x]) + (4*b^2*Sqrt[d*x]*ArcTan[c^(1/4)
*Sqrt[x]]*Log[2/(1 - I*c^(1/4)*Sqrt[x])])/(3*c^(3/4)*Sqrt[x]) - (2*b^2*Sqrt
[d*x]*ArcTan[c^(1/4)*Sqrt[x]]*Log[(-2*c^(1/4)*(1 - Sqrt[-Sqrt[-c]]*Sqrt[x])
)/((I*Sqrt[-Sqrt[-c]] - c^(1/4))*(1 - I*c^(1/4)*Sqrt[x]))])/(3*c^(3/4)*Sqrt
[x]) - (2*b^2*Sqrt[d*x]*ArcTan[c^(1/4)*Sqrt[x]]*Log[(2*c^(1/4)*(1 + Sqrt[-S
qrt[-c]]*Sqrt[x]))/((I*Sqrt[-Sqrt[-c]] + c^(1/4))*(1 - I*c^(1/4)*Sqrt[x]))]
)/(3*c^(3/4)*Sqrt[x]) - (2*b^2*Sqrt[d*x]*ArcTan[c^(1/4)*Sqrt[x]]*Log[(-2*c^
(1/4)*(1 - (-c)^(1/4)*Sqrt[x]))/((I*(-c)^(1/4) - c^(1/4))*(1 - I*c^(1/4)*Sq
rt[x]))])/(3*c^(3/4)*Sqrt[x]) - (2*b^2*Sqrt[d*x]*ArcTan[c^(1/4)*Sqrt[x]]*Lo
g[(2*c^(1/4)*(1 + (-c)^(1/4)*Sqrt[x]))/((I*(-c)^(1/4) + c^(1/4))*(1 - I*c^(
1/4)*Sqrt[x]))])/(3*c^(3/4)*Sqrt[x]) + (2*b^2*Sqrt[d*x]*ArcTan[c^(1/4)*Sqrt
[x]]*Log[((1 + I)*(1 - c^(1/4)*Sqrt[x]))/(1 - I*c^(1/4)*Sqrt[x])])/(3*c^(3/
4)*Sqrt[x]) - (4*b^2*Sqrt[d*x]*ArcTan[c^(1/4)*Sqrt[x]]*Log[2/(1 + I*c^(1/4)
*Sqrt[x])])/(3*c^(3/4)*Sqrt[x]) - (4*b^2*Sqrt[d*x]*ArcTanh[c^(1/4)*Sqrt[x]]
*Log[2/(1 + c^(1/4)*Sqrt[x])])/(3*c^(3/4)*Sqrt[x]) + (2*b^2*Sqrt[d*x]*ArcTa
nh[c^(1/4)*Sqrt[x]]*Log[(-2*c^(1/4)*(1 - Sqrt[-Sqrt[-c]]*Sqrt[x]))/((Sqrt[-
Sqrt[-c]] - c^(1/4))*(1 + c^(1/4)*Sqrt[x]))])/(3*c^(3/4)*Sqrt[x]) + (2*b^2*
Sqrt[d*x]*ArcTanh[c^(1/4)*Sqrt[x]]*Log[(2*c^(1/4)*(1 + Sqrt[-Sqrt[-c]]*Sqrt
```

```
[x]))/((Sqrt[-Sqrt[-c]] + c^(1/4))*(1 + c^(1/4)*Sqrt[x]))])/(3*c^(3/4)*Sqrt
[x]) - (2*b^2*Sqrt[d*x]*ArcTanh[c^(1/4)*Sqrt[x]]*Log[(-2*c^(1/4)*(1 - Sqrt[
-Sqrt[c]]*Sqrt[x]))/((Sqrt[-Sqrt[c]] - c^(1/4))*(1 + c^(1/4)*Sqrt[x]))])/(3
*c^(3/4)*Sqrt[x]) - (2*b^2*Sqrt[d*x]*ArcTanh[c^(1/4)*Sqrt[x]]*Log[(2*c^(1/4
)*(1 + Sqrt[-Sqrt[c]]*Sqrt[x]))/((Sqrt[-Sqrt[c]] + c^(1/4))*(1 + c^(1/4)*Sq
rt[x]))])/(3*c^(3/4)*Sqrt[x]) + (2*b^2*Sqrt[d*x]*ArcTanh[c^(1/4)*Sqrt[x]]*L
og[(-2*c^(1/4)*(1 - (-c)^(1/4)*Sqrt[x]))/(((-c)^(1/4) - c^(1/4))*(1 + c^(1/
4)*Sqrt[x]))])/(3*c^(3/4)*Sqrt[x]) + (2*b^2*Sqrt[d*x]*ArcTanh[c^(1/4)*Sqrt[
x]]*Log[(2*c^(1/4)*(1 + (-c)^(1/4)*Sqrt[x]))/(((-c)^(1/4) + c^(1/4))*(1 + c
^(1/4)*Sqrt[x]))])/(3*c^(3/4)*Sqrt[x]) - (2*b^2*Sqrt[d*x]*ArcTan[(-c)^(1/4)
*Sqrt[x]]*Log[(2*(-c)^(1/4)*(1 + c^(1/4)*Sqrt[x]))/(((-c)^(1/4) + I*c^(1/4)
)*(1 - I*(-c)^(1/4)*Sqrt[x]))])/(3*(-c)^(3/4)*Sqrt[x]) + (2*b^2*Sqrt[d*x]*A
rcTanh[(-c)^(1/4)*Sqrt[x]]*Log[(2*(-c)^(1/4)*(1 + c^(1/4)*Sqrt[x]))/(((-c)^
(1/4) + c^(1/4))*(1 + (-c)^(1/4)*Sqrt[x]))])/(3*(-c)^(3/4)*Sqrt[x]) + (2*b^
2*Sqrt[d*x]*ArcTan[c^(1/4)*Sqrt[x]]*Log[((1 - I)*(1 + c^(1/4)*Sqrt[x]))/(1
- I*c^(1/4)*Sqrt[x])])/(3*c^(3/4)*Sqrt[x]) + (Sqrt[2]*a*b*Sqrt[d*x]*Log[1 -
 Sqrt[2]*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(3*c^(3/4)*Sqrt[x]) - (Sqrt[2]*a*b*S
qrt[d*x]*Log[1 + Sqrt[2]*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(3*c^(3/4)*Sqrt[x])
+ (4*b^2*x*Sqrt[d*x]*Log[1 - c*x^2])/9 + (2*b^2*Sqrt[d*x]*ArcTan[(-c)^(1/4)
*Sqrt[x]]*Log[1 - c*x^2])/(3*(-c)^(3/4)*Sqrt[x]) - (2*b^2*Sqrt[d*x]*ArcTanh
[(-c)^(1/4)*Sqrt[x]]*Log[1 - c*x^2])/(3*(-c)^(3/4)*Sqrt[x]) + (4*b*x*Sqrt[d
*x]*(2*a - b*Log[1 - c*x^2]))/9 + (2*b*Sqrt[d*x]*ArcTan[c^(1/4)*Sqrt[x]]*(2
*a - b*Log[1 - c*x^2]))/(3*c^(3/4)*Sqrt[x]) - (2*b*Sqrt[d*x]*ArcTanh[c^(1/4
)*Sqrt[x]]*(2*a - b*Log[1 - c*x^2]))/(3*c^(3/4)*Sqrt[x]) + (x*Sqrt[d*x]*(2*
a - b*Log[1 - c*x^2])^2)/6 + (2*a*b*x*Sqrt[d*x]*Log[1 + c*x^2])/3 - (2*b^2*
Sqrt[d*x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[1 + c*x^2])/(3*(-c)^(3/4)*Sqrt[x])
 + (2*b^2*Sqrt[d*x]*ArcTan[c^(1/4)*Sqrt[x]]*Log[1 + c*x^2])/(3*c^(3/4)*Sqrt
[x]) + (2*b^2*Sqrt[d*x]*ArcTanh[(-c)^(1/4)*Sqrt[x]]*Log[1 + c*x^2])/(3*(-c)
^(3/4)*Sqrt[x]) - (2*b^2*Sqrt[d*x]*ArcTanh[c^(1/4)*Sqrt[x]]*Log[1 + c*x^2])
/(3*c^(3/4)*Sqrt[x]) - (b^2*x*Sqrt[d*x]*Log[1 - c*x^2]*Log[1 + c*x^2])/3 +
(b^2*x*Sqrt[d*x]*Log[1 + c*x^2]^2)/6 + (2*b^2*Sqrt[d*x]*PolyLog[2, 1 - 2/(1
 - (-c)^(1/4)*Sqrt[x])])/(3*(-c)^(3/4)*Sqrt[x]) - (((2*I)/3)*b^2*Sqrt[d*x]*
PolyLog[2, 1 - 2/(1 - I*(-c)^(1/4)*Sqrt[x])])/((-c)^(3/4)*Sqrt[x]) + ((I/3)
*b^2*Sqrt[d*x]*PolyLog[2, 1 + (2*(-c)^(1/4)*(1 - Sqrt[-Sqrt[c]]*Sqrt[x]))/(
(I*Sqrt[-Sqrt[c]] - (-c)^(1/4))*(1 - I*(-c)^(1/4)*Sqrt[x]))])/((-c)^(3/4)*S
qrt[x]) + ((I/3)*b^2*Sqrt[d*x]*PolyLog[2, 1 - (2*(-c)^(1/4)*(1 + Sqrt[-Sqrt
[c]]*Sqrt[x]))/((I*Sqrt[-Sqrt[c]] + (-c)^(1/4))*(1 - I*(-c)^(1/4)*Sqrt[x]))
])/((-c)^(3/4)*Sqrt[x]) - ((I/3)*b^2*Sqrt[d*x]*PolyLog[2, 1 - ((1 + I)*(1 -
 (-c)^(1/4)*Sqrt[x]))/(1 - I*(-c)^(1/4)*Sqrt[x])])/((-c)^(3/4)*Sqrt[x]) - (
((2*I)/3)*b^2*Sqrt[d*x]*PolyLog[2, 1 - 2/(1 + I*(-c)^(1/4)*Sqrt[x])])/((-c)
^(3/4)*Sqrt[x]) + (2*b^2*Sqrt[d*x]*PolyLog[2, 1 - 2/(1 + (-c)^(1/4)*Sqrt[x]
)])/(3*(-c)^(3/4)*Sqrt[x]) + (b^2*Sqrt[d*x]*PolyLog[2, 1 + (2*(-c)^(1/4)*(1
 - Sqrt[-Sqrt[-c]]*Sqrt[x]))/((Sqrt[-Sqrt[-c]] - (-c)^(1/4))*(1 + (-c)^(1/4
)*Sqrt[x]))])/(3*(-c)^(3/4)*Sqrt[x]) + (b^2*Sqrt[d*x]*PolyLog[2, 1 - (2*(-c
)^(1/4)*(1 + Sqrt[-Sqrt[-c]]*Sqrt[x]))/((Sqrt[-Sqrt[-c]] + (-c)^(1/4))*(1 +
 (-c)^(1/4)*Sqrt[x]))])/(3*(-c)^(3/4)*Sqrt[x]) - (b^2*Sqrt[d*x]*PolyLog[2,
1 + (2*(-c)^(1/4)*(1 - Sqrt[-Sqrt[c]]*Sqrt[x]))/((Sqrt[-Sqrt[c]] - (-c)^(1/
4))*(1 + (-c)^(1/4)*Sqrt[x]))])/(3*(-c)^(3/4)*Sqrt[x]) - (b^2*Sqrt[d*x]*Pol
yLog[2, 1 - (2*(-c)^(1/4)*(1 + Sqrt[-Sqrt[c]]*Sqrt[x]))/((Sqrt[-Sqrt[c]] +
(-c)^(1/4))*(1 + (-c)^(1/4)*Sqrt[x]))])/(3*(-c)^(3/4)*Sqrt[x]) - ((I/3)*b^2
*Sqrt[d*x]*PolyLog[2, 1 - ((1 - I)*(1 + (-c)^(1/4)*Sqrt[x]))/(1 - I*(-c)^(1
/4)*Sqrt[x])])/((-c)^(3/4)*Sqrt[x]) + (2*b^2*Sqrt[d*x]*PolyLog[2, 1 - 2/(1
- c^(1/4)*Sqrt[x])])/(3*c^(3/4)*Sqrt[x]) + ((I/3)*b^2*Sqrt[d*x]*PolyLog[2,
1 - (2*(-c)^(1/4)*(1 - c^(1/4)*Sqrt[x]))/(((-c)^(1/4) - I*c^(1/4))*(1 - I*(
-c)^(1/4)*Sqrt[x]))])/((-c)^(3/4)*Sqrt[x]) - (b^2*Sqrt[d*x]*PolyLog[2, 1 -
(2*(-c)^(1/4)*(1 - c^(1/4)*Sqrt[x]))/(((-c)^(1/4) - c^(1/4))*(1 + (-c)^(1/4
)*Sqrt[x]))])/(3*(-c)^(3/4)*Sqrt[x]) - (((2*I)/3)*b^2*Sqrt[d*x]*PolyLog[2,
1 - 2/(1 - I*c^(1/4)*Sqrt[x])])/(c^(3/4)*Sqrt[x]) + ((I/3)*b^2*Sqrt[d*x]*Po
lyLog[2, 1 + (2*c^(1/4)*(1 - Sqrt[-Sqrt[-c]]*Sqrt[x]))/((I*Sqrt[-Sqrt[-c]]
```

```
- c^(1/4))*(1 - I*c^(1/4)*Sqrt[x]))])/(c^(3/4)*Sqrt[x]) + ((I/3)*b^2*Sqrt[d
*x]*PolyLog[2, 1 - (2*c^(1/4)*(1 + Sqrt[-Sqrt[-c]]*Sqrt[x]))/((I*Sqrt[-Sqrt
[-c]] + c^(1/4))*(1 - I*c^(1/4)*Sqrt[x]))])/(c^(3/4)*Sqrt[x]) + ((I/3)*b^2*
Sqrt[d*x]*PolyLog[2, 1 + (2*c^(1/4)*(1 - (-c)^(1/4)*Sqrt[x]))/((I*(-c)^(1/4
) - c^(1/4))*(1 - I*c^(1/4)*Sqrt[x]))])/(c^(3/4)*Sqrt[x]) + ((I/3)*b^2*Sqrt
[d*x]*PolyLog[2, 1 - (2*c^(1/4)*(1 + (-c)^(1/4)*Sqrt[x]))/((I*(-c)^(1/4) +
c^(1/4))*(1 - I*c^(1/4)*Sqrt[x]))])/(c^(3/4)*Sqrt[x]) - ((I/3)*b^2*Sqrt[d*x
]*PolyLog[2, 1 - ((1 + I)*(1 - c^(1/4)*Sqrt[x]))/(1 - I*c^(1/4)*Sqrt[x])])/
(c^(3/4)*Sqrt[x]) - (((2*I)/3)*b^2*Sqrt[d*x]*PolyLog[2, 1 - 2/(1 + I*c^(1/4
)*Sqrt[x])])/(c^(3/4)*Sqrt[x]) + (2*b^2*Sqrt[d*x]*PolyLog[2, 1 - 2/(1 + c^(
1/4)*Sqrt[x])])/(3*c^(3/4)*Sqrt[x]) - (b^2*Sqrt[d*x]*PolyLog[2, 1 + (2*c^(1
/4)*(1 - Sqrt[-Sqrt[-c]]*Sqrt[x]))/((Sqrt[-Sqrt[-c]] - c^(1/4))*(1 + c^(1/4
)*Sqrt[x]))])/(3*c^(3/4)*Sqrt[x]) - (b^2*Sqrt[d*x]*PolyLog[2, 1 - (2*c^(1/4
)*(1 + Sqrt[-Sqrt[-c]]*Sqrt[x]))/((Sqrt[-Sqrt[-c]] + c^(1/4))*(1 + c^(1/4)*
Sqrt[x]))])/(3*c^(3/4)*Sqrt[x]) + (b^2*Sqrt[d*x]*PolyLog[2, 1 + (2*c^(1/4)*
(1 - Sqrt[-Sqrt[c]]*Sqrt[x]))/((Sqrt[-Sqrt[c]] - c^(1/4))*(1 + c^(1/4)*Sqrt
[x]))])/(3*c^(3/4)*Sqrt[x]) + (b^2*Sqrt[d*x]*PolyLog[2, 1 - (2*c^(1/4)*(1 +
 Sqrt[-Sqrt[c]]*Sqrt[x]))/((Sqrt[-Sqrt[c]] + c^(1/4))*(1 + c^(1/4)*Sqrt[x])
)])/(3*c^(3/4)*Sqrt[x]) - (b^2*Sqrt[d*x]*PolyLog[2, 1 + (2*c^(1/4)*(1 - (-c
)^(1/4)*Sqrt[x]))/(((-c)^(1/4) - c^(1/4))*(1 + c^(1/4)*Sqrt[x]))])/(3*c^(3/
4)*Sqrt[x]) - (b^2*Sqrt[d*x]*PolyLog[2, 1 - (2*c^(1/4)*(1 + (-c)^(1/4)*Sqrt
[x]))/(((-c)^(1/4) + c^(1/4))*(1 + c^(1/4)*Sqrt[x]))])/(3*c^(3/4)*Sqrt[x])
+ ((I/3)*b^2*Sqrt[d*x]*PolyLog[2, 1 - (2*(-c)^(1/4)*(1 + c^(1/4)*Sqrt[x]))/
(((-c)^(1/4) + I*c^(1/4))*(1 - I*(-c)^(1/4)*Sqrt[x]))])/((-c)^(3/4)*Sqrt[x]
) - (b^2*Sqrt[d*x]*PolyLog[2, 1 - (2*(-c)^(1/4)*(1 + c^(1/4)*Sqrt[x]))/(((-
c)^(1/4) + c^(1/4))*(1 + (-c)^(1/4)*Sqrt[x]))])/(3*(-c)^(3/4)*Sqrt[x]) - ((
I/3)*b^2*Sqrt[d*x]*PolyLog[2, 1 - ((1 - I)*(1 + c^(1/4)*Sqrt[x]))/(1 - I*c^
(1/4)*Sqrt[x])])/(c^(3/4)*Sqrt[x])
```

Rubi [F] time = 0.0256037, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \sqrt{dx}\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2 dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[d*x]*(a + b*ArcTanh[c*x^2])^2,x]
```

```
[Out] Defer[Int][Sqrt[d*x]*(a + b*ArcTanh[c*x^2])^2, x]
```

Rubi steps

$$\int \sqrt{dx}\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2 dx=\int \sqrt{dx}\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2 dx$$

Mathematica [F] time = 61.9463, size = 0, normalized size = 0.

$$\int \sqrt{dx}\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2 dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[d*x]*(a + b*ArcTanh[c*x^2])^2,x]
```

```
[Out] Integrate[Sqrt[d*x]*(a + b*ArcTanh[c*x^2])^2, x]
```

Maple [F] time = 0.313, size = 0, normalized size = 0.

$$\int \sqrt{dx}\left(a+b\mathrm{Artanh}\left(cx^{2}\right)\right)^{2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)*(a+b*arctanh(c*x^2))^2,x)
```

```
[Out] int((d*x)^(1/2)*(a+b*arctanh(c*x^2))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*(a+b*arctanh(c*x^2))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b^{2}\,\mathrm{artanh}\left(cx^{2}\right)^{2}+2\,ab\,\mathrm{artanh}\left(cx^{2}\right)+a^{2}\right)\sqrt{dx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*(a+b*arctanh(c*x^2))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x^2)^2 + 2*a*b*arctanh(c*x^2) + a^2)*sqrt(d*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)*(a+b*atanh(c*x**2))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dx}\left(b\,\mathrm{artanh}\left(cx^{2}\right)+a\right)^{2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*(a+b*arctanh(c*x^2))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x)*(b*arctanh(c*x^2) + a)^2, x)
```

3.91 $\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{\sqrt{dx}}\,dx$

Optimal. Leaf size=6177

result too large to display

```
[Out] (2*a^2*x)/Sqrt[d*x] - (2*Sqrt[2]*a*b*Sqrt[x]*ArcTan[1 - Sqrt[2]*c^(1/4)*Sqr
t[x]])/(c^(1/4)*Sqrt[d*x]) + (2*Sqrt[2]*a*b*Sqrt[x]*ArcTan[1 + Sqrt[2]*c^(1
/4)*Sqrt[x]])/(c^(1/4)*Sqrt[d*x]) + ((2*I)*b^2*Sqrt[x]*ArcTan[(-c)^(1/4)*Sq
rt[x]]^2)/((-c)^(1/4)*Sqrt[d*x]) - (4*a*b*Sqrt[x]*ArcTan[c^(1/4)*Sqrt[x]])/
(c^(1/4)*Sqrt[d*x]) + ((2*I)*b^2*Sqrt[x]*ArcTan[c^(1/4)*Sqrt[x]]^2)/(c^(1/4
)*Sqrt[d*x]) - (2*b^2*Sqrt[x]*ArcTanh[(-c)^(1/4)*Sqrt[x]]^2)/((-c)^(1/4)*Sq
rt[d*x]) - (4*a*b*Sqrt[x]*ArcTanh[c^(1/4)*Sqrt[x]])/(c^(1/4)*Sqrt[d*x]) - (
2*b^2*Sqrt[x]*ArcTanh[c^(1/4)*Sqrt[x]]^2)/(c^(1/4)*Sqrt[d*x]) + (4*b^2*Sqrt
[x]*ArcTanh[(-c)^(1/4)*Sqrt[x]]*Log[2/(1 - (-c)^(1/4)*Sqrt[x])])/((-c)^(1/4
)*Sqrt[d*x]) - (4*b^2*Sqrt[x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[2/(1 - I*(-c)^
(1/4)*Sqrt[x])])/((-c)^(1/4)*Sqrt[d*x]) + (2*b^2*Sqrt[x]*ArcTan[(-c)^(1/4)*
Sqrt[x]]*Log[(-2*(-c)^(1/4)*(1 - Sqrt[-Sqrt[c]]*Sqrt[x]))/((I*Sqrt[-Sqrt[c]
] - (-c)^(1/4))*(1 - I*(-c)^(1/4)*Sqrt[x]))])/((-c)^(1/4)*Sqrt[d*x]) + (2*b
^2*Sqrt[x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[(2*(-c)^(1/4)*(1 + Sqrt[-Sqrt[c]]
*Sqrt[x]))/((I*Sqrt[-Sqrt[c]] + (-c)^(1/4))*(1 - I*(-c)^(1/4)*Sqrt[x]))])/(
(-c)^(1/4)*Sqrt[d*x]) - (2*b^2*Sqrt[x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[((1 +
 I)*(1 - (-c)^(1/4)*Sqrt[x]))/(1 - I*(-c)^(1/4)*Sqrt[x])])/((-c)^(1/4)*Sqrt
[d*x]) + (4*b^2*Sqrt[x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[2/(1 + I*(-c)^(1/4)*
Sqrt[x])])/((-c)^(1/4)*Sqrt[d*x]) - (4*b^2*Sqrt[x]*ArcTanh[(-c)^(1/4)*Sqrt[
x]]*Log[2/(1 + (-c)^(1/4)*Sqrt[x])])/((-c)^(1/4)*Sqrt[d*x]) - (2*b^2*Sqrt[x
]*ArcTanh[(-c)^(1/4)*Sqrt[x]]*Log[(-2*(-c)^(1/4)*(1 - Sqrt[-Sqrt[-c]]*Sqrt[
x]))/((Sqrt[-Sqrt[-c]] - (-c)^(1/4))*(1 + (-c)^(1/4)*Sqrt[x]))])/((-c)^(1/4
)*Sqrt[d*x]) - (2*b^2*Sqrt[x]*ArcTanh[(-c)^(1/4)*Sqrt[x]]*Log[(2*(-c)^(1/4)
*(1 + Sqrt[-Sqrt[-c]]*Sqrt[x]))/((Sqrt[-Sqrt[-c]] + (-c)^(1/4))*(1 + (-c)^(
1/4)*Sqrt[x]))])/((-c)^(1/4)*Sqrt[d*x]) + (2*b^2*Sqrt[x]*ArcTanh[(-c)^(1/4)
*Sqrt[x]]*Log[(-2*(-c)^(1/4)*(1 - Sqrt[-Sqrt[c]]*Sqrt[x]))/((Sqrt[-Sqrt[c]]
 - (-c)^(1/4))*(1 + (-c)^(1/4)*Sqrt[x]))])/((-c)^(1/4)*Sqrt[d*x]) + (2*b^2*
Sqrt[x]*ArcTanh[(-c)^(1/4)*Sqrt[x]]*Log[(2*(-c)^(1/4)*(1 + Sqrt[-Sqrt[c]]*S
qrt[x]))/((Sqrt[-Sqrt[c]] + (-c)^(1/4))*(1 + (-c)^(1/4)*Sqrt[x]))])/((-c)^(
1/4)*Sqrt[d*x]) - (2*b^2*Sqrt[x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[((1 - I)*(1
 + (-c)^(1/4)*Sqrt[x]))/(1 - I*(-c)^(1/4)*Sqrt[x])])/((-c)^(1/4)*Sqrt[d*x])
 + (4*b^2*Sqrt[x]*ArcTanh[c^(1/4)*Sqrt[x]]*Log[2/(1 - c^(1/4)*Sqrt[x])])/(c
^(1/4)*Sqrt[d*x]) + (2*b^2*Sqrt[x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[(2*(-c)^(
1/4)*(1 - c^(1/4)*Sqrt[x]))/(((-c)^(1/4) - I*c^(1/4))*(1 - I*(-c)^(1/4)*Sqr
t[x]))])/((-c)^(1/4)*Sqrt[d*x]) + (2*b^2*Sqrt[x]*ArcTanh[(-c)^(1/4)*Sqrt[x]
]*Log[(2*(-c)^(1/4)*(1 - c^(1/4)*Sqrt[x]))/(((-c)^(1/4) - c^(1/4))*(1 + (-c
)^(1/4)*Sqrt[x]))])/((-c)^(1/4)*Sqrt[d*x]) - (4*b^2*Sqrt[x]*ArcTan[c^(1/4)*
Sqrt[x]]*Log[2/(1 - I*c^(1/4)*Sqrt[x])])/(c^(1/4)*Sqrt[d*x]) + (2*b^2*Sqrt[
x]*ArcTan[c^(1/4)*Sqrt[x]]*Log[(-2*c^(1/4)*(1 - Sqrt[-Sqrt[-c]]*Sqrt[x]))/(
(I*Sqrt[-Sqrt[-c]] - c^(1/4))*(1 - I*c^(1/4)*Sqrt[x]))])/(c^(1/4)*Sqrt[d*x]
) + (2*b^2*Sqrt[x]*ArcTan[c^(1/4)*Sqrt[x]]*Log[(2*c^(1/4)*(1 + Sqrt[-Sqrt[-
c]]*Sqrt[x]))/((I*Sqrt[-Sqrt[-c]] + c^(1/4))*(1 - I*c^(1/4)*Sqrt[x]))])/(c^
(1/4)*Sqrt[d*x]) + (2*b^2*Sqrt[x]*ArcTan[c^(1/4)*Sqrt[x]]*Log[(-2*c^(1/4)*(
1 - (-c)^(1/4)*Sqrt[x]))/((I*(-c)^(1/4) - c^(1/4))*(1 - I*c^(1/4)*Sqrt[x]))
])/(c^(1/4)*Sqrt[d*x]) + (2*b^2*Sqrt[x]*ArcTan[c^(1/4)*Sqrt[x]]*Log[(2*c^(1
/4)*(1 + (-c)^(1/4)*Sqrt[x]))/((I*(-c)^(1/4) + c^(1/4))*(1 - I*c^(1/4)*Sqrt
[x]))])/(c^(1/4)*Sqrt[d*x]) - (2*b^2*Sqrt[x]*ArcTan[c^(1/4)*Sqrt[x]]*Log[((
1 + I)*(1 - c^(1/4)*Sqrt[x]))/(1 - I*c^(1/4)*Sqrt[x])])/(c^(1/4)*Sqrt[d*x])
 + (4*b^2*Sqrt[x]*ArcTan[c^(1/4)*Sqrt[x]]*Log[2/(1 + I*c^(1/4)*Sqrt[x])])/(
c^(1/4)*Sqrt[d*x]) - (4*b^2*Sqrt[x]*ArcTanh[c^(1/4)*Sqrt[x]]*Log[2/(1 + c^(
1/4)*Sqrt[x])])/(c^(1/4)*Sqrt[d*x]) + (2*b^2*Sqrt[x]*ArcTanh[c^(1/4)*Sqrt[x
```

```
]]*Log[(-2*c^(1/4)*(1 - Sqrt[-Sqrt[-c]]*Sqrt[x]))/((Sqrt[-Sqrt[-c]] - c^(1/
4))*(1 + c^(1/4)*Sqrt[x]))])/(c^(1/4)*Sqrt[d*x]) + (2*b^2*Sqrt[x]*ArcTanh[c
^(1/4)*Sqrt[x]]*Log[(2*c^(1/4)*(1 + Sqrt[-Sqrt[-c]]*Sqrt[x]))/((Sqrt[-Sqrt[
-c]] + c^(1/4))*(1 + c^(1/4)*Sqrt[x]))])/(c^(1/4)*Sqrt[d*x]) - (2*b^2*Sqrt[
x]*ArcTanh[c^(1/4)*Sqrt[x]]*Log[(-2*c^(1/4)*(1 - Sqrt[-Sqrt[c]]*Sqrt[x]))/(
(Sqrt[-Sqrt[c]] - c^(1/4))*(1 + c^(1/4)*Sqrt[x]))])/(c^(1/4)*Sqrt[d*x]) - (
2*b^2*Sqrt[x]*ArcTanh[c^(1/4)*Sqrt[x]]*Log[(2*c^(1/4)*(1 + Sqrt[-Sqrt[c]]*S
qrt[x]))/((Sqrt[-Sqrt[c]] + c^(1/4))*(1 + c^(1/4)*Sqrt[x]))])/(c^(1/4)*Sqrt
[d*x]) + (2*b^2*Sqrt[x]*ArcTanh[c^(1/4)*Sqrt[x]]*Log[(-2*c^(1/4)*(1 - (-c)^
(1/4)*Sqrt[x]))/(((-c)^(1/4) - c^(1/4))*(1 + c^(1/4)*Sqrt[x]))])/(c^(1/4)*S
qrt[d*x]) + (2*b^2*Sqrt[x]*ArcTanh[c^(1/4)*Sqrt[x]]*Log[(2*c^(1/4)*(1 + (-c
)^(1/4)*Sqrt[x]))/(((-c)^(1/4) + c^(1/4))*(1 + c^(1/4)*Sqrt[x]))])/(c^(1/4)
*Sqrt[d*x]) + (2*b^2*Sqrt[x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[(2*(-c)^(1/4)*(
1 + c^(1/4)*Sqrt[x]))/(((-c)^(1/4) + I*c^(1/4))*(1 - I*(-c)^(1/4)*Sqrt[x]))
])/((-c)^(1/4)*Sqrt[d*x]) + (2*b^2*Sqrt[x]*ArcTanh[(-c)^(1/4)*Sqrt[x]]*Log[
(2*(-c)^(1/4)*(1 + c^(1/4)*Sqrt[x]))/(((-c)^(1/4) + c^(1/4))*(1 + (-c)^(1/4
)*Sqrt[x]))])/((-c)^(1/4)*Sqrt[d*x]) - (2*b^2*Sqrt[x]*ArcTan[c^(1/4)*Sqrt[x
]]*Log[((1 - I)*(1 + c^(1/4)*Sqrt[x]))/(1 - I*c^(1/4)*Sqrt[x])])/(c^(1/4)*S
qrt[d*x]) - (Sqrt[2]*a*b*Sqrt[x]*Log[1 - Sqrt[2]*c^(1/4)*Sqrt[x] + Sqrt[c]*
x])/(c^(1/4)*Sqrt[d*x]) + (Sqrt[2]*a*b*Sqrt[x]*Log[1 + Sqrt[2]*c^(1/4)*Sqrt
[x] + Sqrt[c]*x])/(c^(1/4)*Sqrt[d*x]) - (2*a*b*x*Log[1 - c*x^2])/Sqrt[d*x]
- (2*b^2*Sqrt[x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[1 - c*x^2])/((-c)^(1/4)*Sqr
t[d*x]) + (2*b^2*Sqrt[x]*ArcTan[c^(1/4)*Sqrt[x]]*Log[1 - c*x^2])/(c^(1/4)*S
qrt[d*x]) - (2*b^2*Sqrt[x]*ArcTanh[(-c)^(1/4)*Sqrt[x]]*Log[1 - c*x^2])/((-c
)^(1/4)*Sqrt[d*x]) + (2*b^2*Sqrt[x]*ArcTanh[c^(1/4)*Sqrt[x]]*Log[1 - c*x^2]
)/(c^(1/4)*Sqrt[d*x]) + (b^2*x*Log[1 - c*x^2]^2)/(2*Sqrt[d*x]) + (2*a*b*x*L
og[1 + c*x^2])/Sqrt[d*x] + (2*b^2*Sqrt[x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[1
+ c*x^2])/((-c)^(1/4)*Sqrt[d*x]) - (2*b^2*Sqrt[x]*ArcTan[c^(1/4)*Sqrt[x]]*L
og[1 + c*x^2])/(c^(1/4)*Sqrt[d*x]) + (2*b^2*Sqrt[x]*ArcTanh[(-c)^(1/4)*Sqrt
[x]]*Log[1 + c*x^2])/((-c)^(1/4)*Sqrt[d*x]) - (2*b^2*Sqrt[x]*ArcTanh[c^(1/4
)*Sqrt[x]]*Log[1 + c*x^2])/(c^(1/4)*Sqrt[d*x]) - (b^2*x*Log[1 - c*x^2]*Log[
1 + c*x^2])/Sqrt[d*x] + (b^2*x*Log[1 + c*x^2]^2)/(2*Sqrt[d*x]) + (2*b^2*Sqr
t[x]*PolyLog[2, 1 - 2/(1 - (-c)^(1/4)*Sqrt[x])])/((-c)^(1/4)*Sqrt[d*x]) + (
(2*I)*b^2*Sqrt[x]*PolyLog[2, 1 - 2/(1 - I*(-c)^(1/4)*Sqrt[x])])/((-c)^(1/4)
*Sqrt[d*x]) - (I*b^2*Sqrt[x]*PolyLog[2, 1 + (2*(-c)^(1/4)*(1 - Sqrt[-Sqrt[c
]]*Sqrt[x]))/((I*Sqrt[-Sqrt[c]] - (-c)^(1/4))*(1 - I*(-c)^(1/4)*Sqrt[x]))])
/((-c)^(1/4)*Sqrt[d*x]) - (I*b^2*Sqrt[x]*PolyLog[2, 1 - (2*(-c)^(1/4)*(1 +
Sqrt[-Sqrt[c]]*Sqrt[x]))/((I*Sqrt[-Sqrt[c]] + (-c)^(1/4))*(1 - I*(-c)^(1/4)
*Sqrt[x]))])/((-c)^(1/4)*Sqrt[d*x]) + (I*b^2*Sqrt[x]*PolyLog[2, 1 - ((1 + I
)*(1 - (-c)^(1/4)*Sqrt[x]))/(1 - I*(-c)^(1/4)*Sqrt[x])])/((-c)^(1/4)*Sqrt[d
*x]) + ((2*I)*b^2*Sqrt[x]*PolyLog[2, 1 - 2/(1 + I*(-c)^(1/4)*Sqrt[x])])/((-
c)^(1/4)*Sqrt[d*x]) + (2*b^2*Sqrt[x]*PolyLog[2, 1 - 2/(1 + (-c)^(1/4)*Sqrt[
x])])/((-c)^(1/4)*Sqrt[d*x]) + (b^2*Sqrt[x]*PolyLog[2, 1 + (2*(-c)^(1/4)*(1
 - Sqrt[-Sqrt[-c]]*Sqrt[x]))/((Sqrt[-Sqrt[-c]] - (-c)^(1/4))*(1 + (-c)^(1/4
)*Sqrt[x]))])/((-c)^(1/4)*Sqrt[d*x]) + (b^2*Sqrt[x]*PolyLog[2, 1 - (2*(-c)^
(1/4)*(1 + Sqrt[-Sqrt[-c]]*Sqrt[x]))/((Sqrt[-Sqrt[-c]] + (-c)^(1/4))*(1 + (
-c)^(1/4)*Sqrt[x]))])/((-c)^(1/4)*Sqrt[d*x]) - (b^2*Sqrt[x]*PolyLog[2, 1 +
(2*(-c)^(1/4)*(1 - Sqrt[-Sqrt[c]]*Sqrt[x]))/((Sqrt[-Sqrt[c]] - (-c)^(1/4))*
(1 + (-c)^(1/4)*Sqrt[x]))])/((-c)^(1/4)*Sqrt[d*x]) - (b^2*Sqrt[x]*PolyLog[2
, 1 - (2*(-c)^(1/4)*(1 + Sqrt[-Sqrt[c]]*Sqrt[x]))/((Sqrt[-Sqrt[c]] + (-c)^(
1/4))*(1 + (-c)^(1/4)*Sqrt[x]))])/((-c)^(1/4)*Sqrt[d*x]) + (I*b^2*Sqrt[x]*P
olyLog[2, 1 - ((1 - I)*(1 + (-c)^(1/4)*Sqrt[x]))/(1 - I*(-c)^(1/4)*Sqrt[x])
])/((-c)^(1/4)*Sqrt[d*x]) + (2*b^2*Sqrt[x]*PolyLog[2, 1 - 2/(1 - c^(1/4)*Sq
rt[x])])/(c^(1/4)*Sqrt[d*x]) - (I*b^2*Sqrt[x]*PolyLog[2, 1 - (2*(-c)^(1/4)*
(1 - c^(1/4)*Sqrt[x]))/(((-c)^(1/4) - I*c^(1/4))*(1 - I*(-c)^(1/4)*Sqrt[x])
)])/((-c)^(1/4)*Sqrt[d*x]) - (b^2*Sqrt[x]*PolyLog[2, 1 - (2*(-c)^(1/4)*(1 -
 c^(1/4)*Sqrt[x]))/(((-c)^(1/4) - c^(1/4))*(1 + (-c)^(1/4)*Sqrt[x]))])/((-c
)^(1/4)*Sqrt[d*x]) + ((2*I)*b^2*Sqrt[x]*PolyLog[2, 1 - 2/(1 - I*c^(1/4)*Sqr
t[x])])/(c^(1/4)*Sqrt[d*x]) - (I*b^2*Sqrt[x]*PolyLog[2, 1 + (2*c^(1/4)*(1 -
```

```
 Sqrt[-Sqrt[-c]]*Sqrt[x]))/((I*Sqrt[-Sqrt[-c]] - c^(1/4))*(1 - I*c^(1/4)*Sq
rt[x]))])/(c^(1/4)*Sqrt[d*x]) - (I*b^2*Sqrt[x]*PolyLog[2, 1 - (2*c^(1/4)*(1
 + Sqrt[-Sqrt[-c]]*Sqrt[x]))/((I*Sqrt[-Sqrt[-c]] + c^(1/4))*(1 - I*c^(1/4)*
Sqrt[x]))])/(c^(1/4)*Sqrt[d*x]) - (I*b^2*Sqrt[x]*PolyLog[2, 1 + (2*c^(1/4)*
(1 - (-c)^(1/4)*Sqrt[x]))/((I*(-c)^(1/4) - c^(1/4))*(1 - I*c^(1/4)*Sqrt[x])
)])/(c^(1/4)*Sqrt[d*x]) - (I*b^2*Sqrt[x]*PolyLog[2, 1 - (2*c^(1/4)*(1 + (-c
)^(1/4)*Sqrt[x]))/((I*(-c)^(1/4) + c^(1/4))*(1 - I*c^(1/4)*Sqrt[x]))])/(c^(
1/4)*Sqrt[d*x]) + (I*b^2*Sqrt[x]*PolyLog[2, 1 - ((1 + I)*(1 - c^(1/4)*Sqrt[
x]))/(1 - I*c^(1/4)*Sqrt[x])])/(c^(1/4)*Sqrt[d*x]) + ((2*I)*b^2*Sqrt[x]*Pol
yLog[2, 1 - 2/(1 + I*c^(1/4)*Sqrt[x])])/(c^(1/4)*Sqrt[d*x]) + (2*b^2*Sqrt[x
]*PolyLog[2, 1 - 2/(1 + c^(1/4)*Sqrt[x])])/(c^(1/4)*Sqrt[d*x]) - (b^2*Sqrt[
x]*PolyLog[2, 1 + (2*c^(1/4)*(1 - Sqrt[-Sqrt[-c]]*Sqrt[x]))/((Sqrt[-Sqrt[-c
]] - c^(1/4))*(1 + c^(1/4)*Sqrt[x]))])/(c^(1/4)*Sqrt[d*x]) - (b^2*Sqrt[x]*P
olyLog[2, 1 - (2*c^(1/4)*(1 + Sqrt[-Sqrt[-c]]*Sqrt[x]))/((Sqrt[-Sqrt[-c]] +
 c^(1/4))*(1 + c^(1/4)*Sqrt[x]))])/(c^(1/4)*Sqrt[d*x]) + (b^2*Sqrt[x]*PolyL
og[2, 1 + (2*c^(1/4)*(1 - Sqrt[-Sqrt[c]]*Sqrt[x]))/((Sqrt[-Sqrt[c]] - c^(1/
4))*(1 + c^(1/4)*Sqrt[x]))])/(c^(1/4)*Sqrt[d*x]) + (b^2*Sqrt[x]*PolyLog[2,
1 - (2*c^(1/4)*(1 + Sqrt[-Sqrt[c]]*Sqrt[x]))/((Sqrt[-Sqrt[c]] + c^(1/4))*(1
 + c^(1/4)*Sqrt[x]))])/(c^(1/4)*Sqrt[d*x]) - (b^2*Sqrt[x]*PolyLog[2, 1 + (2
*c^(1/4)*(1 - (-c)^(1/4)*Sqrt[x]))/(((-c)^(1/4) - c^(1/4))*(1 + c^(1/4)*Sqr
t[x]))])/(c^(1/4)*Sqrt[d*x]) - (b^2*Sqrt[x]*PolyLog[2, 1 - (2*c^(1/4)*(1 +
(-c)^(1/4)*Sqrt[x]))/(((-c)^(1/4) + c^(1/4))*(1 + c^(1/4)*Sqrt[x]))])/(c^(1
/4)*Sqrt[d*x]) - (I*b^2*Sqrt[x]*PolyLog[2, 1 - (2*(-c)^(1/4)*(1 + c^(1/4)*S
qrt[x]))/(((-c)^(1/4) + I*c^(1/4))*(1 - I*(-c)^(1/4)*Sqrt[x]))])/((-c)^(1/4
)*Sqrt[d*x]) - (b^2*Sqrt[x]*PolyLog[2, 1 - (2*(-c)^(1/4)*(1 + c^(1/4)*Sqrt[
x]))/(((-c)^(1/4) + c^(1/4))*(1 + (-c)^(1/4)*Sqrt[x]))])/((-c)^(1/4)*Sqrt[d
*x]) + (I*b^2*Sqrt[x]*PolyLog[2, 1 - ((1 - I)*(1 + c^(1/4)*Sqrt[x]))/(1 - I
*c^(1/4)*Sqrt[x])])/(c^(1/4)*Sqrt[d*x])
```

Rubi [F] time = 0.025645, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\left(a + b\tanh^{-1}\left(cx^2\right)\right)^2}{\sqrt{dx}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcTanh[c*x^2])^2/Sqrt[d*x],x]

[Out] Defer[Int][(a + b*ArcTanh[c*x^2])^2/Sqrt[d*x], x]
```

Rubi steps

$$\int \frac{\left(a + b\tanh^{-1}\left(cx^2\right)\right)^2}{\sqrt{dx}}\,dx = \int \frac{\left(a + b\tanh^{-1}\left(cx^2\right)\right)^2}{\sqrt{dx}}\,dx$$

Mathematica [F] time = 59.3506, size = 0, normalized size = 0.

$$\int \frac{\left(a + b\tanh^{-1}\left(cx^2\right)\right)^2}{\sqrt{dx}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])^2/Sqrt[d*x],x]
```

```
[Out] Integrate[(a + b*ArcTanh[c*x^2])^2/Sqrt[d*x], x]
```

Maple [F] time = 0.213, size = 0, normalized size = 0.

$$\int \left(a + b\text{Artanh}\left(cx^2\right)\right)^2 \frac{1}{\sqrt{dx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))^2/(d*x)^(1/2),x)
```

```
[Out] int((a+b*arctanh(c*x^2))^2/(d*x)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2\operatorname{artanh}\left(cx^2\right)^2 + 2\,ab\operatorname{artanh}\left(cx^2\right) + a^2\right)\sqrt{dx}}{dx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x^2)^2 + 2*a*b*arctanh(c*x^2) + a^2)*sqrt(d*x)/(d*x
), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + b\operatorname{atanh}\left(cx^2\right)\right)^2}{\sqrt{dx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))**2/(d*x)**(1/2),x)
```

```
[Out] Integral((a + b*atanh(c*x**2))**2/sqrt(d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\,\text{artanh}\left(cx^2\right)+a\right)^2}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^2) + a)^2/sqrt(d*x), x)
```

3.92 $\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{(dx)^{3/2}}\,dx$

Optimal. Leaf size=6334

result too large to display

```
[Out] (-2*Sqrt[2]*a*b*c^(1/4)*Sqrt[x]*ArcTan[1 - Sqrt[2]*c^(1/4)*Sqrt[x]])/(d*Sqr
t[d*x]) + (2*Sqrt[2]*a*b*c^(1/4)*Sqrt[x]*ArcTan[1 + Sqrt[2]*c^(1/4)*Sqrt[x]
])/(d*Sqrt[d*x]) + ((2*I)*b^2*(-c)^(1/4)*Sqrt[x]*ArcTan[(-c)^(1/4)*Sqrt[x]]
^2)/(d*Sqrt[d*x]) + ((2*I)*b^2*c^(1/4)*Sqrt[x]*ArcTan[c^(1/4)*Sqrt[x]]^2)/(
d*Sqrt[d*x]) + (2*b^2*(-c)^(1/4)*Sqrt[x]*ArcTanh[(-c)^(1/4)*Sqrt[x]]^2)/(d*
Sqrt[d*x]) + (2*b^2*c^(1/4)*Sqrt[x]*ArcTanh[c^(1/4)*Sqrt[x]]^2)/(d*Sqrt[d*x
]) - (4*b^2*(-c)^(1/4)*Sqrt[x]*ArcTanh[(-c)^(1/4)*Sqrt[x]]*Log[2/(1 - (-c)^
(1/4)*Sqrt[x])])/(d*Sqrt[d*x]) - (4*b^2*(-c)^(1/4)*Sqrt[x]*ArcTan[(-c)^(1/4
)*Sqrt[x]]*Log[2/(1 - I*(-c)^(1/4)*Sqrt[x])])/(d*Sqrt[d*x]) + (2*b^2*(-c)^(
1/4)*Sqrt[x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[(-2*(-c)^(1/4)*(1 - Sqrt[-Sqrt[
c]]*Sqrt[x]))/((I*Sqrt[-Sqrt[c]] - (-c)^(1/4))*(1 - I*(-c)^(1/4)*Sqrt[x]))]
)/(d*Sqrt[d*x]) + (2*b^2*(-c)^(1/4)*Sqrt[x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[
(2*(-c)^(1/4)*(1 + Sqrt[-Sqrt[c]]*Sqrt[x]))/((I*Sqrt[-Sqrt[c]] + (-c)^(1/4)
)*(1 - I*(-c)^(1/4)*Sqrt[x]))])/(d*Sqrt[d*x]) - (2*b^2*(-c)^(1/4)*Sqrt[x]*A
rcTan[(-c)^(1/4)*Sqrt[x]]*Log[((1 + I)*(1 - (-c)^(1/4)*Sqrt[x]))/(1 - I*(-c
)^(1/4)*Sqrt[x])])/(d*Sqrt[d*x]) + (4*b^2*(-c)^(1/4)*Sqrt[x]*ArcTan[(-c)^(1
/4)*Sqrt[x]]*Log[2/(1 + I*(-c)^(1/4)*Sqrt[x])])/(d*Sqrt[d*x]) + (4*b^2*(-c)
^(1/4)*Sqrt[x]*ArcTanh[(-c)^(1/4)*Sqrt[x]]*Log[2/(1 + (-c)^(1/4)*Sqrt[x])])
/(d*Sqrt[d*x]) + (2*b^2*(-c)^(1/4)*Sqrt[x]*ArcTanh[(-c)^(1/4)*Sqrt[x]]*Log[
(-2*(-c)^(1/4)*(1 - Sqrt[-Sqrt[-c]]*Sqrt[x]))/((Sqrt[-Sqrt[-c]] - (-c)^(1/4
))*(1 + (-c)^(1/4)*Sqrt[x]))])/(d*Sqrt[d*x]) + (2*b^2*(-c)^(1/4)*Sqrt[x]*Ar
cTanh[(-c)^(1/4)*Sqrt[x]]*Log[(2*(-c)^(1/4)*(1 + Sqrt[-Sqrt[-c]]*Sqrt[x]))/
((Sqrt[-Sqrt[-c]] + (-c)^(1/4))*(1 + (-c)^(1/4)*Sqrt[x]))])/(d*Sqrt[d*x]) -
 (2*b^2*(-c)^(1/4)*Sqrt[x]*ArcTanh[(-c)^(1/4)*Sqrt[x]]*Log[(-2*(-c)^(1/4)*(
1 - Sqrt[-Sqrt[c]]*Sqrt[x]))/((Sqrt[-Sqrt[c]] - (-c)^(1/4))*(1 + (-c)^(1/4)
*Sqrt[x]))])/(d*Sqrt[d*x]) - (2*b^2*(-c)^(1/4)*Sqrt[x]*ArcTanh[(-c)^(1/4)*S
qrt[x]]*Log[(2*(-c)^(1/4)*(1 + Sqrt[-Sqrt[c]]*Sqrt[x]))/((Sqrt[-Sqrt[c]] +
(-c)^(1/4))*(1 + (-c)^(1/4)*Sqrt[x]))])/(d*Sqrt[d*x]) - (2*b^2*(-c)^(1/4)*S
qrt[x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[((1 - I)*(1 + (-c)^(1/4)*Sqrt[x]))/(1
 - I*(-c)^(1/4)*Sqrt[x])])/(d*Sqrt[d*x]) - (4*b^2*c^(1/4)*Sqrt[x]*ArcTanh[c
^(1/4)*Sqrt[x]]*Log[2/(1 - c^(1/4)*Sqrt[x])])/(d*Sqrt[d*x]) + (2*b^2*(-c)^(
1/4)*Sqrt[x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[(2*(-c)^(1/4)*(1 - c^(1/4)*Sqrt
[x]))/(((-c)^(1/4) - I*c^(1/4))*(1 - I*(-c)^(1/4)*Sqrt[x]))])/(d*Sqrt[d*x])
 - (2*b^2*(-c)^(1/4)*Sqrt[x]*ArcTanh[(-c)^(1/4)*Sqrt[x]]*Log[(2*(-c)^(1/4)*
(1 - c^(1/4)*Sqrt[x]))/(((-c)^(1/4) - c^(1/4))*(1 + (-c)^(1/4)*Sqrt[x]))])/
(d*Sqrt[d*x]) - (4*b^2*c^(1/4)*Sqrt[x]*ArcTan[c^(1/4)*Sqrt[x]]*Log[2/(1 - I
*c^(1/4)*Sqrt[x])])/(d*Sqrt[d*x]) + (2*b^2*c^(1/4)*Sqrt[x]*ArcTan[c^(1/4)*S
qrt[x]]*Log[(-2*c^(1/4)*(1 - Sqrt[-Sqrt[-c]]*Sqrt[x]))/((I*Sqrt[-Sqrt[-c]]
 - c^(1/4))*(1 - I*c^(1/4)*Sqrt[x]))])/(d*Sqrt[d*x]) + (2*b^2*c^(1/4)*Sqrt[x
]*ArcTan[c^(1/4)*Sqrt[x]]*Log[(2*c^(1/4)*(1 + Sqrt[-Sqrt[-c]]*Sqrt[x]))/((I
*Sqrt[-Sqrt[-c]] + c^(1/4))*(1 - I*c^(1/4)*Sqrt[x]))])/(d*Sqrt[d*x]) + (2*b
^2*c^(1/4)*Sqrt[x]*ArcTan[c^(1/4)*Sqrt[x]]*Log[(-2*c^(1/4)*(1 - (-c)^(1/4)*
Sqrt[x]))/((I*(-c)^(1/4) - c^(1/4))*(1 - I*c^(1/4)*Sqrt[x]))])/(d*Sqrt[d*x]
) + (2*b^2*c^(1/4)*Sqrt[x]*ArcTan[c^(1/4)*Sqrt[x]]*Log[(2*c^(1/4)*(1 + (-c)
^(1/4)*Sqrt[x]))/((I*(-c)^(1/4) + c^(1/4))*(1 - I*c^(1/4)*Sqrt[x]))])/(d*Sq
rt[d*x]) - (2*b^2*c^(1/4)*Sqrt[x]*ArcTan[c^(1/4)*Sqrt[x]]*Log[((1 + I)*(1 -
 c^(1/4)*Sqrt[x]))/(1 - I*c^(1/4)*Sqrt[x])])/(d*Sqrt[d*x]) + (4*b^2*c^(1/4)
*Sqrt[x]*ArcTan[c^(1/4)*Sqrt[x]]*Log[2/(1 + I*c^(1/4)*Sqrt[x])])/(d*Sqrt[d*
x]) + (4*b^2*c^(1/4)*Sqrt[x]*ArcTanh[c^(1/4)*Sqrt[x]]*Log[2/(1 + c^(1/4)*Sq
rt[x])])/(d*Sqrt[d*x]) - (2*b^2*c^(1/4)*Sqrt[x]*ArcTanh[c^(1/4)*Sqrt[x]]*Lo
g[(-2*c^(1/4)*(1 - Sqrt[-Sqrt[-c]]*Sqrt[x]))/((Sqrt[-Sqrt[-c]] - c^(1/4))*(
```

```
1 + c^(1/4)*Sqrt[x]))])/(d*Sqrt[d*x]) - (2*b^2*c^(1/4)*Sqrt[x]*ArcTanh[c^(1
/4)*Sqrt[x]]*Log[(2*c^(1/4)*(1 + Sqrt[-Sqrt[-c]]*Sqrt[x]))/((Sqrt[-Sqrt[-c]
] + c^(1/4))*(1 + c^(1/4)*Sqrt[x]))])/(d*Sqrt[d*x]) + (2*b^2*c^(1/4)*Sqrt[x
]*ArcTanh[c^(1/4)*Sqrt[x]]*Log[(-2*c^(1/4)*(1 - Sqrt[-Sqrt[c]]*Sqrt[x]))/((
Sqrt[-Sqrt[c]] - c^(1/4))*(1 + c^(1/4)*Sqrt[x]))])/(d*Sqrt[d*x]) + (2*b^2*c
^(1/4)*Sqrt[x]*ArcTanh[c^(1/4)*Sqrt[x]]*Log[(2*c^(1/4)*(1 + Sqrt[-Sqrt[c]]*
Sqrt[x]))/((Sqrt[-Sqrt[c]] + c^(1/4))*(1 + c^(1/4)*Sqrt[x]))])/(d*Sqrt[d*x]
) - (2*b^2*c^(1/4)*Sqrt[x]*ArcTanh[c^(1/4)*Sqrt[x]]*Log[(-2*c^(1/4)*(1 - (-
c)^(1/4)*Sqrt[x]))/(((-c)^(1/4) - c^(1/4))*(1 + c^(1/4)*Sqrt[x]))])/(d*Sqrt
[d*x]) - (2*b^2*c^(1/4)*Sqrt[x]*ArcTanh[c^(1/4)*Sqrt[x]]*Log[(2*c^(1/4)*(1
+ (-c)^(1/4)*Sqrt[x]))/(((-c)^(1/4) + c^(1/4))*(1 + c^(1/4)*Sqrt[x]))])/(d*
Sqrt[d*x]) + (2*b^2*(-c)^(1/4)*Sqrt[x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[(2*(-
c)^(1/4)*(1 + c^(1/4)*Sqrt[x]))/(((-c)^(1/4) + I*c^(1/4))*(1 - I*(-c)^(1/4)
*Sqrt[x]))])/(d*Sqrt[d*x]) - (2*b^2*(-c)^(1/4)*Sqrt[x]*ArcTanh[(-c)^(1/4)*S
qrt[x]]*Log[(2*(-c)^(1/4)*(1 + c^(1/4)*Sqrt[x]))/(((-c)^(1/4) + c^(1/4))*(1
 + (-c)^(1/4)*Sqrt[x]))])/(d*Sqrt[d*x]) - (2*b^2*c^(1/4)*Sqrt[x]*ArcTan[c^(
1/4)*Sqrt[x]]*Log[((1 - I)*(1 + c^(1/4)*Sqrt[x]))/(1 - I*c^(1/4)*Sqrt[x])])
/(d*Sqrt[d*x]) + (Sqrt[2]*a*b*c^(1/4)*Sqrt[x]*Log[1 - Sqrt[2]*c^(1/4)*Sqrt[
x] + Sqrt[c]*x])/(d*Sqrt[d*x]) - (Sqrt[2]*a*b*c^(1/4)*Sqrt[x]*Log[1 + Sqrt[
2]*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(d*Sqrt[d*x]) - (2*b^2*(-c)^(1/4)*Sqrt[x]*
ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[1 - c*x^2])/(d*Sqrt[d*x]) + (2*b^2*(-c)^(1/4
)*Sqrt[x]*ArcTanh[(-c)^(1/4)*Sqrt[x]]*Log[1 - c*x^2])/(d*Sqrt[d*x]) - (2*b*
c^(1/4)*Sqrt[x]*ArcTan[c^(1/4)*Sqrt[x]]*(2*a - b*Log[1 - c*x^2]))/(d*Sqrt[d
*x]) + (2*b*c^(1/4)*Sqrt[x]*ArcTanh[c^(1/4)*Sqrt[x]]*(2*a - b*Log[1 - c*x^2
]))/(d*Sqrt[d*x]) - (2*a - b*Log[1 - c*x^2])^2/(2*d*Sqrt[d*x]) - (2*a*b*Log
[1 + c*x^2])/(d*Sqrt[d*x]) + (2*b^2*(-c)^(1/4)*Sqrt[x]*ArcTan[(-c)^(1/4)*Sq
rt[x]]*Log[1 + c*x^2])/(d*Sqrt[d*x]) - (2*b^2*c^(1/4)*Sqrt[x]*ArcTan[c^(1/4
)*Sqrt[x]]*Log[1 + c*x^2])/(d*Sqrt[d*x]) - (2*b^2*(-c)^(1/4)*Sqrt[x]*ArcTan
h[(-c)^(1/4)*Sqrt[x]]*Log[1 + c*x^2])/(d*Sqrt[d*x]) + (2*b^2*c^(1/4)*Sqrt[x
]*ArcTanh[c^(1/4)*Sqrt[x]]*Log[1 + c*x^2])/(d*Sqrt[d*x]) + (b^2*Log[1 - c*x
^2]*Log[1 + c*x^2])/(d*Sqrt[d*x]) - (b^2*Log[1 + c*x^2]^2)/(2*d*Sqrt[d*x])
- (2*b^2*(-c)^(1/4)*Sqrt[x]*PolyLog[2, 1 - 2/(1 - (-c)^(1/4)*Sqrt[x])])/(d*
Sqrt[d*x]) + ((2*I)*b^2*(-c)^(1/4)*Sqrt[x]*PolyLog[2, 1 - 2/(1 - I*(-c)^(1/
4)*Sqrt[x])])/(d*Sqrt[d*x]) - (I*b^2*(-c)^(1/4)*Sqrt[x]*PolyLog[2, 1 + (2*(
-c)^(1/4)*(1 - Sqrt[-Sqrt[c]]*Sqrt[x]))/((I*Sqrt[-Sqrt[c]] - (-c)^(1/4))*(1
 - I*(-c)^(1/4)*Sqrt[x]))])/(d*Sqrt[d*x]) - (I*b^2*(-c)^(1/4)*Sqrt[x]*PolyL
og[2, 1 - (2*(-c)^(1/4)*(1 + Sqrt[-Sqrt[c]]*Sqrt[x]))/((I*Sqrt[-Sqrt[c]] +
(-c)^(1/4))*(1 - I*(-c)^(1/4)*Sqrt[x]))])/(d*Sqrt[d*x]) + (I*b^2*(-c)^(1/4)
*Sqrt[x]*PolyLog[2, 1 - ((1 + I)*(1 - (-c)^(1/4)*Sqrt[x]))/(1 - I*(-c)^(1/4
)*Sqrt[x])])/(d*Sqrt[d*x]) + ((2*I)*b^2*(-c)^(1/4)*Sqrt[x]*PolyLog[2, 1 - 2
/(1 + I*(-c)^(1/4)*Sqrt[x])])/(d*Sqrt[d*x]) - (2*b^2*(-c)^(1/4)*Sqrt[x]*Pol
yLog[2, 1 - 2/(1 + (-c)^(1/4)*Sqrt[x])])/(d*Sqrt[d*x]) - (b^2*(-c)^(1/4)*Sq
rt[x]*PolyLog[2, 1 + (2*(-c)^(1/4)*(1 - Sqrt[-Sqrt[-c]]*Sqrt[x]))/((Sqrt[-S
qrt[-c]] - (-c)^(1/4))*(1 + (-c)^(1/4)*Sqrt[x]))])/(d*Sqrt[d*x]) - (b^2*(-c
)^(1/4)*Sqrt[x]*PolyLog[2, 1 - (2*(-c)^(1/4)*(1 + Sqrt[-Sqrt[-c]]*Sqrt[x]))
/((Sqrt[-Sqrt[-c]] + (-c)^(1/4))*(1 + (-c)^(1/4)*Sqrt[x]))])/(d*Sqrt[d*x])
+ (b^2*(-c)^(1/4)*Sqrt[x]*PolyLog[2, 1 + (2*(-c)^(1/4)*(1 - Sqrt[-Sqrt[c]]*
Sqrt[x]))/((Sqrt[-Sqrt[c]] - (-c)^(1/4))*(1 + (-c)^(1/4)*Sqrt[x]))])/(d*Sqr
t[d*x]) + (b^2*(-c)^(1/4)*Sqrt[x]*PolyLog[2, 1 - (2*(-c)^(1/4)*(1 + Sqrt[-S
qrt[c]]*Sqrt[x]))/((Sqrt[-Sqrt[c]] + (-c)^(1/4))*(1 + (-c)^(1/4)*Sqrt[x]))]
)/(d*Sqrt[d*x]) + (I*b^2*(-c)^(1/4)*Sqrt[x]*PolyLog[2, 1 - ((1 - I)*(1 + (-
c)^(1/4)*Sqrt[x]))/(1 - I*(-c)^(1/4)*Sqrt[x])])/(d*Sqrt[d*x]) - (2*b^2*c^(1
/4)*Sqrt[x]*PolyLog[2, 1 - 2/(1 - c^(1/4)*Sqrt[x])])/(d*Sqrt[d*x]) - (I*b^2
*(-c)^(1/4)*Sqrt[x]*PolyLog[2, 1 - (2*(-c)^(1/4)*(1 - c^(1/4)*Sqrt[x]))/(((
-c)^(1/4) - I*c^(1/4))*(1 - I*(-c)^(1/4)*Sqrt[x]))])/(d*Sqrt[d*x]) + (b^2*(
-c)^(1/4)*Sqrt[x]*PolyLog[2, 1 - (2*(-c)^(1/4)*(1 - c^(1/4)*Sqrt[x]))/(((-c
)^(1/4) - c^(1/4))*(1 + (-c)^(1/4)*Sqrt[x]))])/(d*Sqrt[d*x]) + ((2*I)*b^2*c
^(1/4)*Sqrt[x]*PolyLog[2, 1 - 2/(1 - I*c^(1/4)*Sqrt[x])])/(d*Sqrt[d*x]) - (
I*b^2*c^(1/4)*Sqrt[x]*PolyLog[2, 1 + (2*c^(1/4)*(1 - Sqrt[-Sqrt[-c]]*Sqrt[x
```

```
]))/((I*Sqrt[-Sqrt[-c]] - c^(1/4))*(1 - I*c^(1/4)*Sqrt[x]))])/(d*Sqrt[d*x])
 - (I*b^2*c^(1/4)*Sqrt[x]*PolyLog[2, 1 - (2*c^(1/4)*(1 + Sqrt[-Sqrt[-c]]*Sq
rt[x]))/((I*Sqrt[-Sqrt[-c]] + c^(1/4))*(1 - I*c^(1/4)*Sqrt[x]))])/(d*Sqrt[d
*x]) - (I*b^2*c^(1/4)*Sqrt[x]*PolyLog[2, 1 + (2*c^(1/4)*(1 - (-c)^(1/4)*Sqr
t[x]))/((I*(-c)^(1/4) - c^(1/4))*(1 - I*c^(1/4)*Sqrt[x]))])/(d*Sqrt[d*x]) -
 (I*b^2*c^(1/4)*Sqrt[x]*PolyLog[2, 1 - (2*c^(1/4)*(1 + (-c)^(1/4)*Sqrt[x]))
/((I*(-c)^(1/4) + c^(1/4))*(1 - I*c^(1/4)*Sqrt[x]))])/(d*Sqrt[d*x]) + (I*b^
2*c^(1/4)*Sqrt[x]*PolyLog[2, 1 - ((1 + I)*(1 - c^(1/4)*Sqrt[x]))/(1 - I*c^(
1/4)*Sqrt[x])])/(d*Sqrt[d*x]) + ((2*I)*b^2*c^(1/4)*Sqrt[x]*PolyLog[2, 1 - 2
/(1 + I*c^(1/4)*Sqrt[x])])/(d*Sqrt[d*x]) - (2*b^2*c^(1/4)*Sqrt[x]*PolyLog[2
, 1 - 2/(1 + c^(1/4)*Sqrt[x])])/(d*Sqrt[d*x]) + (b^2*c^(1/4)*Sqrt[x]*PolyLo
g[2, 1 + (2*c^(1/4)*(1 - Sqrt[-Sqrt[-c]]*Sqrt[x]))/((Sqrt[-Sqrt[-c]] - c^(1
/4))*(1 + c^(1/4)*Sqrt[x]))])/(d*Sqrt[d*x]) + (b^2*c^(1/4)*Sqrt[x]*PolyLog[
2, 1 - (2*c^(1/4)*(1 + Sqrt[-Sqrt[-c]]*Sqrt[x]))/((Sqrt[-Sqrt[-c]] + c^(1/4
))*(1 + c^(1/4)*Sqrt[x]))])/(d*Sqrt[d*x]) - (b^2*c^(1/4)*Sqrt[x]*PolyLog[2,
 1 + (2*c^(1/4)*(1 - Sqrt[-Sqrt[c]]*Sqrt[x]))/((Sqrt[-Sqrt[c]] - c^(1/4))*(
1 + c^(1/4)*Sqrt[x]))])/(d*Sqrt[d*x]) - (b^2*c^(1/4)*Sqrt[x]*PolyLog[2, 1 -
 (2*c^(1/4)*(1 + Sqrt[-Sqrt[c]]*Sqrt[x]))/((Sqrt[-Sqrt[c]] + c^(1/4))*(1 +
c^(1/4)*Sqrt[x]))])/(d*Sqrt[d*x]) + (b^2*c^(1/4)*Sqrt[x]*PolyLog[2, 1 + (2*
c^(1/4)*(1 - (-c)^(1/4)*Sqrt[x]))/(((-c)^(1/4) - c^(1/4))*(1 + c^(1/4)*Sqrt
[x]))])/(d*Sqrt[d*x]) + (b^2*c^(1/4)*Sqrt[x]*PolyLog[2, 1 - (2*c^(1/4)*(1 +
 (-c)^(1/4)*Sqrt[x]))/(((-c)^(1/4) + c^(1/4))*(1 + c^(1/4)*Sqrt[x]))])/(d*S
qrt[d*x]) - (I*b^2*(-c)^(1/4)*Sqrt[x]*PolyLog[2, 1 - (2*(-c)^(1/4)*(1 + c^(
1/4)*Sqrt[x]))/(((-c)^(1/4) + I*c^(1/4))*(1 - I*(-c)^(1/4)*Sqrt[x]))])/(d*S
qrt[d*x]) + (b^2*(-c)^(1/4)*Sqrt[x]*PolyLog[2, 1 - (2*(-c)^(1/4)*(1 + c^(1/
4)*Sqrt[x]))/(((-c)^(1/4) + c^(1/4))*(1 + (-c)^(1/4)*Sqrt[x]))])/(d*Sqrt[d*
x]) + (I*b^2*c^(1/4)*Sqrt[x]*PolyLog[2, 1 - ((1 - I)*(1 + c^(1/4)*Sqrt[x]))
/(1 - I*c^(1/4)*Sqrt[x])])/(d*Sqrt[d*x])
```

Rubi [F] time = 0.0292299, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{(dx)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcTanh[c*x^2])^2/(d*x)^(3/2),x]
```

```
[Out] Defer[Int][(a + b*ArcTanh[c*x^2])^2/(d*x)^(3/2), x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{(dx)^{3/2}}\,dx=\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{(dx)^{3/2}}\,dx$$

Mathematica [F] time = 94.0147, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{(dx)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])^2/(d*x)^(3/2),x]
```

```
[Out] Integrate[(a + b*ArcTanh[c*x^2])^2/(d*x)^(3/2), x]
```

Maple [F] time = 0.272, size = 0, normalized size = 0.

$$\int \left(a + b\mathrm{Artanh}\left(cx^2\right)\right)^2 (dx)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))^2/(d*x)^(3/2),x)
```

```
[Out] int((a+b*arctanh(c*x^2))^2/(d*x)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/(d*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b^2\,\mathrm{artanh}\left(cx^2\right)^2 + 2\,ab\,\mathrm{artanh}\left(cx^2\right) + a^2\right)\sqrt{dx}}{d^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x^2)^2 + 2*a*b*arctanh(c*x^2) + a^2)*sqrt(d*x)/(d^2
*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))**2/(d*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \operatorname{artanh}\left(cx^2\right) + a\right)^2}{(dx)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^2) + a)^2/(d*x)^(3/2), x)
```

3.93 $\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{(dx)^{5/2}}\,dx$

Optimal. Leaf size=6520

result too large to display

```
[Out] (-2*Sqrt[2]*a*b*c^(3/4)*Sqrt[x]*ArcTan[1 - Sqrt[2]*c^(1/4)*Sqrt[x]])/(3*d^2
*Sqrt[d*x]) + (2*Sqrt[2]*a*b*c^(3/4)*Sqrt[x]*ArcTan[1 + Sqrt[2]*c^(1/4)*Sqr
t[x]])/(3*d^2*Sqrt[d*x]) - (((2*I)/3)*b^2*(-c)^(3/4)*Sqrt[x]*ArcTan[(-c)^(1
/4)*Sqrt[x]]^2)/(d^2*Sqrt[d*x]) - (((2*I)/3)*b^2*c^(3/4)*Sqrt[x]*ArcTan[c^(
1/4)*Sqrt[x]]^2)/(d^2*Sqrt[d*x]) + (2*b^2*(-c)^(3/4)*Sqrt[x]*ArcTanh[(-c)^(
1/4)*Sqrt[x]]^2)/(3*d^2*Sqrt[d*x]) + (2*b^2*c^(3/4)*Sqrt[x]*ArcTanh[c^(1/4)
*Sqrt[x]]^2)/(3*d^2*Sqrt[d*x]) - (4*b^2*(-c)^(3/4)*Sqrt[x]*ArcTanh[(-c)^(1/
4)*Sqrt[x]]*Log[2/(1 - (-c)^(1/4)*Sqrt[x])])/(3*d^2*Sqrt[d*x]) + (4*b^2*(-c
)^(3/4)*Sqrt[x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[2/(1 - I*(-c)^(1/4)*Sqrt[x])
])/(3*d^2*Sqrt[d*x]) - (2*b^2*(-c)^(3/4)*Sqrt[x]*ArcTan[(-c)^(1/4)*Sqrt[x]]
*Log[(-2*(-c)^(1/4)*(1 - Sqrt[-Sqrt[c]]*Sqrt[x]))/((I*Sqrt[-Sqrt[c]] - (-c)
^(1/4))*(1 - I*(-c)^(1/4)*Sqrt[x]))])/(3*d^2*Sqrt[d*x]) - (2*b^2*(-c)^(3/4)
*Sqrt[x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[(2*(-c)^(1/4)*(1 + Sqrt[-Sqrt[c]]*S
qrt[x]))/((I*Sqrt[-Sqrt[c]] + (-c)^(1/4))*(1 - I*(-c)^(1/4)*Sqrt[x]))])/(3*
d^2*Sqrt[d*x]) + (2*b^2*(-c)^(3/4)*Sqrt[x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[(
(1 + I)*(1 - (-c)^(1/4)*Sqrt[x]))/(1 - I*(-c)^(1/4)*Sqrt[x])])/(3*d^2*Sqrt[
d*x]) - (4*b^2*(-c)^(3/4)*Sqrt[x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[2/(1 + I*(
-c)^(1/4)*Sqrt[x])])/(3*d^2*Sqrt[d*x]) + (4*b^2*(-c)^(3/4)*Sqrt[x]*ArcTanh[
(-c)^(1/4)*Sqrt[x]]*Log[2/(1 + (-c)^(1/4)*Sqrt[x])])/(3*d^2*Sqrt[d*x]) + (2
*b^2*(-c)^(3/4)*Sqrt[x]*ArcTanh[(-c)^(1/4)*Sqrt[x]]*Log[(-2*(-c)^(1/4)*(1 -
 Sqrt[-Sqrt[-c]]*Sqrt[x]))/((Sqrt[-Sqrt[-c]] - (-c)^(1/4))*(1 + (-c)^(1/4)*
Sqrt[x]))])/(3*d^2*Sqrt[d*x]) + (2*b^2*(-c)^(3/4)*Sqrt[x]*ArcTanh[(-c)^(1/4
)*Sqrt[x]]*Log[(2*(-c)^(1/4)*(1 + Sqrt[-Sqrt[-c]]*Sqrt[x]))/((Sqrt[-Sqrt[-c
]] + (-c)^(1/4))*(1 + (-c)^(1/4)*Sqrt[x]))])/(3*d^2*Sqrt[d*x]) - (2*b^2*(-c
)^(3/4)*Sqrt[x]*ArcTanh[(-c)^(1/4)*Sqrt[x]]*Log[(-2*(-c)^(1/4)*(1 - Sqrt[-S
qrt[c]]*Sqrt[x]))/((Sqrt[-Sqrt[c]] - (-c)^(1/4))*(1 + (-c)^(1/4)*Sqrt[x]))]
)/(3*d^2*Sqrt[d*x]) - (2*b^2*(-c)^(3/4)*Sqrt[x]*ArcTanh[(-c)^(1/4)*Sqrt[x]]
*Log[(2*(-c)^(1/4)*(1 + Sqrt[-Sqrt[c]]*Sqrt[x]))/((Sqrt[-Sqrt[c]] + (-c)^(1
/4))*(1 + (-c)^(1/4)*Sqrt[x]))])/(3*d^2*Sqrt[d*x]) + (2*b^2*(-c)^(3/4)*Sqrt
[x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[((1 - I)*(1 + (-c)^(1/4)*Sqrt[x]))/(1 -
I*(-c)^(1/4)*Sqrt[x])])/(3*d^2*Sqrt[d*x]) - (4*b^2*c^(3/4)*Sqrt[x]*ArcTanh[
c^(1/4)*Sqrt[x]]*Log[2/(1 - c^(1/4)*Sqrt[x])])/(3*d^2*Sqrt[d*x]) - (2*b^2*(
-c)^(3/4)*Sqrt[x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[(2*(-c)^(1/4)*(1 - c^(1/4)
*Sqrt[x]))/(((-c)^(1/4) - I*c^(1/4))*(1 - I*(-c)^(1/4)*Sqrt[x]))])/(3*d^2*S
qrt[d*x]) - (2*b^2*(-c)^(3/4)*Sqrt[x]*ArcTanh[(-c)^(1/4)*Sqrt[x]]*Log[(2*(-
c)^(1/4)*(1 - c^(1/4)*Sqrt[x]))/(((-c)^(1/4) - c^(1/4))*(1 + (-c)^(1/4)*Sqr
t[x]))])/(3*d^2*Sqrt[d*x]) + (4*b^2*c^(3/4)*Sqrt[x]*ArcTan[c^(1/4)*Sqrt[x]]
*Log[2/(1 - I*c^(1/4)*Sqrt[x])])/(3*d^2*Sqrt[d*x]) - (2*b^2*c^(3/4)*Sqrt[x]
*ArcTan[c^(1/4)*Sqrt[x]]*Log[(-2*c^(1/4)*(1 - Sqrt[-Sqrt[-c]]*Sqrt[x]))/((I
*Sqrt[-Sqrt[-c]] - c^(1/4))*(1 - I*c^(1/4)*Sqrt[x]))])/(3*d^2*Sqrt[d*x]) -
(2*b^2*c^(3/4)*Sqrt[x]*ArcTan[c^(1/4)*Sqrt[x]]*Log[(2*c^(1/4)*(1 + Sqrt[-Sq
rt[-c]]*Sqrt[x]))/((I*Sqrt[-Sqrt[-c]] + c^(1/4))*(1 - I*c^(1/4)*Sqrt[x]))])
/(3*d^2*Sqrt[d*x]) - (2*b^2*c^(3/4)*Sqrt[x]*ArcTan[c^(1/4)*Sqrt[x]]*Log[(-2
*c^(1/4)*(1 - (-c)^(1/4)*Sqrt[x]))/((I*(-c)^(1/4) - c^(1/4))*(1 - I*c^(1/4)
*Sqrt[x]))])/(3*d^2*Sqrt[d*x]) - (2*b^2*c^(3/4)*Sqrt[x]*ArcTan[c^(1/4)*Sqrt
[x]]*Log[(2*c^(1/4)*(1 + (-c)^(1/4)*Sqrt[x]))/((I*(-c)^(1/4) + c^(1/4))*(1
- I*c^(1/4)*Sqrt[x]))])/(3*d^2*Sqrt[d*x]) + (2*b^2*c^(3/4)*Sqrt[x]*ArcTan[c
^(1/4)*Sqrt[x]]*Log[((1 + I)*(1 - c^(1/4)*Sqrt[x]))/(1 - I*c^(1/4)*Sqrt[x])
])/(3*d^2*Sqrt[d*x]) - (4*b^2*c^(3/4)*Sqrt[x]*ArcTan[c^(1/4)*Sqrt[x]]*Log[2
/(1 + I*c^(1/4)*Sqrt[x])])/(3*d^2*Sqrt[d*x]) + (4*b^2*c^(3/4)*Sqrt[x]*ArcTa
nh[c^(1/4)*Sqrt[x]]*Log[2/(1 + c^(1/4)*Sqrt[x])])/(3*d^2*Sqrt[d*x]) - (2*b^
```

2*c^(3/4)*Sqrt[x]*ArcTanh[c^(1/4)*Sqrt[x]]*Log[(-2*c^(1/4)*(1 - Sqrt[-Sqrt[-c]]*Sqrt[x]))/((Sqrt[-Sqrt[-c]] - c^(1/4))*(1 + c^(1/4)*Sqrt[x]))]])/(3*d^2*Sqrt[d*x]) - (2*b^2*c^(3/4)*Sqrt[x]*ArcTanh[c^(1/4)*Sqrt[x]]*Log[(2*c^(1/4)*(1 + Sqrt[-Sqrt[-c]]*Sqrt[x]))/((Sqrt[-Sqrt[-c]] + c^(1/4))*(1 + c^(1/4)*Sqrt[x]))]])/(3*d^2*Sqrt[d*x]) + (2*b^2*c^(3/4)*Sqrt[x]*ArcTanh[c^(1/4)*Sqrt[x]]*Log[(-2*c^(1/4)*(1 - Sqrt[-Sqrt[c]]*Sqrt[x]))/((Sqrt[-Sqrt[c]] - c^(1/4))*(1 + c^(1/4)*Sqrt[x]))]])/(3*d^2*Sqrt[d*x]) + (2*b^2*c^(3/4)*Sqrt[x]*ArcTanh[c^(1/4)*Sqrt[x]]*Log[(2*c^(1/4)*(1 + Sqrt[-Sqrt[c]]*Sqrt[x]))/((Sqrt[-Sqrt[c]] + c^(1/4))*(1 + c^(1/4)*Sqrt[x]))]])/(3*d^2*Sqrt[d*x]) - (2*b^2*c^(3/4)*Sqrt[x]*ArcTanh[c^(1/4)*Sqrt[x]]*Log[(-2*c^(1/4)*(1 - (-c)^(1/4)*Sqrt[x]))/(((-c)^(1/4) - c^(1/4))*(1 + c^(1/4)*Sqrt[x]))]])/(3*d^2*Sqrt[d*x]) - (2*b^2*c^(3/4)*Sqrt[x]*ArcTanh[c^(1/4)*Sqrt[x]]*Log[(2*c^(1/4)*(1 + (-c)^(1/4)*Sqrt[x]))/(((-c)^(1/4) + c^(1/4))*(1 + c^(1/4)*Sqrt[x]))]])/(3*d^2*Sqrt[d*x]) - (2*b^2*(-c)^(3/4)*Sqrt[x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[(2*(-c)^(1/4)*(1 + c^(1/4)*Sqrt[x]))/(((-c)^(1/4) + I*c^(1/4))*(1 - I*(-c)^(1/4)*Sqrt[x]))]])/(3*d^2*Sqrt[d*x]) - (2*b^2*(-c)^(3/4)*Sqrt[x]*ArcTanh[(-c)^(1/4)*Sqrt[x]]*Log[(2*(-c)^(1/4)*(1 + c^(1/4)*Sqrt[x]))/(((-c)^(1/4) + c^(1/4))*(1 + (-c)^(1/4)*Sqrt[x]))]])/(3*d^2*Sqrt[d*x]) + (2*b^2*c^(3/4)*Sqrt[x]*ArcTan[c^(1/4)*Sqrt[x]]*Log[((1 - I)*(1 + c^(1/4)*Sqrt[x]))/(1 - I*c^(1/4)*Sqrt[x])]])/(3*d^2*Sqrt[d*x]) - (Sqrt[2]*a*b*c^(3/4)*Sqrt[x]*Log[1 - Sqrt[2]*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(3*d^2*Sqrt[d*x]) + (Sqrt[2]*a*b*c^(3/4)*Sqrt[x]*Log[1 + Sqrt[2]*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(3*d^2*Sqrt[d*x]) + (2*b^2*(-c)^(3/4)*Sqrt[x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[1 - c*x^2])/(3*d^2*Sqrt[d*x]) + (2*b^2*(-c)^(3/4)*Sqrt[x]*ArcTanh[(-c)^(1/4)*Sqrt[x]]*Log[1 - c*x^2])/(3*d^2*Sqrt[d*x]) + (2*b*c^(3/4)*Sqrt[x]*ArcTan[c^(1/4)*Sqrt[x]]*(2*a - b*Log[1 - c*x^2]))/(3*d^2*Sqrt[d*x]) + (2*b*c^(3/4)*Sqrt[x]*ArcTanh[c^(1/4)*Sqrt[x]]*(2*a - b*Log[1 - c*x^2]))/(3*d^2*Sqrt[d*x]) - (2*a - b*Log[1 - c*x^2])^2/(6*d^2*x*Sqrt[d*x]) - (2*a*b*Log[1 + c*x^2])/(3*d^2*x*Sqrt[d*x]) - (2*b^2*(-c)^(3/4)*Sqrt[x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[1 + c*x^2])/(3*d^2*Sqrt[d*x]) + (2*b^2*c^(3/4)*Sqrt[x]*ArcTan[c^(1/4)*Sqrt[x]]*Log[1 + c*x^2])/(3*d^2*Sqrt[d*x]) - (2*b^2*(-c)^(3/4)*Sqrt[x]*ArcTanh[(-c)^(1/4)*Sqrt[x]]*Log[1 + c*x^2])/(3*d^2*Sqrt[d*x]) + (2*b^2*c^(3/4)*Sqrt[x]*ArcTanh[c^(1/4)*Sqrt[x]]*Log[1 + c*x^2])/(3*d^2*Sqrt[d*x]) + (b^2*Log[1 - c*x^2]*Log[1 + c*x^2])/(3*d^2*x*Sqrt[d*x]) - (b^2*Log[1 + c*x^2]^2)/(6*d^2*x*Sqrt[d*x]) - (2*b^2*(-c)^(3/4)*Sqrt[x]*PolyLog[2, 1 - 2/(1 - (-c)^(1/4)*Sqrt[x])])/(3*d^2*Sqrt[d*x]) - (((2*I)/3)*b^2*(-c)^(3/4)*Sqrt[x]*PolyLog[2, 1 - 2/(1 - I*(-c)^(1/4)*Sqrt[x])])/(d^2*Sqrt[d*x]) + ((I/3)*b^2*(-c)^(3/4)*Sqrt[x]*PolyLog[2, 1 + (2*(-c)^(1/4)*(1 - Sqrt[-Sqrt[c]]*Sqrt[x]))/((I*Sqrt[-Sqrt[c]] - (-c)^(1/4))*(1 - I*(-c)^(1/4)*Sqrt[x]))])/(d^2*Sqrt[d*x]) + ((I/3)*b^2*(-c)^(3/4)*Sqrt[x]*PolyLog[2, 1 - (2*(-c)^(1/4)*(1 + Sqrt[-Sqrt[c]]*Sqrt[x]))/((I*Sqrt[-Sqrt[c]] + (-c)^(1/4))*(1 - I*(-c)^(1/4)*Sqrt[x]))])/(d^2*Sqrt[d*x]) - ((I/3)*b^2*(-c)^(3/4)*Sqrt[x]*PolyLog[2, 1 - ((1 + I)*(1 - (-c)^(1/4)*Sqrt[x]))/(1 - I*(-c)^(1/4)*Sqrt[x])])/(d^2*Sqrt[d*x]) - (((2*I)/3)*b^2*(-c)^(3/4)*Sqrt[x]*PolyLog[2, 1 - 2/(1 + I*(-c)^(1/4)*Sqrt[x])])/(d^2*Sqrt[d*x]) - (2*b^2*(-c)^(3/4)*Sqrt[x]*PolyLog[2, 1 - 2/(1 + (-c)^(1/4)*Sqrt[x])])/(3*d^2*Sqrt[d*x]) - (b^2*(-c)^(3/4)*Sqrt[x]*PolyLog[2, 1 + (2*(-c)^(1/4)*(1 - Sqrt[-Sqrt[-c]]*Sqrt[x]))/((Sqrt[-Sqrt[-c]] - (-c)^(1/4))*(1 + (-c)^(1/4)*Sqrt[x]))])/(3*d^2*Sqrt[d*x]) - (b^2*(-c)^(3/4)*Sqrt[x]*PolyLog[2, 1 - (2*(-c)^(1/4)*(1 + Sqrt[-Sqrt[-c]]*Sqrt[x]))/((Sqrt[-Sqrt[-c]] + (-c)^(1/4))*(1 + (-c)^(1/4)*Sqrt[x]))])/(3*d^2*Sqrt[d*x]) + (b^2*(-c)^(3/4)*Sqrt[x]*PolyLog[2, 1 + (2*(-c)^(1/4)*(1 - Sqrt[-Sqrt[c]]*Sqrt[x]))/((Sqrt[-Sqrt[c]] - (-c)^(1/4))*(1 + (-c)^(1/4)*Sqrt[x]))])/(3*d^2*Sqrt[d*x]) + (b^2*(-c)^(3/4)*Sqrt[x]*PolyLog[2, 1 - (2*(-c)^(1/4)*(1 + Sqrt[-Sqrt[c]]*Sqrt[x]))/((Sqrt[-Sqrt[c]] + (-c)^(1/4))*(1 + (-c)^(1/4)*Sqrt[x]))])/(3*d^2*Sqrt[d*x]) - ((I/3)*b^2*(-c)^(3/4)*Sqrt[x]*PolyLog[2, 1 - ((1 - I)*(1 + (-c)^(1/4)*Sqrt[x]))/(1 - I*(-c)^(1/4)*Sqrt[x])])/(d^2*Sqrt[d*x]) - (2*b^2*c^(3/4)*Sqrt[x]*PolyLog[2, 1 - 2/(1 - c^(1/4)*Sqrt[x])])/(3*d^2*Sqrt[d*x]) + ((I/3)*b^2*(-c)^(3/4)*Sqrt[x]*PolyLog[2, 1 - (2*(-c)^(1/4)*(1 - c^(1/4)*Sqrt[x]))/(((-c)^(1/4) - I*c^(1/4))*(1 - I*(-c)^(1/4)*Sqrt[x]))])/(d^2*Sqrt[d*x]) + (b^2*(-c)^(3/4)*Sqrt

```
[x]*PolyLog[2, 1 - (2*(-c)^(1/4)*(1 - c^(1/4)*Sqrt[x]))/(((-c)^(1/4) - c^(1
/4))*(1 + (-c)^(1/4)*Sqrt[x]))])/(3*d^2*Sqrt[d*x]) - (((2*I)/3)*b^2*c^(3/4)
*Sqrt[x]*PolyLog[2, 1 - 2/(1 - I*c^(1/4)*Sqrt[x])])/(d^2*Sqrt[d*x]) + ((I/3
)*b^2*c^(3/4)*Sqrt[x]*PolyLog[2, 1 + (2*c^(1/4)*(1 - Sqrt[-Sqrt[-c]]*Sqrt[x
]))/((I*Sqrt[-Sqrt[-c]] - c^(1/4))*(1 - I*c^(1/4)*Sqrt[x]))])/(d^2*Sqrt[d*x
]) + ((I/3)*b^2*c^(3/4)*Sqrt[x]*PolyLog[2, 1 - (2*c^(1/4)*(1 + Sqrt[-Sqrt[-
c]]*Sqrt[x]))/((I*Sqrt[-Sqrt[-c]] + c^(1/4))*(1 - I*c^(1/4)*Sqrt[x]))])/(d^
2*Sqrt[d*x]) + ((I/3)*b^2*c^(3/4)*Sqrt[x]*PolyLog[2, 1 + (2*c^(1/4)*(1 - (-
c)^(1/4)*Sqrt[x]))/((I*(-c)^(1/4) - c^(1/4))*(1 - I*c^(1/4)*Sqrt[x]))])/(d^
2*Sqrt[d*x]) + ((I/3)*b^2*c^(3/4)*Sqrt[x]*PolyLog[2, 1 - (2*c^(1/4)*(1 + (-
c)^(1/4)*Sqrt[x]))/((I*(-c)^(1/4) + c^(1/4))*(1 - I*c^(1/4)*Sqrt[x]))])/(d^
2*Sqrt[d*x]) - ((I/3)*b^2*c^(3/4)*Sqrt[x]*PolyLog[2, 1 - ((1 + I)*(1 - c^(1
/4)*Sqrt[x]))/(1 - I*c^(1/4)*Sqrt[x])])/(d^2*Sqrt[d*x]) - (((2*I)/3)*b^2*c^
(3/4)*Sqrt[x]*PolyLog[2, 1 - 2/(1 + I*c^(1/4)*Sqrt[x])])/(d^2*Sqrt[d*x]) -
(2*b^2*c^(3/4)*Sqrt[x]*PolyLog[2, 1 - 2/(1 + c^(1/4)*Sqrt[x])])/(3*d^2*Sqrt
[d*x]) + (b^2*c^(3/4)*Sqrt[x]*PolyLog[2, 1 + (2*c^(1/4)*(1 - Sqrt[-Sqrt[-c]
]*Sqrt[x]))/((Sqrt[-Sqrt[-c]] - c^(1/4))*(1 + c^(1/4)*Sqrt[x]))])/(3*d^2*Sq
rt[d*x]) + (b^2*c^(3/4)*Sqrt[x]*PolyLog[2, 1 - (2*c^(1/4)*(1 + Sqrt[-Sqrt[-
c]]*Sqrt[x]))/((Sqrt[-Sqrt[-c]] + c^(1/4))*(1 + c^(1/4)*Sqrt[x]))])/(3*d^2*
Sqrt[d*x]) - (b^2*c^(3/4)*Sqrt[x]*PolyLog[2, 1 + (2*c^(1/4)*(1 - Sqrt[-Sqrt
[c]]*Sqrt[x]))/((Sqrt[-Sqrt[c]] - c^(1/4))*(1 + c^(1/4)*Sqrt[x]))])/(3*d^2*
Sqrt[d*x]) - (b^2*c^(3/4)*Sqrt[x]*PolyLog[2, 1 - (2*c^(1/4)*(1 + Sqrt[-Sqrt
[c]]*Sqrt[x]))/((Sqrt[-Sqrt[c]] + c^(1/4))*(1 + c^(1/4)*Sqrt[x]))])/(3*d^2*
Sqrt[d*x]) + (b^2*c^(3/4)*Sqrt[x]*PolyLog[2, 1 + (2*c^(1/4)*(1 - (-c)^(1/4)
*Sqrt[x]))/(((-c)^(1/4) - c^(1/4))*(1 + c^(1/4)*Sqrt[x]))])/(3*d^2*Sqrt[d*x
]) + (b^2*c^(3/4)*Sqrt[x]*PolyLog[2, 1 - (2*c^(1/4)*(1 + (-c)^(1/4)*Sqrt[x]
))/(((-c)^(1/4) + c^(1/4))*(1 + c^(1/4)*Sqrt[x]))])/(3*d^2*Sqrt[d*x]) + ((I
/3)*b^2*(-c)^(3/4)*Sqrt[x]*PolyLog[2, 1 - (2*(-c)^(1/4)*(1 + c^(1/4)*Sqrt[x
]))/(((-c)^(1/4) + I*c^(1/4))*(1 - I*(-c)^(1/4)*Sqrt[x]))])/(d^2*Sqrt[d*x])
 + (b^2*(-c)^(3/4)*Sqrt[x]*PolyLog[2, 1 - (2*(-c)^(1/4)*(1 + c^(1/4)*Sqrt[x
]))/(((-c)^(1/4) + c^(1/4))*(1 + (-c)^(1/4)*Sqrt[x]))])/(3*d^2*Sqrt[d*x]) -
 ((I/3)*b^2*c^(3/4)*Sqrt[x]*PolyLog[2, 1 - ((1 - I)*(1 + c^(1/4)*Sqrt[x]))/
(1 - I*c^(1/4)*Sqrt[x])])/(d^2*Sqrt[d*x])
```

Rubi [F] time = 0.030478, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{(dx)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcTanh[c*x^2])^2/(d*x)^(5/2),x]
```

```
[Out] Defer[Int][(a + b*ArcTanh[c*x^2])^2/(d*x)^(5/2), x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{(dx)^{5/2}}\,dx=\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{(dx)^{5/2}}\,dx$$

Mathematica [F] time = 50.7384, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2}{(dx)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])^2/(d*x)^(5/2),x]
```

```
[Out] Integrate[(a + b*ArcTanh[c*x^2])^2/(d*x)^(5/2), x]
```

Maple [F] time = 0.275, size = 0, normalized size = 0.

$$\int \left(a+b\mathrm{Artanh}\left(cx^2\right)\right)^2 (dx)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))^2/(d*x)^(5/2),x)
```

```
[Out] int((a+b*arctanh(c*x^2))^2/(d*x)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/(d*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b^2\,\mathrm{artanh}\left(cx^2\right)^2+2\,ab\,\mathrm{artanh}\left(cx^2\right)+a^2\right)\sqrt{dx}}{d^3x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/(d*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x^2)^2 + 2*a*b*arctanh(c*x^2) + a^2)*sqrt(d*x)/(d^3
*x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))**2/(d*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \operatorname{artanh}\left(cx^2\right)+a\right)^2}{(dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/(d*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^2) + a)^2/(d*x)^(5/2), x)
```

3.94 $\int (dx)^m \left(a+b\tanh^{-1}\left(cx^2\right)\right)^3 \, dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left((dx)^m \left(a+b\tanh^{-1}\left(cx^2\right)\right)^3,x\right)$$

```
[Out] Unintegrable[(d*x)^m*(a + b*ArcTanh[c*x^2])^3, x]
```

Rubi [A] time = 0.0239834, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (dx)^m \left(a+b\tanh^{-1}\left(cx^2\right)\right)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m*(a + b*ArcTanh[c*x^2])^3,x]
```

```
[Out] Defer[Int][(d*x)^m*(a + b*ArcTanh[c*x^2])^3, x]
```

Rubi steps

$$\int (dx)^m \left(a+b\tanh^{-1}\left(cx^2\right)\right)^3 \, dx=\int (dx)^m \left(a+b\tanh^{-1}\left(cx^2\right)\right)^3 \, dx$$

Mathematica [A] time = 1.75129, size = 0, normalized size = 0.

$$\int (dx)^m \left(a+b\tanh^{-1}\left(cx^2\right)\right)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m*(a + b*ArcTanh[c*x^2])^3,x]
```

```
[Out] Integrate[(d*x)^m*(a + b*ArcTanh[c*x^2])^3, x]
```

Maple [A] time = 0.122, size = 0, normalized size = 0.

$$\int (dx)^m \left(a+b\text{Artanh}\left(cx^2\right)\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctanh(c*x^2))^3,x)
```

```
[Out] int((d*x)^m*(a+b*arctanh(c*x^2))^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^2))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^3 \operatorname{artanh}\left(cx^2\right)^3 + 3\,ab^2 \operatorname{artanh}\left(cx^2\right)^2 + 3\,a^2b \operatorname{artanh}\left(cx^2\right) + a^3\right)(dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^2))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctanh(c*x^2)^3 + 3*a*b^2*arctanh(c*x^2)^2 + 3*a^2*b*arctanh
(c*x^2) + a^3)*(d*x)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atanh(c*x**2))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b \operatorname{artanh}\left(cx^2\right) + a\right)^3 (dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^2))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^2) + a)^3*(d*x)^m, x)
```

3.95 $\int (dx)^m \left(a+b\tanh^{-1}\left(cx^2\right)\right)^2 dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left((dx)^m \left(a+b\tanh^{-1}\left(cx^2\right)\right)^2,x\right)$$

```
[Out] Unintegrable[(d*x)^m*(a + b*ArcTanh[c*x^2])^2, x]
```

Rubi [A] time = 0.0237911, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (dx)^m \left(a+b\tanh^{-1}\left(cx^2\right)\right)^2 dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m*(a + b*ArcTanh[c*x^2])^2,x]
```

```
[Out] Defer[Int][(d*x)^m*(a + b*ArcTanh[c*x^2])^2, x]
```

Rubi steps

$$\int (dx)^m \left(a+b\tanh^{-1}\left(cx^2\right)\right)^2 dx=\int (dx)^m \left(a+b\tanh^{-1}\left(cx^2\right)\right)^2 dx$$

Mathematica [A] time = 1.16718, size = 0, normalized size = 0.

$$\int (dx)^m \left(a+b\tanh^{-1}\left(cx^2\right)\right)^2 dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m*(a + b*ArcTanh[c*x^2])^2,x]
```

```
[Out] Integrate[(d*x)^m*(a + b*ArcTanh[c*x^2])^2, x]
```

Maple [A] time = 0.119, size = 0, normalized size = 0.

$$\int (dx)^m \left(a+b\text{Artanh}\left(cx^2\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctanh(c*x^2))^2,x)
```

```
[Out] int((d*x)^m*(a+b*arctanh(c*x^2))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^2))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2 \operatorname{artanh}\left(cx^2\right)^2 + 2\,ab \operatorname{artanh}\left(cx^2\right) + a^2\right)(dx)^m\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^2))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x^2)^2 + 2*a*b*arctanh(c*x^2) + a^2)*(d*x)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atanh(c*x**2))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b \operatorname{artanh}\left(cx^2\right) + a\right)^2 (dx)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^2))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^2) + a)^2*(d*x)^m, x)
```

3.96 $\int (dx)^m \left(a+b\tanh^{-1}\left(cx^2\right)\right) dx$

Optimal. Leaf size=74

$$\frac{(dx)^{m+1}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{d(m+1)}-\frac{2bc(dx)^{m+3}\text{Hypergeometric2F1}\left(1,\frac{m+3}{4},\frac{m+7}{4},c^2x^4\right)}{d^3(m+1)(m+3)}$$

```
[Out] ((d*x)^(1 + m)*(a + b*ArcTanh[c*x^2]))/(d*(1 + m)) - (2*b*c*(d*x)^(3 + m)*H
ypergeometric2F1[1, (3 + m)/4, (7 + m)/4, c^2*x^4])/(d^3*(1 + m)*(3 + m))
```

Rubi [A] time = 0.0397164, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {6097, 16, 364}

$$\frac{(dx)^{m+1}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{d(m+1)}-\frac{2bc(dx)^{m+3}\,{}_2F_1\left(1,\frac{m+3}{4};\frac{m+7}{4};c^2x^4\right)}{d^3(m+1)(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*ArcTanh[c*x^2]),x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*ArcTanh[c*x^2]))/(d*(1 + m)) - (2*b*c*(d*x)^(3 + m)*H
ypergeometric2F1[1, (3 + m)/4, (7 + m)/4, c^2*x^4])/(d^3*(1 + m)*(3 + m))
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int (dx)^m\left(a+b\tanh^{-1}\left(cx^2\right)\right) dx &= \frac{(dx)^{1+m}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{d(1+m)}-\frac{(2bc)\int \frac{x(dx)^{1+m}}{1-c^2x^4}\,dx}{d(1+m)}\\
&= \frac{(dx)^{1+m}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{d(1+m)}-\frac{(2bc)\int \frac{(dx)^{2+m}}{1-c^2x^4}\,dx}{d^2(1+m)}\\
&= \frac{(dx)^{1+m}\left(a+b\tanh^{-1}\left(cx^2\right)\right)}{d(1+m)}-\frac{2bc(dx)^{3+m}\,{}_2F_1\left(1,\frac{3+m}{4};\frac{7+m}{4};c^2x^4\right)}{d^3(1+m)(3+m)}
\end{aligned}$$

Mathematica [A] time = 0.0659015, size = 64, normalized size = 0.86

$$-\frac{x(dx)^m\left(2bcx^2\text{Hypergeometric2F1}\left(1,\frac{m+3}{4},\frac{m+7}{4},c^2x^4\right)-(m+3)\left(a+b\tanh^{-1}\left(cx^2\right)\right)\right)}{(m+1)(m+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a + b*ArcTanh[c*x^2]),x]
```

```
[Out] -((x*(d*x)^m*(-((3 + m)*(a + b*ArcTanh[c*x^2])) + 2*b*c*x^2*Hypergeometric2
F1[1, (3 + m)/4, (7 + m)/4, c^2*x^4]))/((1 + m)*(3 + m)))
```

Maple [F] time = 0.117, size = 0, normalized size = 0.

$$\int (dx)^m\left(a+b\text{Artanh}\left(cx^2\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctanh(c*x^2)),x)
```

```
[Out] int((d*x)^m*(a+b*arctanh(c*x^2)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\,\text{artanh}\left(cx^2\right)+a\right)(dx)^m\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^2)),x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*x^2) + a)*(d*x)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atanh(c*x**2)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \operatorname{artanh}\left(cx^2\right) + a\right) (dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^2)),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^2) + a)*(d*x)^m, x)
```

3.97 $\int \frac{(dx)^m}{a+b\tanh^{-1}(cx^2)}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{(dx)^m}{a+b\tanh^{-1}\left(cx^2\right)},x\right)$$

```
[Out] Unintegrable[(d*x)^m/(a + b*ArcTanh[c*x^2]), x]
```

Rubi [A] time = 0.0287267, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(dx)^m}{a+b\tanh^{-1}\left(cx^2\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m/(a + b*ArcTanh[c*x^2]),x]
```

```
[Out] Defer[Int][(d*x)^m/(a + b*ArcTanh[c*x^2]), x]
```

Rubi steps

$$\int \frac{(dx)^m}{a+b\tanh^{-1}\left(cx^2\right)}\,dx=\int \frac{(dx)^m}{a+b\tanh^{-1}\left(cx^2\right)}\,dx$$

Mathematica [A] time = 0.348211, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{a+b\tanh^{-1}\left(cx^2\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m/(a + b*ArcTanh[c*x^2]),x]
```

```
[Out] Integrate[(d*x)^m/(a + b*ArcTanh[c*x^2]), x]
```

Maple [A] time = 0.063, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{a+b\text{Artanh}\left(cx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a+b*arctanh(c*x^2)),x)
```

```
[Out] int((d*x)^m/(a+b*arctanh(c*x^2)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{b \operatorname{artanh}(cx^2) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c*x^2)),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/(b*arctanh(c*x^2) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{(dx)^m}{b \operatorname{artanh}(cx^2) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c*x^2)),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(b*arctanh(c*x^2) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*atanh(c*x**2)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{b \operatorname{artanh}(cx^2) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c*x^2)),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b*arctanh(c*x^2) + a), x)
```

3.98 $\int \frac{(dx)^m}{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2} dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{(dx)^m}{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2},x\right)$$

```
[Out] Unintegrable[(d*x)^m/(a + b*ArcTanh[c*x^2])^2, x]
```

Rubi [A] time = 0.0271365, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(dx)^m}{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m/(a + b*ArcTanh[c*x^2])^2,x]
```

```
[Out] Defer[Int][(d*x)^m/(a + b*ArcTanh[c*x^2])^2, x]
```

Rubi steps

$$\int \frac{(dx)^m}{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2} dx = \int \frac{(dx)^m}{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2} dx$$

Mathematica [A] time = 0.413779, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left(a+b\tanh^{-1}\left(cx^2\right)\right)^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m/(a + b*ArcTanh[c*x^2])^2,x]
```

```
[Out] Integrate[(d*x)^m/(a + b*ArcTanh[c*x^2])^2, x]
```

Maple [A] time = 0.068, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left(a+b\mathrm{Artanh}\left(cx^2\right)\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a+b*arctanh(c*x^2))^2,x)
```

```
[Out] int((d*x)^m/(a+b*arctanh(c*x^2))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{\left(c^2 d^m x^4 - d^m\right)x^m}{b^2 cx \log\left(cx^2+1\right) - b^2 cx \log\left(-cx^2+1\right) + 2\,abcx} + \int -\frac{\left(c^2 d^m (m+3)x^4 - d^m (m-1)\right)x^m}{b^2 cx^2 \log\left(cx^2+1\right) - b^2 cx^2 \log\left(-cx^2+1\right) + 2\,abcx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c*x^2))^2,x, algorithm="maxima")
```

```
[Out] (c^2*d^m*x^4 - d^m)*x^m/(b^2*c*x*log(c*x^2 + 1) - b^2*c*x*log(-c*x^2 + 1) +
 2*a*b*c*x) + integrate(-(c^2*d^m*(m + 3)*x^4 - d^m*(m - 1))*x^m/(b^2*c*x^2
*log(c*x^2 + 1) - b^2*c*x^2*log(-c*x^2 + 1) + 2*a*b*c*x^2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{(dx)^m}{b^2 \operatorname{artanh}\left(cx^2\right)^2 + 2\,ab \operatorname{artanh}\left(cx^2\right) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c*x^2))^2,x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(b^2*arctanh(c*x^2)^2 + 2*a*b*arctanh(c*x^2) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*atanh(c*x**2))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left(b \operatorname{artanh}\left(cx^2\right) + a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c*x^2))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b*arctanh(c*x^2) + a)^2, x)
```

3.99 $\int x^{11} \left(a + b \tanh^{-1}\left(cx^3\right)\right) dx$

Optimal. Leaf size=54

$$\frac{1}{12} x^{12} \left(a + b \tanh^{-1}\left(cx^3\right)\right) + \frac{bx^3}{12c^3} - \frac{b \tanh^{-1}\left(cx^3\right)}{12c^4} + \frac{bx^9}{36c}$$

```
[Out] (b*x^3)/(12*c^3) + (b*x^9)/(36*c) - (b*ArcTanh[c*x^3])/(12*c^4) + (x^12*(a
+ b*ArcTanh[c*x^3]))/12
```

Rubi [A] time = 0.0382175, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6097, 275, 302, 206}

$$\frac{1}{12} x^{12} \left(a + b \tanh^{-1}\left(cx^3\right)\right) + \frac{bx^3}{12c^3} - \frac{b \tanh^{-1}\left(cx^3\right)}{12c^4} + \frac{bx^9}{36c}$$

Antiderivative was successfully verified.

```
[In] Int[x^11*(a + b*ArcTanh[c*x^3]),x]
```


```
[Out] (b*x^3)/(12*c^3) + (b*x^9)/(36*c) - (b*ArcTanh[c*x^3])/(12*c^4) + (x^12*(a
+ b*ArcTanh[c*x^3]))/12
```


Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int x^{11}\left(a+b\tanh^{-1}\left(cx^3\right)\right)dx &= \frac{1}{12}x^{12}\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{1}{4}(bc)\int\frac{x^{14}}{1-c^2x^6}\,dx\\ &= \frac{1}{12}x^{12}\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{1}{12}(bc)\,\text{Subst}\left(\int\frac{x^4}{1-c^2x^2}\,dx,x,x^3\right)\\ &= \frac{1}{12}x^{12}\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{1}{12}(bc)\,\text{Subst}\left(\int\left(-\frac{1}{c^4}-\frac{x^2}{c^2}+\frac{1}{c^4\left(1-c^2x^2\right)}\right)dx,x,x^3\right)\\ &= \frac{bx^3}{12c^3}+\frac{bx^9}{36c}+\frac{1}{12}x^{12}\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{b\,\text{Subst}\left(\int\frac{1}{1-c^2x^2}\,dx,x,x^3\right)}{12c^3}\\ &= \frac{bx^3}{12c^3}+\frac{bx^9}{36c}-\frac{b\tanh^{-1}\left(cx^3\right)}{12c^4}+\frac{1}{12}x^{12}\left(a+b\tanh^{-1}\left(cx^3\right)\right)\end{aligned}$$

Mathematica [A] time = 0.0200856, size = 78, normalized size = 1.44

$$\frac{ax^{12}}{12}+\frac{bx^3}{12c^3}+\frac{b\log\left(1-cx^3\right)}{24c^4}-\frac{b\log\left(cx^3+1\right)}{24c^4}+\frac{bx^9}{36c}+\frac{1}{12}bx^{12}\tanh^{-1}\left(cx^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11*(a + b*ArcTanh[c*x^3]),x]
```

```
[Out] (b*x^3)/(12*c^3) + (b*x^9)/(36*c) + (a*x^12)/12 + (b*x^12*ArcTanh[c*x^3])/1
2 + (b*Log[1 - c*x^3])/(24*c^4) - (b*Log[1 + c*x^3])/(24*c^4)
```

Maple [A] time = 0.009, size = 66, normalized size = 1.2

$$\frac{x^{12}a}{12}+\frac{bx^{12}\text{Artanh}\left(cx^3\right)}{12}+\frac{bx^9}{36\,c}+\frac{bx^3}{12\,c^3}+\frac{b\ln\left(cx^3-1\right)}{24\,c^4}-\frac{b\ln\left(cx^3+1\right)}{24\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(a+b*arctanh(c*x^3)),x)
```

```
[Out] 1/12*x^12*a+1/12*b*x^12*arctanh(c*x^3)+1/36*b*x^9/c+1/12*b*x^3/c^3+1/24*b/c
^4*ln(c*x^3-1)-1/24*b/c^4*ln(c*x^3+1)
```

Maxima [A] time = 0.953359, size = 93, normalized size = 1.72

$$\frac{1}{12}\,ax^{12}+\frac{1}{72}\left(6\,x^{12}\,\text{artanh}\left(cx^3\right)+c\left(\frac{2\left(c^2x^9+3\,x^3\right)}{c^4}-\frac{3\,\log\left(cx^3+1\right)}{c^5}+\frac{3\,\log\left(cx^3-1\right)}{c^5}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(a+b*arctanh(c*x^3)),x, algorithm="maxima")
```

```
[Out] 1/12*a*x^12 + 1/72*(6*x^12*arctanh(c*x^3) + c*(2*(c^2*x^9 + 3*x^3)/c^4 - 3*
log(c*x^3 + 1)/c^5 + 3*log(c*x^3 - 1)/c^5))*b
```

Fricas [A] time = 2.04169, size = 138, normalized size = 2.56

$$\frac{6\,ac^4x^{12} + 2\,bc^3x^9 + 6\,bcx^3 + 3\,\left(bc^4x^{12} - b\right)\log\left(-\frac{cx^3+1}{cx^3-1}\right)}{72\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(a+b*arctanh(c*x^3)),x, algorithm="fricas")
```

```
[Out] 1/72*(6*a*c^4*x^12 + 2*b*c^3*x^9 + 6*b*c*x^3 + 3*(b*c^4*x^12 - b)*log(-(c*x
^3 + 1)/(c*x^3 - 1)))/c^4
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: KeyError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(a+b*atanh(c*x**3)),x)
```

```
[Out] Exception raised: KeyError
```

Giac [A] time = 1.20153, size = 105, normalized size = 1.94

$$\frac{1}{24}\,bx^{12}\log\left(-\frac{cx^3+1}{cx^3-1}\right) + \frac{1}{12}\,ax^{12} + \frac{bx^9}{36\,c} + \frac{bx^3}{12\,c^3} - \frac{b\log\left(cx^3+1\right)}{24\,c^4} + \frac{b\log\left(cx^3-1\right)}{24\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(a+b*arctanh(c*x^3)),x, algorithm="giac")
```

```
[Out] 1/24*b*x^12*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 1/12*a*x^12 + 1/36*b*x^9/c + 1/
12*b*x^3/c^3 - 1/24*b*log(c*x^3 + 1)/c^4 + 1/24*b*log(c*x^3 - 1)/c^4
```

3.100 $\int x^8 \left(a+b \tanh^{-1}\left(cx^3\right)\right) \, dx$

Optimal. Leaf size=48

$$\frac{1}{9}x^9\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{b\log\left(1-c^2x^6\right)}{18c^3}+\frac{bx^6}{18c}$$

```
[Out] (b*x^6)/(18*c) + (x^9*(a + b*ArcTanh[c*x^3]))/9 + (b*Log[1 - c^2*x^6])/(18*
c^3)
```

Rubi [A] time = 0.0359128, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {6097, 266, 43}

$$\frac{1}{9}x^9\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{b\log\left(1-c^2x^6\right)}{18c^3}+\frac{bx^6}{18c}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a + b*ArcTanh[c*x^3]),x]
```

```
[Out] (b*x^6)/(18*c) + (x^9*(a + b*ArcTanh[c*x^3]))/9 + (b*Log[1 - c^2*x^6])/(18*
c^3)
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x^8\left(a+b\tanh^{-1}\left(cx^3\right)\right)\,dx &= \frac{1}{9}x^9\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{1}{3}(bc)\int\frac{x^{11}}{1-c^2x^6}\,dx\\
&= \frac{1}{9}x^9\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{1}{18}(bc)\,\text{Subst}\left(\int\frac{x}{1-c^2x}\,dx,x,x^6\right)\\
&= \frac{1}{9}x^9\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{1}{18}(bc)\,\text{Subst}\left(\int\left(-\frac{1}{c^2}-\frac{1}{c^2\left(-1+c^2x\right)}\right)dx,x,x^6\right)\\
&= \frac{bx^6}{18c}+\frac{1}{9}x^9\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{b\log\left(1-c^2x^6\right)}{18c^3}
\end{aligned}$$

Mathematica [A] time = 0.0158537, size = 53, normalized size = 1.1

$$\frac{ax^9}{9}+\frac{b\log\left(1-c^2x^6\right)}{18c^3}+\frac{bx^6}{18c}+\frac{1}{9}bx^9\tanh^{-1}\left(cx^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a + b*ArcTanh[c*x^3]),x]
```

```
[Out] (b*x^6)/(18*c) + (a*x^9)/9 + (b*x^9*ArcTanh[c*x^3])/9 + (b*Log[1 - c^2*x^6]
)/(18*c^3)
```

Maple [A] time = 0.008, size = 45, normalized size = 0.9

$$\frac{x^9a}{9}+\frac{bx^9\operatorname{Artanh}\left(cx^3\right)}{9}+\frac{bx^6}{18c}+\frac{b\ln\left(c^2x^6-1\right)}{18c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a+b*arctanh(c*x^3)),x)
```

```
[Out] 1/9*x^9*a+1/9*b*x^9*arctanh(c*x^3)+1/18*b*x^6/c+1/18*b/c^3*ln(c^2*x^6-1)
```

Maxima [A] time = 1.02131, size = 62, normalized size = 1.29

$$\frac{1}{9}ax^9+\frac{1}{18}\left(2x^9\operatorname{artanh}\left(cx^3\right)+\left(\frac{x^6}{c^2}+\frac{\log\left(c^2x^6-1\right)}{c^4}\right)c\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(a+b*arctanh(c*x^3)),x, algorithm="maxima")
```

```
[Out] 1/9*a*x^9 + 1/18*(2*x^9*arctanh(c*x^3) + (x^6/c^2 + log(c^2*x^6 - 1)/c^4)*c
)*b
```

Fricas [A] time = 1.91646, size = 134, normalized size = 2.79

$$\frac{bc^3x^9\log\left(-\frac{cx^3+1}{cx^3-1}\right)+2ac^3x^9+bc^2x^6+b\log\left(c^2x^6-1\right)}{18c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(a+b*arctanh(c*x^3)),x, algorithm="fricas")
```

```
[Out] 1/18*(b*c^3*x^9*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 2*a*c^3*x^9 + b*c^2*x^6 + b
*log(c^2*x^6 - 1))/c^3
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: KeyError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(a+b*atanh(c*x**3)),x)
```

```
[Out] Exception raised: KeyError
```

Giac [A] time = 1.19606, size = 77, normalized size = 1.6

$$\frac{1}{18}\,bx^9\log\left(-\frac{cx^3+1}{cx^3-1}\right)+\frac{1}{9}\,ax^9+\frac{bx^6}{18\,c}+\frac{b\log\left(c^2x^6-1\right)}{18\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(a+b*arctanh(c*x^3)),x, algorithm="giac")
```

```
[Out] 1/18*b*x^9*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 1/9*a*x^9 + 1/18*b*x^6/c + 1/18*
b*log(c^2*x^6 - 1)/c^3
```

3.101 $\int x^5 \left(a + b \tanh^{-1}\left(cx^3\right)\right) dx$

Optimal. Leaf size=43

$$\frac{1}{6}x^6\left(a + b\tanh^{-1}\left(cx^3\right)\right) - \frac{b\tanh^{-1}\left(cx^3\right)}{6c^2} + \frac{bx^3}{6c}$$

```
[Out] (b*x^3)/(6*c) - (b*ArcTanh[c*x^3])/(6*c^2) + (x^6*(a + b*ArcTanh[c*x^3]))/6
```

Rubi [A] time = 0.030026, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6097, 275, 321, 206}

$$\frac{1}{6}x^6\left(a + b\tanh^{-1}\left(cx^3\right)\right) - \frac{b\tanh^{-1}\left(cx^3\right)}{6c^2} + \frac{bx^3}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*ArcTanh[c*x^3]),x]
```

```
[Out] (b*x^3)/(6*c) - (b*ArcTanh[c*x^3])/(6*c^2) + (x^6*(a + b*ArcTanh[c*x^3]))/6
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^5 \left(a+b\tanh^{-1}\left(cx^3\right)\right) dx = \frac{1}{6}x^6\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{1}{2}(bc)\int \frac{x^8}{1-c^2x^6}\,dx$$
$$=\frac{1}{6}x^6\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{1}{6}(bc)\,\text{Subst}\left(\int \frac{x^2}{1-c^2x^2}\,dx,x,x^3\right)$$
$$=\frac{bx^3}{6c}+\frac{1}{6}x^6\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{b\,\text{Subst}\left(\int \frac{1}{1-c^2x^2}\,dx,x,x^3\right)}{6c}$$
$$=\frac{bx^3}{6c}-\frac{b\tanh^{-1}\left(cx^3\right)}{6c^2}+\frac{1}{6}x^6\left(a+b\tanh^{-1}\left(cx^3\right)\right)$$

Mathematica [A] time = 0.0156988, size = 67, normalized size = 1.56

$$\frac{ax^6}{6}+\frac{b\log\left(1-cx^3\right)}{12c^2}-\frac{b\log\left(cx^3+1\right)}{12c^2}+\frac{bx^3}{6c}+\frac{1}{6}bx^6\tanh^{-1}\left(cx^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*ArcTanh[c*x^3]),x]
```

```
[Out] (b*x^3)/(6*c) + (a*x^6)/6 + (b*x^6*ArcTanh[c*x^3])/6 + (b*Log[1 - c*x^3])/(
12*c^2) - (b*Log[1 + c*x^3])/(12*c^2)
```

Maple [A] time = 0.012, size = 57, normalized size = 1.3

$$\frac{x^6a}{6}+\frac{bx^6\text{Artanh}\left(cx^3\right)}{6}+\frac{bx^3}{6c}+\frac{b\ln\left(cx^3-1\right)}{12c^2}-\frac{b\ln\left(cx^3+1\right)}{12c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arctanh(c*x^3)),x)
```

```
[Out] 1/6*x^6*a+1/6*b*x^6*arctanh(c*x^3)+1/6*b*x^3/c+1/12*b/c^2*ln(c*x^3-1)-1/12*
b/c^2*ln(c*x^3+1)
```

Maxima [A] time = 0.979988, size = 78, normalized size = 1.81

$$\frac{1}{6}ax^6+\frac{1}{12}\left(2x^6\,\text{artanh}\left(cx^3\right)+c\left(\frac{2x^3}{c^2}-\frac{\log\left(cx^3+1\right)}{c^3}+\frac{\log\left(cx^3-1\right)}{c^3}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x^3)),x, algorithm="maxima")
```

```
[Out] 1/6*a*x^6 + 1/12*(2*x^6*arctanh(c*x^3) + c*(2*x^3/c^2 - log(c*x^3 + 1)/c^3
+ log(c*x^3 - 1)/c^3))*b
```

Fricas [A] time = 1.79863, size = 113, normalized size = 2.63

$$\frac{2\,ac^2x^6+2\,bcx^3+\left(bc^2x^6-b\right)\log\left(-\frac{cx^3+1}{cx^3-1}\right)}{12\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x^3)),x, algorithm="fricas")
```

```
[Out] 1/12*(2*a*c^2*x^6 + 2*b*c*x^3 + (b*c^2*x^6 - b)*log(-(c*x^3 + 1)/(c*x^3 - 1
)))/c^2
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: KeyError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*atanh(c*x**3)),x)
```

```
[Out] Exception raised: KeyError
```

Giac [A] time = 1.14918, size = 93, normalized size = 2.16

$$\frac{1}{12}\,bx^6\log\left(-\frac{cx^3+1}{cx^3-1}\right)+\frac{1}{6}\,ax^6+\frac{bx^3}{6\,c}-\frac{b\log\left(cx^3+1\right)}{12\,c^2}+\frac{b\log\left(cx^3-1\right)}{12\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x^3)),x, algorithm="giac")
```

```
[Out] 1/12*b*x^6*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 1/6*a*x^6 + 1/6*b*x^3/c - 1/12*b
*log(c*x^3 + 1)/c^2 + 1/12*b*log(c*x^3 - 1)/c^2
```

3.102 $\int x^2 \left(a+b\tanh^{-1}\left(cx^3\right)\right) dx$

Optimal. Leaf size=37

$$\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{b\log\left(1-c^2x^6\right)}{6c}$$

```
[Out] (x^3*(a + b*ArcTanh[c*x^3]))/3 + (b*Log[1 - c^2*x^6])/(6*c)
```

Rubi [A] time = 0.020476, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6097, 260}

$$\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{b\log\left(1-c^2x^6\right)}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcTanh[c*x^3]),x]
```

```
[Out] (x^3*(a + b*ArcTanh[c*x^3]))/3 + (b*Log[1 - c^2*x^6])/(6*c)
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+b\tanh^{-1}\left(cx^3\right)\right)dx &= \frac{1}{3}x^3\left(a+b\tanh^{-1}\left(cx^3\right)\right)-(bc)\int\frac{x^5}{1-c^2x^6}\,dx\\ &= \frac{1}{3}x^3\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{b\log\left(1-c^2x^6\right)}{6c}\end{aligned}$$

Mathematica [A] time = 0.008322, size = 42, normalized size = 1.14

$$\frac{ax^3}{3}+\frac{b\log\left(1-c^2x^6\right)}{6c}+\frac{1}{3}bx^3\tanh^{-1}\left(cx^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcTanh[c*x^3]),x]
```

```
[Out] (a*x^3)/3 + (b*x^3*ArcTanh[c*x^3])/3 + (b*Log[1 - c^2*x^6])/(6*c)
```

Maple [A] time = 0.003, size = 37, normalized size = 1.

$$\frac{x^3 a}{3}+\frac{b x^3 \operatorname{Artanh}\left(c x^3\right)}{3}+\frac{b \ln\left(-c^2 x^6+1\right)}{6 c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c*x^3)),x)
```

```
[Out] 1/3*x^3*a+1/3*b*x^3*arctanh(c*x^3)+1/6*b*ln(-c^2*x^6+1)/c
```

Maxima [A] time = 1.01133, size = 50, normalized size = 1.35

$$\frac{1}{3}ax^3+\frac{\left(2 c x^3 \operatorname{artanh}\left(c x^3\right)+\log\left(-c^2 x^6+1\right)\right) b}{6 c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^3)),x, algorithm="maxima")
```

```
[Out] 1/3*a*x^3 + 1/6*(2*c*x^3*arctanh(c*x^3) + log(-c^2*x^6 + 1))*b/c
```

Fricas [A] time = 1.9613, size = 108, normalized size = 2.92

$$\frac{b c x^3 \log\left(-\frac{c x^3+1}{c x^3-1}\right)+2 a c x^3+b \log\left(c^2 x^6-1\right)}{6 c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^3)),x, algorithm="fricas")
```

```
[Out] 1/6*(b*c*x^3*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 2*a*c*x^3 + b*log(c^2*x^6 - 1)
)/c
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: KeyError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c*x**3)),x)
```

```
[Out] Exception raised: KeyError
```

Giac [A] time = 1.1694, size = 66, normalized size = 1.78

$$\frac{1}{3}ax^3+\frac{1}{6}\left(x^3 \log\left(-\frac{c x^3+1}{c x^3-1}\right)+\frac{\log\left(\left|c^2 x^6-1\right|\right)}{c}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^3)),x, algorithm="giac")
```

```
[Out] 1/3*a*x^3 + 1/6*(x^3*log(-(c*x^3 + 1)/(c*x^3 - 1)) + log(abs(c^2*x^6 - 1))/
c)*b
```

3.103 $\int \frac{a+b\tanh^{-1}(cx^3)}{x}\,dx$

Optimal. Leaf size=30

$$-\frac{1}{6}b\text{PolyLog}\left(2,-cx^3\right)+\frac{1}{6}b\text{PolyLog}\left(2,cx^3\right)+a\log(x)$$

```
[Out] a*Log[x] - (b*PolyLog[2, -(c*x^3)])/6 + (b*PolyLog[2, c*x^3])/6
```

Rubi [A] time = 0.0337, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6095, 5912}

$$-\frac{1}{6}b\text{PolyLog}\left(2,-cx^3\right)+\frac{1}{6}b\text{PolyLog}\left(2,cx^3\right)+a\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^3])/x,x]
```

```
[Out] a*Log[x] - (b*PolyLog[2, -(c*x^3)])/6 + (b*PolyLog[2, c*x^3])/6
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[
1/n, Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
 n}, x] && IGtQ[p, 0]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}\left(cx^3\right)}{x}\,dx&=\frac{1}{3}\text{Subst}\left(\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx,x,x^3\right)\\&=a\log(x)-\frac{1}{6}b\text{Li}_2\left(-cx^3\right)+\frac{1}{6}b\text{Li}_2\left(cx^3\right)\end{aligned}$$

Mathematica [A] time = 0.0136819, size = 28, normalized size = 0.93

$$\frac{1}{6}b\left(\text{PolyLog}\left(2,cx^3\right)-\text{PolyLog}\left(2,-cx^3\right)\right)+a\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^3])/x,x]
```

```
[Out] a*Log[x] + (b*(-PolyLog[2, -(c*x^3)] + PolyLog[2, c*x^3]))/6
```

Maple [C] time = 0.029, size = 92, normalized size = 3.1

$$a\ln(x)+b\ln(x)\operatorname{Artanh}\left(cx^3\right)+\frac{b}{2}\sum_{\_R1=\operatorname{RootOf}\left(c\_Z^3-1\right)}\ln(x)\ln\left(\frac{\_R1-x}{\_R1}\right)+\operatorname{dilog}\left(\frac{\_R1-x}{\_R1}\right)-\frac{b}{2}\sum_{\_R1=\operatorname{RootOf}\left(c\_Z^3+1\right)}\ln($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^3))/x,x)
```

```
[Out] a*ln(x)+b*ln(x)*arctanh(c*x^3)+1/2*b*sum(ln(x)*ln((_R1-x)/_R1)+dilog((_R1-x
)/_R1),_R1=RootOf(_Z^3*c-1))-1/2*b*sum(ln(x)*ln((_R1-x)/_R1)+dilog((_R1-x)/
_R1),_R1=RootOf(_Z^3*c+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}b\int\frac{\log\left(cx^3+1\right)-\log\left(-cx^3+1\right)}{x}\,dx+a\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/x,x, algorithm="maxima")
```

```
[Out] 1/2*b*integrate((log(c*x^3 + 1) - log(-c*x^3 + 1))/x, x) + a*log(x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{b\operatorname{artanh}\left(cx^3\right)+a}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/x,x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*x^3) + a)/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**3))/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{b\operatorname{artanh}\left(cx^3\right)+a}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^3) + a)/x, x)
```

3.104 $\int \frac{a+b\tanh^{-1}(cx^3)}{x^4}\,dx$

Optimal. Leaf size=40

$$-\frac{a+b\tanh^{-1}(cx^3)}{3x^3}-\frac{1}{6}bc\log(1-c^2x^6)+bc\log(x)$$

```
[Out] -(a + b*ArcTanh[c*x^3])/(3*x^3) + b*c*Log[x] - (b*c*Log[1 - c^2*x^6])/6
```

Rubi [A] time = 0.0270264, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6097, 266, 36, 29, 31}

$$-\frac{a+b\tanh^{-1}(cx^3)}{3x^3}-\frac{1}{6}bc\log(1-c^2x^6)+bc\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^3])/x^4,x]
```

```
[Out] -(a + b*ArcTanh[c*x^3])/(3*x^3) + b*c*Log[x] - (b*c*Log[1 - c^2*x^6])/6
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}\left(cx^3\right)}{x^4}\,dx = -\frac{a+b\tanh^{-1}\left(cx^3\right)}{3x^3} + (bc)\int \frac{1}{x\left(1-c^2x^6\right)}\,dx$$
$$= -\frac{a+b\tanh^{-1}\left(cx^3\right)}{3x^3} + \frac{1}{6}(bc)\,\text{Subst}\left(\int \frac{1}{x\left(1-c^2x\right)}\,dx, x, x^6\right)$$
$$= -\frac{a+b\tanh^{-1}\left(cx^3\right)}{3x^3} + \frac{1}{6}(bc)\,\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^6\right) + \frac{1}{6}\left(bc^3\right)\text{Subst}\left(\int \frac{1}{1-c^2x}\,dx, x, x^6\right)$$
$$= -\frac{a+b\tanh^{-1}\left(cx^3\right)}{3x^3} + bc\log(x) - \frac{1}{6}bc\log\left(1-c^2x^6\right)$$

Mathematica [A] time = 0.0122482, size = 45, normalized size = 1.12

$$-\frac{a}{3x^3} - \frac{1}{6}bc\log\left(1-c^2x^6\right) - \frac{b\tanh^{-1}\left(cx^3\right)}{3x^3} + bc\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^3])/x^4,x]
```

```
[Out] -a/(3*x^3) - (b*ArcTanh[c*x^3])/(3*x^3) + b*c*Log[x] - (b*c*Log[1 - c^2*x^6
])/6
```

Maple [A] time = 0.013, size = 49, normalized size = 1.2

$$-\frac{a}{3\,x^3} - \frac{b\text{Artanh}\left(cx^3\right)}{3\,x^3} + bc\ln(x) - \frac{bc\ln\left(cx^3-1\right)}{6} - \frac{bc\ln\left(cx^3+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^3))/x^4,x)
```

```
[Out] -1/3*a/x^3-1/3*b/x^3*arctanh(c*x^3)+b*c*ln(x)-1/6*b*c*ln(c*x^3-1)-1/6*b*c*l
n(c*x^3+1)
```

Maxima [A] time = 1.02135, size = 55, normalized size = 1.38

$$-\frac{1}{6}\left(c\left(\log\left(c^2x^6-1\right) - \log\left(x^6\right)\right) + \frac{2\,\text{artanh}\left(cx^3\right)}{x^3}\right)b - \frac{a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/x^4,x, algorithm="maxima")
```

```
[Out] -1/6*(c*(log(c^2*x^6 - 1) - log(x^6)) + 2*arctanh(c*x^3)/x^3)*b - 1/3*a/x^3
```

Fricas [A] time = 1.98002, size = 130, normalized size = 3.25

$$-\frac{bcx^3\log\left(c^2x^6-1\right) - 6\,bcx^3\log(x) + b\log\left(-\frac{cx^3+1}{cx^3-1}\right) + 2\,a}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(b*c*x^3*log(c^2*x^6 - 1) - 6*b*c*x^3*log(x) + b*log(-(c*x^3 + 1)/(c*x
^3 - 1)) + 2*a)/x^3
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: KeyError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**3))/x**4,x)
```

```
[Out] Exception raised: KeyError
```

Giac [A] time = 1.13407, size = 69, normalized size = 1.72

$$-\frac{1}{6}\,bc\log\left(c^2x^6-1\right)+bc\log\left(x\right)-\frac{b\log\left(-\frac{cx^3+1}{cx^3-1}\right)}{6\,x^3}-\frac{a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/x^4,x, algorithm="giac")
```

```
[Out] -1/6*b*c*log(c^2*x^6 - 1) + b*c*log(x) - 1/6*b*log(-(c*x^3 + 1)/(c*x^3 - 1)
)/x^3 - 1/3*a/x^3
```

3.105 $\int \frac{a+b\tanh^{-1}(cx^3)}{x^7}\,dx$

Optimal. Leaf size=41

$$-\frac{a+b\tanh^{-1}\left(cx^3\right)}{6x^6}+\frac{1}{6}bc^2\tanh^{-1}\left(cx^3\right)-\frac{bc}{6x^3}$$

```
[Out] -(b*c)/(6*x^3) + (b*c^2*ArcTanh[c*x^3])/6 - (a + b*ArcTanh[c*x^3])/(6*x^6)
```

Rubi [A] time = 0.0270646, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6097, 275, 325, 206}

$$-\frac{a+b\tanh^{-1}\left(cx^3\right)}{6x^6}+\frac{1}{6}bc^2\tanh^{-1}\left(cx^3\right)-\frac{bc}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^3])/x^7,x]
```

```
[Out] -(b*c)/(6*x^3) + (b*c^2*ArcTanh[c*x^3])/6 - (a + b*ArcTanh[c*x^3])/(6*x^6)
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}\left(cx^3\right)}{x^7}\,dx = -\frac{a+b\tanh^{-1}\left(cx^3\right)}{6x^6} + \frac{1}{2}(bc)\int \frac{1}{x^4\left(1-c^2x^6\right)}\,dx$$
$$= -\frac{a+b\tanh^{-1}\left(cx^3\right)}{6x^6} + \frac{1}{6}(bc)\,\text{Subst}\left(\int \frac{1}{x^2\left(1-c^2x^2\right)}\,dx, x, x^3\right)$$
$$= -\frac{bc}{6x^3} - \frac{a+b\tanh^{-1}\left(cx^3\right)}{6x^6} + \frac{1}{6}\left(bc^3\right)\text{Subst}\left(\int \frac{1}{1-c^2x^2}\,dx, x, x^3\right)$$
$$= -\frac{bc}{6x^3} + \frac{1}{6}bc^2\tanh^{-1}\left(cx^3\right) - \frac{a+b\tanh^{-1}\left(cx^3\right)}{6x^6}$$

Mathematica [A] time = 0.0182664, size = 65, normalized size = 1.59

$$-\frac{a}{6x^6} - \frac{1}{12}bc^2\log\left(1-cx^3\right) + \frac{1}{12}bc^2\log\left(cx^3+1\right) - \frac{bc}{6x^3} - \frac{b\tanh^{-1}\left(cx^3\right)}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^3])/x^7,x]
```

```
[Out] -a/(6*x^6) - (b*c)/(6*x^3) - (b*ArcTanh[c*x^3])/(6*x^6) - (b*c^2*Log[1 - c*
x^3])/12 + (b*c^2*Log[1 + c*x^3])/12
```

Maple [A] time = 0.013, size = 55, normalized size = 1.3

$$-\frac{a}{6\,x^6} - \frac{b\text{Artanh}\left(cx^3\right)}{6\,x^6} - \frac{bc^2\ln\left(cx^3-1\right)}{12} + \frac{bc^2\ln\left(cx^3+1\right)}{12} - \frac{bc}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^3))/x^7,x)
```

```
[Out] -1/6*a/x^6-1/6*b/x^6*arctanh(c*x^3)-1/12*b*c^2*ln(c*x^3-1)+1/12*b*c^2*ln(c*
x^3+1)-1/6*b*c/x^3
```

Maxima [A] time = 1.01283, size = 69, normalized size = 1.68

$$\frac{1}{12}\left(\left(c\log\left(cx^3+1\right) - c\log\left(cx^3-1\right) - \frac{2}{x^3}\right)c - \frac{2\,\text{artanh}\left(cx^3\right)}{x^6}\right)b - \frac{a}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/x^7,x, algorithm="maxima")
```

```
[Out] 1/12*((c*log(c*x^3 + 1) - c*log(c*x^3 - 1) - 2/x^3)*c - 2*arctanh(c*x^3)/x^
6)*b - 1/6*a/x^6
```

Fricas [A] time = 1.93639, size = 104, normalized size = 2.54

$$-\frac{2\,bcx^3 - \left(bc^2x^6 - b\right)\log\left(-\frac{cx^3+1}{cx^3-1}\right) + 2\,a}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/x^7,x, algorithm="fricas")
```

```
[Out] -1/12*(2*b*c*x^3 - (b*c^2*x^6 - b)*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 2*a)/x^6
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**3))/x**7,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16228, size = 90, normalized size = 2.2

$$\frac{1}{12}\,bc^2\log\left(cx^3+1\right) - \frac{1}{12}\,bc^2\log\left(cx^3-1\right) - \frac{b\log\left(-\frac{cx^3+1}{cx^3-1}\right)}{12\,x^6} - \frac{bcx^3+a}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/x^7,x, algorithm="giac")
```

```
[Out] 1/12*b*c^2*log(c*x^3 + 1) - 1/12*b*c^2*log(c*x^3 - 1) - 1/12*b*log(-(c*x^3
+ 1)/(c*x^3 - 1))/x^6 - 1/6*(b*c*x^3 + a)/x^6
```

3.106 $\int \frac{a+b\tanh^{-1}(cx^3)}{x^{10}} dx$

Optimal. Leaf size=56

$$-\frac{a+b\tanh^{-1}\left(cx^3\right)}{9x^9}-\frac{1}{18}bc^3\log\left(1-c^2x^6\right)+\frac{1}{3}bc^3\log(x)-\frac{bc}{18x^6}$$

```
[Out] -(b*c)/(18*x^6) - (a + b*ArcTanh[c*x^3])/(9*x^9) + (b*c^3*Log[x])/3 - (b*c^
3*Log[1 - c^2*x^6])/18
```

Rubi [A] time = 0.0363813, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {6097, 266, 44}

$$-\frac{a+b\tanh^{-1}\left(cx^3\right)}{9x^9}-\frac{1}{18}bc^3\log\left(1-c^2x^6\right)+\frac{1}{3}bc^3\log(x)-\frac{bc}{18x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^3])/x^10,x]
```

```
[Out] -(b*c)/(18*x^6) - (a + b*ArcTanh[c*x^3])/(9*x^9) + (b*c^3*Log[x])/3 - (b*c^
3*Log[1 - c^2*x^6])/18
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\tanh^{-1}\left(cx^3\right)}{x^{10}}\,dx &= -\frac{a+b\tanh^{-1}\left(cx^3\right)}{9x^9}+\frac{1}{3}(bc)\int \frac{1}{x^7\left(1-c^2x^6\right)}\,dx \\
&= -\frac{a+b\tanh^{-1}\left(cx^3\right)}{9x^9}+\frac{1}{18}(bc)\,\text{Subst}\left(\int \frac{1}{x^2\left(1-c^2x\right)}\,dx,x,x^6\right) \\
&= -\frac{a+b\tanh^{-1}\left(cx^3\right)}{9x^9}+\frac{1}{18}(bc)\,\text{Subst}\left(\int \left(\frac{1}{x^2}+\frac{c^2}{x}-\frac{c^4}{-1+c^2x}\right)dx,x,x^6\right) \\
&= -\frac{bc}{18x^6}-\frac{a+b\tanh^{-1}\left(cx^3\right)}{9x^9}+\frac{1}{3}bc^3\log(x)-\frac{1}{18}bc^3\log\left(1-c^2x^6\right)
\end{aligned}$$

Mathematica [A] time = 0.0117102, size = 61, normalized size = 1.09

$$-\frac{a}{9x^9}-\frac{1}{18}bc^3\log\left(1-c^2x^6\right)+\frac{1}{3}bc^3\log(x)-\frac{bc}{18x^6}-\frac{b\tanh^{-1}\left(cx^3\right)}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^3])/x^10,x]
```

```
[Out] -a/(9*x^9) - (b*c)/(18*x^6) - (b*ArcTanh[c*x^3])/(9*x^9) + (b*c^3*Log[x])/3
 - (b*c^3*Log[1 - c^2*x^6])/18
```

Maple [A] time = 0.014, size = 63, normalized size = 1.1

$$-\frac{a}{9\,x^9}-\frac{b\text{Artanh}\left(cx^3\right)}{9\,x^9}-\frac{bc}{18\,x^6}+\frac{bc^3\ln(x)}{3}-\frac{bc^3\ln\left(cx^3-1\right)}{18}-\frac{bc^3\ln\left(cx^3+1\right)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^3))/x^10,x)
```

```
[Out] -1/9*a/x^9-1/9*b/x^9*arctanh(c*x^3)-1/18*b*c/x^6+1/3*b*c^3*ln(x)-1/18*b*c^3
*ln(c*x^3-1)-1/18*b*c^3*ln(c*x^3+1)
```

Maxima [A] time = 0.964576, size = 69, normalized size = 1.23

$$-\frac{1}{18}\left(\left(c^2\log\left(c^2x^6-1\right)-c^2\log\left(x^6\right)+\frac{1}{x^6}\right)c+\frac{2\,\text{artanh}\left(cx^3\right)}{x^9}\right)b-\frac{a}{9\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/x^10,x, algorithm="maxima")
```

```
[Out] -1/18*((c^2*log(c^2*x^6 - 1) - c^2*log(x^6) + 1/x^6)*c + 2*arctanh(c*x^3)/x
^9)*b - 1/9*a/x^9
```

Fricas [A] time = 2.15736, size = 150, normalized size = 2.68

$$-\frac{bc^3x^9\log\left(c^2x^6-1\right)-6\,bc^3x^9\log\left(x\right)+bcx^3+b\log\left(-\frac{cx^3+1}{cx^3-1}\right)+2\,a}{18\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/x^10,x, algorithm="fricas")
```

```
[Out] -1/18*(b*c^3*x^9*log(c^2*x^6 - 1) - 6*b*c^3*x^9*log(x) + b*c*x^3 + b*log(-(
c*x^3 + 1)/(c*x^3 - 1)) + 2*a)/x^9
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**3))/x**10,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17147, size = 88, normalized size = 1.57

$$-\frac{1}{18}\,bc^3\log\left(c^2x^6-1\right)+\frac{1}{3}\,bc^3\log\left(x\right)-\frac{b\log\left(-\frac{cx^3+1}{cx^3-1}\right)}{18\,x^9}-\frac{bcx^3+2\,a}{18\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/x^10,x, algorithm="giac")
```

```
[Out] -1/18*b*c^3*log(c^2*x^6 - 1) + 1/3*b*c^3*log(x) - 1/18*b*log(-(c*x^3 + 1)/(
c*x^3 - 1))/x^9 - 1/18*(b*c*x^3 + 2*a)/x^9
```

3.107 $\int x^3 \left(a + b \tanh^{-1}\left(cx^3\right)\right) dx$

Optimal. Leaf size=174

$$\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{b\log\left(c^{2/3}x^2-\sqrt[3]{c}x+1\right)}{16c^{4/3}}-\frac{b\log\left(c^{2/3}x^2+\sqrt[3]{c}x+1\right)}{16c^{4/3}}+\frac{\sqrt{3}b\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right)}{8c^{4/3}}-\frac{\sqrt{3}b\tan$$

```
[Out] (3*b*x)/(4*c) + (Sqrt[3]*b*ArcTan[1/Sqrt[3] - (2*c^(1/3)*x)/Sqrt[3]])/(8*c^
(4/3)) - (Sqrt[3]*b*ArcTan[1/Sqrt[3] + (2*c^(1/3)*x)/Sqrt[3]])/(8*c^(4/3))
- (b*ArcTanh[c^(1/3)*x])/(4*c^(4/3)) + (x^4*(a + b*ArcTanh[c*x^3]))/4 + (b*
Log[1 - c^(1/3)*x + c^(2/3)*x^2])/(16*c^(4/3)) - (b*Log[1 + c^(1/3)*x + c^(
2/3)*x^2])/(16*c^(4/3))
```

Rubi [A] time = 0.219734, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6097, 321, 210, 634, 618, 204, 628, 206}

$$\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{b\log\left(c^{2/3}x^2-\sqrt[3]{c}x+1\right)}{16c^{4/3}}-\frac{b\log\left(c^{2/3}x^2+\sqrt[3]{c}x+1\right)}{16c^{4/3}}+\frac{\sqrt{3}b\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right)}{8c^{4/3}}-\frac{\sqrt{3}b\tan$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcTanh[c*x^3]),x]
```

```
[Out] (3*b*x)/(4*c) + (Sqrt[3]*b*ArcTan[1/Sqrt[3] - (2*c^(1/3)*x)/Sqrt[3]])/(8*c^
(4/3)) - (Sqrt[3]*b*ArcTan[1/Sqrt[3] + (2*c^(1/3)*x)/Sqrt[3]])/(8*c^(4/3))
- (b*ArcTanh[c^(1/3)*x])/(4*c^(4/3)) + (x^4*(a + b*ArcTanh[c*x^3]))/4 + (b*
Log[1 - c^(1/3)*x + c^(2/3)*x^2])/(16*c^(4/3)) - (b*Log[1 + c^(1/3)*x + c^(
2/3)*x^2])/(16*c^(4/3))
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^3\left(a+b\tanh^{-1}\left(cx^3\right)\right)dx=\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{1}{4}(3bc)\int\frac{x^6}{1-c^2x^6}\,dx$$

$$=\frac{3bx}{4c}+\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{(3b)\int\frac{1}{1-c^2x^6}\,dx}{4c}$$

$$=\frac{3bx}{4c}+\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{b\int\frac{1}{1-c^{2/3}x^2}\,dx}{4c}-\frac{b\int\frac{1-\frac{\sqrt[3]{c}x}{2}}{1-\sqrt[3]{c}x+c^{2/3}x^2}\,dx}{4c}-\frac{b\int\frac{1+\frac{\sqrt[3]{c}x}{2}}{1+\sqrt[3]{c}x+c^{2/3}x^2}}{4c}$$

$$=\frac{3bx}{4c}-\frac{b\tanh^{-1}\left(\sqrt[3]{c}x\right)}{4c^{4/3}}+\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{b\int\frac{-\sqrt[3]{c}+2c^{2/3}x}{1-\sqrt[3]{c}x+c^{2/3}x^2}\,dx}{16c^{4/3}}-\frac{b\int\frac{\sqrt[3]{c}+2c^{2/3}x}{1+\sqrt[3]{c}x+c^{2/3}}}{16c^{4/3}}$$

$$=\frac{3bx}{4c}-\frac{b\tanh^{-1}\left(\sqrt[3]{c}x\right)}{4c^{4/3}}+\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{b\log\left(1-\sqrt[3]{c}x+c^{2/3}x^2\right)}{16c^{4/3}}-\frac{b\log(1}{}$$

$$=\frac{3bx}{4c}+\frac{\sqrt{3}b\tan^{-1}\left(\frac{1-2\sqrt[3]{c}x}{\sqrt{3}}\right)}{8c^{4/3}}-\frac{\sqrt{3}b\tan^{-1}\left(\frac{1+2\sqrt[3]{c}x}{\sqrt{3}}\right)}{8c^{4/3}}-\frac{b\tanh^{-1}\left(\sqrt[3]{c}x\right)}{4c^{4/3}}+\frac{1}{4}x^4\left(a+b\tanh\right.$$

Mathematica [A] time = 0.0424553, size = 196, normalized size = 1.13

$$\frac{ax^4}{4}+\frac{b\log\left(c^{2/3}x^2-\sqrt[3]{c}x+1\right)}{16c^{4/3}}-\frac{b\log\left(c^{2/3}x^2+\sqrt[3]{c}x+1\right)}{16c^{4/3}}+\frac{b\log\left(1-\sqrt[3]{c}x\right)}{8c^{4/3}}-\frac{b\log\left(\sqrt[3]{c}x+1\right)}{8c^{4/3}}-\frac{\sqrt{3}b\tan^{-1}\left(\frac{2\sqrt[3]{c}x-1}{\sqrt{3}}\right)}{8c^{4/3}}-$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcTanh[c*x^3]),x]
```

```
[Out] (3*b*x)/(4*c) + (a*x^4)/4 - (Sqrt[3]*b*ArcTan[(-1 + 2*c^(1/3)*x)/Sqrt[3]])/
(8*c^(4/3)) - (Sqrt[3]*b*ArcTan[(1 + 2*c^(1/3)*x)/Sqrt[3]])/(8*c^(4/3)) + (
b*x^4*ArcTanh[c*x^3])/4 + (b*Log[1 - c^(1/3)*x])/(8*c^(4/3)) - (b*Log[1 + c
^(1/3)*x])/(8*c^(4/3)) + (b*Log[1 - c^(1/3)*x + c^(2/3)*x^2])/(16*c^(4/3))
- (b*Log[1 + c^(1/3)*x + c^(2/3)*x^2])/(16*c^(4/3))
```

Maple [A] time = 0.015, size = 184, normalized size = 1.1

$$\frac{x^4 a}{4}+\frac{b x^4 \operatorname{Artanh}\left(c x^3\right)}{4}+\frac{3 b x}{4 c}+\frac{b}{8 c^2} \ln\left(x-\sqrt[3]{c^{-1}}\right)\left(c^{-1}\right)^{-\frac{2}{3}}-\frac{b}{16 c^2} \ln\left(x^2+\sqrt[3]{c^{-1}}x+\left(c^{-1}\right)^{\frac{2}{3}}\right)\left(c^{-1}\right)^{-\frac{2}{3}}-\frac{b\sqrt{3}}{8 c^2} \operatorname{arctan}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c*x^3)),x)
```

```
[Out] 1/4*x^4*a+1/4*b*x^4*arctanh(c*x^3)+3/4*b*x/c+1/8*b/c^2/(1/c)^(2/3)*ln(x-(1/
c)^(1/3))-1/16*b/c^2/(1/c)^(2/3)*ln(x^2+(1/c)^(1/3)*x+(1/c)^(2/3))-1/8*b/c^
2/(1/c)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x+1))-1/8*b/c^2/(1/
c)^(2/3)*ln(x+(1/c)^(1/3))+1/16*b/c^2/(1/c)^(2/3)*ln(x^2-(1/c)^(1/3)*x+(1/c
)^(2/3))-1/8*b/c^2/(1/c)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x-
1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^3)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.07704, size = 2611, normalized size = 15.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^3)),x, algorithm="fricas")
```

```
[Out] [1/16*(2*b*c^2*x^4*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 4*a*c^2*x^4 + sqrt(3)*b*
c*sqrt((-c)^(1/3)/c)*log((2*c*x^3 - sqrt(3)*(2*c*x^2 + (-c)^(2/3)*x + (-c)^
(1/3))*sqrt((-c)^(1/3)/c) + 3*(-c)^(1/3)*x - 1)/(c*x^3 + 1)) + sqrt(3)*b*c*
sqrt(-1/c^(2/3))*log((2*c*x^3 - sqrt(3)*(2*c*x^2 - c^(2/3)*x - c^(1/3))*sqr
t(-1/c^(2/3)) - 3*c^(1/3)*x + 1)/(c*x^3 - 1)) + 12*b*c*x + b*(-c)^(2/3)*log
(c*x^2 - (-c)^(2/3)*x - (-c)^(1/3)) - b*c^(2/3)*log(c*x^2 + c^(2/3)*x + c^(
1/3)) - 2*b*(-c)^(2/3)*log(c*x + (-c)^(2/3)) + 2*b*c^(2/3)*log(c*x - c^(2/3
)))/c^2, 1/16*(2*b*c^2*x^4*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 4*a*c^2*x^4 - 2*
sqrt(3)*b*c*sqrt(-(-c)^(1/3)/c)*arctan(1/3*sqrt(3)*(2*(-c)^(2/3)*x + (-c)^(
1/3))*sqrt(-(-c)^(1/3)/c)) + sqrt(3)*b*c*sqrt(-1/c^(2/3))*log((2*c*x^3 - sq
rt(3)*(2*c*x^2 - c^(2/3)*x - c^(1/3))*sqrt(-1/c^(2/3)) - 3*c^(1/3)*x + 1)/(
c*x^3 - 1)) + 12*b*c*x + b*(-c)^(2/3)*log(c*x^2 - (-c)^(2/3)*x - (-c)^(1/3)
```

```
) - b*c^(2/3)*log(c*x^2 + c^(2/3)*x + c^(1/3)) - 2*b*(-c)^(2/3)*log(c*x + (
-c)^(2/3)) + 2*b*c^(2/3)*log(c*x - c^(2/3)))/c^2, 1/16*(2*b*c^2*x^4*log(-(c
*x^3 + 1)/(c*x^3 - 1)) + 4*a*c^2*x^4 + sqrt(3)*b*c*sqrt((-c)^(1/3)/c)*log((
2*c*x^3 - sqrt(3)*(2*c*x^2 + (-c)^(2/3)*x + (-c)^(1/3))*sqrt((-c)^(1/3)/c)
+ 3*(-c)^(1/3)*x - 1)/(c*x^3 + 1)) - 2*sqrt(3)*b*c^(2/3)*arctan(1/3*sqrt(3)
*(2*c^(2/3)*x + c^(1/3))/c^(1/3)) + 12*b*c*x + b*(-c)^(2/3)*log(c*x^2 - (-c
)^(2/3)*x - (-c)^(1/3)) - b*c^(2/3)*log(c*x^2 + c^(2/3)*x + c^(1/3)) - 2*b*
(-c)^(2/3)*log(c*x + (-c)^(2/3)) + 2*b*c^(2/3)*log(c*x - c^(2/3)))/c^2, 1/1
6*(2*b*c^2*x^4*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 4*a*c^2*x^4 - 2*sqrt(3)*b*c*
sqrt(-(-c)^(1/3)/c)*arctan(1/3*sqrt(3)*(2*(-c)^(2/3)*x + (-c)^(1/3))*sqrt(-
(-c)^(1/3)/c)) - 2*sqrt(3)*b*c^(2/3)*arctan(1/3*sqrt(3)*(2*c^(2/3)*x + c^(1
/3))/c^(1/3)) + 12*b*c*x + b*(-c)^(2/3)*log(c*x^2 - (-c)^(2/3)*x - (-c)^(1/
3)) - b*c^(2/3)*log(c*x^2 + c^(2/3)*x + c^(1/3)) - 2*b*(-c)^(2/3)*log(c*x +
 (-c)^(2/3)) + 2*b*c^(2/3)*log(c*x - c^(2/3)))/c^2]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: KeyError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c*x**3)),x)
```

```
[Out] Exception raised: KeyError
```

Giac [A] time = 1.31662, size = 279, normalized size = 1.6

$$\frac{1}{16}bc^7\left(\frac{2\left(-\frac{1}{c}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{1}{c}\right)^{\frac{1}{3}}\right|\right)}{c^8}-\frac{2\sqrt{3}|c|^{\frac{2}{3}}\arctan\left(\frac{1}{3}\sqrt{3}c^{\frac{1}{3}}\left(2x+\frac{1}{c^{\frac{1}{3}}}\right)\right)}{c^9}-\frac{2\sqrt{3}\left(-c^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{1}{c}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{1}{c}\right)^{\frac{1}{3}}}\right)}{c^9}-\frac{|c|^{\frac{2}{3}}\log \ldots}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^3)),x, algorithm="giac")
```

```
[Out] 1/16*b*c^7*(2*(-1/c)^(1/3)*log(abs(x - (-1/c)^(1/3)))/c^8 - 2*sqrt(3)*abs(c
)^(2/3)*arctan(1/3*sqrt(3)*c^(1/3)*(2*x + 1/c^(1/3)))/c^9 - 2*sqrt(3)*(-c^2
)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-1/c)^(1/3))/(-1/c)^(1/3))/c^9 - abs(c)^
(2/3)*log(x^2 + x/c^(1/3) + 1/c^(2/3))/c^9 + 2*log(abs(x - 1/c^(1/3)))/c^(2
5/3) - (-c^2)^(1/3)*log(x^2 + x*(-1/c)^(1/3) + (-1/c)^(2/3))/c^9) + 1/8*b*x
^4*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 1/4*a*x^4 + 3/4*b*x/c
```

3.108 $\int \left(a+b \tanh^{-1}\left(c x^3\right)\right) \, dx$

Optimal. Leaf size=101

$$ax+\frac{b \log \left(1-c^{2/3} x^2\right)}{2 \sqrt[3]{c}}-\frac{b \log \left(c^{4/3} x^4+c^{2/3} x^2+1\right)}{4 \sqrt[3]{c}}+\frac{\sqrt{3} b \tan^{-1}\left(\frac{2 c^{2/3} x^2+1}{\sqrt{3}}\right)}{2 \sqrt[3]{c}}+b x \tanh^{-1}\left(c x^3\right)$$

```
[Out] a*x + (Sqrt[3]*b*ArcTan[(1 + 2*c^(2/3)*x^2)/Sqrt[3]])/(2*c^(1/3)) + b*x*Arc
Tanh[c*x^3] + (b*Log[1 - c^(2/3)*x^2])/(2*c^(1/3)) - (b*Log[1 + c^(2/3)*x^2
 + c^(4/3)*x^4])/(4*c^(1/3))
```

Rubi [A] time = 0.0966456, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.8, Rules used = {6091, 275, 292, 31, 634, 617, 204, 628}

$$ax+\frac{b \log \left(1-c^{2/3} x^2\right)}{2 \sqrt[3]{c}}-\frac{b \log \left(c^{4/3} x^4+c^{2/3} x^2+1\right)}{4 \sqrt[3]{c}}+\frac{\sqrt{3} b \tan^{-1}\left(\frac{2 c^{2/3} x^2+1}{\sqrt{3}}\right)}{2 \sqrt[3]{c}}+b x \tanh^{-1}\left(c x^3\right)$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcTanh[c*x^3],x]
```

```
[Out] a*x + (Sqrt[3]*b*ArcTan[(1 + 2*c^(2/3)*x^2)/Sqrt[3]])/(2*c^(1/3)) + b*x*Arc
Tanh[c*x^3] + (b*Log[1 - c^(2/3)*x^2])/(2*c^(1/3)) - (b*Log[1 + c^(2/3)*x^2
 + c^(4/3)*x^4])/(4*c^(1/3))
```

Rule 6091

```
Int[ArcTanh[(c_.)*(x_)^(n_)], x_Symbol] :> Simp[x*ArcTanh[c*x^n], x] - Dist
[c*n, Int[x^n/(1 - c^2*x^(2*n)), x], x] /; FreeQ[{c, n}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \left(a+b\tanh^{-1}\left(cx^3\right)\right)dx = ax+b\int \tanh^{-1}\left(cx^3\right)dx$$
$$= ax+bx\tanh^{-1}\left(cx^3\right)-(3bc)\int \frac{x^3}{1-c^2x^6}\,dx$$
$$= ax+bx\tanh^{-1}\left(cx^3\right)-\frac{1}{2}(3bc)\,\text{Subst}\left(\int \frac{x}{1-c^2x^3}\,dx,x,x^2\right)$$
$$= ax+bx\tanh^{-1}\left(cx^3\right)-\frac{1}{2}\left(b\sqrt[3]{c}\right)\text{Subst}\left(\int \frac{1}{1-c^{2/3}x}\,dx,x,x^2\right)+\frac{1}{2}\left(b\sqrt[3]{c}\right)\text{Subst}\left(\int \frac{1-}{1+c^{2/3}}\right.$$
$$= ax+bx\tanh^{-1}\left(cx^3\right)+\frac{b\log\left(1-c^{2/3}x^2\right)}{2\sqrt[3]{c}}-\frac{b\,\text{Subst}\left(\int \frac{c^{2/3}+2c^{4/3}x}{1+c^{2/3}x+c^{4/3}x^2}\,dx,x,x^2\right)}{4\sqrt[3]{c}}+\frac{1}{4}\left(3b\sqrt[3]{c}\right)\text{S}$$
$$= ax+bx\tanh^{-1}\left(cx^3\right)+\frac{b\log\left(1-c^{2/3}x^2\right)}{2\sqrt[3]{c}}-\frac{b\log\left(1+c^{2/3}x^2+c^{4/3}x^4\right)}{4\sqrt[3]{c}}-\frac{(3b)\,\text{Subst}\left(\int \frac{1}{-3-x}\right.}{2}$$
$$= ax+\frac{\sqrt{3}b\tan^{-1}\left(\frac{1+2c^{2/3}x^2}{\sqrt{3}}\right)}{2\sqrt[3]{c}}+bx\tanh^{-1}\left(cx^3\right)+\frac{b\log\left(1-c^{2/3}x^2\right)}{2\sqrt[3]{c}}-\frac{b\log\left(1+c^{2/3}x^2+c^{4/3}\right.}{4\sqrt[3]{c}}$$

Mathematica [A] time = 0.042819, size = 136, normalized size = 1.35

$$ax-\frac{b\left(\log\left(c^{2/3}x^2-\sqrt[3]{c}x+1\right)+\log\left(c^{2/3}x^2+\sqrt[3]{c}x+1\right)-2\log\left(1-\sqrt[3]{c}x\right)-2\log\left(\sqrt[3]{c}x+1\right)-2\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{c}x-1}{\sqrt{3}}\right)+2\sqrt{3}\right.}{4\sqrt[3]{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcTanh[c*x^3],x]
```

```
[Out] a*x + b*x*ArcTanh[c*x^3] - (b*(-2*Sqrt[3]*ArcTan[(-1 + 2*c^(1/3)*x)/Sqrt[3]
] + 2*Sqrt[3]*ArcTan[(1 + 2*c^(1/3)*x)/Sqrt[3]] - 2*Log[1 - c^(1/3)*x] - 2*
Log[1 + c^(1/3)*x] + Log[1 - c^(1/3)*x + c^(2/3)*x^2] + Log[1 + c^(1/3)*x +
 c^(2/3)*x^2]))/(4*c^(1/3))
```

Maple [A] time = 0.006, size = 99, normalized size = 1.

$$ax+bx\mathrm{Artanh}\left(cx^{3}\right)+\frac{b}{2c}\ln\left(x^{2}-\sqrt[3]{c^{-2}}\right)\frac{1}{\sqrt[3]{c^{-2}}}-\frac{b}{4c}\ln\left(x^{4}+\sqrt[3]{c^{-2}}x^{2}+\left(c^{-2}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{c^{-2}}}+\frac{b\sqrt{3}}{2c}\arctan\left(\frac{\sqrt{3}}{3}\left(2\frac{x^{2}}{\sqrt[3]{c^{-2}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arctanh(c*x^3),x)
```

```
[Out] a*x+b*x*arctanh(c*x^3)+1/2*b/c/(1/c^2)^(1/3)*ln(x^2-(1/c^2)^(1/3))-1/4*b/c/
(1/c^2)^(1/3)*ln(x^4+(1/c^2)^(1/3)*x^2+(1/c^2)^(2/3))+1/2*b*3^(1/2)/c/(1/c^
2)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/c^2)^(1/3)*x^2+1))
```

Maxima [A] time = 1.4484, size = 143, normalized size = 1.42

$$\frac{1}{4}\left(c\left(\frac{2\sqrt{3}\left(c^{2}\right)^{\frac{1}{3}}\arctan\left(\frac{1}{3}\sqrt{3}\left(c^{2}\right)^{\frac{1}{3}}\left(2x^{2}+\frac{1}{c^{2}}^{\frac{1}{3}}\right)\right)}{c^{2}}-\frac{\left(c^{2}\right)^{\frac{1}{3}}\log\left(x^{4}+\frac{1}{c^{2}}^{\frac{1}{3}}x^{2}+\frac{1}{c^{2}}^{\frac{2}{3}}\right)}{c^{2}}+\frac{2\left(c^{2}\right)^{\frac{1}{3}}\log\left(x^{2}-\frac{1}{c^{2}}^{\frac{1}{3}}\right)}{c^{2}}\right)+4x\,\mathrm{arta}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c*x^3),x, algorithm="maxima")
```

```
[Out] 1/4*(c*(2*sqrt(3)*(c^2)^(1/3)*arctan(1/3*sqrt(3)*(c^2)^(1/3)*(2*x^2 + (c^(-
2))^(1/3)))/c^2 - (c^2)^(1/3)*log(x^4 + (c^(-2))^(1/3)*x^2 + (c^(-2))^(2/3)
)/c^2 + 2*(c^2)^(1/3)*log(x^2 - (c^(-2))^(1/3))/c^2) + 4*x*arctanh(c*x^3))*
b + a*x
```

Fricas [A] time = 1.86935, size = 674, normalized size = 6.67

$$\left[\frac{\sqrt{3}bc\sqrt{-\frac{1}{c^{\frac{2}{3}}}}\log\left(\frac{2c^{2}x^{6}-3c^{\frac{2}{3}}x^{2}+\sqrt{3}\left(2c^{\frac{5}{3}}x^{4}-cx^{2}-c^{\frac{1}{3}}\right)\sqrt{-\frac{1}{c^{\frac{2}{3}}}}+1}{c^{2}x^{6}-1}\right)+2bcx\log\left(-\frac{cx^{3}+1}{cx^{3}-1}\right)+4acx-bc^{\frac{2}{3}}\log\left(c^{2}x^{4}+c^{\frac{4}{3}}x^{2}+c^{\frac{2}{3}}\right)+2}{4c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c*x^3),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(3)*b*c*sqrt(-1/c^(2/3))*log((2*c^2*x^6 - 3*c^(2/3)*x^2 + sqrt(3)
*(2*c^(5/3)*x^4 - c*x^2 - c^(1/3))*sqrt(-1/c^(2/3)) + 1)/(c^2*x^6 - 1)) + 2
*b*c*x*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 4*a*c*x - b*c^(2/3)*log(c^2*x^4 + c^
(4/3)*x^2 + c^(2/3)) + 2*b*c^(2/3)*log(c*x^2 - c^(1/3)))/c, 1/4*(2*b*c*x*lo
g(-(c*x^3 + 1)/(c*x^3 - 1)) + 2*sqrt(3)*b*c^(2/3)*arctan(1/3*sqrt(3)*(2*c*x
^2 + c^(1/3))/c^(1/3)) + 4*a*c*x - b*c^(2/3)*log(c^2*x^4 + c^(4/3)*x^2 + c^
(2/3)) + 2*b*c^(2/3)*log(c*x^2 - c^(1/3)))/c]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*atanh(c*x**3),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18351, size = 147, normalized size = 1.46

$$\frac{1}{4}\left(c\left(\frac{2\sqrt{3}|c|^{\frac{2}{3}}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+\frac{1}{|c|^{\frac{2}{3}}}\right)|c|^{\frac{2}{3}}\right)}{c^2}-\frac{|c|^{\frac{2}{3}}\log\left(x^4+\frac{x^2}{|c|^{\frac{2}{3}}}+\frac{1}{|c|^{\frac{4}{3}}}\right)}{c^2}+\frac{2\log\left(\left|x^2-\frac{1}{|c|^{\frac{2}{3}}}\right|\right)}{|c|^{\frac{4}{3}}}\right)+2x\log\left(-\frac{cx^3+1}{cx^3-1}\right)\right)b+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c*x^3),x, algorithm="giac")
```

```
[Out] 1/4*(c*(2*sqrt(3)*abs(c)^(2/3)*arctan(1/3*sqrt(3)*(2*x^2 + 1/abs(c)^(2/3))*
abs(c)^(2/3))/c^2 - abs(c)^(2/3)*log(x^4 + x^2/abs(c)^(2/3) + 1/abs(c)^(4/3
))/c^2 + 2*log(abs(x^2 - 1/abs(c)^(2/3)))/abs(c)^(4/3)) + 2*x*log(-(c*x^3 +
 1)/(c*x^3 - 1)))*b + a*x
```

3.109 $\int \frac{a+b\tanh^{-1}(cx^3)}{x^3}\,dx$

Optimal. Leaf size=165

$$-\frac{a+b\tanh^{-1}\left(cx^3\right)}{2x^2}-\frac{1}{8}bc^{2/3}\log\left(c^{2/3}x^2-\sqrt[3]{c}x+1\right)+\frac{1}{8}bc^{2/3}\log\left(c^{2/3}x^2+\sqrt[3]{c}x+1\right)-\frac{1}{4}\sqrt{3}bc^{2/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right)$$

```
[Out] -(Sqrt[3]*b*c^(2/3)*ArcTan[1/Sqrt[3] - (2*c^(1/3)*x)/Sqrt[3]])/4 + (Sqrt[3]
*b*c^(2/3)*ArcTan[1/Sqrt[3] + (2*c^(1/3)*x)/Sqrt[3]])/4 + (b*c^(2/3)*ArcTan
h[c^(1/3)*x])/2 - (a + b*ArcTanh[c*x^3])/(2*x^2) - (b*c^(2/3)*Log[1 - c^(1/
3)*x + c^(2/3)*x^2])/8 + (b*c^(2/3)*Log[1 + c^(1/3)*x + c^(2/3)*x^2])/8
```

Rubi [A] time = 0.197406, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6097, 210, 634, 618, 204, 628, 206}

$$-\frac{a+b\tanh^{-1}\left(cx^3\right)}{2x^2}-\frac{1}{8}bc^{2/3}\log\left(c^{2/3}x^2-\sqrt[3]{c}x+1\right)+\frac{1}{8}bc^{2/3}\log\left(c^{2/3}x^2+\sqrt[3]{c}x+1\right)-\frac{1}{4}\sqrt{3}bc^{2/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^3])/x^3,x]
```


```
[Out] -(Sqrt[3]*b*c^(2/3)*ArcTan[1/Sqrt[3] - (2*c^(1/3)*x)/Sqrt[3]])/4 + (Sqrt[3]
*b*c^(2/3)*ArcTan[1/Sqrt[3] + (2*c^(1/3)*x)/Sqrt[3]])/4 + (b*c^(2/3)*ArcTan
h[c^(1/3)*x])/2 - (a + b*ArcTanh[c*x^3])/(2*x^2) - (b*c^(2/3)*Log[1 - c^(1/
3)*x + c^(2/3)*x^2])/8 + (b*c^(2/3)*Log[1 + c^(1/3)*x + c^(2/3)*x^2])/8
```


Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}\left(cx^3\right)}{x^3}\,dx = -\frac{a+b\tanh^{-1}\left(cx^3\right)}{2x^2}+\frac{1}{2}(3bc)\int \frac{1}{1-c^2x^6}\,dx$$

$$= -\frac{a+b\tanh^{-1}\left(cx^3\right)}{2x^2}+\frac{1}{2}(bc)\int \frac{1}{1-c^{2/3}x^2}\,dx+\frac{1}{2}(bc)\int \frac{1-\frac{\sqrt[3]{c}x}{2}}{1-\sqrt[3]{c}x+c^{2/3}x^2}\,dx+\frac{1}{2}(bc)\int \frac{}{1+}$$

$$= \frac{1}{2}bc^{2/3}\tanh^{-1}\left(\sqrt[3]{c}x\right)-\frac{a+b\tanh^{-1}\left(cx^3\right)}{2x^2}-\frac{1}{8}\left(bc^{2/3}\right)\int \frac{-\sqrt[3]{c}+2c^{2/3}x}{1-\sqrt[3]{c}x+c^{2/3}x^2}\,dx+\frac{1}{8}\left(bc^{2/3}\right)\int \frac{}{1}$$

$$= \frac{1}{2}bc^{2/3}\tanh^{-1}\left(\sqrt[3]{c}x\right)-\frac{a+b\tanh^{-1}\left(cx^3\right)}{2x^2}-\frac{1}{8}bc^{2/3}\log\left(1-\sqrt[3]{c}x+c^{2/3}x^2\right)+\frac{1}{8}bc^{2/3}\log\left(1+\right.$$

$$= -\frac{1}{4}\sqrt{3}bc^{2/3}\tan^{-1}\left(\frac{1-2\sqrt[3]{c}x}{\sqrt{3}}\right)+\frac{1}{4}\sqrt{3}bc^{2/3}\tan^{-1}\left(\frac{1+2\sqrt[3]{c}x}{\sqrt{3}}\right)+\frac{1}{2}bc^{2/3}\tanh^{-1}\left(\sqrt[3]{c}x\right)-\frac{a+b}{}$$

Mathematica [A] time = 0.0523718, size = 187, normalized size = 1.13

$$-\frac{a}{2x^2}-\frac{1}{8}bc^{2/3}\log\left(c^{2/3}x^2-\sqrt[3]{c}x+1\right)+\frac{1}{8}bc^{2/3}\log\left(c^{2/3}x^2+\sqrt[3]{c}x+1\right)-\frac{1}{4}bc^{2/3}\log\left(1-\sqrt[3]{c}x\right)+\frac{1}{4}bc^{2/3}\log\left(\sqrt[3]{c}x+1\right)+\frac{1}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^3])/x^3,x]
```

```
[Out] -a/(2*x^2) + (Sqrt[3]*b*c^(2/3)*ArcTan[(-1 + 2*c^(1/3)*x)/Sqrt[3]])/4 + (Sq
rt[3]*b*c^(2/3)*ArcTan[(1 + 2*c^(1/3)*x)/Sqrt[3]])/4 - (b*ArcTanh[c*x^3])/(
2*x^2) - (b*c^(2/3)*Log[1 - c^(1/3)*x])/4 + (b*c^(2/3)*Log[1 + c^(1/3)*x])/
4 - (b*c^(2/3)*Log[1 - c^(1/3)*x + c^(2/3)*x^2])/8 + (b*c^(2/3)*Log[1 + c^(
1/3)*x + c^(2/3)*x^2])/8
```

Maple [A] time = 0.01, size = 159, normalized size = 1.

$$-\frac{a}{2\,x^2}-\frac{b\mathrm{Artanh}\left(cx^3\right)}{2\,x^2}-\frac{b}{4}\ln\left(x-\sqrt[3]{c^{-1}}\right)\left(c^{-1}\right)^{-\frac{2}{3}}+\frac{b}{8}\ln\left(x^2+\sqrt[3]{c^{-1}}x+\left(c^{-1}\right)^{\frac{2}{3}}\right)\left(c^{-1}\right)^{-\frac{2}{3}}+\frac{b\sqrt{3}}{4}\arctan\left(\frac{\sqrt{3}}{3}\left(2\frac{x}{\sqrt[3]{c^{-1}}}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^3))/x^3,x)
```

```
[Out] -1/2*a/x^2-1/2*b/x^2*arctanh(c*x^3)-1/4*b/(1/c)^(2/3)*ln(x-(1/c)^(1/3))+1/8
*b/(1/c)^(2/3)*ln(x^2+(1/c)^(1/3)*x+(1/c)^(2/3))+1/4*b/(1/c)^(2/3)*3^(1/2)*
arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x+1))+1/4*b/(1/c)^(2/3)*ln(x+(1/c)^(1/3))
-1/8*b/(1/c)^(2/3)*ln(x^2-(1/c)^(1/3)*x+(1/c)^(2/3))+1/4*b/(1/c)^(2/3)*3^(1
/2)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.9275, size = 598, normalized size = 3.62

$$-\frac{2\sqrt{3}\left(-c^2\right)^{\frac{1}{3}}bx^2\arctan\left(\frac{2\sqrt{3}\left(-c^2\right)^{\frac{2}{3}}x+\sqrt{3}c}{3c}\right)-2\sqrt{3}b\left(c^2\right)^{\frac{1}{3}}x^2\arctan\left(\frac{2\sqrt{3}\left(c^2\right)^{\frac{2}{3}}x-\sqrt{3}c}{3c}\right)+\left(-c^2\right)^{\frac{1}{3}}bx^2\log\left(c^2x^2-\left(-c^2\right)^{\frac{1}{3}}cx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/x^3,x, algorithm="fricas")
```

```
[Out] -1/8*(2*sqrt(3)*(-c^2)^(1/3)*b*x^2*arctan(1/3*(2*sqrt(3)*(-c^2)^(2/3)*x + s
qrt(3)*c)/c) - 2*sqrt(3)*b*(c^2)^(1/3)*x^2*arctan(1/3*(2*sqrt(3)*(c^2)^(2/3
)*x - sqrt(3)*c)/c) + (-c^2)^(1/3)*b*x^2*log(c^2*x^2 - (-c^2)^(1/3)*c*x + (
-c^2)^(2/3)) + b*(c^2)^(1/3)*x^2*log(c^2*x^2 - (c^2)^(1/3)*c*x + (c^2)^(2/3
)) - 2*(-c^2)^(1/3)*b*x^2*log(c*x + (-c^2)^(1/3)) - 2*b*(c^2)^(1/3)*x^2*log
(c*x + (c^2)^(1/3)) + 2*b*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 4*a)/x^2
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: KeyError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**3))/x**3,x)
```

```
[Out] Exception raised: KeyError
```

Giac [A] time = 1.29789, size = 223, normalized size = 1.35

$$\frac{1}{8}\left(\frac{2\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x+\frac{1}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{|c|^{\frac{1}{3}}}+\frac{2\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x-\frac{1}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{|c|^{\frac{1}{3}}}+\frac{\log\left(x^2+\frac{x}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{|c|^{\frac{1}{3}}}-\frac{\log\left(x^2-\frac{x}{|c|^{\frac{1}{3}}}+\frac{1}{|c|\right.}}{|c|^{\frac{1}{3}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/x^3,x, algorithm="giac")
```

```
[Out] 1/8*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1/abs(c)^(1/3))*abs(c)^(1/3))/abs(
c)^(1/3) + 2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1/abs(c)^(1/3))*abs(c)^(1/3)
)/abs(c)^(1/3) + log(x^2 + x/abs(c)^(1/3) + 1/abs(c)^(2/3))/abs(c)^(1/3) -
log(x^2 - x/abs(c)^(1/3) + 1/abs(c)^(2/3))/abs(c)^(1/3) + 2*log(abs(x + 1/a
bs(c)^(1/3)))/abs(c)^(1/3) - 2*log(abs(x - 1/abs(c)^(1/3)))/abs(c)^(1/3))*b
*c - 1/4*b*log(-(c*x^3 + 1)/(c*x^3 - 1))/x^2 - 1/2*a/x^2
```

3.110 $\int \frac{a+b\tanh^{-1}(cx^3)}{x^6}\,dx$

Optimal. Leaf size=115

$$-\frac{a+b\tanh^{-1}(cx^3)}{5x^5}-\frac{1}{10}bc^{5/3}\log\left(1-c^{2/3}x^2\right)+\frac{1}{20}bc^{5/3}\log\left(c^{4/3}x^4+c^{2/3}x^2+1\right)-\frac{1}{10}\sqrt{3}bc^{5/3}\tan^{-1}\left(\frac{2c^{2/3}x^2+1}{\sqrt{3}}\right)-$$

```
[Out] (-3*b*c)/(10*x^2) - (Sqrt[3]*b*c^(5/3)*ArcTan[(1 + 2*c^(2/3)*x^2)/Sqrt[3]])
/10 - (a + b*ArcTanh[c*x^3])/(5*x^5) - (b*c^(5/3)*Log[1 - c^(2/3)*x^2])/10
+ (b*c^(5/3)*Log[1 + c^(2/3)*x^2 + c^(4/3)*x^4])/20
```

Rubi [A] time = 0.0945154, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {6097, 275, 325, 292, 31, 634, 617, 204, 628}

$$-\frac{a+b\tanh^{-1}(cx^3)}{5x^5}-\frac{1}{10}bc^{5/3}\log\left(1-c^{2/3}x^2\right)+\frac{1}{20}bc^{5/3}\log\left(c^{4/3}x^4+c^{2/3}x^2+1\right)-\frac{1}{10}\sqrt{3}bc^{5/3}\tan^{-1}\left(\frac{2c^{2/3}x^2+1}{\sqrt{3}}\right)-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^3])/x^6,x]
```

```
[Out] (-3*b*c)/(10*x^2) - (Sqrt[3]*b*c^(5/3)*ArcTan[(1 + 2*c^(2/3)*x^2)/Sqrt[3]])
/10 - (a + b*ArcTanh[c*x^3])/(5*x^5) - (b*c^(5/3)*Log[1 - c^(2/3)*x^2])/10
+ (b*c^(5/3)*Log[1 + c^(2/3)*x^2 + c^(4/3)*x^4])/20
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\tanh^{-1}\left(cx^3\right)}{x^6}\,dx &= -\frac{a+b\tanh^{-1}\left(cx^3\right)}{5x^5}+\frac{1}{5}(3bc)\int\frac{1}{x^3\left(1-c^2x^6\right)}\,dx\\
&= -\frac{a+b\tanh^{-1}\left(cx^3\right)}{5x^5}+\frac{1}{10}(3bc)\,\text{Subst}\left(\int\frac{1}{x^2\left(1-c^2x^3\right)}\,dx,x,x^2\right)\\
&= -\frac{3bc}{10x^2}-\frac{a+b\tanh^{-1}\left(cx^3\right)}{5x^5}+\frac{1}{10}\left(3bc^3\right)\text{Subst}\left(\int\frac{x}{1-c^2x^3}\,dx,x,x^2\right)\\
&= -\frac{3bc}{10x^2}-\frac{a+b\tanh^{-1}\left(cx^3\right)}{5x^5}+\frac{1}{10}\left(bc^{7/3}\right)\text{Subst}\left(\int\frac{1}{1-c^{2/3}x}\,dx,x,x^2\right)-\frac{1}{10}\left(bc^{7/3}\right)\text{Subst}\Bigg(\int\\
&= -\frac{3bc}{10x^2}-\frac{a+b\tanh^{-1}\left(cx^3\right)}{5x^5}-\frac{1}{10}bc^{5/3}\log\left(1-c^{2/3}x^2\right)+\frac{1}{20}\left(bc^{5/3}\right)\text{Subst}\Bigg(\int\frac{c^{2/3}+2c^{4/3}x}{1+c^{2/3}x+c^{4/3}}\\
&= -\frac{3bc}{10x^2}-\frac{a+b\tanh^{-1}\left(cx^3\right)}{5x^5}-\frac{1}{10}bc^{5/3}\log\left(1-c^{2/3}x^2\right)+\frac{1}{20}bc^{5/3}\log\left(1+c^{2/3}x^2+c^{4/3}x^4\right)+\frac{1}{1}\\
&= -\frac{3bc}{10x^2}-\frac{1}{10}\sqrt{3}bc^{5/3}\tan^{-1}\left(\frac{1+2c^{2/3}x^2}{\sqrt{3}}\right)-\frac{a+b\tanh^{-1}\left(cx^3\right)}{5x^5}-\frac{1}{10}bc^{5/3}\log\left(1-c^{2/3}x^2\right)+\frac{1}{20}
\end{aligned}
$$

Mathematica [A] time = 0.0496929, size = 196, normalized size = 1.7

$$
-\frac{a}{5x^5}+\frac{1}{20}bc^{5/3}\log\left(c^{2/3}x^2-\sqrt[3]{c}x+1\right)+\frac{1}{20}bc^{5/3}\log\left(c^{2/3}x^2+\sqrt[3]{c}x+1\right)-\frac{1}{10}bc^{5/3}\log\left(1-\sqrt[3]{c}x\right)-\frac{1}{10}bc^{5/3}\log\left(\sqrt[3]{c}x+1\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^3])/x^6,x]
```

```
[Out] -a/(5*x^5) - (3*b*c)/(10*x^2) - (Sqrt[3]*b*c^(5/3)*ArcTan[(-1 + 2*c^(1/3)*x
)/Sqrt[3]])/10 + (Sqrt[3]*b*c^(5/3)*ArcTan[(1 + 2*c^(1/3)*x)/Sqrt[3]])/10 -
 (b*ArcTanh[c*x^3])/(5*x^5) - (b*c^(5/3)*Log[1 - c^(1/3)*x])/10 - (b*c^(5/3
)*Log[1 + c^(1/3)*x])/10 + (b*c^(5/3)*Log[1 - c^(1/3)*x + c^(2/3)*x^2])/20
+ (b*c^(5/3)*Log[1 + c^(1/3)*x + c^(2/3)*x^2])/20
```

Maple [A] time = 0.014, size = 172, normalized size = 1.5

$$-\frac{a}{5x^5} - \frac{b\mathrm{Artanh}\left(cx^3\right)}{5x^5} - \frac{3bc}{10x^2} - \frac{bc}{10}\ln\left(x-\sqrt[3]{c^{-1}}\right)\left(c^{-1}\right)^{-\frac{2}{3}} + \frac{bc}{20}\ln\left(x^2+\sqrt[3]{c^{-1}}x+\left(c^{-1}\right)^{\frac{2}{3}}\right)\left(c^{-1}\right)^{-\frac{2}{3}} + \frac{bc\sqrt{3}}{10}\arctan\Big($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^3))/x^6,x)
```

```
[Out] -1/5*a/x^5-1/5*b/x^5*arctanh(c*x^3)-3/10*b*c/x^2-1/10*b*c/(1/c)^(2/3)*ln(x-
(1/c)^(1/3))+1/20*b*c/(1/c)^(2/3)*ln(x^2+(1/c)^(1/3)*x+(1/c)^(2/3))+1/10*b*
c/(1/c)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x+1))-1/10*b*c/(1/c
)^(2/3)*ln(x+(1/c)^(1/3))+1/20*b*c/(1/c)^(2/3)*ln(x^2-(1/c)^(1/3)*x+(1/c)^(
2/3))-1/10*b*c/(1/c)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x-1))
```

Maxima [A] time = 1.44875, size = 144, normalized size = 1.25

$$-\frac{1}{20}\left(\left(2\sqrt{3}\left(c^2\right)^{\frac{1}{3}}\arctan\left(\frac{1}{3}\sqrt{3}\left(c^2\right)^{\frac{1}{3}}\left(2x^2+\left(\frac{1}{c^2}\right)^{\frac{1}{3}}\right)\right)-\left(c^2\right)^{\frac{1}{3}}\log\left(x^4+\left(\frac{1}{c^2}\right)^{\frac{1}{3}}x^2+\left(\frac{1}{c^2}\right)^{\frac{2}{3}}\right)+2\left(c^2\right)^{\frac{1}{3}}\log\left(x^2-\left(\frac{1}{c^2}\right)^{\frac{1}{3}}\right)+\frac{6}{x^2}\right)c+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/x^6,x, algorithm="maxima")
```

```
[Out] -1/20*((2*sqrt(3)*(c^2)^(1/3)*arctan(1/3*sqrt(3)*(c^2)^(1/3)*(2*x^2 + (c^(-
2))^(1/3))) - (c^2)^(1/3)*log(x^4 + (c^(-2))^(1/3)*x^2 + (c^(-2))^(2/3)) +
2*(c^2)^(1/3)*log(x^2 - (c^(-2))^(1/3)) + 6/x^2)*c + 4*arctanh(c*x^3)/x^5)*
b - 1/5*a/x^5
```

Fricas [A] time = 1.76675, size = 367, normalized size = 3.19

$$-\frac{2\sqrt{3}\left(-c^2\right)^{\frac{1}{3}}bcx^5\arctan\left(\frac{2}{3}\sqrt{3}\left(-c^2\right)^{\frac{1}{3}}x^2-\frac{1}{3}\sqrt{3}\right)+\left(-c^2\right)^{\frac{1}{3}}bcx^5\log\left(c^2x^4+\left(-c^2\right)^{\frac{2}{3}}x^2-\left(-c^2\right)^{\frac{1}{3}}\right)-2\left(-c^2\right)^{\frac{1}{3}}bcx^5\,\mathrm{lo}}{20x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/x^6,x, algorithm="fricas")
```

```
[Out] -1/20*(2*sqrt(3)*(-c^2)^(1/3)*b*c*x^5*arctan(2/3*sqrt(3)*(-c^2)^(1/3)*x^2 -
 1/3*sqrt(3)) + (-c^2)^(1/3)*b*c*x^5*log(c^2*x^4 + (-c^2)^(2/3)*x^2 - (-c^2
)^(1/3)) - 2*(-c^2)^(1/3)*b*c*x^5*log(c^2*x^2 - (-c^2)^(2/3)) + 6*b*c*x^3 +
```

```
2*b*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 4*a)/x^5
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**3))/x**6,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21764, size = 169, normalized size = 1.47

$$-\frac{1}{20}bc^3\left(\frac{2\sqrt{3}|c|^{\frac{2}{3}}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+\frac{1}{|c|^{\frac{2}{3}}}\right)|c|^{\frac{2}{3}}\right)}{c^2}-\frac{|c|^{\frac{2}{3}}\log\left(x^4+\frac{x^2}{|c|^{\frac{2}{3}}}+\frac{1}{|c|^{\frac{4}{3}}}\right)}{c^2}+\frac{2\log\left(\left|x^2-\frac{1}{|c|^{\frac{2}{3}}}\right|\right)}{|c|^{\frac{4}{3}}}\right)-\frac{b\log\left(-\frac{cx^3+1}{cx^3-1}\right)}{10x^5}-\frac{3b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/x^6,x, algorithm="giac")
```

```
[Out] -1/20*b*c^3*(2*sqrt(3)*abs(c)^(2/3)*arctan(1/3*sqrt(3)*(2*x^2 + 1/abs(c)^(2
/3))*abs(c)^(2/3))/c^2 - abs(c)^(2/3)*log(x^4 + x^2/abs(c)^(2/3) + 1/abs(c)
^(4/3))/c^2 + 2*log(abs(x^2 - 1/abs(c)^(2/3)))/abs(c)^(4/3)) - 1/10*b*log(-
(c*x^3 + 1)/(c*x^3 - 1))/x^5 - 1/10*(3*b*c*x^3 + 2*a)/x^5
```

3.111 $\int x^7 \left(a+b\tanh^{-1}\left(cx^3\right)\right) dx$

Optimal. Leaf size=176

$$\frac{1}{8}x^8\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{b\log\left(c^{2/3}x^2-\sqrt[3]{c}x+1\right)}{32c^{8/3}}-\frac{b\log\left(c^{2/3}x^2+\sqrt[3]{c}x+1\right)}{32c^{8/3}}-\frac{\sqrt{3}b\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right)}{16c^{8/3}}+\frac{\sqrt{3}b\tan^{-1}}{}$$

```
[Out] (3*b*x^5)/(40*c) - (Sqrt[3]*b*ArcTan[1/Sqrt[3] - (2*c^(1/3)*x)/Sqrt[3]])/(1
6*c^(8/3)) + (Sqrt[3]*b*ArcTan[1/Sqrt[3] + (2*c^(1/3)*x)/Sqrt[3]])/(16*c^(8
/3)) - (b*ArcTanh[c^(1/3)*x])/(8*c^(8/3)) + (x^8*(a + b*ArcTanh[c*x^3]))/8
+ (b*Log[1 - c^(1/3)*x + c^(2/3)*x^2])/(32*c^(8/3)) - (b*Log[1 + c^(1/3)*x
+ c^(2/3)*x^2])/(32*c^(8/3))
```

Rubi [A] time = 0.283719, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6097, 321, 296, 634, 618, 204, 628, 206}

$$\frac{1}{8}x^8\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{b\log\left(c^{2/3}x^2-\sqrt[3]{c}x+1\right)}{32c^{8/3}}-\frac{b\log\left(c^{2/3}x^2+\sqrt[3]{c}x+1\right)}{32c^{8/3}}-\frac{\sqrt{3}b\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right)}{16c^{8/3}}+\frac{\sqrt{3}b\tan^{-1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*ArcTanh[c*x^3]),x]
```


```
[Out] (3*b*x^5)/(40*c) - (Sqrt[3]*b*ArcTan[1/Sqrt[3] - (2*c^(1/3)*x)/Sqrt[3]])/(1
6*c^(8/3)) + (Sqrt[3]*b*ArcTan[1/Sqrt[3] + (2*c^(1/3)*x)/Sqrt[3]])/(16*c^(8
/3)) - (b*ArcTanh[c^(1/3)*x])/(8*c^(8/3)) + (x^8*(a + b*ArcTanh[c*x^3]))/8
+ (b*Log[1 - c^(1/3)*x + c^(2/3)*x^2])/(32*c^(8/3)) - (b*Log[1 + c^(1/3)*x
+ c^(2/3)*x^2])/(32*c^(8/3))
```


Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int x^7\left(a+b\tanh^{-1}\left(cx^3\right)\right)dx &= \frac{1}{8}x^8\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{1}{8}(3bc)\int\frac{x^{10}}{1-c^2x^6}\,dx\\
&= \frac{3bx^5}{40c}+\frac{1}{8}x^8\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{(3b)\int\frac{x^4}{1-c^2x^6}\,dx}{8c}\\
&= \frac{3bx^5}{40c}+\frac{1}{8}x^8\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{b\int\frac{1}{1-c^{2/3}x^2}\,dx}{8c^{7/3}}-\frac{b\int\frac{-\frac{1}{2}-\frac{\sqrt[3]{c}x}{2}}{1-\sqrt[3]{c}x+c^{2/3}x^2}\,dx}{8c^{7/3}}-\frac{b\int\frac{-\frac{1}{2}+\frac{\sqrt[3]{c}x}{2}}{1+\sqrt[3]{c}x+c^{2/3}x}}{8c^{7/3}}\\
&= \frac{3bx^5}{40c}-\frac{b\tanh^{-1}\left(\sqrt[3]{c}x\right)}{8c^{8/3}}+\frac{1}{8}x^8\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{b\int\frac{-\sqrt[3]{c}+2c^{2/3}x}{1-\sqrt[3]{c}x+c^{2/3}x^2}\,dx}{32c^{8/3}}-\frac{b\int\frac{\sqrt[3]{c}+2c^{2/3}}{1+\sqrt[3]{c}x+c^{2/}}}{32c^{8/3}}\\
&= \frac{3bx^5}{40c}-\frac{b\tanh^{-1}\left(\sqrt[3]{c}x\right)}{8c^{8/3}}+\frac{1}{8}x^8\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{b\log\left(1-\sqrt[3]{c}x+c^{2/3}x^2\right)}{32c^{8/3}}-\frac{b\log(1}{}\\
&= \frac{3bx^5}{40c}-\frac{\sqrt{3}b\tan^{-1}\left(\frac{1-2\sqrt[3]{c}x}{\sqrt{3}}\right)}{16c^{8/3}}+\frac{\sqrt{3}b\tan^{-1}\left(\frac{1+2\sqrt[3]{c}x}{\sqrt{3}}\right)}{16c^{8/3}}-\frac{b\tanh^{-1}\left(\sqrt[3]{c}x\right)}{8c^{8/3}}+\frac{1}{8}x^8\left(a+b\tanh\right.
\end{aligned}$$

Mathematica [A] time = 0.0302007, size = 198, normalized size = 1.12

$$\frac{ax^8}{8}+\frac{b\log\left(c^{2/3}x^2-\sqrt[3]{c}x+1\right)}{32c^{8/3}}-\frac{b\log\left(c^{2/3}x^2+\sqrt[3]{c}x+1\right)}{32c^{8/3}}+\frac{b\log\left(1-\sqrt[3]{c}x\right)}{16c^{8/3}}-\frac{b\log\left(\sqrt[3]{c}x+1\right)}{16c^{8/3}}+\frac{\sqrt{3}b\tan^{-1}\left(\frac{2\sqrt[3]{c}x-1}{\sqrt{3}}\right)}{16c^{8/3}}+$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*ArcTanh[c*x^3]),x]
```

```
[Out] (3*b*x^5)/(40*c) + (a*x^8)/8 + (Sqrt[3]*b*ArcTan[(-1 + 2*c^(1/3)*x)/Sqrt[3]
])/(16*c^(8/3)) + (Sqrt[3]*b*ArcTan[(1 + 2*c^(1/3)*x)/Sqrt[3]])/(16*c^(8/3)
) + (b*x^8*ArcTanh[c*x^3])/8 + (b*Log[1 - c^(1/3)*x])/(16*c^(8/3)) - (b*Log
[1 + c^(1/3)*x])/(16*c^(8/3)) + (b*Log[1 - c^(1/3)*x + c^(2/3)*x^2])/(32*c^
(8/3)) - (b*Log[1 + c^(1/3)*x + c^(2/3)*x^2])/(32*c^(8/3))
```

Maple [A] time = 0.013, size = 186, normalized size = 1.1

$$\frac{x^8a}{8}+\frac{x^8b\mathrm{Artanh}\left(cx^3\right)}{8}+\frac{3\,bx^5}{40\,c}+\frac{b}{16\,c^3}\ln\left(x-\sqrt[3]{c^{-1}}\right)\frac{1}{\sqrt[3]{c^{-1}}}-\frac{b}{32\,c^3}\ln\left(x^2+\sqrt[3]{c^{-1}}x+\left(c^{-1}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{c^{-1}}}+\frac{b\sqrt{3}}{16\,c^3}\arctan\Big($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a+b*arctanh(c*x^3)),x)
```

```
[Out] 1/8*x^8*a+1/8*x^8*b*arctanh(c*x^3)+3/40*b*x^5/c+1/16*b/c^3/(1/c)^(1/3)*ln(x
-(1/c)^(1/3))-1/32*b/c^3/(1/c)^(1/3)*ln(x^2+(1/c)^(1/3)*x+(1/c)^(2/3))+1/16
*b/c^3*3^(1/2)/(1/c)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x+1))-1/16*b/c
^3/(1/c)^(1/3)*ln(x+(1/c)^(1/3))+1/32*b/c^3/(1/c)^(1/3)*ln(x^2-(1/c)^(1/3)*
x+(1/c)^(2/3))+1/16*b/c^3*3^(1/2)/(1/c)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/c)^(
1/3)*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arctanh(c*x^3)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.88365, size = 659, normalized size = 3.74

$$10\,bc^4x^8\log\left(-\frac{cx^3+1}{cx^3-1}\right)+20\,ac^4x^8+12\,bc^3x^5+10\,\sqrt{3}bc\sqrt{-\left(-c^2\right)^{\frac{1}{3}}}\arctan\left(\frac{\sqrt{3}\left(2\,cx+\left(-c^2\right)^{\frac{1}{3}}\right)\sqrt{-\left(-c^2\right)^{\frac{1}{3}}}}{3\,c}\right)+10\,\sqrt{3}b\left(c^2\right)^{\frac{1}{6}}c\,\mathrm{arc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arctanh(c*x^3)),x, algorithm="fricas")
```

```
[Out] 1/160*(10*b*c^4*x^8*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 20*a*c^4*x^8 + 12*b*c^3
*x^5 + 10*sqrt(3)*b*c*sqrt(-(-c^2)^(1/3))*arctan(1/3*sqrt(3)*(2*c*x + (-c^2
)^(1/3))*sqrt(-(-c^2)^(1/3))/c) + 10*sqrt(3)*b*(c^2)^(1/6)*c*arctan(1/3*sqr
t(3)*(c^2)^(1/6)*(2*c*x + (c^2)^(1/3))/c) + 5*(-c^2)^(2/3)*b*log(c^2*x^2 +
(-c^2)^(1/3)*c*x + (-c^2)^(2/3)) - 5*b*(c^2)^(2/3)*log(c^2*x^2 + (c^2)^(1/3
```

```
)*c*x + (c^2)^(2/3)) - 10*(-c^2)^(2/3)*b*log(c*x - (-c^2)^(1/3)) + 10*b*(c^
2)^(2/3)*log(c*x - (c^2)^(1/3)))/c^4
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: KeyError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(a+b*atanh(c*x**3)),x)
```

```
[Out] Exception raised: KeyError
```

Giac [A] time = 1.49266, size = 281, normalized size = 1.6

$$-\frac{1}{32}bc^{15}\left(\frac{2\left(-\frac{1}{c}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{1}{c}\right)^{\frac{1}{3}}\right|\right)}{c^{17}}-\frac{2\sqrt{3}|c|^{\frac{4}{3}}\arctan\left(\frac{1}{3}\sqrt{3}c^{\frac{1}{3}}\left(2x+\frac{1}{c^{\frac{1}{3}}}\right)\right)}{c^{19}}+\frac{|c|^{\frac{4}{3}}\log\left(x^2+\frac{x}{c^{\frac{1}{3}}}+\frac{1}{c^{\frac{2}{3}}}\right)}{c^{19}}-\frac{2\log\left(\left|x-\frac{1}{c^{\frac{1}{3}}}\right|\right)}{c^{\frac{53}{3}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arctanh(c*x^3)),x, algorithm="giac")
```

```
[Out] -1/32*b*c^15*(2*(-1/c)^(2/3)*log(abs(x - (-1/c)^(1/3)))/c^17 - 2*sqrt(3)*ab
s(c)^(4/3)*arctan(1/3*sqrt(3)*c^(1/3)*(2*x + 1/c^(1/3)))/c^19 + abs(c)^(4/3
)*log(x^2 + x/c^(1/3) + 1/c^(2/3))/c^19 - 2*log(abs(x - 1/c^(1/3)))/c^(53/3
) + 2*sqrt(3)*(-c^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-1/c)^(1/3))/(-1/c)^(
1/3))/c^19 - (-c^2)^(2/3)*log(x^2 + x*(-1/c)^(1/3) + (-1/c)^(2/3))/c^19) +
1/16*b*x^8*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 1/8*a*x^8 + 3/40*b*x^5/c
```

3.112 $\int x^4 \left(a + b \tanh^{-1}\left(cx^3\right)\right) dx$

Optimal. Leaf size=117

$$\frac{1}{5}x^5\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{b\log\left(1-c^{2/3}x^2\right)}{10c^{5/3}}-\frac{b\log\left(c^{4/3}x^4+c^{2/3}x^2+1\right)}{20c^{5/3}}-\frac{\sqrt{3}b\tan^{-1}\left(\frac{2c^{2/3}x^2+1}{\sqrt{3}}\right)}{10c^{5/3}}+\frac{3bx^2}{10c}$$

```
[Out] (3*b*x^2)/(10*c) - (Sqrt[3]*b*ArcTan[(1 + 2*c^(2/3)*x^2)/Sqrt[3]])/(10*c^(5
/3)) + (x^5*(a + b*ArcTanh[c*x^3]))/5 + (b*Log[1 - c^(2/3)*x^2])/(10*c^(5/3
)) - (b*Log[1 + c^(2/3)*x^2 + c^(4/3)*x^4])/(20*c^(5/3))
```

Rubi [A] time = 0.0972647, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {6097, 275, 321, 200, 31, 634, 617, 204, 628}

$$\frac{1}{5}x^5\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{b\log\left(1-c^{2/3}x^2\right)}{10c^{5/3}}-\frac{b\log\left(c^{4/3}x^4+c^{2/3}x^2+1\right)}{20c^{5/3}}-\frac{\sqrt{3}b\tan^{-1}\left(\frac{2c^{2/3}x^2+1}{\sqrt{3}}\right)}{10c^{5/3}}+\frac{3bx^2}{10c}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*ArcTanh[c*x^3]),x]
```



```
[Out] (3*b*x^2)/(10*c) - (Sqrt[3]*b*ArcTan[(1 + 2*c^(2/3)*x^2)/Sqrt[3]])/(10*c^(5
/3)) + (x^5*(a + b*ArcTanh[c*x^3]))/5 + (b*Log[1 - c^(2/3)*x^2])/(10*c^(5/3
)) - (b*Log[1 + c^(2/3)*x^2 + c^(4/3)*x^4])/(20*c^(5/3))
```


Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int x^4\left(a+b\tanh^{-1}\left(cx^3\right)\right)dx &= \frac{1}{5}x^5\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{1}{5}(3bc)\int\frac{x^7}{1-c^2x^6}\,dx\\
&= \frac{1}{5}x^5\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{1}{10}(3bc)\,\mathrm{Subst}\left(\int\frac{x^3}{1-c^2x^3}\,dx,x,x^2\right)\\
&= \frac{3bx^2}{10c}+\frac{1}{5}x^5\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{(3b)\,\mathrm{Subst}\left(\int\frac{1}{1-c^2x^3}\,dx,x,x^2\right)}{10c}\\
&= \frac{3bx^2}{10c}+\frac{1}{5}x^5\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{b\,\mathrm{Subst}\left(\int\frac{1}{1-c^{2/3}x}\,dx,x,x^2\right)}{10c}-\frac{b\,\mathrm{Subst}\left(\int\frac{2+c^{2/3}x}{1+c^{2/3}x+c^{4/3}\right.}}{10c}\\
&= \frac{3bx^2}{10c}+\frac{1}{5}x^5\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{b\log\left(1-c^{2/3}x^2\right)}{10c^{5/3}}-\frac{b\,\mathrm{Subst}\left(\int\frac{c^{2/3}+2c^{4/3}x}{1+c^{2/3}x+c^{4/3}x^2}\,dx,x,x^2\right.}{20c^{5/3}}\\
&= \frac{3bx^2}{10c}+\frac{1}{5}x^5\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{b\log\left(1-c^{2/3}x^2\right)}{10c^{5/3}}-\frac{b\log\left(1+c^{2/3}x^2+c^{4/3}x^4\right)}{20c^{5/3}}+\frac{(3b)}{}\\
&= \frac{3bx^2}{10c}-\frac{\sqrt{3}b\tan^{-1}\left(\frac{1+2c^{2/3}x^2}{\sqrt{3}}\right)}{10c^{5/3}}+\frac{1}{5}x^5\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{b\log\left(1-c^{2/3}x^2\right)}{10c^{5/3}}-\frac{b\log\left(1\right.}{}
\end{aligned}$$

Mathematica [A] time = 0.0282765, size = 198, normalized size = 1.69

$$\frac{ax^5}{5}-\frac{b\log\left(c^{2/3}x^2-\sqrt[3]{c}x+1\right)}{20c^{5/3}}-\frac{b\log\left(c^{2/3}x^2+\sqrt[3]{c}x+1\right)}{20c^{5/3}}+\frac{b\log\left(1-\sqrt[3]{c}x\right)}{10c^{5/3}}+\frac{b\log\left(\sqrt[3]{c}x+1\right)}{10c^{5/3}}-\frac{\sqrt{3}b\tan^{-1}\left(\frac{2\sqrt[3]{c}x-1}{\sqrt{3}}\right)}{10c^{5/3}}+$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*ArcTanh[c*x^3]),x]
```

```
[Out] (3*b*x^2)/(10*c) + (a*x^5)/5 - (Sqrt[3]*b*ArcTan[(-1 + 2*c^(1/3)*x)/Sqrt[3]
])/(10*c^(5/3)) + (Sqrt[3]*b*ArcTan[(1 + 2*c^(1/3)*x)/Sqrt[3]])/(10*c^(5/3)
) + (b*x^5*ArcTanh[c*x^3])/5 + (b*Log[1 - c^(1/3)*x])/(10*c^(5/3)) + (b*Log
[1 + c^(1/3)*x])/(10*c^(5/3)) - (b*Log[1 - c^(1/3)*x + c^(2/3)*x^2])/(20*c^
(5/3)) - (b*Log[1 + c^(1/3)*x + c^(2/3)*x^2])/(20*c^(5/3))
```

Maple [A] time = 0.007, size = 114, normalized size = 1.

$$\frac{ax^5}{5} + \frac{x^5 b\text{Artanh}\left(cx^3\right)}{5} + \frac{3\,bx^2}{10\,c} + \frac{b}{10\,c^3}\ln\left(x^2 - \sqrt[3]{c^{-2}}\right)\left(c^{-2}\right)^{-\frac{2}{3}} - \frac{b}{20\,c^3}\ln\left(x^4 + \sqrt[3]{c^{-2}}x^2 + \left(c^{-2}\right)^{\frac{2}{3}}\right)\left(c^{-2}\right)^{-\frac{2}{3}} - \frac{b\sqrt{3}}{10\,c^3}\text{ar}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arctanh(c*x^3)),x)
```

```
[Out] 1/5*a*x^5+1/5*x^5*b*arctanh(c*x^3)+3/10*b*x^2/c+1/10*b/c^3/(1/c^2)^(2/3)*ln
(x^2-(1/c^2)^(1/3))-1/20*b/c^3/(1/c^2)^(2/3)*ln(x^4+(1/c^2)^(1/3)*x^2+(1/c^
2)^(2/3))-1/10*b/c^3/(1/c^2)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/c^2)^(1
/3)*x^2+1))
```

Maxima [A] time = 1.50079, size = 161, normalized size = 1.38

$$\frac{1}{5}\,ax^5 + \frac{1}{20}\left(4\,x^5\,\text{artanh}\left(cx^3\right) + c\left(\frac{6\,x^2}{c^2} - \frac{2\,\sqrt{3}\left(c^2\right)^{\frac{2}{3}}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(c^2\right)^{\frac{1}{3}}\left(2\,x^2 + \frac{1}{c^2}^{\frac{1}{3}}\right)\right)}{c^4} - \frac{\left(c^2\right)^{\frac{2}{3}}\log\left(x^4 + \frac{1}{c^2}^{\frac{1}{3}}x^2 + \frac{1}{c^2}^{\frac{2}{3}}\right)}{c^4}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x^3)),x, algorithm="maxima")
```

```
[Out] 1/5*a*x^5 + 1/20*(4*x^5*arctanh(c*x^3) + c*(6*x^2/c^2 - 2*sqrt(3)*(c^2)^(2/
3)*arctan(1/3*sqrt(3)*(c^2)^(1/3)*(2*x^2 + (c^(-2))^(1/3)))/c^4 - (c^2)^(2/
3)*log(x^4 + (c^(-2))^(1/3)*x^2 + (c^(-2))^(2/3))/c^4 + 2*(c^2)^(2/3)*log(x
^2 - (c^(-2))^(1/3))/c^4))*b
```

Fricas [A] time = 1.79069, size = 414, normalized size = 3.54

$$\frac{2\,bc^3x^5\log\left(-\frac{cx^3+1}{cx^3-1}\right) + 4\,ac^3x^5 + 6\,bc^2x^2 - 2\,\sqrt{3}b\left(c^2\right)^{\frac{1}{6}}c\arctan\left(-\frac{\sqrt{3}\left(4\,c^2x^4 - 2\left(c^2\right)^{\frac{2}{3}}x^2 + \left(c^2\right)^{\frac{1}{3}}\right)\left(c^2\right)^{\frac{1}{6}}}{8\,c^3x^6 + c}\right) - b\left(c^2\right)^{\frac{2}{3}}\log\left(c^2x^4 + \left(c\right.\right.}{20\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x^3)),x, algorithm="fricas")
```

```
[Out] 1/20*(2*b*c^3*x^5*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 4*a*c^3*x^5 + 6*b*c^2*x^2
 - 2*sqrt(3)*b*(c^2)^(1/6)*c*arctan(-sqrt(3)*(4*c^2*x^4 - 2*(c^2)^(2/3)*x^2
```

```
 + (c^2)^(1/3))*(c^2)^(1/6)/(8*c^3*x^6 + c)) - b*(c^2)^(2/3)*log(c^2*x^4 +
(c^2)^(2/3)*x^2 + (c^2)^(1/3)) + 2*b*(c^2)^(2/3)*log(c^2*x^2 - (c^2)^(2/3))
)/c^3
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: KeyError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*atanh(c*x**3)),x)
```

```
[Out] Exception raised: KeyError
```

Giac [A] time = 1.18607, size = 170, normalized size = 1.45

$$-\frac{1}{20}\,bc^9\left(\frac{2\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^2+\frac{1}{|c|^{\frac{2}{3}}}\right)|c|^{\frac{2}{3}}\right)}{c^{10}|c|^{\frac{2}{3}}}+\frac{\log\left(x^4+\frac{x^2}{|c|^{\frac{2}{3}}}+\frac{1}{|c|^{\frac{4}{3}}}\right)}{c^{10}|c|^{\frac{2}{3}}}-\frac{2\,\log\left(\left|x^2-\frac{1}{|c|^{\frac{2}{3}}}\right|\right)}{c^{10}|c|^{\frac{2}{3}}}\right)+\frac{1}{10}\,bx^5\log\left(-\frac{cx^3+1}{cx^3-1}\right)+\frac{1}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x^3)),x, algorithm="giac")
```

```
[Out] -1/20*b*c^9*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1/abs(c)^(2/3))*abs(c)^(
2/3))/(c^10*abs(c)^(2/3)) + log(x^4 + x^2/abs(c)^(2/3) + 1/abs(c)^(4/3))/(c
^10*abs(c)^(2/3)) - 2*log(abs(x^2 - 1/abs(c)^(2/3)))/(c^10*abs(c)^(2/3))) +
 1/10*b*x^5*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 1/5*a*x^5 + 3/10*b*x^2/c
```

3.113 $\int x \left(a+b\tanh^{-1}\left(cx^3\right)\right) dx$

Optimal. Leaf size=165

$$\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{b\log\left(c^{2/3}x^2-\sqrt[3]{c}x+1\right)}{8c^{2/3}}-\frac{b\log\left(c^{2/3}x^2+\sqrt[3]{c}x+1\right)}{8c^{2/3}}-\frac{\sqrt{3}b\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right)}{4c^{2/3}}+\frac{\sqrt{3}b\tan^{-1}}{}$$

```
[Out] -(Sqrt[3]*b*ArcTan[1/Sqrt[3] - (2*c^(1/3)*x)/Sqrt[3]])/(4*c^(2/3)) + (Sqrt[
3]*b*ArcTan[1/Sqrt[3] + (2*c^(1/3)*x)/Sqrt[3]])/(4*c^(2/3)) - (b*ArcTanh[c^
(1/3)*x])/(2*c^(2/3)) + (x^2*(a + b*ArcTanh[c*x^3]))/2 + (b*Log[1 - c^(1/3)
*x + c^(2/3)*x^2])/(8*c^(2/3)) - (b*Log[1 + c^(1/3)*x + c^(2/3)*x^2])/(8*c^
(2/3))
```

Rubi [A] time = 0.248606, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6097, 296, 634, 618, 204, 628, 206}

$$\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{b\log\left(c^{2/3}x^2-\sqrt[3]{c}x+1\right)}{8c^{2/3}}-\frac{b\log\left(c^{2/3}x^2+\sqrt[3]{c}x+1\right)}{8c^{2/3}}-\frac{\sqrt{3}b\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right)}{4c^{2/3}}+\frac{\sqrt{3}b\tan^{-1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcTanh[c*x^3]),x]
```

```
[Out] -(Sqrt[3]*b*ArcTan[1/Sqrt[3] - (2*c^(1/3)*x)/Sqrt[3]])/(4*c^(2/3)) + (Sqrt[
3]*b*ArcTan[1/Sqrt[3] + (2*c^(1/3)*x)/Sqrt[3]])/(4*c^(2/3)) - (b*ArcTanh[c^
(1/3)*x])/(2*c^(2/3)) + (x^2*(a + b*ArcTanh[c*x^3]))/2 + (b*Log[1 - c^(1/3)
*x + c^(2/3)*x^2])/(8*c^(2/3)) - (b*Log[1 + c^(1/3)*x + c^(2/3)*x^2])/(8*c^
(2/3))
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x\left(a+b\tanh^{-1}\left(cx^3\right)\right)dx=\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{1}{2}(3bc)\int\frac{x^4}{1-c^2x^6}\,dx$$

$$=\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{b\int\frac{1}{1-c^{2/3}x^2}\,dx}{2\sqrt[3]{c}}-\frac{b\int\frac{-\frac{1}{2}-\frac{\sqrt[3]{c}x}{2}}{1-\sqrt[3]{c}x+c^{2/3}x^2}\,dx}{2\sqrt[3]{c}}-\frac{b\int\frac{-\frac{1}{2}+\frac{\sqrt[3]{c}x}{2}}{1+\sqrt[3]{c}x+c^{2/3}x^2}\,dx}{2\sqrt[3]{c}}$$

$$=-\frac{b\tanh^{-1}\left(\sqrt[3]{c}x\right)}{2c^{2/3}}+\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{b\int\frac{-\sqrt[3]{c}+2c^{2/3}x}{1-\sqrt[3]{c}x+c^{2/3}x^2}\,dx}{8c^{2/3}}-\frac{b\int\frac{\sqrt[3]{c}+2c^{2/3}x}{1+\sqrt[3]{c}x+c^{2/3}x^2}\,dx}{8c^{2/3}}-$$

$$=-\frac{b\tanh^{-1}\left(\sqrt[3]{c}x\right)}{2c^{2/3}}+\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{b\log\left(1-\sqrt[3]{c}x+c^{2/3}x^2\right)}{8c^{2/3}}-\frac{b\log\left(1+\sqrt[3]{c}x\right.}{8c^{2/3}}$$

$$=-\frac{\sqrt{3}b\tan^{-1}\left(\frac{1-2\sqrt[3]{c}x}{\sqrt{3}}\right)}{4c^{2/3}}+\frac{\sqrt{3}b\tan^{-1}\left(\frac{1+2\sqrt[3]{c}x}{\sqrt{3}}\right)}{4c^{2/3}}-\frac{b\tanh^{-1}\left(\sqrt[3]{c}x\right)}{2c^{2/3}}+\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(cx^3\right.\right.$$

Mathematica [A] time = 0.0255684, size = 187, normalized size = 1.13

$$\frac{ax^2}{2}+\frac{b\log\left(c^{2/3}x^2-\sqrt[3]{c}x+1\right)}{8c^{2/3}}-\frac{b\log\left(c^{2/3}x^2+\sqrt[3]{c}x+1\right)}{8c^{2/3}}+\frac{b\log\left(1-\sqrt[3]{c}x\right)}{4c^{2/3}}-\frac{b\log\left(\sqrt[3]{c}x+1\right)}{4c^{2/3}}+\frac{\sqrt{3}b\tan^{-1}\left(\frac{2\sqrt[3]{c}x-1}{\sqrt{3}}\right)}{4c^{2/3}}+$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcTanh[c*x^3]),x]
```

```
[Out] (a*x^2)/2 + (Sqrt[3]*b*ArcTan[(-1 + 2*c^(1/3)*x)/Sqrt[3]])/(4*c^(2/3)) + (S
qrt[3]*b*ArcTan[(1 + 2*c^(1/3)*x)/Sqrt[3]])/(4*c^(2/3)) + (b*x^2*ArcTanh[c*
x^3])/2 + (b*Log[1 - c^(1/3)*x])/(4*c^(2/3)) - (b*Log[1 + c^(1/3)*x])/(4*c^
(2/3)) + (b*Log[1 - c^(1/3)*x + c^(2/3)*x^2])/(8*c^(2/3)) - (b*Log[1 + c^(1
/3)*x + c^(2/3)*x^2])/(8*c^(2/3))
```

Maple [A] time = 0.01, size = 177, normalized size = 1.1

$$\frac{ax^2}{2}+\frac{bx^2\operatorname{Artanh}\left(cx^3\right)}{2}+\frac{b}{4c}\ln\left(x-\sqrt[3]{c^{-1}}\right)\frac{1}{\sqrt[3]{c^{-1}}}-\frac{b}{8c}\ln\left(x^2+\sqrt[3]{c^{-1}}x+\left(c^{-1}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{c^{-1}}}+\frac{b\sqrt{3}}{4c}\arctan\left(\frac{\sqrt{3}}{3}\left(2\frac{x}{\sqrt[3]{c^{-1}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c*x^3)),x)
```

```
[Out] 1/2*a*x^2+1/2*b*x^2*arctanh(c*x^3)+1/4*b/c/(1/c)^(1/3)*ln(x-(1/c)^(1/3))-1/
8*b/c/(1/c)^(1/3)*ln(x^2+(1/c)^(1/3)*x+(1/c)^(2/3))+1/4*b*3^(1/2)/c/(1/c)^(
1/3)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x+1))-1/4*b/c/(1/c)^(1/3)*ln(x+(1/c)
^(1/3))+1/8*b/c/(1/c)^(1/3)*ln(x^2-(1/c)^(1/3)*x+(1/c)^(2/3))+1/4*b*3^(1/2)
/c/(1/c)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^3)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.85752, size = 622, normalized size = 3.77

$$2\,bc^2x^2\log\left(-\frac{cx^3+1}{cx^3-1}\right)+4\,ac^2x^2+2\,\sqrt{3}bc\sqrt{-\left(-c^2\right)^{\frac{1}{3}}}\arctan\left(\frac{\sqrt{3}\left(2\,cx+\left(-c^2\right)^{\frac{1}{3}}\right)\sqrt{-\left(-c^2\right)^{\frac{1}{3}}}}{3\,c}\right)+2\,\sqrt{3}b\left(c^2\right)^{\frac{1}{6}}c\arctan\left(\frac{\sqrt{3}\left(c^2\right)^{\frac{1}{6}}\left(2\,c\right.}{3\,c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^3)),x, algorithm="fricas")
```

```
[Out] 1/8*(2*b*c^2*x^2*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 4*a*c^2*x^2 + 2*sqrt(3)*b*
c*sqrt(-(-c^2)^(1/3))*arctan(1/3*sqrt(3)*(2*c*x + (-c^2)^(1/3))*sqrt(-(-c^2
)^(1/3))/c) + 2*sqrt(3)*b*(c^2)^(1/6)*c*arctan(1/3*sqrt(3)*(c^2)^(1/6)*(2*c
*x + (c^2)^(1/3))/c) + (-c^2)^(2/3)*b*log(c^2*x^2 + (-c^2)^(1/3)*c*x + (-c^
2)^(2/3)) - b*(c^2)^(2/3)*log(c^2*x^2 + (c^2)^(1/3)*c*x + (c^2)^(2/3)) - 2*
(-c^2)^(2/3)*b*log(c*x - (-c^2)^(1/3)) + 2*b*(c^2)^(2/3)*log(c*x - (c^2)^(1
/3)))/c^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c*x**3)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.31032, size = 269, normalized size = 1.63

$$-\frac{1}{8}\,bc^{5}\left(\frac{2\left(-\frac{1}{c}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{1}{c}\right)^{\frac{1}{3}}\right|\right)}{c^{5}}-\frac{2\sqrt{3}|c|^{\frac{4}{3}}\arctan\left(\frac{1}{3}\sqrt{3}c^{\frac{1}{3}}\left(2x+\frac{1}{c^{\frac{1}{3}}}\right)\right)}{c^{7}}+\frac{|c|^{\frac{4}{3}}\log\left(x^{2}+\frac{x}{c^{\frac{1}{3}}}+\frac{1}{c^{\frac{2}{3}}}\right)}{c^{7}}-\frac{2\log\left(\left|x-\frac{1}{c^{\frac{1}{3}}}\right|\right)}{c^{\frac{17}{3}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^3)),x, algorithm="giac")
```

```
[Out] -1/8*b*c^5*(2*(-1/c)^(2/3)*log(abs(x - (-1/c)^(1/3)))/c^5 - 2*sqrt(3)*abs(c
)^(4/3)*arctan(1/3*sqrt(3)*c^(1/3)*(2*x + 1/c^(1/3)))/c^7 + abs(c)^(4/3)*lo
g(x^2 + x/c^(1/3) + 1/c^(2/3))/c^7 - 2*log(abs(x - 1/c^(1/3)))/c^(17/3) + 2
*sqrt(3)*(-c^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-1/c)^(1/3))/(-1/c)^(1/3))
/c^7 - (-c^2)^(2/3)*log(x^2 + x*(-1/c)^(1/3) + (-1/c)^(2/3))/c^7) + 1/4*b*x
^2*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 1/2*a*x^2
```

3.114 $\int \frac{a+b\tanh^{-1}(cx^3)}{x^2}\,dx$

Optimal. Leaf size=104

$$-\frac{a+b\tanh^{-1}\left(cx^3\right)}{x}-\frac{1}{2}b\sqrt[3]{c}\log\left(1-c^{2/3}x^2\right)+\frac{1}{4}b\sqrt[3]{c}\log\left(c^{4/3}x^4+c^{2/3}x^2+1\right)+\frac{1}{2}\sqrt{3}b\sqrt[3]{c}\tan^{-1}\left(\frac{2c^{2/3}x^2+1}{\sqrt{3}}\right)$$

```
[Out] (Sqrt[3]*b*c^(1/3)*ArcTan[(1 + 2*c^(2/3)*x^2)/Sqrt[3]])/2 - (a + b*ArcTanh[
c*x^3])/x - (b*c^(1/3)*Log[1 - c^(2/3)*x^2])/2 + (b*c^(1/3)*Log[1 + c^(2/3)
*x^2 + c^(4/3)*x^4])/4
```

Rubi [A] time = 0.0827732, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6097, 275, 200, 31, 634, 617, 204, 628}

$$-\frac{a+b\tanh^{-1}\left(cx^3\right)}{x}-\frac{1}{2}b\sqrt[3]{c}\log\left(1-c^{2/3}x^2\right)+\frac{1}{4}b\sqrt[3]{c}\log\left(c^{4/3}x^4+c^{2/3}x^2+1\right)+\frac{1}{2}\sqrt{3}b\sqrt[3]{c}\tan^{-1}\left(\frac{2c^{2/3}x^2+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^3])/x^2,x]
```


```
[Out] (Sqrt[3]*b*c^(1/3)*ArcTan[(1 + 2*c^(2/3)*x^2)/Sqrt[3]])/2 - (a + b*ArcTanh[
c*x^3])/x - (b*c^(1/3)*Log[1 - c^(2/3)*x^2])/2 + (b*c^(1/3)*Log[1 + c^(2/3)
*x^2 + c^(4/3)*x^4])/4
```


Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}\left(cx^3\right)}{x^2}\,dx &= -\frac{a+b\tanh^{-1}\left(cx^3\right)}{x}+(3bc)\int \frac{x}{1-c^2x^6}\,dx\\ &= -\frac{a+b\tanh^{-1}\left(cx^3\right)}{x}+\frac{1}{2}(3bc)\,\text{Subst}\left(\int \frac{1}{1-c^2x^3}\,dx,x,x^2\right)\\ &= -\frac{a+b\tanh^{-1}\left(cx^3\right)}{x}+\frac{1}{2}(bc)\,\text{Subst}\left(\int \frac{1}{1-c^{2/3}x}\,dx,x,x^2\right)+\frac{1}{2}(bc)\,\text{Subst}\left(\int \frac{2+c^{2/3}x}{1+c^{2/3}x+c^{4/3}\cdots}\right.\\ &= -\frac{a+b\tanh^{-1}\left(cx^3\right)}{x}-\frac{1}{2}b\sqrt[3]{c}\log\left(1-c^{2/3}x^2\right)+\frac{1}{4}\left(b\sqrt[3]{c}\right)\text{Subst}\left(\int \frac{c^{2/3}+2c^{4/3}x}{1+c^{2/3}x+c^{4/3}x^2}\,dx,x,x^2\cdots\right.\\ &= -\frac{a+b\tanh^{-1}\left(cx^3\right)}{x}-\frac{1}{2}b\sqrt[3]{c}\log\left(1-c^{2/3}x^2\right)+\frac{1}{4}b\sqrt[3]{c}\log\left(1+c^{2/3}x^2+c^{4/3}x^4\right)-\frac{1}{2}\left(3b\sqrt[3]{c}\right)\text{Su}\cdots\\ &= \frac{1}{2}\sqrt{3}b\sqrt[3]{c}\tan^{-1}\left(\frac{1+2c^{2/3}x^2}{\sqrt{3}}\right)-\frac{a+b\tanh^{-1}\left(cx^3\right)}{x}-\frac{1}{2}b\sqrt[3]{c}\log\left(1-c^{2/3}x^2\right)+\frac{1}{4}b\sqrt[3]{c}\log\left(1+\cdots\right.\end{aligned}$$

Mathematica [A] time = 0.0290422, size = 183, normalized size = 1.76

$$-\frac{a}{x}+\frac{1}{4}b\sqrt[3]{c}\log\left(c^{2/3}x^2-\sqrt[3]{c}x+1\right)+\frac{1}{4}b\sqrt[3]{c}\log\left(c^{2/3}x^2+\sqrt[3]{c}x+1\right)-\frac{b\tanh^{-1}\left(cx^3\right)}{x}-\frac{1}{2}b\sqrt[3]{c}\log\left(1-\sqrt[3]{c}x\right)-\frac{1}{2}b\sqrt[3]{c}\log\left(\cdots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^3])/x^2,x]
```

```
[Out] -(a/x) + (Sqrt[3]*b*c^(1/3)*ArcTan[(-1 + 2*c^(1/3)*x)/Sqrt[3]])/2 - (Sqrt[3
]*b*c^(1/3)*ArcTan[(1 + 2*c^(1/3)*x)/Sqrt[3]])/2 - (b*ArcTanh[c*x^3])/x - (
b*c^(1/3)*Log[1 - c^(1/3)*x])/2 - (b*c^(1/3)*Log[1 + c^(1/3)*x])/2 + (b*c^(
1/3)*Log[1 - c^(1/3)*x + c^(2/3)*x^2])/4 + (b*c^(1/3)*Log[1 + c^(1/3)*x + c
^(2/3)*x^2])/4
```

Maple [A] time = 0.005, size = 105, normalized size = 1.

$$-\frac{a}{x}-\frac{b\mathrm{Artanh}\left(cx^3\right)}{x}-\frac{b}{2c}\ln\left(x^2-\sqrt[3]{c^{-2}}\right)\left(c^{-2}\right)^{-\frac{2}{3}}+\frac{b}{4c}\ln\left(x^4+\sqrt[3]{c^{-2}}x^2+\left(c^{-2}\right)^{\frac{2}{3}}\right)\left(c^{-2}\right)^{-\frac{2}{3}}+\frac{b\sqrt{3}}{2c}\arctan\left(\frac{\sqrt{3}}{3}\left(2\frac{x}{\sqrt[3]{c}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^3))/x^2,x)
```

```
[Out] -a/x-b/x*arctanh(c*x^3)-1/2*b/c/(1/c^2)^(2/3)*ln(x^2-(1/c^2)^(1/3))+1/4*b/c
/(1/c^2)^(2/3)*ln(x^4+(1/c^2)^(1/3)*x^2+(1/c^2)^(2/3))+1/2*b/c/(1/c^2)^(2/3
)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/c^2)^(1/3)*x^2+1))
```

Maxima [A] time = 1.52792, size = 149, normalized size = 1.43

$$\frac{1}{4}\left(c\left(\frac{2\sqrt{3}\left(c^2\right)^{\frac{2}{3}}\arctan\left(\frac{1}{3}\sqrt{3}\left(c^2\right)^{\frac{1}{3}}\left(2x^2+\frac{1}{c^2}^{\frac{1}{3}}\right)\right)}{c^2}+\frac{\left(c^2\right)^{\frac{2}{3}}\log\left(x^4+\frac{1}{c^2}^{\frac{1}{3}}x^2+\frac{1}{c^2}^{\frac{2}{3}}\right)}{c^2}-\frac{2\left(c^2\right)^{\frac{2}{3}}\log\left(x^2-\frac{1}{c^2}^{\frac{1}{3}}\right)}{c^2}\right)-\frac{4\ \mathrm{artan}}{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*(c*(2*sqrt(3)*(c^2)^(2/3)*arctan(1/3*sqrt(3)*(c^2)^(1/3)*(2*x^2 + (c^(-
2))^(1/3)))/c^2 + (c^2)^(2/3)*log(x^4 + (c^(-2))^(1/3)*x^2 + (c^(-2))^(2/3)
)/c^2 - 2*(c^2)^(2/3)*log(x^2 - (c^(-2))^(1/3))/c^2) - 4*arctanh(c*x^3)/x)*
b - a/x
```

Fricas [A] time = 1.82913, size = 312, normalized size = 3.

$$-\frac{2\sqrt{3}b\left(-c\right)^{\frac{1}{3}}x\arctan\left(\frac{2}{3}\sqrt{3}\left(-c\right)^{\frac{2}{3}}x^2+\frac{1}{3}\sqrt{3}\right)+b\left(-c\right)^{\frac{1}{3}}x\log\left(c^2x^4-\left(-c\right)^{\frac{1}{3}}cx^2+\left(-c\right)^{\frac{2}{3}}\right)-2b\left(-c\right)^{\frac{1}{3}}x\log\left(cx^2+\left(-c\right)\right.}{4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/x^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(3)*b*(-c)^(1/3)*x*arctan(2/3*sqrt(3)*(-c)^(2/3)*x^2 + 1/3*sqrt
(3)) + b*(-c)^(1/3)*x*log(c^2*x^4 - (-c)^(1/3)*c*x^2 + (-c)^(2/3)) - 2*b*(-
c)^(1/3)*x*log(c*x^2 + (-c)^(1/3)) + 2*b*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 4*
a)/x
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: KeyError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**3))/x**2,x)
```

```
[Out] Exception raised: KeyError
```

Giac [A] time = 1.18253, size = 143, normalized size = 1.38

$$\frac{1}{4}\,bc\left(\frac{2\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+\frac{1}{|c|^{\frac{2}{3}}}\right)|c|^{\frac{2}{3}}\right)}{|c|^{\frac{2}{3}}}+\frac{\log\left(x^4+\frac{x^2}{|c|^{\frac{2}{3}}}+\frac{1}{|c|^{\frac{4}{3}}}\right)}{|c|^{\frac{2}{3}}}-\frac{2\log\left(\left|x^2-\frac{1}{|c|^{\frac{2}{3}}}\right|\right)}{|c|^{\frac{2}{3}}}\right)-\frac{b\log\left(-\frac{cx^3+1}{cx^3-1}\right)}{2x}-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/x^2,x, algorithm="giac")
```

```
[Out] 1/4*b*c*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1/abs(c)^(2/3))*abs(c)^(2/3)
)/abs(c)^(2/3) + log(x^4 + x^2/abs(c)^(2/3) + 1/abs(c)^(4/3))/abs(c)^(2/3)
- 2*log(abs(x^2 - 1/abs(c)^(2/3)))/abs(c)^(2/3)) - 1/2*b*log(-(c*x^3 + 1)/(
c*x^3 - 1))/x - a/x
```

3.115 $\int \frac{a+b\tanh^{-1}(cx^3)}{x^5}\,dx$

Optimal. Leaf size=174

$$-\frac{a+b\tanh^{-1}(cx^3)}{4x^4}-\frac{1}{16}bc^{4/3}\log\left(c^{2/3}x^2-\sqrt[3]{c}x+1\right)+\frac{1}{16}bc^{4/3}\log\left(c^{2/3}x^2+\sqrt[3]{c}x+1\right)+\frac{1}{8}\sqrt{3}bc^{4/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{c}}{\sqrt{3}}\right.$$

```
[Out] (-3*b*c)/(4*x) + (Sqrt[3]*b*c^(4/3)*ArcTan[1/Sqrt[3] - (2*c^(1/3)*x)/Sqrt[3
]])/8 - (Sqrt[3]*b*c^(4/3)*ArcTan[1/Sqrt[3] + (2*c^(1/3)*x)/Sqrt[3]])/8 + (
b*c^(4/3)*ArcTanh[c^(1/3)*x])/4 - (a + b*ArcTanh[c*x^3])/(4*x^4) - (b*c^(4/
3)*Log[1 - c^(1/3)*x + c^(2/3)*x^2])/16 + (b*c^(4/3)*Log[1 + c^(1/3)*x + c^
(2/3)*x^2])/16
```

Rubi [A] time = 0.267174, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6097, 325, 296, 634, 618, 204, 628, 206}

$$-\frac{a+b\tanh^{-1}(cx^3)}{4x^4}-\frac{1}{16}bc^{4/3}\log\left(c^{2/3}x^2-\sqrt[3]{c}x+1\right)+\frac{1}{16}bc^{4/3}\log\left(c^{2/3}x^2+\sqrt[3]{c}x+1\right)+\frac{1}{8}\sqrt{3}bc^{4/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{c}}{\sqrt{3}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^3])/x^5,x]
```

```
[Out] (-3*b*c)/(4*x) + (Sqrt[3]*b*c^(4/3)*ArcTan[1/Sqrt[3] - (2*c^(1/3)*x)/Sqrt[3
]])/8 - (Sqrt[3]*b*c^(4/3)*ArcTan[1/Sqrt[3] + (2*c^(1/3)*x)/Sqrt[3]])/8 + (
b*c^(4/3)*ArcTanh[c^(1/3)*x])/4 - (a + b*ArcTanh[c*x^3])/(4*x^4) - (b*c^(4/
3)*Log[1 - c^(1/3)*x + c^(2/3)*x^2])/16 + (b*c^(4/3)*Log[1 + c^(1/3)*x + c^
(2/3)*x^2])/16
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\tanh^{-1}\left(cx^3\right)}{x^5}\,dx &= -\frac{a+b\tanh^{-1}\left(cx^3\right)}{4x^4}+\frac{1}{4}(3bc)\int \frac{1}{x^2\left(1-c^2x^6\right)}\,dx\\
&= -\frac{3bc}{4x}-\frac{a+b\tanh^{-1}\left(cx^3\right)}{4x^4}+\frac{1}{4}\left(3bc^3\right)\int \frac{x^4}{1-c^2x^6}\,dx\\
&= -\frac{3bc}{4x}-\frac{a+b\tanh^{-1}\left(cx^3\right)}{4x^4}+\frac{1}{4}\left(bc^{5/3}\right)\int \frac{1}{1-c^{2/3}x^2}\,dx+\frac{1}{4}\left(bc^{5/3}\right)\int \frac{-\frac{1}{2}-\frac{\sqrt[3]{c}x}{2}}{1-\sqrt[3]{c}x+c^{2/3}x^2}\,dx+\\
&= -\frac{3bc}{4x}+\frac{1}{4}bc^{4/3}\tanh^{-1}\left(\sqrt[3]{c}x\right)-\frac{a+b\tanh^{-1}\left(cx^3\right)}{4x^4}-\frac{1}{16}\left(bc^{4/3}\right)\int \frac{-\sqrt[3]{c}+2c^{2/3}x}{1-\sqrt[3]{c}x+c^{2/3}x^2}\,dx+\frac{1}{16}\Big(\\
&= -\frac{3bc}{4x}+\frac{1}{4}bc^{4/3}\tanh^{-1}\left(\sqrt[3]{c}x\right)-\frac{a+b\tanh^{-1}\left(cx^3\right)}{4x^4}-\frac{1}{16}bc^{4/3}\log\left(1-\sqrt[3]{c}x+c^{2/3}x^2\right)+\frac{1}{16}bc^{4/3}\\
&= -\frac{3bc}{4x}+\frac{1}{8}\sqrt{3}bc^{4/3}\tan^{-1}\left(\frac{1-2\sqrt[3]{c}x}{\sqrt{3}}\right)-\frac{1}{8}\sqrt{3}bc^{4/3}\tan^{-1}\left(\frac{1+2\sqrt[3]{c}x}{\sqrt{3}}\right)+\frac{1}{4}bc^{4/3}\tanh^{-1}\left(\sqrt[3]{c}x\right)-
\end{aligned}
$$

Mathematica [A] time = 0.0516986, size = 196, normalized size = 1.13

$$
-\frac{a}{4x^4}-\frac{1}{16}bc^{4/3}\log\left(c^{2/3}x^2-\sqrt[3]{c}x+1\right)+\frac{1}{16}bc^{4/3}\log\left(c^{2/3}x^2+\sqrt[3]{c}x+1\right)-\frac{1}{8}bc^{4/3}\log\left(1-\sqrt[3]{c}x\right)+\frac{1}{8}bc^{4/3}\log\left(\sqrt[3]{c}x+1\right)-
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^3])/x^5,x]
```

```
[Out] -a/(4*x^4) - (3*b*c)/(4*x) - (Sqrt[3]*b*c^(4/3)*ArcTan[(-1 + 2*c^(1/3)*x)/S
qrt[3]])/8 - (Sqrt[3]*b*c^(4/3)*ArcTan[(1 + 2*c^(1/3)*x)/Sqrt[3]])/8 - (b*A
rcTanh[c*x^3])/(4*x^4) - (b*c^(4/3)*Log[1 - c^(1/3)*x])/8 + (b*c^(4/3)*Log[
1 + c^(1/3)*x])/8 - (b*c^(4/3)*Log[1 - c^(1/3)*x + c^(2/3)*x^2])/16 + (b*c^
(4/3)*Log[1 + c^(1/3)*x + c^(2/3)*x^2])/16
```

Maple [A] time = 0.013, size = 172, normalized size = 1.

$$-\frac{a}{4x^4}-\frac{b\mathrm{Artanh}\left(cx^3\right)}{4x^4}-\frac{3bc}{4x}-\frac{bc}{8}\ln\left(x-\sqrt[3]{c^{-1}}\right)\frac{1}{\sqrt[3]{c^{-1}}}+\frac{bc}{16}\ln\left(x^2+\sqrt[3]{c^{-1}}x+\left(c^{-1}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{c^{-1}}}-\frac{bc\sqrt{3}}{8}\arctan\left(\frac{\sqrt{3}}{3}\left(2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^3))/x^5,x)
```

```
[Out] -1/4*a/x^4-1/4*b/x^4*arctanh(c*x^3)-3/4*b*c/x-1/8*b*c/(1/c)^(1/3)*ln(x-(1/c
)^(1/3))+1/16*b*c/(1/c)^(1/3)*ln(x^2+(1/c)^(1/3)*x+(1/c)^(2/3))-1/8*b*c*3^(
1/2)/(1/c)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x+1))+1/8*b*c/(1/c)^(1/3
)*ln(x+(1/c)^(1/3))-1/16*b*c/(1/c)^(1/3)*ln(x^2-(1/c)^(1/3)*x+(1/c)^(2/3))-
1/8*b*c*3^(1/2)/(1/c)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.67314, size = 547, normalized size = 3.14

$$-\frac{2\sqrt{3}b\left(-c\right)^{\frac{1}{3}}cx^4\arctan\left(\frac{2}{3}\sqrt{3}\left(-c\right)^{\frac{1}{3}}x-\frac{1}{3}\sqrt{3}\right)+2\sqrt{3}bc^{\frac{4}{3}}x^4\arctan\left(\frac{2}{3}\sqrt{3}c^{\frac{1}{3}}x-\frac{1}{3}\sqrt{3}\right)+b\left(-c\right)^{\frac{1}{3}}cx^4\log\left(cx^2+\left(-c\right)^{\frac{2}{3}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/x^5,x, algorithm="fricas")
```

```
[Out] -1/16*(2*sqrt(3)*b*(-c)^(1/3)*c*x^4*arctan(2/3*sqrt(3)*(-c)^(1/3)*x - 1/3*s
qrt(3)) + 2*sqrt(3)*b*c^(4/3)*x^4*arctan(2/3*sqrt(3)*c^(1/3)*x - 1/3*sqrt(3
)) + b*(-c)^(1/3)*c*x^4*log(c*x^2 + (-c)^(2/3)*x - (-c)^(1/3)) + b*c^(4/3)*
x^4*log(c*x^2 - c^(2/3)*x + c^(1/3)) - 2*b*(-c)^(1/3)*c*x^4*log(c*x - (-c)^
(2/3)) - 2*b*c^(4/3)*x^4*log(c*x + c^(2/3)) + 12*b*c*x^3 + 2*b*log(-(c*x^3
+ 1)/(c*x^3 - 1)) + 4*a)/x^4
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: KeyError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**3))/x**5,x)
```

```
[Out] Exception raised: KeyError
```

Giac [A] time = 2.08691, size = 261, normalized size = 1.5

$$-\frac{1}{16}bc^3\left(\frac{2\sqrt{3}|c|^{\frac{1}{3}}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x+\frac{1}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{c^2}+\frac{2\sqrt{3}|c|^{\frac{1}{3}}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x-\frac{1}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{c^2}-\frac{|c|^{\frac{1}{3}}\log\left(x^2+\frac{x}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{c^2}+\frac{|c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/x^5,x, algorithm="giac")
```

```
[Out] -1/16*b*c^3*(2*sqrt(3)*abs(c)^(1/3)*arctan(1/3*sqrt(3)*(2*x + 1/abs(c)^(1/3
))*abs(c)^(1/3))/c^2 + 2*sqrt(3)*abs(c)^(1/3)*arctan(1/3*sqrt(3)*(2*x - 1/a
bs(c)^(1/3))*abs(c)^(1/3))/c^2 - abs(c)^(1/3)*log(x^2 + x/abs(c)^(1/3) + 1/
abs(c)^(2/3))/c^2 + abs(c)^(1/3)*log(x^2 - x/abs(c)^(1/3) + 1/abs(c)^(2/3))
/c^2 - 2*abs(c)^(1/3)*log(abs(x + 1/abs(c)^(1/3)))/c^2 + 2*abs(c)^(1/3)*log
(abs(x - 1/abs(c)^(1/3)))/c^2) - 1/8*b*log(-(c*x^3 + 1)/(c*x^3 - 1))/x^4 -
1/4*(3*b*c*x^3 + a)/x^4
```

3.116 $\int x^{11} \left(a + b \tanh^{-1}\left(cx^3\right)\right)^2 dx$

Optimal. Leaf size=125

$$\frac{abx^3}{6c^3} - \frac{\left(a + b\tanh^{-1}\left(cx^3\right)\right)^2}{12c^4} + \frac{1}{12}x^{12}\left(a + b\tanh^{-1}\left(cx^3\right)\right)^2 + \frac{bx^9\left(a + b\tanh^{-1}\left(cx^3\right)\right)}{18c} + \frac{b^2x^6}{36c^2} + \frac{b^2\log\left(1 - c^2x^6\right)}{9c^4} +$$

```
[Out] (a*b*x^3)/(6*c^3) + (b^2*x^6)/(36*c^2) + (b^2*x^3*ArcTanh[c*x^3])/(6*c^3) +
 (b*x^9*(a + b*ArcTanh[c*x^3]))/(18*c) - (a + b*ArcTanh[c*x^3])^2/(12*c^4)
+ (x^12*(a + b*ArcTanh[c*x^3])^2)/12 + (b^2*Log[1 - c^2*x^6])/(9*c^4)
```

Rubi [C] time = 1.54585, antiderivative size = 636, normalized size of antiderivative = 5.09, number of steps used = 62, number of rules used = 19, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.187, Rules used = {6099, 2454, 2398, 2411, 43, 2334, 12, 14, 2301, 2395, 2439, 2416, 2389, 2295, 2394, 2393, 2391, 2410, 2390}

$$\frac{b^2\text{PolyLog}\left(2, \frac{1}{2}\left(1 - cx^3\right)\right)}{24c^4} + \frac{b^2\text{PolyLog}\left(2, \frac{1}{2}\left(cx^3 + 1\right)\right)}{24c^4} + \frac{abx^3}{12c^3} - \frac{bx^6\left(2a - b\log\left(1 - cx^3\right)\right)}{48c^2} - \frac{1}{288}b\left(-\frac{3\left(1 - cx^3\right)^4}{c^4}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^11*(a + b*ArcTanh[c*x^3])^2,x]
```

```
[Out] (a*b*x^3)/(12*c^3) + (23*b^2*x^3)/(288*c^3) + (b^2*x^6)/(192*c^2) - (7*b^2*
x^9)/(864*c) - (b^2*x^12)/384 + (b^2*(1 - c*x^3)^2)/(16*c^4) - (b^2*(1 - c*
x^3)^3)/(54*c^4) + (b^2*(1 - c*x^3)^4)/(384*c^4) - (5*b^2*Log[1 - c*x^3])/(
288*c^4) + (b^2*(1 - c*x^3)*Log[1 - c*x^3])/(24*c^4) + (b^2*Log[1 - c*x^3]^
2)/(48*c^4) - (b*x^6*(2*a - b*Log[1 - c*x^3]))/(48*c^2) + (b*x^9*(2*a - b*L
og[1 - c*x^3]))/(72*c) - (b*x^12*(2*a - b*Log[1 - c*x^3]))/96 + (x^12*(2*a
- b*Log[1 - c*x^3])^2)/48 - (b*(2*a - b*Log[1 - c*x^3])*((48*(1 - c*x^3))/c
^4 - (36*(1 - c*x^3)^2)/c^4 + (16*(1 - c*x^3)^3)/c^4 - (3*(1 - c*x^3)^4)/c^
4 - (12*Log[1 - c*x^3])/c^4))/288 - (b*(2*a - b*Log[1 - c*x^3])*Log[(1 + c*
x^3)/2])/(24*c^4) + (b^2*Log[1 + c*x^3])/(36*c^4) + (b^2*x^9*Log[1 + c*x^3]
)/(36*c) + (b^2*(1 + c*x^3)*Log[1 + c*x^3])/(12*c^4) + (b^2*Log[(1 - c*x^3)
/2]*Log[1 + c*x^3])/(24*c^4) + (b*x^12*(2*a - b*Log[1 - c*x^3])*Log[1 + c*x
^3])/24 - (b^2*Log[1 + c*x^3]^2)/(48*c^4) + (b^2*x^12*Log[1 + c*x^3]^2)/48
+ (b^2*PolyLog[2, (1 - c*x^3)/2])/(24*c^4) + (b^2*PolyLog[2, (1 + c*x^3)/2]
)/(24*c^4)
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1
] && EqQ[m, -1])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[(x^
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
```

```
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
 + j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rubi steps

$$\int x^{11}\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2 dx = \int \left(\frac{1}{4}x^{11}\left(2a-b\log\left(1-cx^3\right)\right)^2-\frac{1}{2}bx^{11}\left(-2a+b\log\left(1-cx^3\right)\right)\log\left(1+cx^3\right)+\frac{1}{4}b^2x^{1}$$

$$=\frac{1}{4}\int x^{11}\left(2a-b\log\left(1-cx^3\right)\right)^2 dx-\frac{1}{2}b\int x^{11}\left(-2a+b\log\left(1-cx^3\right)\right)\log\left(1+cx^3\right) dx$$

$$=\frac{1}{12}\text{Subst}\left(\int x^3(2a-b\log(1-cx))^2 dx, x, x^3\right)-\frac{1}{6}b\,\text{Subst}\left(\int x^3(-2a+b\log(1-cx))\right.$$

$$=\frac{1}{48}x^{12}\left(2a-b\log\left(1-cx^3\right)\right)^2+\frac{1}{24}bx^{12}\left(2a-b\log\left(1-cx^3\right)\right)\log\left(1+cx^3\right)+\frac{1}{48}b^2x^{12}\,\mathrm{l}$$

$$=\frac{1}{48}x^{12}\left(2a-b\log\left(1-cx^3\right)\right)^2+\frac{1}{24}bx^{12}\left(2a-b\log\left(1-cx^3\right)\right)\log\left(1+cx^3\right)+\frac{1}{48}b^2x^{12}\,\mathrm{l}$$

$$=\frac{1}{48}x^{12}\left(2a-b\log\left(1-cx^3\right)\right)^2-\frac{1}{288}b\left(2a-b\log\left(1-cx^3\right)\right)\left(\frac{48\left(1-cx^3\right)}{c^4}-\frac{36\left(1-cx^3\right.}{c^4}\right.$$

$$=\frac{abx^3}{12c^3}-\frac{bx^6\left(2a-b\log\left(1-cx^3\right)\right)}{48c^2}+\frac{bx^9\left(2a-b\log\left(1-cx^3\right)\right)}{72c}-\frac{1}{96}bx^{12}\left(2a-b\log\left(1\right.\right.$$

$$=\frac{abx^3}{12c^3}-\frac{bx^6\left(2a-b\log\left(1-cx^3\right)\right)}{48c^2}+\frac{bx^9\left(2a-b\log\left(1-cx^3\right)\right)}{72c}-\frac{1}{96}bx^{12}\left(2a-b\log\left(1\right.\right.$$

$$=\frac{abx^3}{12c^3}+\frac{55b^2x^3}{288c^3}-\frac{5b^2x^6}{576c^2}+\frac{b^2x^9}{864c}-\frac{b^2x^{12}}{384}+\frac{b^2\left(1-cx^3\right)^2}{16c^4}-\frac{b^2\left(1-cx^3\right)^3}{54c^4}+\frac{b^2\left(1-cx^3\right.}{384c^4}$$

$$=\frac{abx^3}{12c^3}+\frac{55b^2x^3}{288c^3}-\frac{5b^2x^6}{576c^2}+\frac{b^2x^9}{864c}-\frac{b^2x^{12}}{384}+\frac{b^2\left(1-cx^3\right)^2}{16c^4}-\frac{b^2\left(1-cx^3\right)^3}{54c^4}+\frac{b^2\left(1-cx^3\right.}{384c^4}$$

Mathematica [A] time = 0.0761196, size = 146, normalized size = 1.17

$$\frac{3a^2c^4x^{12}+2abc^3x^9+2bcx^3\tanh^{-1}\left(cx^3\right)\left(3ac^3x^9+b\left(c^2x^6+3\right)\right)+6abcx^3+b(3a+4b)\log\left(1-cx^3\right)-3ab\log\left(cx^3+1\right)}{36c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11*(a + b*ArcTanh[c*x^3])^2,x]
```

```
[Out] (6*a*b*c*x^3 + b^2*c^2*x^6 + 2*a*b*c^3*x^9 + 3*a^2*c^4*x^12 + 2*b*c*x^3*(3*
a*c^3*x^9 + b*(3 + c^2*x^6))*ArcTanh[c*x^3] + 3*b^2*(-1 + c^4*x^12)*ArcTanh
[c*x^3]^2 + b*(3*a + 4*b)*Log[1 - c*x^3] - 3*a*b*Log[1 + c*x^3] + 4*b^2*Log
[1 + c*x^3])/(36*c^4)
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int x^{11}\left(a+b\text{Artanh}\left(cx^3\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(a+b*arctanh(c*x^3))^2,x)
```

```
[Out] int(x^11*(a+b*arctanh(c*x^3))^2,x)
```

Maxima [A] time = 0.985566, size = 293, normalized size = 2.34

$$\frac{1}{12}\,b^2x^{12}\,\mathrm{artanh}\left(cx^3\right)^2+\frac{1}{12}\,a^2x^{12}+\frac{1}{36}\left(6\,x^{12}\,\mathrm{artanh}\left(cx^3\right)+c\left(\frac{2\left(c^2x^9+3\,x^3\right)}{c^4}-\frac{3\,\log\left(cx^3+1\right)}{c^5}+\frac{3\,\log\left(cx^3-1\right)}{c^5}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(a+b*arctanh(c*x^3))^2,x, algorithm="maxima")
```

```
[Out] 1/12*b^2*x^12*arctanh(c*x^3)^2 + 1/12*a^2*x^12 + 1/36*(6*x^12*arctanh(c*x^3
) + c*(2*(c^2*x^9 + 3*x^3)/c^4 - 3*log(c*x^3 + 1)/c^5 + 3*log(c*x^3 - 1)/c^
5))*a*b + 1/144*(4*c*(2*(c^2*x^9 + 3*x^3)/c^4 - 3*log(c*x^3 + 1)/c^5 + 3*lo
g(c*x^3 - 1)/c^5)*arctanh(c*x^3) + (4*c^2*x^6 - 2*(3*log(c*x^3 - 1) - 8)*lo
g(c*x^3 + 1) + 3*log(c*x^3 + 1)^2 + 3*log(c*x^3 - 1)^2 + 16*log(c*x^3 - 1))
/c^4)*b^2
```

Fricas [A] time = 1.85454, size = 381, normalized size = 3.05

$$\frac{12\,a^2c^4x^{12}+8\,abc^3x^9+4\,b^2c^2x^6+24\,abcx^3+3\left(b^2c^4x^{12}-b^2\right)\log\left(-\frac{cx^3+1}{cx^3-1}\right)^2-4\left(3\,ab-4\,b^2\right)\log\left(cx^3+1\right)+4\left(3\,ab+\right.}{144\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(a+b*arctanh(c*x^3))^2,x, algorithm="fricas")
```

```
[Out] 1/144*(12*a^2*c^4*x^12 + 8*a*b*c^3*x^9 + 4*b^2*c^2*x^6 + 24*a*b*c*x^3 + 3*(
b^2*c^4*x^12 - b^2)*log(-(c*x^3 + 1)/(c*x^3 - 1))^2 - 4*(3*a*b - 4*b^2)*log
(c*x^3 + 1) + 4*(3*a*b + 4*b^2)*log(c*x^3 - 1) + 4*(3*a*b*c^4*x^12 + b^2*c^
3*x^9 + 3*b^2*c*x^3)*log(-(c*x^3 + 1)/(c*x^3 - 1)))/c^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(a+b*atanh(c*x**3))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28881, size = 236, normalized size = 1.89

$$\frac{1}{12}\,a^2x^{12}+\frac{abx^9}{18\,c}+\frac{b^2x^6}{36\,c^2}+\frac{1}{48}\left(b^2x^{12}-\frac{b^2}{c^4}\right)\log\left(-\frac{cx^3+1}{cx^3-1}\right)^2+\frac{abx^3}{6\,c^3}+\frac{1}{36}\left(3\,abx^{12}+\frac{b^2x^9}{c}+\frac{3\,b^2x^3}{c^3}\right)\log\left(-\frac{cx^3+1}{cx^3-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(a+b*arctanh(c*x^3))^2,x, algorithm="giac")
```

```
[Out] 1/12*a^2*x^12 + 1/18*a*b*x^9/c + 1/36*b^2*x^6/c^2 + 1/48*(b^2*x^12 - b^2/c^
4)*log(-(c*x^3 + 1)/(c*x^3 - 1))^2 + 1/6*a*b*x^3/c^3 + 1/36*(3*a*b*x^12 + b
^2*x^9/c + 3*b^2*x^3/c^3)*log(-(c*x^3 + 1)/(c*x^3 - 1)) - 1/36*(3*a*b - 4*b
^2)*log(c*x^3 + 1)/c^4 + 1/36*(3*a*b + 4*b^2)*log(c*x^3 - 1)/c^4
```

3.117 $\int x^8 \left(a+b\tanh^{-1}\left(cx^3\right)\right)^2 dx$

Optimal. Leaf size=146

$$-\frac{b^2\text{PolyLog}\left(2,1-\frac{2}{1-cx^3}\right)}{9c^3}+\frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2}{9c^3}-\frac{2b\log\left(\frac{2}{1-cx^3}\right)\left(a+b\tanh^{-1}\left(cx^3\right)\right)}{9c^3}+\frac{1}{9}x^9\left(a+b\tanh^{-1}\left(cx^3\right)\right)$$

```
[Out] (b^2*x^3)/(9*c^2) - (b^2*ArcTanh[c*x^3])/(9*c^3) + (b*x^6*(a + b*ArcTanh[c*
x^3]))/(9*c) + (a + b*ArcTanh[c*x^3])^2/(9*c^3) + (x^9*(a + b*ArcTanh[c*x^3
])^2)/9 - (2*b*(a + b*ArcTanh[c*x^3])*Log[2/(1 - c*x^3)])/(9*c^3) - (b^2*Po
lyLog[2, 1 - 2/(1 - c*x^3)])/(9*c^3)
```

Rubi [B] time = 1.30934, antiderivative size = 536, normalized size of antiderivative = 3.67, number of steps used = 53, number of rules used = 19, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.187, Rules used = {6099, 2454, 2398, 2411, 43, 2334, 12, 14, 2301, 2395, 2439, 2416, 2389, 2295, 2394, 2393, 2391, 2410, 2390}

$$-\frac{b^2\text{PolyLog}\left(2,\frac{1}{2}\left(1-cx^3\right)\right)}{18c^3}+\frac{b^2\text{PolyLog}\left(2,\frac{1}{2}\left(cx^3+1\right)\right)}{18c^3}-\frac{abx^3}{9c^2}-\frac{1}{108}b\left(\frac{2\left(1-cx^3\right)^3}{c^3}-\frac{9\left(1-cx^3\right)^2}{c^3}+\frac{18\left(1-cx^3\right)}{c^3}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^8*(a + b*ArcTanh[c*x^3])^2,x]
```

```
[Out] -(a*b*x^3)/(9*c^2) + (19*b^2*x^3)/(108*c^2) - (5*b^2*x^6)/(216*c) - (b^2*x^
9)/162 + (b^2*(1 - c*x^3)^2)/(24*c^3) - (b^2*(1 - c*x^3)^3)/(162*c^3) + (b^
2*Log[1 - c*x^3])/(108*c^3) - (b^2*(1 - c*x^3)*Log[1 - c*x^3])/(18*c^3) + (
b^2*Log[1 - c*x^3]^2)/(36*c^3) + (b*x^6*(2*a - b*Log[1 - c*x^3]))/(36*c) -
(b*x^9*(2*a - b*Log[1 - c*x^3]))/54 + (x^9*(2*a - b*Log[1 - c*x^3])^2)/36 -
 (b*(2*a - b*Log[1 - c*x^3])*((18*(1 - c*x^3))/c^3 - (9*(1 - c*x^3)^2)/c^3
+ (2*(1 - c*x^3)^3)/c^3 - (6*Log[1 - c*x^3])/c^3))/108 + (b*(2*a - b*Log[1
- c*x^3])*Log[(1 + c*x^3)/2])/(18*c^3) - (b^2*Log[1 + c*x^3])/(18*c^3) + (b
^2*x^6*Log[1 + c*x^3])/(18*c) + (b^2*Log[(1 - c*x^3)/2]*Log[1 + c*x^3])/(18
*c^3) + (b*x^9*(2*a - b*Log[1 - c*x^3])*Log[1 + c*x^3])/18 + (b^2*Log[1 + c
*x^3]^2)/(36*c^3) + (b^2*x^9*Log[1 + c*x^3]^2)/36 - (b^2*PolyLog[2, (1 - c*
x^3)/2])/(18*c^3) + (b^2*PolyLog[2, (1 + c*x^3)/2])/(18*c^3)
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1
] && EqQ[m, -1])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[(x^
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
```

```
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
 + j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rubi steps

$$\int x^8\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2\,dx=\int\left(\frac{1}{4}x^8\left(2a-b\log\left(1-cx^3\right)\right)^2-\frac{1}{2}bx^8\left(-2a+b\log\left(1-cx^3\right)\right)\log\left(1+cx^3\right)+\frac{1}{4}b^2x^8\log\right.$$

$$=\frac{1}{4}\int x^8\left(2a-b\log\left(1-cx^3\right)\right)^2\,dx-\frac{1}{2}b\int x^8\left(-2a+b\log\left(1-cx^3\right)\right)\log\left(1+cx^3\right)\,dx+$$

$$=\frac{1}{12}\text{Subst}\left(\int x^2(2a-b\log(1-cx))^2\,dx,x,x^3\right)-\frac{1}{6}b\,\text{Subst}\left(\int x^2(-2a+b\log(1-cx))\,\mathrm{l}\right.$$

$$=\frac{1}{36}x^9\left(2a-b\log\left(1-cx^3\right)\right)^2+\frac{1}{18}bx^9\left(2a-b\log\left(1-cx^3\right)\right)\log\left(1+cx^3\right)+\frac{1}{36}b^2x^9\log^2$$

$$=\frac{1}{36}x^9\left(2a-b\log\left(1-cx^3\right)\right)^2+\frac{1}{18}bx^9\left(2a-b\log\left(1-cx^3\right)\right)\log\left(1+cx^3\right)+\frac{1}{36}b^2x^9\log^2$$

$$=\frac{1}{36}x^9\left(2a-b\log\left(1-cx^3\right)\right)^2-\frac{1}{108}b\left(2a-b\log\left(1-cx^3\right)\right)\left(\frac{18\left(1-cx^3\right)}{c^3}-\frac{9\left(1-cx^3\right)^2}{c^3}\right.$$

$$=-\frac{abx^3}{9c^2}+\frac{bx^6\left(2a-b\log\left(1-cx^3\right)\right)}{36c}-\frac{1}{54}bx^9\left(2a-b\log\left(1-cx^3\right)\right)+\frac{1}{36}x^9\left(2a-b\log\left(1\right.\right.$$

$$=-\frac{abx^3}{9c^2}+\frac{bx^6\left(2a-b\log\left(1-cx^3\right)\right)}{36c}-\frac{1}{54}bx^9\left(2a-b\log\left(1-cx^3\right)\right)+\frac{1}{36}x^9\left(2a-b\log\left(1\right.\right.$$

$$=-\frac{abx^3}{9c^2}+\frac{13b^2x^3}{108c^2}+\frac{b^2x^6}{216c}-\frac{b^2x^9}{162}+\frac{b^2\left(1-cx^3\right)^2}{24c^3}-\frac{b^2\left(1-cx^3\right)^3}{162c^3}+\frac{b^2\log\left(1-cx^3\right)}{108c^3}-\frac{b^2}{}$$

$$=-\frac{abx^3}{9c^2}+\frac{13b^2x^3}{108c^2}+\frac{b^2x^6}{216c}-\frac{b^2x^9}{162}+\frac{b^2\left(1-cx^3\right)^2}{24c^3}-\frac{b^2\left(1-cx^3\right)^3}{162c^3}+\frac{b^2\log\left(1-cx^3\right)}{108c^3}-\frac{b^2}{}$$

Mathematica [A] time = 0.271671, size = 132, normalized size = 0.9

$$\frac{b^2\text{PolyLog}\left(2,-e^{-2\tanh^{-1}\left(cx^3\right)}\right)+a^2c^3x^9+abc^2x^6+ab\log\left(c^2x^6-1\right)+b\tanh^{-1}\left(cx^3\right)\left(2ac^3x^9+bc^2x^6-2b\log\left(e^{-2\tanh^{-1}}\right.\right.}{9c^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^8*(a + b*ArcTanh[c*x^3])^2,x]
```

```
[Out] (b^2*c*x^3 + a*b*c^2*x^6 + a^2*c^3*x^9 + b^2*(-1 + c^3*x^9)*ArcTanh[c*x^3]^
2 + b*ArcTanh[c*x^3]*(-b + b*c^2*x^6 + 2*a*c^3*x^9 - 2*b*Log[1 + E^(-2*ArcT
anh[c*x^3])]) + a*b*Log[-1 + c^2*x^6] + b^2*PolyLog[2, -E^(-2*ArcTanh[c*x^3
])])/(9*c^3)
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int x^8\left(a+b\text{Artanh}\left(cx^3\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a+b*arctanh(c*x^3))^2,x)
```

```
[Out] int(x^8*(a+b*arctanh(c*x^3))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{9}a^2x^9 + \frac{1}{9}\left(2x^9 \operatorname{artanh}\left(cx^3\right) + \left(\frac{x^6}{c^2} + \frac{\log\left(c^2x^6-1\right)}{c^4}\right)c\right)ab + \frac{1}{648}\left(18x^9\log\left(-cx^3+1\right)^2 - 2c^4\left(\frac{2\left(c^2x^9+3x^3\right)}{c^6} - \frac{3\,\mathrm{lo}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(a+b*arctanh(c*x^3))^2,x, algorithm="maxima")
```

```
[Out] 1/9*a^2*x^9 + 1/9*(2*x^9*arctanh(c*x^3) + (x^6/c^2 + log(c^2*x^6 - 1)/c^4)*
c)*a*b + 1/648*(18*x^9*log(-c*x^3 + 1)^2 - 2*c^4*(2*(c^2*x^9 + 3*x^3)/c^6 -
 3*log(c*x^3 + 1)/c^7 + 3*log(c*x^3 - 1)/c^7) + 3*(x^6/c^4 + log(c^2*x^6 -
1)/c^6)*c^3 + 1944*c^3*integrate(1/9*x^11*log(c*x^3 + 1)/(c^4*x^6 - c^2), x
) - 9*c^2*(2*x^3/c^4 - log(c*x^3 + 1)/c^5 + log(c*x^3 - 1)/c^5) - 6*c*((2*c
^2*x^9 + 3*c*x^6 + 6*x^3)/c^3 + 6*log(c*x^3 - 1)/c^4)*log(-c*x^3 + 1) + 972
*c*integrate(1/9*x^5*log(c*x^3 + 1)/(c^4*x^6 - c^2), x) + 6*(3*c^3*x^9*log(
c*x^3 + 1)^2 + (2*c^3*x^9 - 3*c^2*x^6 + 6*c*x^3 - 6*(c^3*x^9 + 1)*log(c*x^3
 + 1))*log(-c*x^3 + 1))/c^3 + (4*c^3*x^9 + 15*c^2*x^6 + 66*c*x^3 + 18*log(c
*x^3 - 1)^2 + 66*log(c*x^3 - 1))/c^3 - 18*log(9*c^4*x^6 - 9*c^2)/c^3 + 972*
integrate(1/9*x^2*log(c*x^3 + 1)/(c^4*x^6 - c^2), x))*b^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(b^2x^8 \operatorname{artanh}\left(cx^3\right)^2 + 2\,abx^8 \operatorname{artanh}\left(cx^3\right) + a^2x^8, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(a+b*arctanh(c*x^3))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^8*arctanh(c*x^3)^2 + 2*a*b*x^8*arctanh(c*x^3) + a^2*x^8, x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: KeyError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(a+b*atanh(c*x**3))**2,x)
```

```
[Out] Exception raised: KeyError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \operatorname{artanh}\left(cx^3\right) + a\right)^2 x^8\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(a+b*arctanh(c*x^3))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^3) + a)^2*x^8, x)
```

3.118 $\int x^5 \left(a+b\tanh^{-1}\left(cx^3\right)\right)^2 dx$

Optimal. Leaf size=91

$$-\frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2}{6c^2}+\frac{abx^3}{3c}+\frac{1}{6}x^6\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2+\frac{b^2\log\left(1-c^2x^6\right)}{6c^2}+\frac{b^2x^3\tanh^{-1}\left(cx^3\right)}{3c}$$

```
[Out] (a*b*x^3)/(3*c) + (b^2*x^3*ArcTanh[c*x^3])/(3*c) - (a + b*ArcTanh[c*x^3])^2
/(6*c^2) + (x^6*(a + b*ArcTanh[c*x^3])^2)/6 + (b^2*Log[1 - c^2*x^6])/(6*c^2
)
```

Rubi [C] time = 0.985499, antiderivative size = 524, normalized size of antiderivative = 5.76, number of steps used = 44, number of rules used = 16, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {6099, 2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304, 2395, 43, 2439, 2416, 2394, 2393, 2391}

$$\frac{b^2\text{PolyLog}\left(2,\frac{1}{2}\left(1-cx^3\right)\right)}{12c^2}+\frac{b^2\text{PolyLog}\left(2,\frac{1}{2}\left(cx^3+1\right)\right)}{12c^2}+\frac{\left(1-cx^3\right)^2\left(2a-b\log\left(1-cx^3\right)\right)^2}{24c^2}-\frac{\left(1-cx^3\right)\left(2a-b\log\right.}{12c^2}$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^5*(a + b*ArcTanh[c*x^3])^2,x]
```

```
[Out] (a*b*x^3)/(2*c) - (b^2*x^6)/24 + (b^2*(1 - c*x^3)^2)/(48*c^2) + (b^2*(1 + c
*x^3)^2)/(48*c^2) - (b^2*Log[1 - c*x^3])/(24*c^2) + (b^2*(1 - c*x^3)*Log[1
- c*x^3])/(4*c^2) - (b*x^6*(2*a - b*Log[1 - c*x^3]))/24 + (b*(1 - c*x^3)^2*
(2*a - b*Log[1 - c*x^3]))/(24*c^2) - ((1 - c*x^3)*(2*a - b*Log[1 - c*x^3])^
2)/(12*c^2) + ((1 - c*x^3)^2*(2*a - b*Log[1 - c*x^3])^2)/(24*c^2) - (b*(2*a
 - b*Log[1 - c*x^3])*Log[(1 + c*x^3)/2])/(12*c^2) - (b^2*Log[1 + c*x^3])/(2
4*c^2) + (b^2*x^6*Log[1 + c*x^3])/24 + (b^2*(1 + c*x^3)*Log[1 + c*x^3])/(4*
c^2) - (b^2*(1 + c*x^3)^2*Log[1 + c*x^3])/(24*c^2) + (b^2*Log[(1 - c*x^3)/2
]*Log[1 + c*x^3])/(12*c^2) + (b*x^6*(2*a - b*Log[1 - c*x^3])*Log[1 + c*x^3]
)/12 - (b^2*(1 + c*x^3)*Log[1 + c*x^3]^2)/(12*c^2) + (b^2*(1 + c*x^3)^2*Log
[1 + c*x^3]^2)/(24*c^2) + (b^2*PolyLog[2, (1 - c*x^3)/2])/(12*c^2) + (b^2*P
olyLog[2, (1 + c*x^3)/2])/(12*c^2)
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
```

```
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[(x^
```

```
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
 + j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int x^5 \left(a+b\tanh^{-1}\left(cx^3\right)\right)^2 dx = \int \left(\frac{1}{4}x^5\left(2a-b\log\left(1-cx^3\right)\right)^2-\frac{1}{2}bx^5\left(-2a+b\log\left(1-cx^3\right)\right)\log\left(1+cx^3\right)+\frac{1}{4}b^2x^5\log$$
$$=\frac{1}{4}\int x^5\left(2a-b\log\left(1-cx^3\right)\right)^2 dx-\frac{1}{2}b\int x^5\left(-2a+b\log\left(1-cx^3\right)\right)\log\left(1+cx^3\right)dx+$$
$$=\frac{1}{12}\text{Subst}\left(\int x(2a-b\log(1-cx))^2\,dx,x,x^3\right)-\frac{1}{6}b\,\text{Subst}\left(\int x(-2a+b\log(1-cx))\log\right.$$
$$=\frac{1}{12}bx^6\left(2a-b\log\left(1-cx^3\right)\right)\log\left(1+cx^3\right)+\frac{1}{12}\text{Subst}\left(\int\left(\frac{(2a-b\log(1-cx))^2}{c}-\frac{(1-}{}\right.\right.$$
$$=\frac{1}{12}bx^6\left(2a-b\log\left(1-cx^3\right)\right)\log\left(1+cx^3\right)+\frac{\text{Subst}\left(\int(2a-b\log(1-cx))^2\,dx,x,x^3\right)}{12c}-$$
$$=\frac{1}{12}bx^6\left(2a-b\log\left(1-cx^3\right)\right)\log\left(1+cx^3\right)+\frac{1}{12}b\,\text{Subst}\left(\int x(-2a+b\log(1-cx))\,dx,x,\right.$$
$$=\frac{abx^3}{6c}-\frac{1}{24}bx^6\left(2a-b\log\left(1-cx^3\right)\right)-\frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^2}{12c^2}+\frac{\left(1-cx^3\right)^2(2a}{}$$
$$=\frac{abx^3}{2c}-\frac{b^2x^3}{4c}+\frac{b^2\left(1-cx^3\right)^2}{48c^2}+\frac{b^2\left(1+cx^3\right)^2}{48c^2}-\frac{1}{24}bx^6\left(2a-b\log\left(1-cx^3\right)\right)+\frac{b\left(1-cx\right.}{}$$
$$=\frac{abx^3}{2c}-\frac{b^2x^6}{24}+\frac{b^2\left(1-cx^3\right)^2}{48c^2}+\frac{b^2\left(1+cx^3\right)^2}{48c^2}-\frac{b^2\log\left(1-cx^3\right)}{24c^2}+\frac{b^2\left(1-cx^3\right)\log\left(1-c\right.}{4c^2}$$

Mathematica [A] time = 0.0555193, size = 106, normalized size = 1.16

$$\frac{a^2c^2x^6+2abcx^3+b(a+b)\log\left(1-cx^3\right)-ab\log\left(cx^3+1\right)+2bcx^3\tanh^{-1}\left(cx^3\right)\left(acx^3+b\right)+b^2\left(c^2x^6-1\right)\tanh^{-1}\left(cx^3\right)^2}{6c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*ArcTanh[c*x^3])^2,x]
```

```
[Out] (2*a*b*c*x^3 + a^2*c^2*x^6 + 2*b*c*x^3*(b + a*c*x^3)*ArcTanh[c*x^3] + b^2*(
-1 + c^2*x^6)*ArcTanh[c*x^3]^2 + b*(a + b)*Log[1 - c*x^3] - a*b*Log[1 + c*x
^3] + b^2*Log[1 + c*x^3])/(6*c^2)
```

Maple [B] time = 0.18, size = 247, normalized size = 2.7

$$\frac{b^2\left(c^2x^6-1\right)\left(\ln\left(cx^3+1\right)\right)^2}{24\,c^2}+\frac{b\left(-x^6b\ln\left(-cx^3+1\right)c^2+2\,ac^2x^6+2\,bcx^3+b\ln\left(-cx^3+1\right)\right)\ln\left(cx^3+1\right)}{12\,c^2}+\frac{b^2x^6\left(\ln\left(-cx\right.\right.}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arctanh(c*x^3))^2,x)
```

```
[Out] 1/24*b^2*(c^2*x^6-1)/c^2*ln(c*x^3+1)^2+1/12*b*(-x^6*b*ln(-c*x^3+1)*c^2+2*a*
c^2*x^6+2*b*c*x^3+b*ln(-c*x^3+1))/c^2*ln(c*x^3+1)+1/24*b^2*x^6*ln(-c*x^3+1)
^2-1/6*a*b*x^6*ln(-c*x^3+1)+1/6*x^6*a^2-1/6/c*b^2*x^3*ln(-c*x^3+1)+1/3*a*b*
x^3/c-1/24/c^2*b^2*ln(-c*x^3+1)^2-1/6/c^2*b*ln(-c*x^3-1)*a+1/6/c^2*b^2*ln(-
c*x^3-1)+1/6/c^2*b*ln(-c*x^3+1)*a+1/6/c^2*b^2*ln(-c*x^3+1)
```

Maxima [B] time = 1.12297, size = 251, normalized size = 2.76

$$\frac{1}{6}b^2x^6\operatorname{artanh}\left(cx^3\right)^2+\frac{1}{6}a^2x^6+\frac{1}{6}\left(2x^6\operatorname{artanh}\left(cx^3\right)+c\left(\frac{2x^3}{c^2}-\frac{\log\left(cx^3+1\right)}{c^3}+\frac{\log\left(cx^3-1\right)}{c^3}\right)\right)ab+\frac{1}{24}\left(4c\left(\frac{2x^3}{c^2}-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x^3))^2,x, algorithm="maxima")
```

```
[Out] 1/6*b^2*x^6*arctanh(c*x^3)^2 + 1/6*a^2*x^6 + 1/6*(2*x^6*arctanh(c*x^3) + c*
(2*x^3/c^2 - log(c*x^3 + 1)/c^3 + log(c*x^3 - 1)/c^3))*a*b + 1/24*(4*c*(2*x
^3/c^2 - log(c*x^3 + 1)/c^3 + log(c*x^3 - 1)/c^3)*arctanh(c*x^3) - (2*(log(
c*x^3 - 1) - 2)*log(c*x^3 + 1) - log(c*x^3 + 1)^2 - log(c*x^3 - 1)^2 - 4*lo
g(c*x^3 - 1))/c^2)*b^2
```

Fricas [A] time = 1.6891, size = 292, normalized size = 3.21

$$\frac{4a^2c^2x^6+8abcx^3+\left(b^2c^2x^6-b^2\right)\log\left(-\frac{cx^3+1}{cx^3-1}\right)^2-4\left(ab-b^2\right)\log\left(cx^3+1\right)+4\left(ab+b^2\right)\log\left(cx^3-1\right)+4\left(abc^2x^6+b\right.}{24c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x^3))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(4*a^2*c^2*x^6 + 8*a*b*c*x^3 + (b^2*c^2*x^6 - b^2)*log(-(c*x^3 + 1)/(c
*x^3 - 1))^2 - 4*(a*b - b^2)*log(c*x^3 + 1) + 4*(a*b + b^2)*log(c*x^3 - 1)
+ 4*(a*b*c^2*x^6 + b^2*c*x^3)*log(-(c*x^3 + 1)/(c*x^3 - 1)))/c^2
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: KeyError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*atanh(c*x**3))**2,x)
```

```
[Out] Exception raised: KeyError
```

Giac [A] time = 1.19832, size = 186, normalized size = 2.04

$$\frac{1}{6}a^2x^6+\frac{abx^3}{3c}+\frac{1}{24}\left(b^2x^6-\frac{b^2}{c^2}\right)\log\left(-\frac{cx^3+1}{cx^3-1}\right)^2+\frac{1}{6}\left(abx^6+\frac{b^2x^3}{c}\right)\log\left(-\frac{cx^3+1}{cx^3-1}\right)-\frac{\left(ab-b^2\right)\log\left(cx^3+1\right)}{6c^2}+\frac{(ab-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x^3))^2,x, algorithm="giac")
```

```
[Out] 1/6*a^2*x^6 + 1/3*a*b*x^3/c + 1/24*(b^2*x^6 - b^2/c^2)*log(-(c*x^3 + 1)/(c*
x^3 - 1))^2 + 1/6*(a*b*x^6 + b^2*x^3/c)*log(-(c*x^3 + 1)/(c*x^3 - 1)) - 1/6
*(a*b - b^2)*log(c*x^3 + 1)/c^2 + 1/6*(a*b + b^2)*log(c*x^3 - 1)/c^2
```

3.119 $\int x^2 \left(a+b \tanh^{-1}\left(cx^3\right)\right)^2 dx$

Optimal. Leaf size=96

$$-\frac{b^2 \text{PolyLog}\left(2,1-\frac{2}{1-cx^3}\right)}{3c}+\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2+\frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2}{3c}-\frac{2b\log\left(\frac{2}{1-cx^3}\right)\left(a+b\tanh^{-1}\left(cx^3\right)\right)}{3c}$$

```
[Out] (a + b*ArcTanh[c*x^3])^2/(3*c) + (x^3*(a + b*ArcTanh[c*x^3])^2)/3 - (2*b*(a
 + b*ArcTanh[c*x^3])*Log[2/(1 - c*x^3)])/(3*c) - (b^2*PolyLog[2, 1 - 2/(1 -
 c*x^3)])/(3*c)
```

Rubi [B] time = 0.591888, antiderivative size = 207, normalized size of antiderivative = 2.16, number of steps used = 28, number of rules used = 12, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {6099, 2454, 2389, 2296, 2295, 6715, 2430, 43, 2416, 2394, 2393, 2391}

$$-\frac{b^2\text{PolyLog}\left(2,\frac{1}{2}\left(1-cx^3\right)\right)}{6c}+\frac{b^2\text{PolyLog}\left(2,\frac{1}{2}\left(cx^3+1\right)\right)}{6c}+\frac{1}{6}bx^3\log\left(cx^3+1\right)\left(2a-b\log\left(1-cx^3\right)\right)-\frac{\left(1-cx^3\right)\left(2a-\right.}{1}$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^2*(a + b*ArcTanh[c*x^3])^2,x]
```

```
[Out] -((1 - c*x^3)*(2*a - b*Log[1 - c*x^3])^2)/(12*c) + (b*(2*a - b*Log[1 - c*x^
3])*Log[(1 + c*x^3)/2])/(6*c) + (b^2*Log[(1 - c*x^3)/2]*Log[1 + c*x^3])/(6*
c) + (b*x^3*(2*a - b*Log[1 - c*x^3])*Log[1 + c*x^3])/6 + (b^2*(1 + c*x^3)*L
og[1 + c*x^3]^2)/(12*c) - (b^2*PolyLog[2, (1 - c*x^3)/2])/(6*c) + (b^2*Poly
Log[2, (1 + c*x^3)/2])/(6*c)
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 2430

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.)), x_Symbol] :> Simp[x*(a + b*Log[c
*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]), x] + (-Dist[g*j*m, Int[(x*(a +
 b*Log[c*(d + e*x)^n])^p)/(i + j*x), x], x] - Dist[b*e*n*p, Int[(x*(a + b*L
og[c*(d + e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /
; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int x^2\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2dx=\int\left(\frac{1}{4}x^2\left(2a-b\log\left(1-cx^3\right)\right)^2-\frac{1}{2}bx^2\left(-2a+b\log\left(1-cx^3\right)\right)\log\left(1+cx^3\right)+\frac{1}{4}b^2x^2\log\right.$$
$$=\frac{1}{4}\int x^2\left(2a-b\log\left(1-cx^3\right)\right)^2dx-\frac{1}{2}b\int x^2\left(-2a+b\log\left(1-cx^3\right)\right)\log\left(1+cx^3\right)dx+$$
$$=\frac{1}{12}\text{Subst}\left(\int(2a-b\log(1-cx))^2dx,x,x^3\right)-\frac{1}{6}b\,\text{Subst}\left(\int(-2a+b\log(1-cx))\log(1\right.$$
$$=\frac{1}{6}bx^3\left(2a-b\log\left(1-cx^3\right)\right)\log\left(1+cx^3\right)-\frac{\text{Subst}\left(\int(2a-b\log(x))^2dx,x,1-cx^3\right)}{12c}+$$
$$=-\frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^2}{12c}+\frac{1}{6}bx^3\left(2a-b\log\left(1-cx^3\right)\right)\log\left(1+cx^3\right)+\frac{b^2\left(1+\right.}{}$$
$$=\frac{1}{3}abx^3+\frac{b^2x^3}{6}-\frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^2}{12c}-\frac{b^2\left(1+cx^3\right)\log\left(1+cx^3\right)}{6c}+\frac{1}{6}bx^3\left(\right.$$
$$=\frac{b^2x^3}{3}+\frac{b^2\left(1-cx^3\right)\log\left(1-cx^3\right)}{6c}-\frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^2}{12c}+\frac{b\left(2a-b\log\left(1-\right.\right.}{}$$
$$=\frac{b^2x^3}{6}+\frac{b^2\left(1-cx^3\right)\log\left(1-cx^3\right)}{6c}-\frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^2}{12c}+\frac{b\left(2a-b\log\left(1-\right.\right.}{}$$
$$=-\frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^2}{12c}+\frac{b\left(2a-b\log\left(1-cx^3\right)\right)\log\left(\frac{1}{2}\left(1+cx^3\right)\right)}{6c}+\frac{b^2\log\left(\right.}{}$$

Mathematica [A] time = 0.152375, size = 99, normalized size = 1.03

$$\frac{b^2\text{PolyLog}\left(2,-e^{-2\tanh^{-1}\left(cx^3\right)}\right)+a\left(acx^3+b\log\left(1-c^2x^6\right)\right)+2b\tanh^{-1}\left(cx^3\right)\left(acx^3-b\log\left(e^{-2\tanh^{-1}\left(cx^3\right)}+1\right)\right)+b^2\left(cx\right.}{3c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(a + b*ArcTanh[c*x^3])^2,x]
```

```
[Out] (b^2*(-1 + c*x^3)*ArcTanh[c*x^3]^2 + 2*b*ArcTanh[c*x^3]*(a*c*x^3 - b*Log[1
+ E^(-2*ArcTanh[c*x^3])]) + a*(a*c*x^3 + b*Log[1 - c^2*x^6]) + b^2*PolyLog[
2, -E^(-2*ArcTanh[c*x^3])])/(3*c)
```

Maple [A] time = 0.004, size = 145, normalized size = 1.5

$$\frac{\left(\text{Artanh}\left(cx^3\right)\right)^2x^3b^2}{3}+\frac{2\,abx^3\text{Artanh}\left(cx^3\right)}{3}+\frac{x^3a^2}{3}-\frac{2\,\text{Artanh}\left(cx^3\right)b^2}{3\,c}\ln\left(\frac{\left(cx^3+1\right)^2}{-c^2x^6+1}+1\right)+\frac{b^2\left(\text{Artanh}\left(cx^3\right)\right)^2}{3\,c}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c*x^3))^2,x)
```

```
[Out] 1/3*arctanh(c*x^3)^2*x^3*b^2+2/3*a*b*x^3*arctanh(c*x^3)+1/3*x^3*a^2-2/3/c*a
rctanh(c*x^3)*ln((c*x^3+1)^2/(-c^2*x^6+1)+1)*b^2+1/3/c*b^2*arctanh(c*x^3)^2
+1/3/c*a*b*ln(-c^2*x^6+1)-1/3/c*polylog(2,-(c*x^3+1)^2/(-c^2*x^6+1))*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{3}a^2x^3 + \frac{1}{12}\left(x^3\log\left(-cx^3+1\right)^2 - c^2\left(\frac{2x^3}{c^2} - \frac{\log\left(cx^3+1\right)}{c^3} + \frac{\log\left(cx^3-1\right)}{c^3}\right) - 2\left(\frac{x^3}{c} + \frac{\log\left(cx^3-1\right)}{c^2}\right)c\log\left(-cx^3+1\right) -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^3))^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*x^3 + 1/12*(x^3*log(-c*x^3 + 1)^2 - c^2*(2*x^3/c^2 - log(c*x^3 + 1)
/c^3 + log(c*x^3 - 1)/c^3) - 2*(x^3/c + log(c*x^3 - 1)/c^2)*c*log(-c*x^3 +
1) + 18*c*integrate(x^5*log(c*x^3 + 1)/(c^2*x^6 - 1), x) + (c*x^3*log(c*x^3
 + 1)^2 + 2*(c*x^3 - (c*x^3 + 1)*log(c*x^3 + 1))*log(-c*x^3 + 1))/c + (2*c*
x^3 + log(c*x^3 - 1)^2 + 2*log(c*x^3 - 1))/c - log(c^2*x^6 - 1)/c + 6*integ
rate(x^2*log(c*x^3 + 1)/(c^2*x^6 - 1), x))*b^2 + 1/3*(2*c*x^3*arctanh(c*x^3
) + log(-c^2*x^6 + 1))*a*b/c
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(b^2x^2\,\text{artanh}\left(cx^3\right)^2 + 2\,abx^2\,\text{artanh}\left(cx^3\right) + a^2x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^3))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^2*arctanh(c*x^3)^2 + 2*a*b*x^2*arctanh(c*x^3) + a^2*x^2, x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: KeyError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c*x**3))**2,x)
```

```
[Out] Exception raised: KeyError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\,\text{artanh}\left(cx^3\right) + a\right)^2 x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^3))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^3) + a)^2*x^2, x)
```

3.120 $\int \frac{(a+b\tanh^{-1}(cx^3))^2}{x}\,dx$

Optimal. Leaf size=140

$$-\frac{1}{3}b\text{PolyLog}\left(2,1-\frac{2}{1-cx^3}\right)\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{1}{3}b\text{PolyLog}\left(2,\frac{2}{1-cx^3}-1\right)\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{1}{6}b^2\text{PolyLog}\left(\right.$$

```
[Out] (2*(a + b*ArcTanh[c*x^3])^2*ArcTanh[1 - 2/(1 - c*x^3)])/3 - (b*(a + b*ArcTa
nh[c*x^3])*PolyLog[2, 1 - 2/(1 - c*x^3)])/3 + (b*(a + b*ArcTanh[c*x^3])*Pol
yLog[2, -1 + 2/(1 - c*x^3)])/3 + (b^2*PolyLog[3, 1 - 2/(1 - c*x^3)])/6 - (b
^2*PolyLog[3, -1 + 2/(1 - c*x^3)])/6
```

Rubi [A] time = 0.334869, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6095, 5914, 6052, 5948, 6058, 6610}

$$-\frac{1}{3}b\text{PolyLog}\left(2,1-\frac{2}{1-cx^3}\right)\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{1}{3}b\text{PolyLog}\left(2,\frac{2}{1-cx^3}-1\right)\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{1}{6}b^2\text{PolyLog}\left(\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^3])^2/x,x]
```


```
[Out] (2*(a + b*ArcTanh[c*x^3])^2*ArcTanh[1 - 2/(1 - c*x^3)])/3 - (b*(a + b*ArcTa
nh[c*x^3])*PolyLog[2, 1 - 2/(1 - c*x^3)])/3 + (b*(a + b*ArcTanh[c*x^3])*Pol
yLog[2, -1 + 2/(1 - c*x^3)])/3 + (b^2*PolyLog[3, 1 - 2/(1 - c*x^3)])/6 - (b
^2*PolyLog[3, -1 + 2/(1 - c*x^3)])/6
```


Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[
1/n, Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
 n}, x] && IGtQ[p, 0]
```

Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
 e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2}{x}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x}\,dx, x, x^3\right)$$

$$= \frac{2}{3}\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx^3}\right)-\frac{1}{3}(4bc)\,\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(cx)\right)\text{t}}{1-c^2}\right.$$

$$= \frac{2}{3}\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx^3}\right)+\frac{1}{3}(2bc)\,\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(cx)\right)\text{l}}{1-c^2x^2}\right.$$

$$= \frac{2}{3}\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx^3}\right)-\frac{1}{3}b\left(a+b\tanh^{-1}\left(cx^3\right)\right)\text{Li}_2\left(1-\frac{2}{1-cx}\right.$$

$$= \frac{2}{3}\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx^3}\right)-\frac{1}{3}b\left(a+b\tanh^{-1}\left(cx^3\right)\right)\text{Li}_2\left(1-\frac{2}{1-cx}\right.$$

Mathematica [A] time = 0.111432, size = 143, normalized size = 1.02

$$\frac{1}{6}\left(b\left(2\text{PolyLog}\left(2,\frac{cx^3+1}{1-cx^3}\right)\left(a+b\tanh^{-1}\left(cx^3\right)\right)-2\text{PolyLog}\left(2,\frac{cx^3+1}{cx^3-1}\right)\left(a+b\tanh^{-1}\left(cx^3\right)\right)+b\left(\text{PolyLog}\left(3,\frac{cx}{cx}\right.\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^3])^2/x,x]
```

```
[Out] (4*(a + b*ArcTanh[c*x^3])^2*ArcTanh[1 + 2/(-1 + c*x^3)] + b*(2*(a + b*ArcTa
nh[c*x^3])*PolyLog[2, (1 + c*x^3)/(1 - c*x^3)] - 2*(a + b*ArcTanh[c*x^3])*P
olyLog[2, (1 + c*x^3)/(-1 + c*x^3)] + b*(-PolyLog[3, (1 + c*x^3)/(1 - c*x^3
)] + PolyLog[3, (1 + c*x^3)/(-1 + c*x^3)])))/6
```

Maple [F] time = 0.144, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\text{Artanh}\left(cx^3\right)\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^3))^2/x,x)
```

```
[Out] int((a+b*arctanh(c*x^3))^2/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$a^2\log(x) + \int \frac{b^2\left(\log\left(cx^3+1\right) - \log\left(-cx^3+1\right)\right)^2}{4\,x} + \frac{ab\left(\log\left(cx^3+1\right) - \log\left(-cx^3+1\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))^2/x,x, algorithm="maxima")
```

```
[Out] a^2*log(x) + integrate(1/4*b^2*(log(c*x^3 + 1) - log(-c*x^3 + 1))^2/x + a*b
*(log(c*x^3 + 1) - log(-c*x^3 + 1))/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^2\,\mathrm{artanh}\left(cx^3\right)^2 + 2\,ab\,\mathrm{artanh}\left(cx^3\right) + a^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))^2/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x^3)^2 + 2*a*b*arctanh(c*x^3) + a^2)/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**3))**2/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\,\mathrm{artanh}\left(cx^3\right) + a\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))^2/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^3) + a)^2/x, x)
```

3.121 $\int \frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2}{x^4}\,dx$

Optimal. Leaf size=90

$$-\frac{1}{3}b^2c\text{PolyLog}\left(2,\frac{2}{cx^3+1}-1\right)+\frac{1}{3}c\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2-\frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2}{3x^3}+\frac{2}{3}bc\log\left(2-\frac{2}{cx^3+1}\right)\left(a+b\tanh^{-1}\left(cx^3\right)\right)$$

```
[Out] (c*(a + b*ArcTanh[c*x^3])^2)/3 - (a + b*ArcTanh[c*x^3])^2/(3*x^3) + (2*b*c*
(a + b*ArcTanh[c*x^3])*Log[2 - 2/(1 + c*x^3)])/3 - (b^2*c*PolyLog[2, -1 + 2
/(1 + c*x^3)])/3
```

Rubi [B] time = 0.619188, antiderivative size = 237, normalized size of antiderivative = 2.63, number of steps used = 24, number of rules used = 13, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.812, Rules used = {6099, 2454, 2397, 2392, 2391, 2395, 36, 29, 31, 2439, 2416, 2394, 2393}

$$-\frac{1}{3}b^2c\text{PolyLog}\left(2,-cx^3\right)+\frac{1}{3}b^2c\text{PolyLog}\left(2,cx^3\right)+\frac{1}{6}b^2c\text{PolyLog}\left(2,\frac{1}{2}\left(1-cx^3\right)\right)-\frac{1}{6}b^2c\text{PolyLog}\left(2,\frac{1}{2}\left(cx^3+1\right)\right)\cdots$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcTanh[c*x^3])^2/x^4,x]
```

```
[Out] 2*a*b*c*Log[x] - ((1 - c*x^3)*(2*a - b*Log[1 - c*x^3])^2)/(12*x^3) - (b*c*(
2*a - b*Log[1 - c*x^3])*Log[(1 + c*x^3)/2])/6 - (b^2*c*Log[(1 - c*x^3)/2]*L
og[1 + c*x^3])/6 - (b*(2*a - b*Log[1 - c*x^3])*Log[1 + c*x^3])/(6*x^3) - (b
^2*(1 + c*x^3)*Log[1 + c*x^3]^2)/(12*x^3) - (b^2*c*PolyLog[2, -(c*x^3)])/3
+ (b^2*c*PolyLog[2, c*x^3])/3 + (b^2*c*PolyLog[2, (1 - c*x^3)/2])/6 - (b^2*
c*PolyLog[2, (1 + c*x^3)/2])/6
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2397

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_))^2, x_Symbol] :> Simp[((d + e*x)*(a + b*Log[c*(d + e*x)^n])^p)/((e*f
 - d*g)*(f + g*x)), x] - Dist[(b*e*n*p)/(e*f - d*g), Int[(a + b*Log[c*(d +
e*x)^n])^(p - 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] &&
 NeQ[e*f - d*g, 0] && GtQ[p, 0]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/(x_), x_Symbol] :> Simp[
(a + b*Log[c*d])*Log[x], x] + Dist[b, Int[Log[1 + (e*x)/d]/x, x], x] /; Fre
eQ[{a, b, c, d, e}, x] && GtQ[c*d, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[(x^
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
 + j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2}{x^4}\,dx = \int \left(\frac{\left(2a-b\log\left(1-cx^3\right)\right)^2}{4x^4} - \frac{b\left(-2a+b\log\left(1-cx^3\right)\right)\log\left(1+cx^3\right)}{2x^4} + \frac{b^2\log^2\left(1+cx^3\right.}{4x^4}\right.$$

$$= \frac{1}{4}\int \frac{\left(2a-b\log\left(1-cx^3\right)\right)^2}{x^4}\,dx - \frac{1}{2}b\int \frac{\left(-2a+b\log\left(1-cx^3\right)\right)\log\left(1+cx^3\right)}{x^4}\,dx + \frac{1}{4}b^2$$

$$= \frac{1}{12}\,\text{Subst}\left(\int \frac{(2a-b\log(1-cx))^2}{x^2}\,dx, x, x^3\right) - \frac{1}{6}b\,\text{Subst}\left(\int \frac{(-2a+b\log(1-cx))\log(}{x^2}\right.$$

$$= -\frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^2}{12x^3} - \frac{b\left(2a-b\log\left(1-cx^3\right)\right)\log\left(1+cx^3\right)}{6x^3} - \frac{b^2\left(1+cx^3\right.}{}$$

$$= abc\log(x) - \frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^2}{12x^3} - \frac{b\left(2a-b\log\left(1-cx^3\right)\right)\log\left(1+cx^3\right)}{6x^3} -$$

$$= abc\log(x) - \frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^2}{12x^3} - \frac{b\left(2a-b\log\left(1-cx^3\right)\right)\log\left(1+cx^3\right)}{6x^3} -$$

$$= 2abc\log(x) - \frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^2}{12x^3} - \frac{1}{6}bc\left(2a-b\log\left(1-cx^3\right)\right)\log\left(\frac{1}{2}\left(1+\right.\right.$$

$$= 2abc\log(x) - \frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^2}{12x^3} - \frac{1}{6}bc\left(2a-b\log\left(1-cx^3\right)\right)\log\left(\frac{1}{2}\left(1+\right.\right.$$

$$= 2abc\log(x) - \frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^2}{12x^3} - \frac{1}{6}bc\left(2a-b\log\left(1-cx^3\right)\right)\log\left(\frac{1}{2}\left(1+\right.\right.$$

Mathematica [A] time = 0.158228, size = 117, normalized size = 1.3

$$\frac{-b^2cx^3\text{PolyLog}\left(2,e^{-2\tanh^{-1}\left(cx^3\right)}\right) - a\left(a+bcx^3\log\left(1-c^2x^6\right)-2bcx^3\log\left(cx^3\right)\right) + 2b\tanh^{-1}\left(cx^3\right)\left(bcx^3\log\left(1-e^{-2}\right.\right.}{3x^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x^3])^2/x^4,x]
```

```
[Out] (b^2*(-1 + c*x^3)*ArcTanh[c*x^3]^2 + 2*b*ArcTanh[c*x^3]*(-a + b*c*x^3*Log[1
 - E^(-2*ArcTanh[c*x^3])]) - a*(a - 2*b*c*x^3*Log[c*x^3] + b*c*x^3*Log[1 -
c^2*x^6]) - b^2*c*x^3*PolyLog[2, E^(-2*ArcTanh[c*x^3])])/(3*x^3)
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \frac{\left(a+b\text{Artanh}\left(cx^3\right)\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^3))^2/x^4,x)
```

```
[Out] int((a+b*arctanh(c*x^3))^2/x^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{3}\left(c\left(\log\left(c^2x^6-1\right)-\log\left(x^6\right)\right)+\frac{2\,\operatorname{artanh}\left(cx^3\right)}{x^3}\right)ab-\frac{1}{12}b^2\left(\frac{\log\left(-cx^3+1\right)^2}{x^3}+3\int-\frac{\left(cx^3-1\right)\log\left(cx^3+1\right)^2+2\left(cx^3\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))^2/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*(c*(log(c^2*x^6 - 1) - log(x^6)) + 2*arctanh(c*x^3)/x^3)*a*b - 1/12*b^
2*(log(-c*x^3 + 1)^2/x^3 + 3*integrate(-((c*x^3 - 1)*log(c*x^3 + 1)^2 + 2*(
c*x^3 - (c*x^3 - 1)*log(c*x^3 + 1))*log(-c*x^3 + 1))/(c*x^7 - x^4), x)) - 1
/3*a^2/x^3
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{b^2\operatorname{artanh}\left(cx^3\right)^2+2\,ab\operatorname{artanh}\left(cx^3\right)+a^2}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))^2/x^4,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x^3)^2 + 2*a*b*arctanh(c*x^3) + a^2)/x^4, x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: KeyError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**3))**2/x**4,x)
```

```
[Out] Exception raised: KeyError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(b\operatorname{artanh}\left(cx^3\right)+a\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))^2/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^3) + a)^2/x^4, x)
```

3.122 $\int \frac{\left(a+b\tanh^{-1}(cx^3)\right)^2}{x^7}\,dx$

Optimal. Leaf size=88

$$\frac{1}{6}c^2\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2-\frac{bc\left(a+b\tanh^{-1}\left(cx^3\right)\right)}{3x^3}-\frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2}{6x^6}-\frac{1}{6}b^2c^2\log\left(1-c^2x^6\right)+b^2c^2\log(x)$$

```
[Out] -(b*c*(a + b*ArcTanh[c*x^3]))/(3*x^3) + (c^2*(a + b*ArcTanh[c*x^3])^2)/6 -
(a + b*ArcTanh[c*x^3])^2/(6*x^6) + b^2*c^2*Log[x] - (b^2*c^2*Log[1 - c^2*x^
6])/6
```

Rubi [C] time = 1.06043, antiderivative size = 360, normalized size of antiderivative = 4.09, number of steps used = 46, number of rules used = 23, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.438, Rules used = {6099, 2454, 2398, 2411, 2347, 2344, 2301, 2316, 2315, 2314, 31, 2395, 44, 2439, 2416, 36, 29, 2392, 2391, 2394, 2393, 2410, 2390}

$$-\frac{1}{12}b^2c^2\text{PolyLog}\left(2,\frac{1}{2}\left(1-cx^3\right)\right)-\frac{1}{12}b^2c^2\text{PolyLog}\left(2,\frac{1}{2}\left(cx^3+1\right)\right)+\frac{1}{12}bc^2\log\left(\frac{1}{2}\left(cx^3+1\right)\right)\left(2a-b\log\left(1-cx^3\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcTanh[c*x^3])^2/x^7,x]
```

```
[Out] b^2*c^2*Log[x] - (b^2*c^2*Log[1 - c*x^3])/12 - (b*c*(2*a - b*Log[1 - c*x^3]
))/(12*x^3) - (b*c*(1 - c*x^3)*(2*a - b*Log[1 - c*x^3]))/(12*x^3) + (c^2*(2
*a - b*Log[1 - c*x^3])^2)/24 - (2*a - b*Log[1 - c*x^3])^2/(24*x^6) + (b*c^2
*(2*a - b*Log[1 - c*x^3])*Log[(1 + c*x^3)/2])/12 - (b^2*c^2*Log[1 + c*x^3])
/6 - (b^2*c*Log[1 + c*x^3])/(6*x^3) - (b^2*c^2*Log[(1 - c*x^3)/2]*Log[1 + c
*x^3])/12 - (b*(2*a - b*Log[1 - c*x^3])*Log[1 + c*x^3])/(12*x^6) + (b^2*c^2
*Log[1 + c*x^3]^2)/24 - (b^2*Log[1 + c*x^3]^2)/(24*x^6) - (b^2*c^2*PolyLog[
2, (1 - c*x^3)/2])/12 - (b^2*c^2*PolyLog[2, (1 + c*x^3)/2])/12
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
```

```
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2316

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[
((a + b*Log[-((c*d)/e)])*Log[d + e*x])/e, x] + Dist[b, Int[Log[-((e*x)/d)]/
(d + e*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[-((c*d)/e), 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
```

```
eQ[q, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[(x^
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
 + j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/(x_), x_Symbol] :> Simp[
(a + b*Log[c*d])*Log[x], x] + Dist[b, Int[Log[1 + (e*x)/d]/x, x], x] /; Fre
eQ[{a, b, c, d, e}, x] && GtQ[c*d, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
```

```
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2}{x^7}\,dx=\int\left(\frac{\left(2a-b\log\left(1-cx^3\right)\right)^2}{4x^7}-\frac{b\left(-2a+b\log\left(1-cx^3\right)\right)\log\left(1+cx^3\right)}{2x^7}+\frac{b^2\log^2\left(1+cx^3\right)}{4x^7}\right)$$

$$=\frac{1}{4}\int\frac{\left(2a-b\log\left(1-cx^3\right)\right)^2}{x^7}\,dx-\frac{1}{2}b\int\frac{\left(-2a+b\log\left(1-cx^3\right)\right)\log\left(1+cx^3\right)}{x^7}\,dx+\frac{1}{4}b^2\int$$

$$=\frac{1}{12}\operatorname{Subst}\left(\int\frac{(2a-b\log(1-cx))^2}{x^3}\,dx,x,x^3\right)-\frac{1}{6}b\operatorname{Subst}\left(\int\frac{(-2a+b\log(1-cx))\log(1+}{x^3}\right.$$

$$=-\frac{\left(2a-b\log\left(1-cx^3\right)\right)^2}{24x^6}-\frac{b\left(2a-b\log\left(1-cx^3\right)\right)\log\left(1+cx^3\right)}{12x^6}-\frac{b^2\log^2\left(1+cx^3\right)}{24x^6}+\frac{1}{12}($$

$$=-\frac{\left(2a-b\log\left(1-cx^3\right)\right)^2}{24x^6}-\frac{b\left(2a-b\log\left(1-cx^3\right)\right)\log\left(1+cx^3\right)}{12x^6}-\frac{b^2\log^2\left(1+cx^3\right)}{24x^6}-\frac{1}{12}b$$

$$=-\frac{\left(2a-b\log\left(1-cx^3\right)\right)^2}{24x^6}-\frac{b\left(2a-b\log\left(1-cx^3\right)\right)\log\left(1+cx^3\right)}{12x^6}-\frac{b^2\log^2\left(1+cx^3\right)}{24x^6}-\frac{1}{12}b$$

$$=-\frac{1}{2}abc^2\log(x)-\frac{bc\left(2a-b\log\left(1-cx^3\right)\right)}{12x^3}-\frac{bc\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)}{12x^3}-\frac{\left(2a-b\log\right.}{}$$

$$=\frac{1}{4}b^2c^2\log(x)-\frac{bc\left(2a-b\log\left(1-cx^3\right)\right)}{12x^3}-\frac{bc\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)}{12x^3}+\frac{1}{24}c^2\left(2a-\right.$$

$$=\frac{1}{2}b^2c^2\log(x)-\frac{1}{12}b^2c^2\log\left(1-cx^3\right)-\frac{bc\left(2a-b\log\left(1-cx^3\right)\right)}{12x^3}-\frac{bc\left(1-cx^3\right)\left(2a-b\log\left(\right.\right.}{12x^3}$$

Mathematica [A] time = 0.0831599, size = 111, normalized size = 1.26

$$\frac{1}{6}\left(-\frac{a^2}{x^6}-bc^2(a+b)\log\left(1-cx^3\right)+bc^2(a-b)\log\left(cx^3+1\right)-\frac{2abc}{x^3}-\frac{2b\tanh^{-1}\left(cx^3\right)\left(a+bcx^3\right)}{x^6}+\frac{b^2\left(c^2x^6-1\right)\tanh^{-1}(}{x^6}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^3])^2/x^7,x]
```

```
[Out] (-(a^2/x^6) - (2*a*b*c)/x^3 - (2*b*(a + b*c*x^3)*ArcTanh[c*x^3])/x^6 + (b^2
*(-1 + c^2*x^6)*ArcTanh[c*x^3]^2)/x^6 + 6*b^2*c^2*Log[x] - b*(a + b)*c^2*Lo
g[1 - c*x^3] + (a - b)*b*c^2*Log[1 + c*x^3])/6
```

Maple [B] time = 0.201, size = 257, normalized size = 2.9

$$\frac{b^2\left(c^2x^6-1\right)\left(\ln\left(cx^3+1\right)\right)^2}{24\,x^6}-\frac{b\left(x^6b\ln\left(-cx^3+1\right)c^2+2\,bcx^3-b\ln\left(-cx^3+1\right)+2\,a\right)\ln\left(cx^3+1\right)}{12\,x^6}-\frac{-b^2c^2x^6\left(\ln\left(-cx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^3))^2/x^7,x)
```

```
[Out] 1/24*b^2*(c^2*x^6-1)/x^6*ln(c*x^3+1)^2-1/12*b*(x^6*b*ln(-c*x^3+1)*c^2+2*b*c
*x^3-b*ln(-c*x^3+1)+2*a)/x^6*ln(c*x^3+1)-1/24*(-b^2*c^2*x^6*ln(-c*x^3+1)^2+
4*b*c^2*ln(c*x^3-1)*x^6*a+4*b^2*c^2*ln(c*x^3-1)*x^6-4*b*c^2*ln(c*x^3+1)*x^6
*a+4*b^2*c^2*ln(c*x^3+1)*x^6-24*b^2*c^2*ln(x)*x^6-4*b^2*c*x^3*ln(-c*x^3+1)+
8*a*b*c*x^3+b^2*ln(-c*x^3+1)^2-4*b*ln(-c*x^3+1)*a+4*a^2)/x^6
```

Maxima [B] time = 0.99658, size = 236, normalized size = 2.68

$$\frac{1}{6}\left(\left(c\log\left(cx^3+1\right)-c\log\left(cx^3-1\right)-\frac{2}{x^3}\right)c-\frac{2\,\text{artanh}\left(cx^3\right)}{x^6}\right)ab+\frac{1}{24}\left(\left(2\left(\log\left(cx^3-1\right)-2\right)\log\left(cx^3+1\right)-\log\left(cx^3\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))^2/x^7,x, algorithm="maxima")
```

```
[Out] 1/6*((c*log(c*x^3 + 1) - c*log(c*x^3 - 1) - 2/x^3)*c - 2*arctanh(c*x^3)/x^6
)*a*b + 1/24*((2*(log(c*x^3 - 1) - 2)*log(c*x^3 + 1) - log(c*x^3 + 1)^2 - l
og(c*x^3 - 1)^2 - 4*log(c*x^3 - 1) + 24*log(x))*c^2 + 4*(c*log(c*x^3 + 1) -
 c*log(c*x^3 - 1) - 2/x^3)*c*arctanh(c*x^3))*b^2 - 1/6*b^2*arctanh(c*x^3)^2
/x^6 - 1/6*a^2/x^6
```

Fricas [A] time = 1.77071, size = 324, normalized size = 3.68

$$\frac{24\,b^2c^2x^6\log\left(x\right)+4\left(ab-b^2\right)c^2x^6\log\left(cx^3+1\right)-4\left(ab+b^2\right)c^2x^6\log\left(cx^3-1\right)-8\,abcx^3+\left(b^2c^2x^6-b^2\right)\log\left(-\frac{cx^3+1}{cx^3-1}\right)}{24\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))^2/x^7,x, algorithm="fricas")
```

```
[Out] 1/24*(24*b^2*c^2*x^6*log(x) + 4*(a*b - b^2)*c^2*x^6*log(c*x^3 + 1) - 4*(a*b
 + b^2)*c^2*x^6*log(c*x^3 - 1) - 8*a*b*c*x^3 + (b^2*c^2*x^6 - b^2)*log(-(c*
x^3 + 1)/(c*x^3 - 1))^2 - 4*a^2 - 4*(b^2*c*x^3 + a*b)*log(-(c*x^3 + 1)/(c*x
^3 - 1)))/x^6
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**3))**2/x**7,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\operatorname{artanh}\left(cx^3\right)+a\right)^2}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))^2/x^7,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^3) + a)^2/x^7, x)
```

3.123 $\int \frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2}{x^{10}}\,dx$

Optimal. Leaf size=144

$$-\frac{1}{9}b^2c^3\text{PolyLog}\left(2,\frac{2}{cx^3+1}-1\right)+\frac{1}{9}c^3\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2+\frac{2}{9}bc^3\log\left(2-\frac{2}{cx^3+1}\right)\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{bc\left(a+\right.}{}$$

```
[Out] -(b^2*c^2)/(9*x^3) + (b^2*c^3*ArcTanh[c*x^3])/9 - (b*c*(a + b*ArcTanh[c*x^3
]))/(9*x^6) + (c^3*(a + b*ArcTanh[c*x^3])^2)/9 - (a + b*ArcTanh[c*x^3])^2/(
9*x^9) + (2*b*c^3*(a + b*ArcTanh[c*x^3])*Log[2 - 2/(1 + c*x^3)])/9 - (b^2*c
^3*PolyLog[2, -1 + 2/(1 + c*x^3)])/9
```

Rubi [B] time = 1.31132, antiderivative size = 420, normalized size of antiderivative = 2.92, number of steps used = 59, number of rules used = 24, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.5, Rules used = {6099, 2454, 2398, 2411, 2347, 2344, 2301, 2316, 2315, 2314, 31, 2319, 44, 2395, 2439, 2416, 36, 29, 2392, 2391, 2394, 2393, 2410, 2390}

$$-\frac{1}{9}b^2c^3\text{PolyLog}\left(2,-cx^3\right)+\frac{1}{9}b^2c^3\text{PolyLog}\left(2,cx^3\right)+\frac{1}{18}b^2c^3\text{PolyLog}\left(2,\frac{1}{2}\left(1-cx^3\right)\right)-\frac{1}{18}b^2c^3\text{PolyLog}\left(2,\frac{1}{2}\left(cx^3\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcTanh[c*x^3])^2/x^10,x]
```

```
[Out] -(b^2*c^2)/(9*x^3) + (2*a*b*c^3*Log[x])/3 - (b*c*(2*a - b*Log[1 - c*x^3]))/
(18*x^6) + (b*c^2*(2*a - b*Log[1 - c*x^3]))/(18*x^3) - (b*c^2*(1 - c*x^3)*(
2*a - b*Log[1 - c*x^3]))/(18*x^3) + (c^3*(2*a - b*Log[1 - c*x^3])^2)/36 - (
2*a - b*Log[1 - c*x^3])^2/(36*x^9) - (b*c^3*(2*a - b*Log[1 - c*x^3])*Log[(1
 + c*x^3)/2])/18 + (b^2*c^3*Log[1 + c*x^3])/18 - (b^2*c*Log[1 + c*x^3])/(18
*x^6) - (b^2*c^3*Log[(1 - c*x^3)/2]*Log[1 + c*x^3])/18 - (b*(2*a - b*Log[1
- c*x^3])*Log[1 + c*x^3])/(18*x^9) - (b^2*c^3*Log[1 + c*x^3]^2)/36 - (b^2*L
og[1 + c*x^3]^2)/(36*x^9) - (b^2*c^3*PolyLog[2, -(c*x^3)])/9 + (b^2*c^3*Pol
yLog[2, c*x^3])/9 + (b^2*c^3*PolyLog[2, (1 - c*x^3)/2])/18 - (b^2*c^3*PolyL
og[2, (1 + c*x^3)/2])/18
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
```

```
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2316

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[
((a + b*Log[-((c*d)/e)])*Log[d + e*x])/e, x] + Dist[b, Int[Log[-((e*x)/d)]/
(d + e*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[-((c*d)/e), 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2319

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
```

```
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[(x^
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
 + j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/(x_), x_Symbol] :> Simp[
(a + b*Log[c*d])*Log[x], x] + Dist[b, Int[Log[1 + (e*x)/d]/x, x], x] /; Fre
eQ[{a, b, c, d, e}, x] && GtQ[c*d, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2}{x^{10}}\,dx = \int \left(\frac{\left(2a-b\log\left(1-cx^3\right)\right)^2}{4x^{10}} - \frac{b\left(-2a+b\log\left(1-cx^3\right)\right)\log\left(1+cx^3\right)}{2x^{10}} + \frac{b^2\log^2\left(1+cx^3\right.}{4x^{10}}\right.$$

$$= \frac{1}{4}\int \frac{\left(2a-b\log\left(1-cx^3\right)\right)^2}{x^{10}}\,dx - \frac{1}{2}b\int \frac{\left(-2a+b\log\left(1-cx^3\right)\right)\log\left(1+cx^3\right)}{x^{10}}\,dx + \frac{1}{4}b^2$$

$$= \frac{1}{12}\text{Subst}\left(\int \frac{(2a-b\log(1-cx))^2}{x^4}\,dx, x, x^3\right) - \frac{1}{6}b\,\text{Subst}\left(\int \frac{(-2a+b\log(1-cx))\log(}{x^4}\right.$$

$$= -\frac{\left(2a-b\log\left(1-cx^3\right)\right)^2}{36x^9} - \frac{b\left(2a-b\log\left(1-cx^3\right)\right)\log\left(1+cx^3\right)}{18x^9} - \frac{b^2\log^2\left(1+cx^3\right)}{36x^9} +$$

$$= -\frac{\left(2a-b\log\left(1-cx^3\right)\right)^2}{36x^9} - \frac{b\left(2a-b\log\left(1-cx^3\right)\right)\log\left(1+cx^3\right)}{18x^9} - \frac{b^2\log^2\left(1+cx^3\right)}{36x^9} -$$

$$= -\frac{\left(2a-b\log\left(1-cx^3\right)\right)^2}{36x^9} - \frac{b\left(2a-b\log\left(1-cx^3\right)\right)\log\left(1+cx^3\right)}{18x^9} - \frac{b^2\log^2\left(1+cx^3\right)}{36x^9} -$$

$$= \frac{1}{3}abc^3\log(x) - \frac{bc\left(2a-b\log\left(1-cx^3\right)\right)}{18x^6} + \frac{bc^2\left(2a-b\log\left(1-cx^3\right)\right)}{18x^3} - \frac{\left(2a-b\log\left(1-\right.\right.}{36x^9}$$

$$= \frac{1}{3}abc^3\log(x) - \frac{bc\left(2a-b\log\left(1-cx^3\right)\right)}{18x^6} + \frac{bc^2\left(2a-b\log\left(1-cx^3\right)\right)}{18x^3} - \frac{bc^2\left(1-cx^3\right)\left(2\right.}{1}$$

$$= -\frac{b^2c^2}{18x^3} + \frac{2}{3}abc^3\log(x) + \frac{1}{6}b^2c^3\log(x) - \frac{bc\left(2a-b\log\left(1-cx^3\right)\right)}{18x^6} + \frac{bc^2\left(2a-b\log\left(1-\right.\right.}{18x^3}$$

$$= -\frac{b^2c^2}{18x^3} + \frac{2}{3}abc^3\log(x) + \frac{1}{6}b^2c^3\log(x) - \frac{bc\left(2a-b\log\left(1-cx^3\right)\right)}{18x^6} + \frac{bc^2\left(2a-b\log\left(1-\right.\right.}{18x^3}$$

Mathematica [A] time = 0.362807, size = 159, normalized size = 1.1

$$-\frac{b^2c^3x^9\text{PolyLog}\left(2,e^{-2\tanh^{-1}\left(cx^3\right)}\right) + a^2 - 2abc^3x^9\log\left(cx^3\right) + abc^3x^9\log\left(1-c^2x^6\right) + b\tanh^{-1}\left(cx^3\right)\left(2a-bc^3x^9-2b\right.}{9x^9}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x^3])^2/x^10,x]
```

```
[Out] -(a^2 + a*b*c*x^3 + b^2*c^2*x^6 + b^2*(1 - c^3*x^9)*ArcTanh[c*x^3]^2 + b*Ar
cTanh[c*x^3]*(2*a + b*c*x^3 - b*c^3*x^9 - 2*b*c^3*x^9*Log[1 - E^(-2*ArcTanh
[c*x^3])]) - 2*a*b*c^3*x^9*Log[c*x^3] + a*b*c^3*x^9*Log[1 - c^2*x^6] + b^2*
c^3*x^9*PolyLog[2, E^(-2*ArcTanh[c*x^3])])/(9*x^9)
```

Maple [F] time = 0.181, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\text{Artanh}\left(cx^3\right)\right)^2}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^3))^2/x^10,x)
```

```
[Out] int((a+b*arctanh(c*x^3))^2/x^10,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{9}\left(\left(c^2\log\left(c^2x^6-1\right)-c^2\log\left(x^6\right)+\frac{1}{x^6}\right)c+\frac{2\,\mathrm{artanh}\left(cx^3\right)}{x^9}\right)ab-\frac{1}{36}\,b^2\left(\frac{\log\left(-cx^3+1\right)^2}{x^9}+9\int-\frac{3\left(cx^3-1\right)\log\left(cx^3+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))^2/x^10,x, algorithm="maxima")
```

```
[Out] -1/9*((c^2*log(c^2*x^6 - 1) - c^2*log(x^6) + 1/x^6)*c + 2*arctanh(c*x^3)/x^
9)*a*b - 1/36*b^2*(log(-c*x^3 + 1)^2/x^9 + 9*integrate(-1/3*(3*(c*x^3 - 1)*
log(c*x^3 + 1)^2 + 2*(c*x^3 - 3*(c*x^3 - 1)*log(c*x^3 + 1))*log(-c*x^3 + 1)
)/(c*x^13 - x^10), x)) - 1/9*a^2/x^9
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^2\,\mathrm{artanh}\left(cx^3\right)^2+2\,ab\,\mathrm{artanh}\left(cx^3\right)+a^2}{x^{10}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))^2/x^10,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x^3)^2 + 2*a*b*arctanh(c*x^3) + a^2)/x^10, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**3))**2/x**10,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(b\,\mathrm{artanh}\left(cx^3\right)+a\right)^2}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))^2/x^10,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^3) + a)^2/x^10, x)
```

3.124 $\int x^8 \left(a+b\tanh^{-1}\left(cx^3\right)\right)^3 \, dx$

Optimal. Leaf size=231

$$-\frac{b^2\text{PolyLog}\left(2,1-\frac{2}{1-cx^3}\right)\left(a+b\tanh^{-1}\left(cx^3\right)\right)}{3c^3}+\frac{b^3\text{PolyLog}\left(3,1-\frac{2}{1-cx^3}\right)}{6c^3}+\frac{ab^2x^3}{3c^2}+\frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^3}{9c^3}-\frac{b\left(a\right.}{}$$

```
[Out] (a*b^2*x^3)/(3*c^2) + (b^3*x^3*ArcTanh[c*x^3])/(3*c^2) - (b*(a + b*ArcTanh[
c*x^3])^2)/(6*c^3) + (b*x^6*(a + b*ArcTanh[c*x^3])^2)/(6*c) + (a + b*ArcTan
h[c*x^3])^3/(9*c^3) + (x^9*(a + b*ArcTanh[c*x^3])^3)/9 - (b*(a + b*ArcTanh[
c*x^3])^2*Log[2/(1 - c*x^3)])/(3*c^3) + (b^3*Log[1 - c^2*x^6])/(6*c^3) - (b
^2*(a + b*ArcTanh[c*x^3])*PolyLog[2, 1 - 2/(1 - c*x^3)])/(3*c^3) + (b^3*Pol
yLog[3, 1 - 2/(1 - c*x^3)])/(6*c^3)
```

Rubi [B] time = 6.35017, antiderivative size = 1421, normalized size of antiderivative = 6.15, number of steps used = 239, number of rules used = 32, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 2., Rules used = {6099, 2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304, 2398, 2411, 43, 2334, 12, 14, 2301, 2439, 2416, 2396, 2433, 2374, 6589, 6742, 2430, 2394, 2393, 2391, 2395, 2375, 2317, 2410, 2425}

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Int[x^8*(a + b*ArcTanh[c*x^3])^3,x]
```

```
[Out] (2*a*b^2*x^3)/(3*c^2) - (7*b^3*x^3)/(216*c^2) - (23*b^3*x^6)/(432*c) + (b^3
*x^9)/324 + (b^3*(1 - c*x^3)^2)/(48*c^3) + (b^3*(1 + c*x^3)^2)/(24*c^3) - (
b^3*(1 + c*x^3)^3)/(324*c^3) - (b^3*Log[1 - c*x^3])/(24*c^3) + (b^3*(1 - c*
x^3)*Log[1 - c*x^3])/(3*c^3) - (b^3*Log[1 - c*x^3]^2)/(72*c^3) - (b^2*x^6*(
2*a - b*Log[1 - c*x^3]))/(24*c) + (b^2*(1 - c*x^3)^2*(2*a - b*Log[1 - c*x^3
]))/(12*c^3) - (b^2*(1 - c*x^3)^3*(2*a - b*Log[1 - c*x^3]))/(108*c^3) - (b*
x^9*(2*a - b*Log[1 - c*x^3])^2)/72 - (b*(1 - c*x^3)*(2*a - b*Log[1 - c*x^3]
)^2)/(8*c^3) + (b*(1 - c*x^3)^2*(2*a - b*Log[1 - c*x^3])^2)/(12*c^3) - (b*(
1 - c*x^3)^3*(2*a - b*Log[1 - c*x^3])^2)/(72*c^3) - ((1 - c*x^3)*(2*a - b*L
og[1 - c*x^3])^3)/(24*c^3) + ((1 - c*x^3)^2*(2*a - b*Log[1 - c*x^3])^3)/(24
*c^3) - ((1 - c*x^3)^3*(2*a - b*Log[1 - c*x^3])^3)/(72*c^3) + (b^2*(2*a - b
*Log[1 - c*x^3])*((18*(1 - c*x^3))/c^3 - (9*(1 - c*x^3)^2)/c^3 + (2*(1 - c*
x^3)^3)/c^3 - (6*Log[1 - c*x^3])/c^3))/216 - (b^2*(2*a - b*Log[1 - c*x^3])*
Log[(1 + c*x^3)/2])/(12*c^3) + (b*(2*a - b*Log[1 - c*x^3])^2*Log[(1 + c*x^3
)/2])/(12*c^3) - (7*b^3*Log[1 + c*x^3])/(108*c^3) + (b^3*x^6*Log[1 + c*x^3]
)/(18*c) - (b^3*x^9*Log[1 + c*x^3])/108 + (11*b^3*(1 + c*x^3)*Log[1 + c*x^3
])/(36*c^3) - (b^3*(1 + c*x^3)^2*Log[1 + c*x^3])/(12*c^3) + (b^3*(1 + c*x^3
)^3*Log[1 + c*x^3])/(108*c^3) + (b^3*Log[(1 - c*x^3)/2]*Log[1 + c*x^3])/(12
*c^3) + (b^2*x^6*(2*a - b*Log[1 - c*x^3])*Log[1 + c*x^3])/(12*c) - (b*(2*a
- b*Log[1 - c*x^3])^2*Log[1 + c*x^3])/(24*c^3) + (b*x^9*(2*a - b*Log[1 - c*
x^3])^2*Log[1 + c*x^3])/24 + (b^3*Log[1 + c*x^3]^2)/(72*c^3) + (b^3*x^9*Log
[1 + c*x^3]^2)/72 - (b^3*(1 + c*x^3)*Log[1 + c*x^3]^2)/(8*c^3) + (b^3*(1 +
c*x^3)^2*Log[1 + c*x^3]^2)/(12*c^3) - (b^3*(1 + c*x^3)^3*Log[1 + c*x^3]^2)/
(72*c^3) + (b^3*Log[(1 - c*x^3)/2]*Log[1 + c*x^3]^2)/(12*c^3) + (b^2*(2*a -
 b*Log[1 - c*x^3])*Log[1 + c*x^3]^2)/(24*c^3) + (b^2*x^9*(2*a - b*Log[1 - c
*x^3])*Log[1 + c*x^3]^2)/24 + (b^3*(1 + c*x^3)*Log[1 + c*x^3]^3)/(24*c^3) -
 (b^3*(1 + c*x^3)^2*Log[1 + c*x^3]^3)/(24*c^3) + (b^3*(1 + c*x^3)^3*Log[1 +
 c*x^3]^3)/(72*c^3) + (b^3*PolyLog[2, (1 - c*x^3)/2])/(12*c^3) - (b^2*(2*a
- b*Log[1 - c*x^3])*PolyLog[2, (1 - c*x^3)/2])/(6*c^3) + (b^3*PolyLog[2, (1
 + c*x^3)/2])/(12*c^3) + (b^3*Log[1 + c*x^3]*PolyLog[2, (1 + c*x^3)/2])/(6*
```

```
c^3) - (b^3*PolyLog[3, (1 - c*x^3)/2])/(6*c^3) - (b^3*PolyLog[3, (1 + c*x^3
)/2])/(6*c^3)
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1
] && EqQ[m, -1])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[(x^
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
 + j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
```

```
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2430

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.)), x_Symbol] :> Simp[x*(a + b*Log[c
*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]), x] + (-Dist[g*j*m, Int[(x*(a +
 b*Log[c*(d + e*x)^n])^p)/(i + j*x), x], x] - Dist[b*e*n*p, Int[(x*(a + b*L
og[c*(d + e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /
; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
```

```
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2375

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[(Log[d*(e + f*x^m)^r]*(a + b*Log[
c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[(f*m*r)/(b*n*(p + 1)), Int[(x^(m
- 1)*(a + b*Log[c*x^n])^(p + 1))/(e + f*x^m), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2425

```
Int[(Log[(f_.)*(x_)^(m_.)]*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b
_.)))/(x_), x_Symbol] :> Simp[(Log[f*x^m]^2*(a + b*Log[c*(d + e*x)^n]))/(2*
m), x] - Dist[(b*e*n)/(2*m), Int[Log[f*x^m]^2/(d + e*x), x], x] /; FreeQ[{a
, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int x^8 \left(a+b\tanh^{-1}\left(cx^3\right)\right)^3 dx = \int \left(\frac{1}{8}x^8\left(2a-b\log\left(1-cx^3\right)\right)^3+\frac{3}{8}bx^8\left(-2a+b\log\left(1-cx^3\right)\right)^2\log\left(1+cx^3\right)-\frac{3}{8}b^2x^8\left(\right.\right.$$

$$=\frac{1}{8}\int x^8\left(2a-b\log\left(1-cx^3\right)\right)^3dx+\frac{1}{8}(3b)\int x^8\left(-2a+b\log\left(1-cx^3\right)\right)^2\log\left(1+cx^3\right)d$$

$$=\frac{1}{24}\text{Subst}\left(\int x^2(2a-b\log(1-cx))^3\,dx,x,x^3\right)+\frac{1}{8}b\,\text{Subst}\left(\int x^2(-2a+b\log(1-cx))^2\right.$$

$$=\frac{1}{24}bx^9\left(2a-b\log\left(1-cx^3\right)\right)^2\log\left(1+cx^3\right)+\frac{1}{24}b^2x^9\left(2a-b\log\left(1-cx^3\right)\right)\log^2\left(1+cx\right.$$

$$=\frac{1}{24}bx^9\left(2a-b\log\left(1-cx^3\right)\right)^2\log\left(1+cx^3\right)+\frac{1}{24}b^2x^9\left(2a-b\log\left(1-cx^3\right)\right)\log^2\left(1+cx\right.$$

$$=\frac{1}{24}bx^9\left(2a-b\log\left(1-cx^3\right)\right)^2\log\left(1+cx^3\right)+\frac{1}{24}b^2x^9\left(2a-b\log\left(1-cx^3\right)\right)\log^2\left(1+cx\right.$$

$$=-\frac{1}{72}bx^9\left(2a-b\log\left(1-cx^3\right)\right)^2-\frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^3}{24c^3}+\frac{\left(1-cx^3\right)^2\left(2a-b\right.}{24c^3}$$

$$=-\frac{1}{72}bx^9\left(2a-b\log\left(1-cx^3\right)\right)^2-\frac{b\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^2}{12c^3}+\frac{b\left(1-cx^3\right)^2\left(2a-\right.}{16}$$

$$=\frac{ab^2x^3}{3c^2}-\frac{b^3x^3}{6c^2}+\frac{b^3\left(1-cx^3\right)^2}{32c^3}-\frac{b^3\left(1-cx^3\right)^3}{324c^3}+\frac{b^3\left(1+cx^3\right)^2}{32c^3}-\frac{b^3\left(1+cx^3\right)^3}{324c^3}+\frac{b^2\left(1-\right.}{}$$

$$=\frac{ab^2x^3}{3c^2}+\frac{b^3\left(1-cx^3\right)^2}{32c^3}-\frac{b^3\left(1-cx^3\right)^3}{324c^3}+\frac{b^3\left(1+cx^3\right)^2}{32c^3}-\frac{b^3\left(1+cx^3\right)^3}{324c^3}+\frac{b^3\left(1-cx^3\right)\log}{6c^3}$$

$$=\frac{ab^2x^3}{2c^2}-\frac{b^3x^3}{18c^2}+\frac{b^3\left(1-cx^3\right)^2}{24c^3}-\frac{b^3\left(1-cx^3\right)^3}{324c^3}+\frac{b^3\left(1+cx^3\right)^2}{24c^3}-\frac{b^3\left(1+cx^3\right)^3}{324c^3}+\frac{b^3\left(1-\right.}{}$$

$$=\frac{ab^2x^3}{2c^2}-\frac{7b^3x^3}{216c^2}-\frac{5b^3x^6}{432c}+\frac{b^3x^9}{324}+\frac{b^3\left(1-cx^3\right)^2}{48c^3}+\frac{b^3\left(1+cx^3\right)^2}{24c^3}-\frac{b^3\left(1+cx^3\right)^3}{324c^3}+\frac{b^3\left(1\right.}{}$$

$$=\frac{ab^2x^3}{2c^2}-\frac{7b^3x^3}{216c^2}-\frac{5b^3x^6}{432c}+\frac{b^3x^9}{324}+\frac{b^3\left(1-cx^3\right)^2}{48c^3}+\frac{b^3\left(1+cx^3\right)^2}{24c^3}-\frac{b^3\left(1+cx^3\right)^3}{324c^3}+\frac{b^3\left(1\right.}{}$$

Mathematica [A] time = 0.470237, size = 334, normalized size = 1.45

$$\frac{6b^2\text{PolyLog}\left(2,-e^{-2\tanh^{-1}\left(cx^3\right)}\right)\left(a+b\tanh^{-1}\left(cx^3\right)\right)+3b^3\text{PolyLog}\left(3,-e^{-2\tanh^{-1}\left(cx^3\right)}\right)+3a^2bc^2x^6+3a^2b\log\left(1-c^2x^6\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^8*(a + b*ArcTanh[c*x^3])^3,x]
```

```
[Out] (6*a*b^2*c*x^3 + 3*a^2*b*c^2*x^6 + 2*a^3*c^3*x^9 - 6*a*b^2*ArcTanh[c*x^3] +
 6*b^3*c*x^3*ArcTanh[c*x^3] + 6*a*b^2*c^2*x^6*ArcTanh[c*x^3] + 6*a^2*b*c^3*
x^9*ArcTanh[c*x^3] - 6*a*b^2*ArcTanh[c*x^3]^2 - 3*b^3*ArcTanh[c*x^3]^2 + 3*
```

```
b^3*c^2*x^6*ArcTanh[c*x^3]^2 + 6*a*b^2*c^3*x^9*ArcTanh[c*x^3]^2 - 2*b^3*Arc
Tanh[c*x^3]^3 + 2*b^3*c^3*x^9*ArcTanh[c*x^3]^3 - 12*a*b^2*ArcTanh[c*x^3]*Lo
g[1 + E^(-2*ArcTanh[c*x^3])] - 6*b^3*ArcTanh[c*x^3]^2*Log[1 + E^(-2*ArcTanh
[c*x^3])] + 3*a^2*b*Log[1 - c^2*x^6] + 3*b^3*Log[1 - c^2*x^6] + 6*b^2*(a +
b*ArcTanh[c*x^3])*PolyLog[2, -E^(-2*ArcTanh[c*x^3])] + 3*b^3*PolyLog[3, -E^
(-2*ArcTanh[c*x^3])])/(18*c^3)
```

Maple [F] time = 0.217, size = 0, normalized size = 0.

$$\int x^8 \left(a + b\text{Artanh}\left(cx^3\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a+b*arctanh(c*x^3))^3,x)
```

```
[Out] int(x^8*(a+b*arctanh(c*x^3))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{9}a^3x^9 + \frac{1}{6}\left(2x^9 \operatorname{artanh}\left(cx^3\right) + \left(\frac{x^6}{c^2} + \frac{\log\left(c^2x^6 - 1\right)}{c^4}\right)c\right)a^2b - \frac{\left(b^3c^3x^9 - b^3\right)\log\left(-cx^3 + 1\right)^3 - 3\left(2\,ab^2c^3x^9 + b^3c^2x^6 + \right.}{72\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(a+b*arctanh(c*x^3))^3,x, algorithm="maxima")
```

```
[Out] 1/9*a^3*x^9 + 1/6*(2*x^9*arctanh(c*x^3) + (x^6/c^2 + log(c^2*x^6 - 1)/c^4)*
c)*a^2*b - 1/72*((b^3*c^3*x^9 - b^3)*log(-c*x^3 + 1)^3 - 3*(2*a*b^2*c^3*x^9
 + b^3*c^2*x^6 + (b^3*c^3*x^9 + b^3)*log(c*x^3 + 1))*log(-c*x^3 + 1)^2)/c^3
 - integrate(-1/8*((b^3*c^3*x^11 - b^3*c^2*x^8)*log(c*x^3 + 1)^3 + 6*(a*b^2
*c^3*x^11 - a*b^2*c^2*x^8)*log(c*x^3 + 1)^2 - (4*a*b^2*c^3*x^11 + 2*b^3*c^2
*x^8 + 3*(b^3*c^3*x^11 - b^3*c^2*x^8)*log(c*x^3 + 1)^2 - 2*(6*a*b^2*c^2*x^8
 - (6*a*b^2*c^3 + b^3*c^3)*x^11 - b^3*x^2)*log(c*x^3 + 1))*log(-c*x^3 + 1))
/(c^3*x^3 - c^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(b^3x^8 \operatorname{artanh}\left(cx^3\right)^3 + 3\,ab^2x^8 \operatorname{artanh}\left(cx^3\right)^2 + 3\,a^2bx^8 \operatorname{artanh}\left(cx^3\right) + a^3x^8, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(a+b*arctanh(c*x^3))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^8*arctanh(c*x^3)^3 + 3*a*b^2*x^8*arctanh(c*x^3)^2 + 3*a^2*b*
x^8*arctanh(c*x^3) + a^3*x^8, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(a+b*atanh(c*x**3))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \operatorname{artanh}\left(cx^3\right) + a\right)^3 x^8 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(a+b*arctanh(c*x^3))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^3) + a)^3*x^8, x)
```

3.125 $\int x^5 \left(a + b \tanh^{-1}\left(cx^3\right)\right)^3 dx$

Optimal. Leaf size=139

$$-\frac{b^3 \text{PolyLog}\left(2, 1 - \frac{2}{1-cx^3}\right)}{2c^2} - \frac{b^2 \log\left(\frac{2}{1-cx^3}\right)\left(a + b\tanh^{-1}\left(cx^3\right)\right)}{c^2} - \frac{\left(a + b\tanh^{-1}\left(cx^3\right)\right)^3}{6c^2} + \frac{b\left(a + b\tanh^{-1}\left(cx^3\right)\right)^2}{2c^2} +$$

```
[Out] (b*(a + b*ArcTanh[c*x^3])^2)/(2*c^2) + (b*x^3*(a + b*ArcTanh[c*x^3])^2)/(2*
c) - (a + b*ArcTanh[c*x^3])^3/(6*c^2) + (x^6*(a + b*ArcTanh[c*x^3])^3)/6 -
(b^2*(a + b*ArcTanh[c*x^3])*Log[2/(1 - c*x^3)])/c^2 - (b^3*PolyLog[2, 1 - 2
/(1 - c*x^3)])/(2*c^2)
```

Rubi [B] time = 4.20449, antiderivative size = 479, normalized size of antiderivative = 3.45, number of steps used = 155, number of rules used = 30, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.875, Rules used = {6099, 2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304, 2439, 2416, 2396, 2433, 2374, 6589, 2411, 43, 2334, 12, 14, 2301, 6742, 2430, 2394, 2393, 2391, 2395, 2375, 2317, 2425}

$$-\frac{b^3 \text{PolyLog}\left(2, \frac{1}{2}\left(1 - cx^3\right)\right)}{4c^2} + \frac{b^3 \text{PolyLog}\left(2, \frac{1}{2}\left(cx^3 + 1\right)\right)}{4c^2} - \frac{b^2 \log^2\left(cx^3 + 1\right)\left(2a - b\log\left(1 - cx^3\right)\right)}{16c^2} + \frac{b^2 \log\left(\frac{1}{2}\left(cx^3\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^5*(a + b*ArcTanh[c*x^3])^3,x]
```

```
[Out] -(b*(1 - c*x^3)*(2*a - b*Log[1 - c*x^3])^2)/(8*c^2) - ((1 - c*x^3)*(2*a - b
*Log[1 - c*x^3])^3)/(24*c^2) + ((1 - c*x^3)^2*(2*a - b*Log[1 - c*x^3])^3)/(
48*c^2) + (b^2*(2*a - b*Log[1 - c*x^3])*Log[(1 + c*x^3)/2])/(4*c^2) + (b^3*
Log[(1 - c*x^3)/2]*Log[1 + c*x^3])/(4*c^2) + (b^2*x^3*(2*a - b*Log[1 - c*x^
3])*Log[1 + c*x^3])/(4*c) - (b*(2*a - b*Log[1 - c*x^3])^2*Log[1 + c*x^3])/(
16*c^2) + (b*x^6*(2*a - b*Log[1 - c*x^3])^2*Log[1 + c*x^3])/16 + (b^3*(1 +
c*x^3)*Log[1 + c*x^3]^2)/(8*c^2) - (b^2*(2*a - b*Log[1 - c*x^3])*Log[1 + c*
x^3]^2)/(16*c^2) + (b^2*x^6*(2*a - b*Log[1 - c*x^3])*Log[1 + c*x^3]^2)/16 -
 (b^3*(1 + c*x^3)*Log[1 + c*x^3]^3)/(24*c^2) + (b^3*(1 + c*x^3)^2*Log[1 + c
*x^3]^3)/(48*c^2) - (b^3*PolyLog[2, (1 - c*x^3)/2])/(4*c^2) + (b^3*PolyLog[
2, (1 + c*x^3)/2])/(4*c^2)
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
```

```
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[(x^
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
 + j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1
] && EqQ[m, -1])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
```

```
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2430

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.)), x_Symbol] :> Simp[x*(a + b*Log[c
*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]), x] + (-Dist[g*j*m, Int[(x*(a +
 b*Log[c*(d + e*x)^n])^p)/(i + j*x), x], x] - Dist[b*e*n*p, Int[(x*(a + b*L
og[c*(d + e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /
; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2375

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[(Log[d*(e + f*x^m)^r]*(a + b*Log[
c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[(f*m*r)/(b*n*(p + 1)), Int[(x^(m
- 1)*(a + b*Log[c*x^n])^(p + 1))/(e + f*x^m), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2425

```
Int[(Log[(f_.)*(x_)^(m_.)]*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b
_.)))/(x_), x_Symbol] :> Simp[(Log[f*x^m]^2*(a + b*Log[c*(d + e*x)^n]))/(2*
m), x] - Dist[(b*e*n)/(2*m), Int[Log[f*x^m]^2/(d + e*x), x], x] /; FreeQ[{a
, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int x^5\left(a+b\tanh^{-1}\left(cx^3\right)\right)^3\,dx = \int\left(\frac{1}{8}x^5\left(2a-b\log\left(1-cx^3\right)\right)^3+\frac{3}{8}bx^5\left(-2a+b\log\left(1-cx^3\right)\right)^2\log\left(1+cx^3\right)-\frac{3}{8}b^2x$$

$$=\frac{1}{8}\int x^5\left(2a-b\log\left(1-cx^3\right)\right)^3\,dx+\frac{1}{8}(3b)\int x^5\left(-2a+b\log\left(1-cx^3\right)\right)^2\log\left(1+cx^3$$

$$=\frac{1}{24}\text{Subst}\left(\int x(2a-b\log(1-cx))^3\,dx,x,x^3\right)+\frac{1}{8}b\,\text{Subst}\left(\int x(-2a+b\log(1-cx)\right)$$

$$=\frac{1}{16}bx^6\left(2a-b\log\left(1-cx^3\right)\right)^2\log\left(1+cx^3\right)+\frac{1}{16}b^2x^6\left(2a-b\log\left(1-cx^3\right)\right)\log^2\left(1+\right.$$

$$=\frac{1}{16}bx^6\left(2a-b\log\left(1-cx^3\right)\right)^2\log\left(1+cx^3\right)+\frac{1}{16}b^2x^6\left(2a-b\log\left(1-cx^3\right)\right)\log^2\left(1+\right.$$

$$=\frac{1}{16}bx^6\left(2a-b\log\left(1-cx^3\right)\right)^2\log\left(1+cx^3\right)+\frac{1}{16}b^2x^6\left(2a-b\log\left(1-cx^3\right)\right)\log^2\left(1+\right.$$

$$=-\frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^3}{24c^2}+\frac{\left(1-cx^3\right)^2\left(2a-b\log\left(1-cx^3\right)\right)^3}{48c^2}-\frac{b\left(2a-b\log\right.}{}$$

$$=-\frac{3b\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^2}{16c^2}+\frac{b\left(1-cx^3\right)^2\left(2a-b\log\left(1-cx^3\right)\right)^2}{32c^2}-\frac{\left(1-cx\right.}{}$$

$$=\frac{3ab^2x^3}{4c}+\frac{3b^3x^3}{8c}+\frac{b^3\left(1-cx^3\right)^2}{64c^2}-\frac{b^3\left(1+cx^3\right)^2}{64c^2}+\frac{b^2\left(1-cx^3\right)^2\left(2a-b\log\left(1-cx^3\right)\right.}{32c^2}$$

$$=\frac{3ab^2x^3}{4c}+\frac{3b^3x^3}{4c}+\frac{b^3\left(1-cx^3\right)^2}{64c^2}-\frac{b^3\left(1+cx^3\right)^2}{64c^2}+\frac{3b^3\left(1-cx^3\right)\log\left(1-cx^3\right)}{8c^2}+\frac{b^2\left(\right.}{}$$

$$=\frac{ab^2x^3}{2c}+\frac{5b^3x^3}{8c}+\frac{3b^3\left(1-cx^3\right)\log\left(1-cx^3\right)}{8c^2}-\frac{b\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^2}{8c^2}-$$

$$=\frac{ab^2x^3}{2c}+\frac{b^3x^3}{2c}+\frac{b^3\left(1-cx^3\right)\log\left(1-cx^3\right)}{4c^2}-\frac{b\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^2}{8c^2}-\frac{\left(1\right.}{}$$

Mathematica [A] time = 0.298934, size = 185, normalized size = 1.33

$$\frac{6b^3\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx^3)}\right)+a\left(2a^2c^2x^6+6abcx^3+3ab\log\left(1-cx^3\right)-3ab\log\left(cx^3+1\right)+6b^2\log\left(1-c^2x^6\right)\right)+6b}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^5*(a + b*ArcTanh[c*x^3])^3,x]
```

```
[Out] (6*b^2*(-1 + c*x^3)*(a + b + a*c*x^3)*ArcTanh[c*x^3]^2 + 2*b^3*(-1 + c^2*x^
6)*ArcTanh[c*x^3]^3 + 6*b*ArcTanh[c*x^3]*(a*c*x^3*(2*b + a*c*x^3) - 2*b^2*L
og[1 + E^(-2*ArcTanh[c*x^3])]) + a*(6*a*b*c*x^3 + 2*a^2*c^2*x^6 + 3*a*b*Log
[1 - c*x^3] - 3*a*b*Log[1 + c*x^3] + 6*b^2*Log[1 - c^2*x^6]) + 6*b^3*PolyLo
g[2, -E^(-2*ArcTanh[c*x^3])])/(12*c^2)
```

Maple [C] time = 0.316, size = 750, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arctanh(c*x^3))^3,x)
```

```
[Out] 1/48*b^3*(c^2*x^6-1)/c^2*ln(c*x^3+1)^3+1/16*b^2*(-x^6*b*ln(-c*x^3+1)*c^2+2*
a*c^2*x^6+2*b*c*x^3+b*ln(-c*x^3+1)-2*a+2*b)/c^2*ln(c*x^3+1)^2+(1/16*b^3*(c^
2*x^6-1)/c^2*ln(-c*x^3+1)^2-1/4*x^3*b^2*(a*c*x^3+b)/c*ln(-c*x^3+1)+1/4*b*(a
^2*c^2*x^6+2*a*b*c*x^3+b*ln(-c*x^3+1)*a+b^2*ln(-c*x^3+1))/c^2)*ln(c*x^3+1)-
1/2*a*b^2/c*x^3*ln(-c*x^3+1)+1/4/c^2*b^3*ln(-c*x^3+1)-1/4/c^2*b^3*ln(c*x^3-
1)+3/4*b^2/c*Sum(-2/3*(ln(x-_alpha)*ln(-c*x^3+1)+3*c*(-1/3*ln(x-_alpha)*(ln
((RootOf(_Z^2+_Z*_alpha+_alpha^2,index=1)-x+_alpha)/RootOf(_Z^2+_Z*_alpha+_
alpha^2,index=1))+ln((RootOf(_Z^2+_Z*_alpha+_alpha^2,index=2)-x+_alpha)/Roo
tOf(_Z^2+_Z*_alpha+_alpha^2,index=2))+ln(1/2*(x+_alpha)/_alpha))/c-1/3*(dil
og((RootOf(_Z^2+_Z*_alpha+_alpha^2,index=1)-x+_alpha)/RootOf(_Z^2+_Z*_alpha
+_alpha^2,index=1))+dilog((RootOf(_Z^2+_Z*_alpha+_alpha^2,index=2)-x+_alpha
)/RootOf(_Z^2+_Z*_alpha+_alpha^2,index=2))+dilog(1/2*(x+_alpha)/_alpha))/c)
)*b/c,_alpha=RootOf(_Z^3*c+1))-1/8*b^3/c^2+1/8*b^3/c*x^3*ln(-c*x^3+1)^2-1/4
*a^2*b*x^6*ln(-c*x^3+1)+1/4*a^2*b/c^2*ln(c*x^3-1)+1/8*a*b^2*x^6*ln(-c*x^3+1
)^2+3/8*a*b^2/c^2*ln(-c*x^3+1)-1/8*a*b^2/c^2*ln(-c*x^3+1)^2+1/6*x^6*a^3-1/4
8*b^3*x^6*ln(-c*x^3+1)^3-1/8*b^3/c^2*ln(-c*x^3+1)^2+1/48*b^3/c^2*ln(-c*x^3+
1)^3+1/2*a*b^2/c^2*ln(c*x^3+1)+1/8*b^2/c^2*a*ln(c*x^3-1)-1/4*a^2*b/c^2*ln(c
*x^3+1)+1/2/c*a^2*b*x^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x^3))^3,x, algorithm="maxima")
```

```
[Out] 1/2*a*b^2*x^6*arctanh(c*x^3)^2 + 1/6*a^3*x^6 + 1/4*(2*x^6*arctanh(c*x^3) +
c*(2*x^3/c^2 - log(c*x^3 + 1)/c^3 + log(c*x^3 - 1)/c^3))*a^2*b + 1/8*(4*c*(
2*x^3/c^2 - log(c*x^3 + 1)/c^3 + log(c*x^3 - 1)/c^3)*arctanh(c*x^3) - (2*(l
```

```
og(c*x^3 - 1) - 2)*log(c*x^3 + 1) - log(c*x^3 + 1)^2 - log(c*x^3 - 1)^2 - 4
*log(c*x^3 - 1))/c^2)*a*b^2 - 1/192*(4*x^6*log(-c*x^3 + 1)^3 + 3*(x^6/c^3 +
 log(c^2*x^6 - 1)/c^5)*c^3 - 6*c*((c*x^6 + 2*x^3)/c^2 + 2*log(c*x^3 - 1)/c^
3)*log(-c*x^3 + 1)^2 + 21*c^2*(2*x^3/c^3 - log(c*x^3 + 1)/c^4 + log(c*x^3 -
 1)/c^4) + c*(6*(c^2*x^6 + 6*c*x^3 + 2*log(c*x^3 - 1)^2 + 6*log(c*x^3 - 1))
*log(-c*x^3 + 1)/c^3 - (3*c^2*x^6 + 42*c*x^3 + 4*log(c*x^3 - 1)^3 + 18*log(
c*x^3 - 1)^2 + 42*log(c*x^3 - 1))/c^3) - 1728*c*integrate(1/4*x^5*log(c*x^3
 + 1)/(c^3*x^6 - c), x) - 2*(12*c*x^3*log(c*x^3 + 1)^2 + 2*(c^2*x^6 - 1)*lo
g(c*x^3 + 1)^3 - 3*(c^2*x^6 - 2*c*x^3 - 2*(c^2*x^6 - 1)*log(c*x^3 + 1) + 1)
*log(-c*x^3 + 1)^2 + 3*(c^2*x^6 + 6*c*x^3 - 2*(c^2*x^6 - 1)*log(c*x^3 + 1)^
2 - 8*(c*x^3 + 1)*log(c*x^3 + 1))*log(-c*x^3 + 1))/c^2 + 18*log(4*c^3*x^6 -
 4*c)/c^2 - 576*integrate(1/4*x^2*log(c*x^3 + 1)/(c^3*x^6 - c), x))*b^3
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(b^3x^5 \operatorname{artanh}\left(cx^3\right)^3 + 3\,ab^2x^5 \operatorname{artanh}\left(cx^3\right)^2 + 3\,a^2bx^5 \operatorname{artanh}\left(cx^3\right) + a^3x^5, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x^3))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^5*arctanh(c*x^3)^3 + 3*a*b^2*x^5*arctanh(c*x^3)^2 + 3*a^2*b*
x^5*arctanh(c*x^3) + a^3*x^5, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*atanh(c*x**3))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \operatorname{artanh}\left(cx^3\right) + a\right)^3 x^5\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x^3))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^3) + a)^3*x^5, x)
```

3.126 $\int x^2 \left(a+b\tanh^{-1}\left(cx^3\right)\right)^3 dx$

Optimal. Leaf size=130

$$-\frac{b^2\text{PolyLog}\left(2,1-\frac{2}{1-cx^3}\right)\left(a+b\tanh^{-1}\left(cx^3\right)\right)}{c}+\frac{b^3\text{PolyLog}\left(3,1-\frac{2}{1-cx^3}\right)}{2c}+\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(cx^3\right)\right)^3+\frac{\left(a+b\tanh^{-}\right.}{3c}$$

```
[Out] (a + b*ArcTanh[c*x^3])^3/(3*c) + (x^3*(a + b*ArcTanh[c*x^3])^3)/3 - (b*(a +
 b*ArcTanh[c*x^3])^2*Log[2/(1 - c*x^3)])/c - (b^2*(a + b*ArcTanh[c*x^3])*Po
lyLog[2, 1 - 2/(1 - c*x^3)])/c + (b^3*PolyLog[3, 1 - 2/(1 - c*x^3)])/(2*c)
```

Rubi [B] time = 2.46745, antiderivative size = 390, normalized size of antiderivative = 3., number of steps used = 82, number of rules used = 23, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.438, Rules used = {6099, 2454, 2389, 2296, 2295, 6715, 2430, 2416, 2396, 2433, 2374, 6589, 2411, 2346, 2301, 6742, 43, 2394, 2393, 2391, 2375, 2317, 2425}

$$-\frac{b^2\text{PolyLog}\left(2,\frac{1}{2}\left(1-cx^3\right)\right)\left(2a-b\log\left(1-cx^3\right)\right)}{2c}-\frac{b^3\text{PolyLog}\left(3,\frac{1}{2}\left(1-cx^3\right)\right)}{2c}-\frac{b^3\text{PolyLog}\left(3,\frac{1}{2}\left(cx^3+1\right)\right)}{2c}+\frac{b^3\log}{}$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^2*(a + b*ArcTanh[c*x^3])^3,x]
```

```
[Out] -((1 - c*x^3)*(2*a - b*Log[1 - c*x^3])^3)/(24*c) + (b*(2*a - b*Log[1 - c*x^
3])^2*Log[(1 + c*x^3)/2])/(4*c) - (b*(2*a - b*Log[1 - c*x^3])^2*Log[1 + c*x
^3])/(8*c) + (b*x^3*(2*a - b*Log[1 - c*x^3])^2*Log[1 + c*x^3])/8 + (b^3*Log
[(1 - c*x^3)/2]*Log[1 + c*x^3]^2)/(4*c) + (b^2*(2*a - b*Log[1 - c*x^3])*Log
[1 + c*x^3]^2)/(8*c) + (b^2*x^3*(2*a - b*Log[1 - c*x^3])*Log[1 + c*x^3]^2)/
8 + (b^3*(1 + c*x^3)*Log[1 + c*x^3]^3)/(24*c) - (b^2*(2*a - b*Log[1 - c*x^3
])*PolyLog[2, (1 - c*x^3)/2])/(2*c) + (b^3*Log[1 + c*x^3]*PolyLog[2, (1 + c
*x^3)/2])/(2*c) - (b^3*PolyLog[3, (1 - c*x^3)/2])/(2*c) - (b^3*PolyLog[3, (
1 + c*x^3)/2])/(2*c)
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 2430

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.)), x_Symbol] :> Simp[x*(a + b*Log[c
*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]), x] + (-Dist[g*j*m, Int[(x*(a +
 b*Log[c*(d + e*x)^n])^p)/(i + j*x), x], x] - Dist[b*e*n*p, Int[(x*(a + b*L
og[c*(d + e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /
; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2346

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Dist[d, Int[((d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p)/x,
 x], x] + Dist[e, Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; Fre
eQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2375

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[(Log[d*(e + f*x^m)^r]*(a + b*Log[
c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[(f*m*r)/(b*n*(p + 1)), Int[(x^(m
- 1)*(a + b*Log[c*x^n])^(p + 1))/(e + f*x^m), x], x] /; FreeQ[{a, b, c, d,
```

```
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2425

```
Int[(Log[(f_.)*(x_)^(m_.)]*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b
_.)))/(x_), x_Symbol] :> Simp[(Log[f*x^m]^2*(a + b*Log[c*(d + e*x)^n]))/(2*
m), x] - Dist[(b*e*n)/(2*m), Int[Log[f*x^m]^2/(d + e*x), x], x] /; FreeQ[{a
, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int x^2\left(a+b\tanh^{-1}\left(cx^3\right)\right)^3dx = \int\left(\frac{1}{8}x^2\left(2a-b\log\left(1-cx^3\right)\right)^3+\frac{3}{8}bx^2\left(-2a+b\log\left(1-cx^3\right)\right)^2\log\left(1+cx^3\right)-\frac{3}{8}b^2x\right.$$

$$=\frac{1}{8}\int x^2\left(2a-b\log\left(1-cx^3\right)\right)^3dx+\frac{1}{8}(3b)\int x^2\left(-2a+b\log\left(1-cx^3\right)\right)^2\log\left(1+cx^3\right.$$

$$=\frac{1}{24}\text{Subst}\left(\int(2a-b\log(1-cx))^3dx,x,x^3\right)+\frac{1}{8}b\,\text{Subst}\left(\int(-2a+b\log(1-cx))^2\log\right.$$

$$=\frac{1}{8}bx^3\left(2a-b\log\left(1-cx^3\right)\right)^2\log\left(1+cx^3\right)+\frac{1}{8}b^2x^3\left(2a-b\log\left(1-cx^3\right)\right)\log^2\left(1+cx\right.$$

$$=-\frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^3}{24c}+\frac{1}{8}bx^3\left(2a-b\log\left(1-cx^3\right)\right)^2\log\left(1+cx^3\right)+\frac{1}{8}b^2$$

$$=-\frac{b\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^2}{8c}-\frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^3}{24c}+\frac{1}{8}bx^3\left(2a-\right.$$

$$=\frac{1}{2}ab^2x^3-\frac{b^3x^3}{4}-\frac{b\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^2}{8c}-\frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right.}{24c}$$

$$=\frac{1}{2}ab^2x^3+\frac{b^3\left(1-cx^3\right)\log\left(1-cx^3\right)}{4c}-\frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^3}{24c}+\frac{b\left(2a-b\,\mathrm{lo}\right.}{}$$

$$=\frac{b^3x^3}{4}+\frac{b^3\left(1-cx^3\right)\log\left(1-cx^3\right)}{4c}-\frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^3}{24c}+\frac{b\left(2a-b\log\right.}{}$$

$$=-\frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^3}{24c}+\frac{b\left(2a-b\log\left(1-cx^3\right)\right)^2\log\left(\frac{1}{2}\left(1+cx^3\right)\right)}{4c}-\frac{b\left(\right.}{}$$

$$=-\frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^3}{24c}+\frac{b\left(2a-b\log\left(1-cx^3\right)\right)^2\log\left(\frac{1}{2}\left(1+cx^3\right)\right)}{4c}-\frac{b\left(\right.}{}$$

Mathematica [A] time = 0.281532, size = 191, normalized size = 1.47

$$\frac{6ab^2\left(\text{PolyLog}\left(2,-e^{-2\tanh^{-1}\left(cx^3\right)}\right)+\tanh^{-1}\left(cx^3\right)\left(\left(cx^3-1\right)\tanh^{-1}\left(cx^3\right)-2\log\left(e^{-2\tanh^{-1}\left(cx^3\right)}+1\right)\right)\right)+b^3\left(6\tanh\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(a + b*ArcTanh[c*x^3])^3,x]
```

```
[Out] (2*a^3*c*x^3 + 6*a^2*b*c*x^3*ArcTanh[c*x^3] + 3*a^2*b*Log[1 - c^2*x^6] + 6*
a*b^2*(ArcTanh[c*x^3]*((-1 + c*x^3)*ArcTanh[c*x^3] - 2*Log[1 + E^(-2*ArcTan
h[c*x^3])]) + PolyLog[2, -E^(-2*ArcTanh[c*x^3])]) + b^3*(2*ArcTanh[c*x^3]^2
*((-1 + c*x^3)*ArcTanh[c*x^3] - 3*Log[1 + E^(-2*ArcTanh[c*x^3])]) + 6*ArcTa
nh[c*x^3]*PolyLog[2, -E^(-2*ArcTanh[c*x^3])] + 3*PolyLog[3, -E^(-2*ArcTanh[
c*x^3])]))/(6*c)
```

Maple [B] time = 0.006, size = 295, normalized size = 2.3

$$\frac{x^3a^3}{3}+\frac{b^3x^3\left(\operatorname{Artanh}\left(cx^3\right)\right)^3}{3}+\frac{b^3\left(\operatorname{Artanh}\left(cx^3\right)\right)^3}{3c}-\frac{b^3\left(\operatorname{Artanh}\left(cx^3\right)\right)^2}{c}\ln\left(\frac{\left(cx^3+1\right)^2}{-c^2x^6+1}+1\right)-\frac{b^3\operatorname{Artanh}\left(cx^3\right)}{c}\operatorname{polylo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c*x^3))^3,x)
```

```
[Out] 1/3*x^3*a^3+1/3*b^3*x^3*arctanh(c*x^3)^3+1/3/c*b^3*arctanh(c*x^3)^3-1/c*b^3
*arctanh(c*x^3)^2*ln((c*x^3+1)^2/(-c^2*x^6+1)+1)-1/c*b^3*arctanh(c*x^3)*pol
ylog(2,-(c*x^3+1)^2/(-c^2*x^6+1))+1/2/c*b^3*polylog(3,-(c*x^3+1)^2/(-c^2*x^
6+1))+arctanh(c*x^3)^2*x^3*a*b^2-2/c*ln((c*x^3+1)^2/(-c^2*x^6+1)+1)*arctanh
(c*x^3)*a*b^2+1/c*a*b^2*arctanh(c*x^3)^2-1/c*polylog(2,-(c*x^3+1)^2/(-c^2*x
^6+1))*a*b^2+x^3*a^2*b*arctanh(c*x^3)+1/2/c*a^2*b*ln(-c^2*x^6+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{3}a^3x^3+\frac{\left(2\,cx^3\operatorname{artanh}\left(cx^3\right)+\log\left(-c^2x^6+1\right)\right)a^2b}{2\,c}-\frac{\left(b^3cx^3-b^3\right)\log\left(-cx^3+1\right)^3-3\left(2\,ab^2cx^3+\left(b^3cx^3+b^3\right)\log\left(cx^3\right.\right.}{24\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^3))^3,x, algorithm="maxima")
```

```
[Out] 1/3*a^3*x^3 + 1/2*(2*c*x^3*arctanh(c*x^3) + log(-c^2*x^6 + 1))*a^2*b/c - 1/
24*((b^3*c*x^3 - b^3)*log(-c*x^3 + 1)^3 - 3*(2*a*b^2*c*x^3 + (b^3*c*x^3 + b
^3)*log(c*x^3 + 1))*log(-c*x^3 + 1)^2)/c - integrate(-1/8*((b^3*c*x^5 - b^3
*x^2)*log(c*x^3 + 1)^3 + 6*(a*b^2*c*x^5 - a*b^2*x^2)*log(c*x^3 + 1)^2 - 3*(
4*a*b^2*c*x^5 + (b^3*c*x^5 - b^3*x^2)*log(c*x^3 + 1)^2 + 2*((2*a*b^2*c + b^
3*c)*x^5 - (2*a*b^2 - b^3)*x^2)*log(c*x^3 + 1))*log(-c*x^3 + 1))/(c*x^3 - 1
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(b^3x^2\operatorname{artanh}\left(cx^3\right)^3+3\,ab^2x^2\operatorname{artanh}\left(cx^3\right)^2+3\,a^2bx^2\operatorname{artanh}\left(cx^3\right)+a^3x^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^3))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^2*arctanh(c*x^3)^3 + 3*a*b^2*x^2*arctanh(c*x^3)^2 + 3*a^2*b*
x^2*arctanh(c*x^3) + a^3*x^2, x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: KeyError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c*x**3))**3,x)
```

```
[Out] Exception raised: KeyError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \operatorname{artanh}\left(cx^3\right) + a\right)^3 x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^3))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^3) + a)^3*x^2, x)
```

3.127 $\int \frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^3}{x}\,dx$

Optimal. Leaf size=210

$$\frac{1}{2}b^2\text{PolyLog}\left(3,1-\frac{2}{1-cx^3}\right)\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{1}{2}b^2\text{PolyLog}\left(3,\frac{2}{1-cx^3}-1\right)\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{1}{2}b\text{PolyLog}\left(2\right.$$

```
[Out] (2*(a + b*ArcTanh[c*x^3])^3*ArcTanh[1 - 2/(1 - c*x^3)])/3 - (b*(a + b*ArcTa
nh[c*x^3])^2*PolyLog[2, 1 - 2/(1 - c*x^3)])/2 + (b*(a + b*ArcTanh[c*x^3])^2
*PolyLog[2, -1 + 2/(1 - c*x^3)])/2 + (b^2*(a + b*ArcTanh[c*x^3])*PolyLog[3,
 1 - 2/(1 - c*x^3)])/2 - (b^2*(a + b*ArcTanh[c*x^3])*PolyLog[3, -1 + 2/(1 -
 c*x^3)])/2 - (b^3*PolyLog[4, 1 - 2/(1 - c*x^3)])/4 + (b^3*PolyLog[4, -1 +
2/(1 - c*x^3)])/4
```

Rubi [A] time = 0.564568, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {6095, 5914, 6052, 5948, 6058, 6062, 6610}

$$\frac{1}{2}b^2\text{PolyLog}\left(3,1-\frac{2}{1-cx^3}\right)\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{1}{2}b^2\text{PolyLog}\left(3,\frac{2}{1-cx^3}-1\right)\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{1}{2}b\text{PolyLog}\left(2\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^3])^3/x,x]
```

```
[Out] (2*(a + b*ArcTanh[c*x^3])^3*ArcTanh[1 - 2/(1 - c*x^3)])/3 - (b*(a + b*ArcTa
nh[c*x^3])^2*PolyLog[2, 1 - 2/(1 - c*x^3)])/2 + (b*(a + b*ArcTanh[c*x^3])^2
*PolyLog[2, -1 + 2/(1 - c*x^3)])/2 + (b^2*(a + b*ArcTanh[c*x^3])*PolyLog[3,
 1 - 2/(1 - c*x^3)])/2 - (b^2*(a + b*ArcTanh[c*x^3])*PolyLog[3, -1 + 2/(1 -
 c*x^3)])/2 - (b^3*PolyLog[4, 1 - 2/(1 - c*x^3)])/4 + (b^3*PolyLog[4, -1 +
2/(1 - c*x^3)])/4
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[
1/n, Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
 n}, x] && IGtQ[p, 0]
```

Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
 e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6062

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[k + 1, u])/(2
*c*d), x] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[k + 1,
u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && Eq
Q[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^3}{x}\,dx=\frac{1}{3}\,\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{x}\,dx,x,x^3\right)$$

$$=\frac{2}{3}\left(a+b\tanh^{-1}\left(cx^3\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1-cx^3}\right)-(2bc)\,\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2\,\text{ta}}{1-c^2x}\right.$$

$$=\frac{2}{3}\left(a+b\tanh^{-1}\left(cx^3\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1-cx^3}\right)+(bc)\,\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2\log}{1-c^2x^2}\right.$$

$$=\frac{2}{3}\left(a+b\tanh^{-1}\left(cx^3\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1-cx^3}\right)-\frac{1}{2}b\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2\text{Li}_2\left(1-\frac{2}{1-c}\right.$$

$$=\frac{2}{3}\left(a+b\tanh^{-1}\left(cx^3\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1-cx^3}\right)-\frac{1}{2}b\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2\text{Li}_2\left(1-\frac{2}{1-c}\right.$$

$$=\frac{2}{3}\left(a+b\tanh^{-1}\left(cx^3\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1-cx^3}\right)-\frac{1}{2}b\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2\text{Li}_2\left(1-\frac{2}{1-c}\right.$$

Mathematica [A] time = 0.180065, size = 214, normalized size = 1.02

$$\frac{1}{4}b\left(2\text{PolyLog}\left(2,\frac{cx^3+1}{1-cx^3}\right)\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2-2\text{PolyLog}\left(2,\frac{cx^3+1}{cx^3-1}\right)\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2+b\left(-2\text{PolyLog}\left(3\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^3])^3/x,x]
```

```
[Out] (2*(a + b*ArcTanh[c*x^3])^3*ArcTanh[1 + 2/(-1 + c*x^3)])/3 + (b*(2*(a + b*A
rcTanh[c*x^3])^2*PolyLog[2, (1 + c*x^3)/(1 - c*x^3)] - 2*(a + b*ArcTanh[c*x
^3])^2*PolyLog[2, (1 + c*x^3)/(-1 + c*x^3)] + b*(-2*(a + b*ArcTanh[c*x^3])*
PolyLog[3, (1 + c*x^3)/(1 - c*x^3)] + 2*(a + b*ArcTanh[c*x^3])*PolyLog[3, (
1 + c*x^3)/(-1 + c*x^3)] + b*(PolyLog[4, (1 + c*x^3)/(1 - c*x^3)] - PolyLog
[4, (1 + c*x^3)/(-1 + c*x^3)]))))/4
```

Maple [F] time = 0.151, size = 0, normalized size = 0.

$$\int \frac{\left(a + b\,\mathrm{Artanh}\left(c x^3\right)\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^3))^3/x,x)
```

```
[Out] int((a+b*arctanh(c*x^3))^3/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$a^3\log(x) + \int \frac{b^3\left(\log\left(cx^3+1\right) - \log\left(-cx^3+1\right)\right)^3}{8x} + \frac{3\,ab^2\left(\log\left(cx^3+1\right) - \log\left(-cx^3+1\right)\right)^2}{4x} + \frac{3\,a^2b\left(\log\left(cx^3+1\right) - \log\left(\right.\right.}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))^3/x,x, algorithm="maxima")
```

```
[Out] a^3*log(x) + integrate(1/8*b^3*(log(c*x^3 + 1) - log(-c*x^3 + 1))^3/x + 3/4
*a*b^2*(log(c*x^3 + 1) - log(-c*x^3 + 1))^2/x + 3/2*a^2*b*(log(c*x^3 + 1) -
 log(-c*x^3 + 1))/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^3\,\mathrm{artanh}\left(cx^3\right)^3 + 3\,ab^2\,\mathrm{artanh}\left(cx^3\right)^2 + 3\,a^2b\,\mathrm{artanh}\left(cx^3\right) + a^3}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))^3/x,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctanh(c*x^3)^3 + 3*a*b^2*arctanh(c*x^3)^2 + 3*a^2*b*arctanh
(c*x^3) + a^3)/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**3))**3/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \operatorname{artanh}\left(cx^3\right) + a\right)^3}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))^3/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^3) + a)^3/x, x)
```

3.128 $\int \frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^3}{x^4}\,dx$

Optimal. Leaf size=120

$$-b^2c\,\text{PolyLog}\left(2,\frac{2}{cx^3+1}-1\right)\left(a+b\tanh^{-1}\left(cx^3\right)\right)-\frac{1}{2}b^3c\,\text{PolyLog}\left(3,\frac{2}{cx^3+1}-1\right)+\frac{1}{3}c\left(a+b\tanh^{-1}\left(cx^3\right)\right)^3-\frac{\left(a+b\right.}{}$$

```
[Out] (c*(a + b*ArcTanh[c*x^3])^3)/3 - (a + b*ArcTanh[c*x^3])^3/(3*x^3) + b*c*(a
+ b*ArcTanh[c*x^3])^2*Log[2 - 2/(1 + c*x^3)] - b^2*c*(a + b*ArcTanh[c*x^3])
*PolyLog[2, -1 + 2/(1 + c*x^3)] - (b^3*c*PolyLog[3, -1 + 2/(1 + c*x^3)])/2
```

Rubi [F] time = 0.778839, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^3}{x^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcTanh[c*x^3])^3/x^4,x]
```

```
[Out] (b*c*Log[c*x^3]*(2*a - b*Log[1 - c*x^3])^2)/8 - ((1 - c*x^3)*(2*a - b*Log[1
 - c*x^3])^3)/(24*x^3) + (b^3*c*Log[-(c*x^3)]*Log[1 + c*x^3]^2)/8 - (b^3*(1
 + c*x^3)*Log[1 + c*x^3]^3)/(24*x^3) - (b^2*c*(2*a - b*Log[1 - c*x^3])*Poly
Log[2, 1 - c*x^3])/4 + (b^3*c*Log[1 + c*x^3]*PolyLog[2, 1 + c*x^3])/4 - (b^
3*c*PolyLog[3, 1 - c*x^3])/4 - (b^3*c*PolyLog[3, 1 + c*x^3])/4 + (b*Defer[S
ubst][Defer[Int][((-2*a + b*Log[1 - c*x])^2*Log[1 + c*x])/x^2, x], x, x^3])
/8 - (b^2*Defer[Subst][Defer[Int][((-2*a + b*Log[1 - c*x])*Log[1 + c*x]^2)/
x^2, x], x, x^3])/8
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^3}{x^4}\,dx = \int \left(\frac{\left(2a-b\log\left(1-cx^3\right)\right)^3}{8x^4} + \frac{3b\left(-2a+b\log\left(1-cx^3\right)\right)^2\log\left(1+cx^3\right)}{8x^4} - \frac{3b^2\left(-2a+b\right.}{}\right.$$

$$= \frac{1}{8}\int \frac{\left(2a-b\log\left(1-cx^3\right)\right)^3}{x^4}dx + \frac{1}{8}(3b)\int \frac{\left(-2a+b\log\left(1-cx^3\right)\right)^2\log\left(1+cx^3\right)}{x^4}dx - $$

$$= \frac{1}{24}\text{Subst}\left(\int \frac{(2a-b\log(1-cx))^3}{x^2}dx, x, x^3\right) + \frac{1}{8}b\,\text{Subst}\left(\int \frac{(-2a+b\log(1-cx))^2\log}{x^2}\right.$$

$$= -\frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^3}{24x^3} - \frac{b^3\left(1+cx^3\right)\log^3\left(1+cx^3\right)}{24x^3} + \frac{1}{8}b\,\text{Subst}\left(\int \frac{(-2a+}{}\right.$$

$$= \frac{1}{8}bc\log\left(cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^2 - \frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^3}{24x^3} + \frac{1}{8}b^3c\log\left(-c\right.$$

$$= \frac{1}{8}bc\log\left(cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^2 - \frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^3}{24x^3} + \frac{1}{8}b^3c\log\left(-c\right.$$

$$= \frac{1}{8}bc\log\left(cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^2 - \frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^3}{24x^3} + \frac{1}{8}b^3c\log\left(-c\right.$$

$$= \frac{1}{8}bc\log\left(cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^2 - \frac{\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^3}{24x^3} + \frac{1}{8}b^3c\log\left(-c\right.$$

Mathematica [C] time = 0.394616, size = 223, normalized size = 1.86

$$ab^2c\left(\tanh^{-1}\left(cx^3\right)\left(\left(1-\frac{1}{cx^3}\right)\tanh^{-1}\left(cx^3\right)+2\log\left(1-e^{-2\tanh^{-1}\left(cx^3\right)}\right)\right)-\text{PolyLog}\left(2,e^{-2\tanh^{-1}\left(cx^3\right)}\right)\right)+\frac{1}{3}b^3c\left(3\tan\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x^3])^3/x^4,x]
```

```
[Out] -a^3/(3*x^3) - (a^2*b*ArcTanh[c*x^3])/x^3 + 3*a^2*b*c*Log[x] - (a^2*b*c*Log
[1 - c^2*x^6])/2 + a*b^2*c*(ArcTanh[c*x^3]*((1 - 1/(c*x^3))*ArcTanh[c*x^3]
+ 2*Log[1 - E^(-2*ArcTanh[c*x^3])]) - PolyLog[2, E^(-2*ArcTanh[c*x^3])]) +
(b^3*c*((I/8)*Pi^3 - ArcTanh[c*x^3]^3 - ArcTanh[c*x^3]^3/(c*x^3) + 3*ArcTan
h[c*x^3]^2*Log[1 - E^(2*ArcTanh[c*x^3])] + 3*ArcTanh[c*x^3]*PolyLog[2, E^(2
*ArcTanh[c*x^3])] - (3*PolyLog[3, E^(2*ArcTanh[c*x^3])])/2))/3
```

Maple [F] time = 0.208, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\text{Artanh}\left(cx^3\right)\right)^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^3))^3/x^4,x)
```

```
[Out] int((a+b*arctanh(c*x^3))^3/x^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}\left(c\left(\log\left(c^2x^6-1\right)-\log\left(x^6\right)\right)+\frac{2\,\mathrm{artanh}\left(cx^3\right)}{x^3}\right)a^2b-\frac{a^3}{3\,x^3}-\frac{\left(b^3cx^3-b^3\right)\log\left(-cx^3+1\right)^3+3\left(2\,ab^2+\left(b^3cx^3+b^3\right)\mathrm{lo}\right.}{24\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))^3/x^4,x, algorithm="maxima")
```

```
[Out] -1/2*(c*(log(c^2*x^6 - 1) - log(x^6)) + 2*arctanh(c*x^3)/x^3)*a^2*b - 1/3*a
^3/x^3 - 1/24*((b^3*c*x^3 - b^3)*log(-c*x^3 + 1)^3 + 3*(2*a*b^2 + (b^3*c*x^
3 + b^3)*log(c*x^3 + 1))*log(-c*x^3 + 1)^2)/x^3 - integrate(-1/8*((b^3*c*x^
3 - b^3)*log(c*x^3 + 1)^3 + 6*(a*b^2*c*x^3 - a*b^2)*log(c*x^3 + 1)^2 + 3*(4
*a*b^2*c*x^3 - (b^3*c*x^3 - b^3)*log(c*x^3 + 1)^2 + 2*(b^3*c^2*x^6 - (2*a*b
^2*c - b^3*c)*x^3 + 2*a*b^2)*log(c*x^3 + 1))*log(-c*x^3 + 1))/(c*x^7 - x^4)
, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^3\,\mathrm{artanh}\left(cx^3\right)^3+3\,ab^2\,\mathrm{artanh}\left(cx^3\right)^2+3\,a^2b\,\mathrm{artanh}\left(cx^3\right)+a^3}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))^3/x^4,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctanh(c*x^3)^3 + 3*a*b^2*arctanh(c*x^3)^2 + 3*a^2*b*arctanh
(c*x^3) + a^3)/x^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**3))**3/x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(b\,\mathrm{artanh}\left(cx^3\right)+a\right)^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))^3/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^3) + a)^3/x^4, x)
```

3.129 $\int \frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^3}{x^7} dx$

Optimal. Leaf size=136

$$-\frac{1}{2}b^3c^2\text{PolyLog}\left(2,\frac{2}{cx^3+1}-1\right)+b^2c^2\log\left(2-\frac{2}{cx^3+1}\right)\left(a+b\tanh^{-1}\left(cx^3\right)\right)+\frac{1}{2}bc^2\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2+\frac{1}{6}c^2\left(a\right.$$

```
[Out] (b*c^2*(a + b*ArcTanh[c*x^3])^2)/2 - (b*c*(a + b*ArcTanh[c*x^3])^2)/(2*x^3)
 + (c^2*(a + b*ArcTanh[c*x^3])^3)/6 - (a + b*ArcTanh[c*x^3])^3/(6*x^6) + b^
2*c^2*(a + b*ArcTanh[c*x^3])*Log[2 - 2/(1 + c*x^3)] - (b^3*c^2*PolyLog[2, -
1 + 2/(1 + c*x^3)])/2
```

Rubi [F] time = 1.57491, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^3}{x^7} dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcTanh[c*x^3])^3/x^7,x]
```

```
[Out] (3*a*b^2*c^2*Log[x])/4 - (b*c*(1 - c*x^3)*(2*a - b*Log[1 - c*x^3])^2)/(16*x
^3) + (b*c^2*Log[c*x^3]*(2*a - b*Log[1 - c*x^3])^2)/16 + (c^2*(2*a - b*Log[
1 - c*x^3])^3)/48 - (2*a - b*Log[1 - c*x^3])^3/(48*x^6) - (b^3*c*(1 + c*x^3
)*Log[1 + c*x^3]^2)/(16*x^3) - (b^3*c^2*Log[-(c*x^3)]*Log[1 + c*x^3]^2)/16
+ (b^3*c^2*Log[1 + c*x^3]^3)/48 - (b^3*Log[1 + c*x^3]^3)/(48*x^6) - (b^3*c^
2*PolyLog[2, -(c*x^3)])/8 + (b^3*c^2*PolyLog[2, c*x^3])/8 - (b^2*c^2*(2*a -
 b*Log[1 - c*x^3])*PolyLog[2, 1 - c*x^3])/8 - (b^3*c^2*Log[1 + c*x^3]*PolyL
og[2, 1 + c*x^3])/8 - (b^3*c^2*PolyLog[3, 1 - c*x^3])/8 + (b^3*c^2*PolyLog[
3, 1 + c*x^3])/8 + (b*Defer[Subst][Defer[Int][((-2*a + b*Log[1 - c*x])^2*Lo
g[1 + c*x])/x^3, x], x, x^3])/8 - (b^2*Defer[Subst][Defer[Int][((-2*a + b*L
og[1 - c*x])*Log[1 + c*x]^2)/x^3, x], x, x^3])/8
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^3}{x^7}\,dx = \int \left(\frac{\left(2a-b\log\left(1-cx^3\right)\right)^3}{8x^7} + \frac{3b\left(-2a+b\log\left(1-cx^3\right)\right)^2\log\left(1+cx^3\right)}{8x^7} - \frac{3b^2\left(-2a+b\log\right.}{}\right.$$

$$= \frac{1}{8}\int \frac{\left(2a-b\log\left(1-cx^3\right)\right)^3}{x^7}\,dx + \frac{1}{8}(3b)\int \frac{\left(-2a+b\log\left(1-cx^3\right)\right)^2\log\left(1+cx^3\right)}{x^7}\,dx - \frac{1}{8}\left(\right.$$

$$= \frac{1}{24}\,\mathrm{Subst}\left(\int \frac{(2a-b\log(1-cx))^3}{x^3}\,dx, x, x^3\right) + \frac{1}{8}b\,\mathrm{Subst}\left(\int \frac{(-2a+b\log(1-cx))^2\log(1}{x^3}\right.$$

$$= -\frac{\left(2a-b\log\left(1-cx^3\right)\right)^3}{48x^6} - \frac{b^3\log^3\left(1+cx^3\right)}{48x^6} + \frac{1}{8}b\,\mathrm{Subst}\left(\int \frac{(-2a+b\log(1-cx))^2\log(1}{x^3}\right.$$

$$= -\frac{\left(2a-b\log\left(1-cx^3\right)\right)^3}{48x^6} - \frac{b^3\log^3\left(1+cx^3\right)}{48x^6} - \frac{1}{16}b\,\mathrm{Subst}\left(\int \frac{(2a-b\log(x))^2}{x\left(\frac{1}{c}-\frac{x}{c}\right)^2}\,dx, x, 1-cx\right.$$

$$= -\frac{\left(2a-b\log\left(1-cx^3\right)\right)^3}{48x^6} - \frac{b^3\log^3\left(1+cx^3\right)}{48x^6} - \frac{1}{16}b\,\mathrm{Subst}\left(\int \frac{(2a-b\log(x))^2}{\left(\frac{1}{c}-\frac{x}{c}\right)^2}\,dx, x, 1-cx\right.$$

$$= -\frac{bc\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^2}{16x^3} - \frac{\left(2a-b\log\left(1-cx^3\right)\right)^3}{48x^6} - \frac{b^3c\left(1+cx^3\right)\log^2\left(1+cx\right.}{16x^3}$$

$$= \frac{3}{4}ab^2c^2\log(x) - \frac{bc\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^2}{16x^3} + \frac{1}{16}bc^2\log\left(cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right.$$

$$= \frac{3}{4}ab^2c^2\log(x) - \frac{bc\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^2}{16x^3} + \frac{1}{16}bc^2\log\left(cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right.$$

$$= \frac{3}{4}ab^2c^2\log(x) - \frac{bc\left(1-cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right)^2}{16x^3} + \frac{1}{16}bc^2\log\left(cx^3\right)\left(2a-b\log\left(1-cx^3\right)\right.$$

Mathematica [A] time = 0.27932, size = 218, normalized size = 1.6

$$\frac{-6b^3c^2x^6\mathrm{PolyLog}\left(2, e^{-2\tanh^{-1}\left(cx^3\right)}\right) + a\left(-2a^2 - 3abc^2x^6\log\left(1-cx^3\right) + 3abc^2x^6\log\left(cx^3+1\right) - 6abcx^3 + 12b^2c^2x^6\log\left(\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x^3])^3/x^7,x]
```

```
[Out] (6*b^2*(-1 + c*x^3)*(a + a*c*x^3 + b*c*x^3)*ArcTanh[c*x^3]^2 + 2*b^3*(-1 +
c^2*x^6)*ArcTanh[c*x^3]^3 - 6*b*ArcTanh[c*x^3]*(a^2 + 2*a*b*c*x^3 - 2*b^2*c
^2*x^6*Log[1 - E^(-2*ArcTanh[c*x^3])]) + a*(-2*a^2 - 6*a*b*c*x^3 - 3*a*b*c^
2*x^6*Log[1 - c*x^3] + 3*a*b*c^2*x^6*Log[1 + c*x^3] + 12*b^2*c^2*x^6*Log[(c
*x^3)/Sqrt[1 - c^2*x^6]]) - 6*b^3*c^2*x^6*PolyLog[2, E^(-2*ArcTanh[c*x^3])]
)/(12*x^6)
```

Maple [F] time = 0.23, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\mathrm{Artanh}\left(cx^3\right)\right)^3}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^3))^3/x^7,x)
```

```
[Out] int((a+b*arctanh(c*x^3))^3/x^7,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{4}\left(\left(c\log\left(cx^3+1\right)-c\log\left(cx^3-1\right)-\frac{2}{x^3}\right)c-\frac{2\,\mathrm{artanh}\left(cx^3\right)}{x^6}\right)a^2b+\frac{1}{8}\left(\left(2\left(\log\left(cx^3-1\right)-2\right)\log\left(cx^3+1\right)-\log\left(cx^3\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))^3/x^7,x, algorithm="maxima")
```

```
[Out] 1/4*((c*log(c*x^3 + 1) - c*log(c*x^3 - 1) - 2/x^3)*c - 2*arctanh(c*x^3)/x^6
)*a^2*b + 1/8*((2*(log(c*x^3 - 1) - 2)*log(c*x^3 + 1) - log(c*x^3 + 1)^2 -
log(c*x^3 - 1)^2 - 4*log(c*x^3 - 1) + 24*log(x))*c^2 + 4*(c*log(c*x^3 + 1)
- c*log(c*x^3 - 1) - 2/x^3)*c*arctanh(c*x^3))*a*b^2 - 1/48*b^3*(((c^2*x^6 -
 1)*log(-c*x^3 + 1)^3 + 3*(2*c*x^3 - (c^2*x^6 - 1)*log(c*x^3 + 1))*log(-c*x
^3 + 1)^2)/x^6 + 6*integrate(-((c*x^3 - 1)*log(c*x^3 + 1)^3 + 3*(2*c^2*x^6
- (c*x^3 - 1)*log(c*x^3 + 1)^2 - (c^3*x^9 - c*x^3)*log(c*x^3 + 1))*log(-c*x
^3 + 1))/(c*x^10 - x^7), x)) - 1/2*a*b^2*arctanh(c*x^3)^2/x^6 - 1/6*a^3/x^6
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^3\,\mathrm{artanh}\left(cx^3\right)^3+3\,ab^2\,\mathrm{artanh}\left(cx^3\right)^2+3\,a^2b\,\mathrm{artanh}\left(cx^3\right)+a^3}{x^7},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))^3/x^7,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctanh(c*x^3)^3 + 3*a*b^2*arctanh(c*x^3)^2 + 3*a^2*b*arctanh
(c*x^3) + a^3)/x^7, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**3))**3/x**7,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(b\,\mathrm{artanh}\left(cx^3\right)+a\right)^3}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))^3/x^7,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^3) + a)^3/x^7, x)
```

3.130 $\int (dx)^m \left(a+b \tanh^{-1}\left(cx^3\right)\right)^3 dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left((dx)^m \left(a+b \tanh^{-1}\left(cx^3\right)\right)^3,x\right)$$

```
[Out] Unintegrable[(d*x)^m*(a + b*ArcTanh[c*x^3])^3, x]
```

Rubi [A] time = 0.0253039, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (dx)^m \left(a+b \tanh^{-1}\left(cx^3\right)\right)^3 dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m*(a + b*ArcTanh[c*x^3])^3,x]

[Out] Defer[Int][(d*x)^m*(a + b*ArcTanh[c*x^3])^3, x]
```

Rubi steps

$$\int (dx)^m \left(a+b \tanh^{-1}\left(cx^3\right)\right)^3 dx=\int (dx)^m \left(a+b \tanh^{-1}\left(cx^3\right)\right)^3 dx$$

Mathematica [A] time = 1.80957, size = 0, normalized size = 0.

$$\int (dx)^m \left(a+b \tanh^{-1}\left(cx^3\right)\right)^3 dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m*(a + b*ArcTanh[c*x^3])^3,x]

[Out] Integrate[(d*x)^m*(a + b*ArcTanh[c*x^3])^3, x]
```

Maple [A] time = 0.125, size = 0, normalized size = 0.

$$\int (dx)^m \left(a+b\text{Artanh}\left(cx^3\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctanh(c*x^3))^3,x)

[Out] int((d*x)^m*(a+b*arctanh(c*x^3))^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^3))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^3\,\text{artanh}\left(cx^3\right)^3+3\,ab^2\,\text{artanh}\left(cx^3\right)^2+3\,a^2b\,\text{artanh}\left(cx^3\right)+a^3\right)(dx)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^3))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctanh(c*x^3)^3 + 3*a*b^2*arctanh(c*x^3)^2 + 3*a^2*b*arctanh
(c*x^3) + a^3)*(d*x)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atanh(c*x**3))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\left(b\,\text{artanh}\left(cx^3\right)+a\right)^3(dx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^3))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^3) + a)^3*(d*x)^m, x)
```

3.131 $\int (dx)^m \left(a+b\tanh^{-1}\left(cx^3\right)\right)^2 dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left((dx)^m \left(a+b\tanh^{-1}\left(cx^3\right)\right)^2,x\right)$$

```
[Out] Unintegrable[(d*x)^m*(a + b*ArcTanh[c*x^3])^2, x]
```

Rubi [A] time = 0.025162, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (dx)^m \left(a+b\tanh^{-1}\left(cx^3\right)\right)^2 dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m*(a + b*ArcTanh[c*x^3])^2,x]
```

```
[Out] Defer[Int][(d*x)^m*(a + b*ArcTanh[c*x^3])^2, x]
```

Rubi steps

$$\int (dx)^m \left(a+b\tanh^{-1}\left(cx^3\right)\right)^2 dx = \int (dx)^m \left(a+b\tanh^{-1}\left(cx^3\right)\right)^2 dx$$

Mathematica [A] time = 1.17361, size = 0, normalized size = 0.

$$\int (dx)^m \left(a+b\tanh^{-1}\left(cx^3\right)\right)^2 dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m*(a + b*ArcTanh[c*x^3])^2,x]
```

```
[Out] Integrate[(d*x)^m*(a + b*ArcTanh[c*x^3])^2, x]
```

Maple [A] time = 0.125, size = 0, normalized size = 0.

$$\int (dx)^m \left(a+b\text{Artanh}\left(cx^3\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctanh(c*x^3))^2,x)
```

```
[Out] int((d*x)^m*(a+b*arctanh(c*x^3))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^3))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2\operatorname{artanh}\left(cx^3\right)^2+2\,ab\operatorname{artanh}\left(cx^3\right)+a^2\right)(dx)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^3))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x^3)^2 + 2*a*b*arctanh(c*x^3) + a^2)*(d*x)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atanh(c*x**3))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\left(b\operatorname{artanh}\left(cx^3\right)+a\right)^2(dx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^3))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^3) + a)^2*(d*x)^m, x)
```

3.132 $\int (dx)^m \left(a+b\tanh^{-1}\left(cx^3\right)\right) dx$

Optimal. Leaf size=74

$$\frac{(dx)^{m+1}\left(a+b\tanh^{-1}\left(cx^3\right)\right)}{d(m+1)}-\frac{3bc(dx)^{m+4}\text{Hypergeometric2F1}\left(1,\frac{m+4}{6},\frac{m+10}{6},c^2x^6\right)}{d^4(m+1)(m+4)}$$

```
[Out] ((d*x)^(1 + m)*(a + b*ArcTanh[c*x^3]))/(d*(1 + m)) - (3*b*c*(d*x)^(4 + m)*H
ypergeometric2F1[1, (4 + m)/6, (10 + m)/6, c^2*x^6])/(d^4*(1 + m)*(4 + m))
```

Rubi [A] time = 0.0431427, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {6097, 16, 364}

$$\frac{(dx)^{m+1}\left(a+b\tanh^{-1}\left(cx^3\right)\right)}{d(m+1)}-\frac{3bc(dx)^{m+4}\,_2F_1\left(1,\frac{m+4}{6};\frac{m+10}{6};c^2x^6\right)}{d^4(m+1)(m+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*ArcTanh[c*x^3]),x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*ArcTanh[c*x^3]))/(d*(1 + m)) - (3*b*c*(d*x)^(4 + m)*H
ypergeometric2F1[1, (4 + m)/6, (10 + m)/6, c^2*x^6])/(d^4*(1 + m)*(4 + m))
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (dx)^m\left(a+b\tanh^{-1}\left(cx^3\right)\right)dx &= \frac{(dx)^{1+m}\left(a+b\tanh^{-1}\left(cx^3\right)\right)}{d(1+m)}-\frac{(3bc)\int\frac{x^2(dx)^{1+m}}{1-c^2x^6}\,dx}{d(1+m)}\\ &= \frac{(dx)^{1+m}\left(a+b\tanh^{-1}\left(cx^3\right)\right)}{d(1+m)}-\frac{(3bc)\int\frac{(dx)^{3+m}}{1-c^2x^6}\,dx}{d^3(1+m)}\\ &= \frac{(dx)^{1+m}\left(a+b\tanh^{-1}\left(cx^3\right)\right)}{d(1+m)}-\frac{3bc(dx)^{4+m}\,_2F_1\left(1,\frac{4+m}{6};\frac{10+m}{6};c^2x^6\right)}{d^4(1+m)(4+m)}\end{aligned}$$

Mathematica [A] time = 0.0623695, size = 64, normalized size = 0.86

$$-\frac{x(dx)^m\left(3bcx^3\text{Hypergeometric2F1}\left(1,\frac{m+4}{6},\frac{m+10}{6},c^2x^6\right)-(m+4)\left(a+b\tanh^{-1}\left(cx^3\right)\right)\right)}{(m+1)(m+4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a + b*ArcTanh[c*x^3]),x]
```

```
[Out] -((x*(d*x)^m*(-((4 + m)*(a + b*ArcTanh[c*x^3])) + 3*b*c*x^3*Hypergeometric2
F1[1, (4 + m)/6, (10 + m)/6, c^2*x^6]))/((1 + m)*(4 + m)))
```

Maple [F] time = 0.119, size = 0, normalized size = 0.

$$\int (dx)^m \left(a+b\text{Artanh}\left(cx^3\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctanh(c*x^3)),x)
```

```
[Out] int((d*x)^m*(a+b*arctanh(c*x^3)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^3)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\,\text{artanh}\left(cx^3\right)+a\right)(dx)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^3)),x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*x^3) + a)*(d*x)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atanh(c*x**3)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \operatorname{artanh}\left(cx^3\right) + a\right) (dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^3)),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^3) + a)*(d*x)^m, x)
```

3.133 $\int \frac{(dx)^m}{a+b\tanh^{-1}(cx^3)}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{(dx)^m}{a+b\tanh^{-1}\left(cx^3\right)},x\right)$$

```
[Out] Unintegrable[(d*x)^m/(a + b*ArcTanh[c*x^3]), x]
```

Rubi [A] time = 0.0280181, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(dx)^m}{a+b\tanh^{-1}\left(cx^3\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m/(a + b*ArcTanh[c*x^3]),x]
```

```
[Out] Defer[Int][(d*x)^m/(a + b*ArcTanh[c*x^3]), x]
```

Rubi steps

$$\int \frac{(dx)^m}{a+b\tanh^{-1}\left(cx^3\right)}\,dx=\int \frac{(dx)^m}{a+b\tanh^{-1}\left(cx^3\right)}\,dx$$

Mathematica [A] time = 0.349283, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{a+b\tanh^{-1}\left(cx^3\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m/(a + b*ArcTanh[c*x^3]),x]
```

```
[Out] Integrate[(d*x)^m/(a + b*ArcTanh[c*x^3]), x]
```

Maple [A] time = 0.07, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{a+b\operatorname{Artanh}\left(cx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a+b*arctanh(c*x^3)),x)
```

```
[Out] int((d*x)^m/(a+b*arctanh(c*x^3)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{b\,\mathrm{artanh}\left(cx^3\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c*x^3)),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/(b*arctanh(c*x^3) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(dx)^m}{b\,\mathrm{artanh}\left(cx^3\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c*x^3)),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(b*arctanh(c*x^3) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*atanh(c*x**3)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{b\,\mathrm{artanh}\left(cx^3\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c*x^3)),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b*arctanh(c*x^3) + a), x)
```

3.134 $\int \frac{(dx)^m}{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{(dx)^m}{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2},x\right)$$

```
[Out] Unintegrable[(d*x)^m/(a + b*ArcTanh[c*x^3])^2, x]
```

Rubi [A] time = 0.0274606, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(dx)^m}{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m/(a + b*ArcTanh[c*x^3])^2,x]
```

```
[Out] Defer[Int][(d*x)^m/(a + b*ArcTanh[c*x^3])^2, x]
```

Rubi steps

$$\int \frac{(dx)^m}{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2}\,dx=\int \frac{(dx)^m}{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2}\,dx$$

Mathematica [A] time = 0.361395, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left(a+b\tanh^{-1}\left(cx^3\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m/(a + b*ArcTanh[c*x^3])^2,x]
```

```
[Out] Integrate[(d*x)^m/(a + b*ArcTanh[c*x^3])^2, x]
```

Maple [A] time = 0.069, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left(a+b\text{Artanh}\left(cx^3\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a+b*arctanh(c*x^3))^2,x)
```

```
[Out] int((d*x)^m/(a+b*arctanh(c*x^3))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{2\left(c^2 d^m x^6 - d^m\right)x^m}{3\left(b^2 c x^2 \log\left(cx^3+1\right) - b^2 c x^2 \log\left(-cx^3+1\right) + 2\,abcx^2\right)} + \int -\frac{2\left(c^2 d^m (m+4) x^6 - d^m (m-2)\right)x^m}{3\left(b^2 c x^3 \log\left(cx^3+1\right) - b^2 c x^3 \log\left(-cx^3+1\right) + 2\,abcx^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c*x^3))^2,x, algorithm="maxima")
```

```
[Out] 2/3*(c^2*d^m*x^6 - d^m)*x^m/(b^2*c*x^2*log(c*x^3 + 1) - b^2*c*x^2*log(-c*x^
3 + 1) + 2*a*b*c*x^2) + integrate(-2/3*(c^2*d^m*(m + 4)*x^6 - d^m*(m - 2))*
x^m/(b^2*c*x^3*log(c*x^3 + 1) - b^2*c*x^3*log(-c*x^3 + 1) + 2*a*b*c*x^3), x
)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx)^m}{b^2 \operatorname{artanh}\left(cx^3\right)^2 + 2\,ab \operatorname{artanh}\left(cx^3\right) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c*x^3))^2,x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(b^2*arctanh(c*x^3)^2 + 2*a*b*arctanh(c*x^3) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*atanh(c*x**3))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left(b \operatorname{artanh}\left(cx^3\right) + a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c*x^3))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b*arctanh(c*x^3) + a)^2, x)
```

3.135 $\int x^3 \left(a + b \tanh^{-1}\left(\frac{c}{x}\right)\right) dx$

Optimal. Leaf size=50

$$\frac{1}{4}x^4\left(a + b\tanh^{-1}\left(\frac{c}{x}\right)\right) + \frac{1}{4}bc^3x - \frac{1}{4}bc^4\tanh^{-1}\left(\frac{x}{c}\right) + \frac{1}{12}bcx^3$$

```
[Out] (b*c^3*x)/4 + (b*c*x^3)/12 + (x^4*(a + b*ArcTanh[c/x]))/4 - (b*c^4*ArcTanh[
x/c])/4
```

Rubi [A] time = 0.0319407, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6097, 263, 302, 207}

$$\frac{1}{4}x^4\left(a + b\tanh^{-1}\left(\frac{c}{x}\right)\right) + \frac{1}{4}bc^3x - \frac{1}{4}bc^4\tanh^{-1}\left(\frac{x}{c}\right) + \frac{1}{12}bcx^3$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcTanh[c/x]),x]
```

```
[Out] (b*c^3*x)/4 + (b*c*x^3)/12 + (x^4*(a + b*ArcTanh[c/x]))/4 - (b*c^4*ArcTanh[
x/c])/4
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int x^3\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)dx = \frac{1}{4}x^4\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)+\frac{1}{4}(bc)\int\frac{x^2}{1-\frac{c^2}{x^2}}dx$$
$$=\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)+\frac{1}{4}(bc)\int\frac{x^4}{-c^2+x^2}dx$$
$$=\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)+\frac{1}{4}(bc)\int\left(c^2+x^2+\frac{c^4}{-c^2+x^2}\right)dx$$
$$=\frac{1}{4}bc^3x+\frac{1}{12}bcx^3+\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)+\frac{1}{4}\left(bc^5\right)\int\frac{1}{-c^2+x^2}dx$$
$$=\frac{1}{4}bc^3x+\frac{1}{12}bcx^3+\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)-\frac{1}{4}bc^4\tanh^{-1}\left(\frac{x}{c}\right)$$

Mathematica [A] time = 0.0099763, size = 67, normalized size = 1.34

$$\frac{ax^4}{4}+\frac{1}{4}bc^3x+\frac{1}{8}bc^4\log(x-c)-\frac{1}{8}bc^4\log(c+x)+\frac{1}{12}bcx^3+\frac{1}{4}bx^4\tanh^{-1}\left(\frac{c}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcTanh[c/x]),x]
```

```
[Out] (b*c^3*x)/4 + (b*c*x^3)/12 + (a*x^4)/4 + (b*x^4*ArcTanh[c/x])/4 + (b*c^4*Lo
g[-c + x])/8 - (b*c^4*Log[c + x])/8
```

Maple [A] time = 0.013, size = 62, normalized size = 1.2

$$\frac{x^4a}{4}+\frac{bx^4}{4}\mathrm{Artanh}\left(\frac{c}{x}\right)+\frac{c^4b}{8}\ln\left(\frac{c}{x}-1\right)+\frac{bcx^3}{12}+\frac{bc^3x}{4}-\frac{c^4b}{8}\ln\left(1+\frac{c}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c/x)),x)
```

```
[Out] 1/4*x^4*a+1/4*b*x^4*arctanh(c/x)+1/8*c^4*b*ln(c/x-1)+1/12*b*c*x^3+1/4*b*c^3
*x-1/8*c^4*b*ln(1+c/x)
```

Maxima [A] time = 0.978632, size = 77, normalized size = 1.54

$$\frac{1}{4}ax^4+\frac{1}{24}\left(6x^4\operatorname{artanh}\left(\frac{c}{x}\right)-\left(3c^3\log(c+x)-3c^3\log(-c+x)-6c^2x-2x^3\right)c\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c/x)),x, algorithm="maxima")
```

```
[Out] 1/4*a*x^4 + 1/24*(6*x^4*arctanh(c/x) - (3*c^3*log(c + x) - 3*c^3*log(-c + x
) - 6*c^2*x - 2*x^3)*c)*b
```

Fricas [A] time = 1.66774, size = 113, normalized size = 2.26

$$\frac{1}{4}bc^3x+\frac{1}{12}bcx^3+\frac{1}{4}ax^4-\frac{1}{8}\left(bc^4-bx^4\right)\log\left(-\frac{c+x}{c-x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c/x)),x, algorithm="fricas")
```

```
[Out] 1/4*b*c^3*x + 1/12*b*c*x^3 + 1/4*a*x^4 - 1/8*(b*c^4 - b*x^4)*log(-(c + x)/(
c - x))
```

Sympy [A] time = 1.75908, size = 46, normalized size = 0.92

$$\frac{ax^4}{4} - \frac{bc^4 \operatorname{atanh}\left(\frac{c}{x}\right)}{4} + \frac{bc^3x}{4} + \frac{bcx^3}{12} + \frac{bx^4 \operatorname{atanh}\left(\frac{c}{x}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c/x)),x)
```

```
[Out] a*x**4/4 - b*c**4*atanh(c/x)/4 + b*c**3*x/4 + b*c*x**3/12 + b*x**4*atanh(c/
x)/4
```

Giac [A] time = 1.16809, size = 84, normalized size = 1.68

$$-\frac{1}{8}\,bc^4 \log\left(c+x\right) + \frac{1}{8}\,bc^4 \log\left(c-x\right) + \frac{1}{8}\,bx^4 \log\left(-\frac{c+x}{c-x}\right) + \frac{1}{4}\,bc^3x + \frac{1}{12}\,bcx^3 + \frac{1}{4}\,ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c/x)),x, algorithm="giac")
```

```
[Out] -1/8*b*c^4*log(c + x) + 1/8*b*c^4*log(c - x) + 1/8*b*x^4*log(-(c + x)/(c -
x)) + 1/4*b*c^3*x + 1/12*b*c*x^3 + 1/4*a*x^4
```

3.136 $\int x^2 \left(a+b \tanh^{-1}\left(\frac{c}{x}\right)\right) \, dx$

Optimal. Leaf size=45

$$\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)+\frac{1}{6}bc^3\log\left(c^2-x^2\right)+\frac{1}{6}bcx^2$$

```
[Out] (b*c*x^2)/6 + (x^3*(a + b*ArcTanh[c/x]))/3 + (b*c^3*Log[c^2 - x^2])/6
```

Rubi [A] time = 0.0327064, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6097, 263, 266, 43}

$$\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)+\frac{1}{6}bc^3\log\left(c^2-x^2\right)+\frac{1}{6}bcx^2$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcTanh[c/x]),x]
```

```
[Out] (b*c*x^2)/6 + (x^3*(a + b*ArcTanh[c/x]))/3 + (b*c^3*Log[c^2 - x^2])/6
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)dx&=\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)+\frac{1}{3}(bc)\int\frac{x}{1-\frac{c^2}{x^2}}\,dx\\&=\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)+\frac{1}{3}(bc)\int\frac{x^3}{-c^2+x^2}\,dx\\&=\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)+\frac{1}{6}(bc)\,\mathrm{Subst}\left(\int\frac{x}{-c^2+x}\,dx,x,x^2\right)\\&=\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)+\frac{1}{6}(bc)\,\mathrm{Subst}\left(\int\left(1-\frac{c^2}{c^2-x}\right)dx,x,x^2\right)\\&=\frac{1}{6}bcx^2+\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)+\frac{1}{6}bc^3\log\left(c^2-x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0079073, size = 50, normalized size = 1.11

$$\frac{ax^3}{3}+\frac{1}{6}bc^3\log\left(x^2-c^2\right)+\frac{1}{6}bcx^2+\frac{1}{3}bx^3\tanh^{-1}\left(\frac{c}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcTanh[c/x]),x]
```

```
[Out] (b*c*x^2)/6 + (a*x^3)/3 + (b*x^3*ArcTanh[c/x])/3 + (b*c^3*Log[-c^2 + x^2])/
6
```

Maple [A] time = 0.013, size = 67, normalized size = 1.5

$$\frac{x^3a}{3}+\frac{bx^3}{3}\mathrm{Artanh}\left(\frac{c}{x}\right)+\frac{c^3b}{6}\ln\left(\frac{c}{x}-1\right)+\frac{bcx^2}{6}-\frac{c^3b}{3}\ln\left(\frac{c}{x}\right)+\frac{c^3b}{6}\ln\left(1+\frac{c}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c/x)),x)
```

```
[Out] 1/3*x^3*a+1/3*b*x^3*arctanh(c/x)+1/6*c^3*b*ln(c/x-1)+1/6*b*c*x^2-1/3*c^3*b*
ln(c/x)+1/6*c^3*b*ln(1+c/x)
```

Maxima [A] time = 0.961072, size = 57, normalized size = 1.27

$$\frac{1}{3}ax^3+\frac{1}{6}\left(2x^3\,\mathrm{artanh}\left(\frac{c}{x}\right)+\left(c^2\log\left(-c^2+x^2\right)+x^2\right)c\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c/x)),x, algorithm="maxima")
```

```
[Out] 1/3*a*x^3 + 1/6*(2*x^3*arctanh(c/x) + (c^2*log(-c^2 + x^2) + x^2)*c)*b
```

Fricas [A] time = 1.76935, size = 117, normalized size = 2.6

$$\frac{1}{6}bc^3\log\left(-c^2+x^2\right)+\frac{1}{6}bx^3\log\left(-\frac{c+x}{c-x}\right)+\frac{1}{6}bcx^2+\frac{1}{3}ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c/x)),x, algorithm="fricas")
```

```
[Out] 1/6*b*c^3*log(-c^2 + x^2) + 1/6*b*x^3*log(-(c + x)/(c - x)) + 1/6*b*c*x^2 +
 1/3*a*x^3
```

Sympy [A] time = 1.10867, size = 49, normalized size = 1.09

$$\frac{ax^3}{3} + \frac{bc^3 \log(-c+x)}{3} + \frac{bc^3 \operatorname{atanh}\left(\frac{c}{x}\right)}{3} + \frac{bcx^2}{6} + \frac{bx^3 \operatorname{atanh}\left(\frac{c}{x}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c/x)),x)
```

```
[Out] a*x**3/3 + b*c**3*log(-c + x)/3 + b*c**3*atanh(c/x)/3 + b*c*x**2/6 + b*x**3
*atanh(c/x)/3
```

Giac [A] time = 1.1681, size = 66, normalized size = 1.47

$$\frac{1}{6}\,bc^3 \log\left(-c^2 + x^2\right) + \frac{1}{6}\,bx^3 \log\left(-\frac{c+x}{c-x}\right) + \frac{1}{6}\,bcx^2 + \frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c/x)),x, algorithm="giac")
```

```
[Out] 1/6*b*c^3*log(-c^2 + x^2) + 1/6*b*x^3*log(-(c + x)/(c - x)) + 1/6*b*c*x^2 +
 1/3*a*x^3
```

3.137 $\int x\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)dx$

Optimal. Leaf size=39

$$\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)-\frac{1}{2}bc^2\tanh^{-1}\left(\frac{x}{c}\right)+\frac{bcx}{2}$$

```
[Out] (b*c*x)/2 + (x^2*(a + b*ArcTanh[c/x]))/2 - (b*c^2*ArcTanh[x/c])/2
```

Rubi [A] time = 0.0202438, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6097, 193, 321, 207}

$$\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)-\frac{1}{2}bc^2\tanh^{-1}\left(\frac{x}{c}\right)+\frac{bcx}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcTanh[c/x]),x]
```

```
[Out] (b*c*x)/2 + (x^2*(a + b*ArcTanh[c/x]))/2 - (b*c^2*ArcTanh[x/c])/2
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int x\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)dx &= \frac{1}{2}x^2\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)+\frac{1}{2}(bc)\int\frac{1}{1-\frac{c^2}{x^2}}dx\\
&= \frac{1}{2}x^2\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)+\frac{1}{2}(bc)\int\frac{x^2}{-c^2+x^2}dx\\
&= \frac{bcx}{2}+\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)+\frac{1}{2}\left(bc^3\right)\int\frac{1}{-c^2+x^2}dx\\
&= \frac{bcx}{2}+\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)-\frac{1}{2}bc^2\tanh^{-1}\left(\frac{x}{c}\right)
\end{aligned}
$$

Mathematica [A] time = 0.007661, size = 56, normalized size = 1.44

$$
\frac{ax^2}{2}+\frac{1}{4}bc^2\log(x-c)-\frac{1}{4}bc^2\log(c+x)+\frac{1}{2}bx^2\tanh^{-1}\left(\frac{c}{x}\right)+\frac{bcx}{2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcTanh[c/x]),x]
```

```
[Out] (b*c*x)/2 + (a*x^2)/2 + (b*x^2*ArcTanh[c/x])/2 + (b*c^2*Log[-c + x])/4 - (b
*c^2*Log[c + x])/4
```

Maple [A] time = 0.01, size = 53, normalized size = 1.4

$$
\frac{ax^2}{2}+\frac{bx^2}{2}\mathrm{Artanh}\left(\frac{c}{x}\right)+\frac{xbc}{2}+\frac{c^2b}{4}\ln\left(\frac{c}{x}-1\right)-\frac{c^2b}{4}\ln\left(1+\frac{c}{x}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c/x)),x)
```

```
[Out] 1/2*a*x^2+1/2*arctanh(c/x)*b*x^2+1/2*x*b*c+1/4*c^2*b*ln(c/x-1)-1/4*c^2*b*ln
(1+c/x)
```

Maxima [A] time = 0.969734, size = 59, normalized size = 1.51

$$
\frac{1}{2}ax^2+\frac{1}{4}\left(2x^2\,\mathrm{artanh}\left(\frac{c}{x}\right)-(c\log(c+x)-c\log(-c+x)-2x)c\right)b
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c/x)),x, algorithm="maxima")
```

```
[Out] 1/2*a*x^2 + 1/4*(2*x^2*arctanh(c/x) - (c*log(c + x) - c*log(-c + x) - 2*x)*
c)*b
```

Fricas [A] time = 1.68662, size = 90, normalized size = 2.31

$$
\frac{1}{2}bcx+\frac{1}{2}ax^2-\frac{1}{4}\left(bc^2-bx^2\right)\log\left(-\frac{c+x}{c-x}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c/x)),x, algorithm="fricas")
```

```
[Out] 1/2*b*c*x + 1/2*a*x^2 - 1/4*(b*c^2 - b*x^2)*log(-(c + x)/(c - x))
```

Sympy [A] time = 0.486917, size = 36, normalized size = 0.92

$$\frac{ax^2}{2} - \frac{bc^2 \operatorname{atanh}\left(\frac{c}{x}\right)}{2} + \frac{bcx}{2} + \frac{bx^2 \operatorname{atanh}\left(\frac{c}{x}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c/x)),x)
```

```
[Out] a*x**2/2 - b*c**2*atanh(c/x)/2 + b*c*x/2 + b*x**2*atanh(c/x)/2
```

Giac [A] time = 1.14094, size = 72, normalized size = 1.85

$$-\frac{1}{4}\,bc^2 \log\left(c + x\right) + \frac{1}{4}\,bc^2 \log\left(c - x\right) + \frac{1}{4}\,bx^2 \log\left(-\frac{c + x}{c - x}\right) + \frac{1}{2}\,bcx + \frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c/x)),x, algorithm="giac")
```

```
[Out] -1/4*b*c^2*log(c + x) + 1/4*b*c^2*log(c - x) + 1/4*b*x^2*log(-(c + x)/(c -
x)) + 1/2*b*c*x + 1/2*a*x^2
```

3.138 $\int \left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)\,dx$

Optimal. Leaf size=29

$$ax+\frac{1}{2}bc\log\left(c^2-x^2\right)+bx\tanh^{-1}\left(\frac{c}{x}\right)$$

```
[Out] a*x + b*x*ArcTanh[c/x] + (b*c*Log[c^2 - x^2])/2
```

Rubi [A] time = 0.0133345, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {6091, 263, 260}

$$ax+\frac{1}{2}bc\log\left(c^2-x^2\right)+bx\tanh^{-1}\left(\frac{c}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcTanh[c/x],x]
```

```
[Out] a*x + b*x*ArcTanh[c/x] + (b*c*Log[c^2 - x^2])/2
```

Rule 6091

```
Int[ArcTanh[(c_.)*(x_)^(n_)], x_Symbol] :> Simp[x*ArcTanh[c*x^n], x] - Dist
[c*n, Int[x^n/(1 - c^2*x^(2*n)), x], x] /; FreeQ[{c, n}, x]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)\,dx &= ax+b\int \tanh^{-1}\left(\frac{c}{x}\right)\,dx\\
&= ax+bx\tanh^{-1}\left(\frac{c}{x}\right)+(bc)\int \frac{1}{\left(1-\frac{c^2}{x^2}\right)x}\,dx\\
&= ax+bx\tanh^{-1}\left(\frac{c}{x}\right)+(bc)\int \frac{x}{-c^2+x^2}\,dx\\
&= ax+bx\tanh^{-1}\left(\frac{c}{x}\right)+\frac{1}{2}bc\log\left(c^2-x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0028142, size = 29, normalized size = 1.

$$ax+\frac{1}{2}bc\log\left(c^2-x^2\right)+bx\tanh^{-1}\left(\frac{c}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcTanh[c/x],x]
```

```
[Out] a*x + b*x*ArcTanh[c/x] + (b*c*Log[c^2 - x^2])/2
```

Maple [A] time = 0.01, size = 48, normalized size = 1.7

$$ax + bx\text{Artanh}\left(\frac{c}{x}\right) + \frac{bc}{2}\ln\left(\frac{c}{x} - 1\right) - bc\ln\left(\frac{c}{x}\right) + \frac{bc}{2}\ln\left(1 + \frac{c}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arctanh(c/x),x)
```

```
[Out] a*x+b*x*arctanh(c/x)+1/2*b*c*ln(c/x-1)-b*c*ln(c/x)+1/2*b*c*ln(1+c/x)
```

Maxima [A] time = 0.949895, size = 39, normalized size = 1.34

$$\frac{1}{2}\left(2\,x\,\text{artanh}\left(\frac{c}{x}\right) + c\log\left(-c^2 + x^2\right)\right)b + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c/x),x, algorithm="maxima")
```

```
[Out] 1/2*(2*x*arctanh(c/x) + c*log(-c^2 + x^2))*b + a*x
```

Fricas [A] time = 1.67863, size = 85, normalized size = 2.93

$$\frac{1}{2}\,bc\log\left(-c^2 + x^2\right) + \frac{1}{2}\,bx\log\left(-\frac{c+x}{c-x}\right) + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c/x),x, algorithm="fricas")
```

```
[Out] 1/2*b*c*log(-c^2 + x^2) + 1/2*b*x*log(-(c + x)/(c - x)) + a*x
```

Sympy [A] time = 0.317447, size = 24, normalized size = 0.83

$$ax + b\left(c\log\left(c - x\right) + c\,\text{atanh}\left(\frac{c}{x}\right) + x\,\text{atanh}\left(\frac{c}{x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*atanh(c/x),x)
```

```
[Out] a*x + b*(c*log(c - x) + c*atanh(c/x) + x*atanh(c/x))
```

Giac [A] time = 1.13099, size = 57, normalized size = 1.97

$$\frac{1}{2}\left(x\log\left(-\frac{\frac{c}{x}+1}{\frac{c}{x}-1}\right)+c\log\left(\left|-c^2+x^2\right|\right)\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c/x),x, algorithm="giac")
```

```
[Out] 1/2*(x*log(-(c/x + 1)/(c/x - 1)) + c*log(abs(-c^2 + x^2)))*b + a*x
```

3.139 $\int \frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{x}\,dx$

Optimal. Leaf size=30

$$\frac{1}{2}b\text{PolyLog}\left(2,-\frac{c}{x}\right)-\frac{1}{2}b\text{PolyLog}\left(2,\frac{c}{x}\right)+a\log(x)$$

```
[Out] a*Log[x] + (b*PolyLog[2, -(c/x)])/2 - (b*PolyLog[2, c/x])/2
```

Rubi [A] time = 0.031678, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6095, 5912}

$$\frac{1}{2}b\text{PolyLog}\left(2,-\frac{c}{x}\right)-\frac{1}{2}b\text{PolyLog}\left(2,\frac{c}{x}\right)+a\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c/x])/x,x]
```

```
[Out] a*Log[x] + (b*PolyLog[2, -(c/x)])/2 - (b*PolyLog[2, c/x])/2
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[
1/n, Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
 n}, x] && IGtQ[p, 0]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{x}\,dx &= -\text{Subst}\left(\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx,x,\frac{1}{x}\right)\\ &= a\log(x)+\frac{1}{2}b\text{Li}_2\left(-\frac{c}{x}\right)-\frac{1}{2}b\text{Li}_2\left(\frac{c}{x}\right)\end{aligned}$$

Mathematica [A] time = 0.012366, size = 28, normalized size = 0.93

$$\frac{1}{2}b\left(\text{PolyLog}\left(2,-\frac{c}{x}\right)-\text{PolyLog}\left(2,\frac{c}{x}\right)\right)+a\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c/x])/x,x]
```

```
[Out] a*Log[x] + (b*(PolyLog[2, -(c/x)] - PolyLog[2, c/x]))/2
```

Maple [B] time = 0.014, size = 63, normalized size = 2.1

$$-a\ln\left(\frac{c}{x}\right)-b\ln\left(\frac{c}{x}\right)\operatorname{Artanh}\left(\frac{c}{x}\right)+\frac{b}{2}\operatorname{dilog}\left(\frac{c}{x}\right)+\frac{b}{2}\operatorname{dilog}\left(1+\frac{c}{x}\right)+\frac{b}{2}\ln\left(\frac{c}{x}\right)\ln\left(1+\frac{c}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x))/x,x)
```

```
[Out] -a*ln(c/x)-b*ln(c/x)*arctanh(c/x)+1/2*b*dilog(c/x)+1/2*b*dilog(1+c/x)+1/2*b
*ln(c/x)*ln(1+c/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}b\int\frac{\log\left(\frac{c}{x}+1\right)-\log\left(-\frac{c}{x}+1\right)}{x}\,dx+a\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))/x,x, algorithm="maxima")
```

```
[Out] 1/2*b*integrate((log(c/x + 1) - log(-c/x + 1))/x, x) + a*log(x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{b\operatorname{artanh}\left(\frac{c}{x}\right)+a}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))/x,x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c/x) + a)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{a+b\operatorname{atanh}\left(\frac{c}{x}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x))/x,x)
```

```
[Out] Integral((a + b*atanh(c/x))/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{b\operatorname{artanh}\left(\frac{c}{x}\right)+a}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c/x) + a)/x, x)
```

3.140 $\int \frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{x^2}\,dx$

Optimal. Leaf size=35

$$-\frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{x}-\frac{b\log\left(1-\frac{c^2}{x^2}\right)}{2c}$$

```
[Out] -((a + b*ArcTanh[c/x])/x) - (b*Log[1 - c^2/x^2])/(2*c)
```

Rubi [A] time = 0.0208549, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6097, 260}

$$-\frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{x}-\frac{b\log\left(1-\frac{c^2}{x^2}\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c/x])/x^2,x]
```

```
[Out] -((a + b*ArcTanh[c/x])/x) - (b*Log[1 - c^2/x^2])/(2*c)
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{x^2}\,dx &= -\frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{x}-(bc)\int \frac{1}{\left(1-\frac{c^2}{x^2}\right)x^3}\,dx\\ &= -\frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{x}-\frac{b\log\left(1-\frac{c^2}{x^2}\right)}{2c}\end{aligned}$$

Mathematica [A] time = 0.0082962, size = 38, normalized size = 1.09

$$-\frac{a}{x}-\frac{b\log\left(1-\frac{c^2}{x^2}\right)}{2c}-\frac{b\tanh^{-1}\left(\frac{c}{x}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c/x])/x^2,x]
```

```
[Out] -(a/x) - (b*ArcTanh[c/x])/x - (b*Log[1 - c^2/x^2])/(2*c)
```

Maple [A] time = 0.004, size = 37, normalized size = 1.1

$$-\frac{a}{x}-\frac{b}{x}\mathrm{Artanh}\left(\frac{c}{x}\right)-\frac{b}{2c}\ln\left(1-\frac{c^2}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x))/x^2,x)
```

```
[Out] -a/x-b/x*arctanh(c/x)-1/2*b*ln(1-c^2/x^2)/c
```

Maxima [A] time = 0.956861, size = 50, normalized size = 1.43

$$-\frac{b\left(\frac{2c\,\mathrm{artanh}\left(\frac{c}{x}\right)}{x}+\log\left(-\frac{c^2}{x^2}+1\right)\right)}{2c}-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*b*(2*c*arctanh(c/x)/x + log(-c^2/x^2 + 1))/c - a/x
```

Fricas [A] time = 1.69369, size = 115, normalized size = 3.29

$$-\frac{bx\log\left(-c^2+x^2\right)-2\,bx\log\left(x\right)+bc\log\left(-\frac{c+x}{c-x}\right)+2\,ac}{2\,cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b*x*log(-c^2 + x^2) - 2*b*x*log(x) + b*c*log(-(c + x)/(c - x)) + 2*a*
c)/(c*x)
```

Sympy [A] time = 1.8518, size = 39, normalized size = 1.11

$$\begin{cases} -\frac{a}{x}-\frac{b\,\mathrm{atanh}\left(\frac{c}{x}\right)}{x}+\frac{b\log\left(x\right)}{c}-\frac{b\log\left(-c+x\right)}{c}-\frac{b\,\mathrm{atanh}\left(\frac{c}{x}\right)}{c} & \text{for } c\neq 0\\ -\frac{a}{x} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x))/x**2,x)
```

```
[Out] Piecewise((-a/x - b*atanh(c/x)/x + b*log(x)/c - b*log(-c + x)/c - b*atanh(c
/x)/c, Ne(c, 0)), (-a/x, True))
```

Giac [A] time = 1.13314, size = 66, normalized size = 1.89

$$-\frac{1}{2}b\left(\frac{\log\left(-\frac{\frac{c}{x}+1}{\frac{c}{x}-1}\right)}{x}+\frac{\log\left(\left|\frac{c^2}{x^2}-1\right|\right)}{c}\right)-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))/x^2,x, algorithm="giac")
```

```
[Out] -1/2*b*(log(-(c/x + 1)/(c/x - 1))/x + log(abs(c^2/x^2 - 1))/c) - a/x
```

3.141 $\int \frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{x^3}\,dx$

Optimal. Leaf size=43

$$-\frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{2x^2}+\frac{b\tanh^{-1}\left(\frac{x}{c}\right)}{2c^2}-\frac{b}{2cx}$$

```
[Out] -b/(2*c*x) - (a + b*ArcTanh[c/x])/(2*x^2) + (b*ArcTanh[x/c])/(2*c^2)
```

Rubi [A] time = 0.0266653, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6097, 263, 325, 207}

$$-\frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{2x^2}+\frac{b\tanh^{-1}\left(\frac{x}{c}\right)}{2c^2}-\frac{b}{2cx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c/x])/x^3,x]
```

```
[Out] -b/(2*c*x) - (a + b*ArcTanh[c/x])/(2*x^2) + (b*ArcTanh[x/c])/(2*c^2)
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{x^3}\,dx = -\frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{2x^2} - \frac{1}{2}(bc)\int \frac{1}{\left(1-\frac{c^2}{x^2}\right)x^4}\,dx$$
$$= -\frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{2x^2} - \frac{1}{2}(bc)\int \frac{1}{x^2\left(-c^2+x^2\right)}\,dx$$
$$= -\frac{b}{2cx} - \frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{2x^2} - \frac{b\int \frac{1}{-c^2+x^2}\,dx}{2c}$$
$$= -\frac{b}{2cx} - \frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{2x^2} + \frac{b\tanh^{-1}\left(\frac{x}{c}\right)}{2c^2}$$

Mathematica [A] time = 0.0090805, size = 60, normalized size = 1.4

$$-\frac{a}{2x^2} - \frac{b\log(x-c)}{4c^2} + \frac{b\log(c+x)}{4c^2} - \frac{b\tanh^{-1}\left(\frac{c}{x}\right)}{2x^2} - \frac{b}{2cx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c/x])/x^3,x]
```

```
[Out] -a/(2*x^2) - b/(2*c*x) - (b*ArcTanh[c/x])/(2*x^2) - (b*Log[-c + x])/(4*c^2)
 + (b*Log[c + x])/(4*c^2)
```

Maple [A] time = 0.007, size = 57, normalized size = 1.3

$$-\frac{a}{2\,x^2} - \frac{b}{2\,x^2}\mathrm{Artanh}\left(\frac{c}{x}\right) - \frac{b}{2\,cx} - \frac{b}{4\,c^2}\ln\left(\frac{c}{x}-1\right) + \frac{b}{4\,c^2}\ln\left(1+\frac{c}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x))/x^3,x)
```

```
[Out] -1/2*a/x^2-1/2*b/x^2*arctanh(c/x)-1/2*b/c/x-1/4/c^2*b*ln(c/x-1)+1/4/c^2*b*l
n(1+c/x)
```

Maxima [A] time = 0.95463, size = 70, normalized size = 1.63

$$\frac{1}{4}\left(c\left(\frac{\log(c+x)}{c^3} - \frac{\log(-c+x)}{c^3} - \frac{2}{c^2x}\right) - \frac{2\,\mathrm{artanh}\left(\frac{c}{x}\right)}{x^2}\right)b - \frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*(c*(log(c + x)/c^3 - log(-c + x)/c^3 - 2/(c^2*x)) - 2*arctanh(c/x)/x^2)
*b - 1/2*a/x^2
```

Fricas [A] time = 1.76097, size = 103, normalized size = 2.4

$$-\frac{2\,ac^2+2\,bcx+\left(bc^2-bx^2\right)\log\left(-\frac{c+x}{c-x}\right)}{4\,c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a*c^2 + 2*b*c*x + (b*c^2 - b*x^2)*log(-(c + x)/(c - x)))/(c^2*x^2)
```

Sympy [A] time = 2.11399, size = 44, normalized size = 1.02

$$\begin{cases} -\frac{a}{2x^2}-\frac{b\operatorname{atanh}\left(\frac{c}{x}\right)}{2x^2}-\frac{b}{2cx}+\frac{b\operatorname{atanh}\left(\frac{c}{x}\right)}{2c^2} & \text{for } c\neq 0 \\ -\frac{a}{2x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x))/x**3,x)
```

```
[Out] Piecewise((-a/(2*x**2) - b*atanh(c/x)/(2*x**2) - b/(2*c*x) + b*atanh(c/x)/(
2*c**2), Ne(c, 0)), (-a/(2*x**2), True))
```

Giac [A] time = 1.16435, size = 77, normalized size = 1.79

$$\frac{b\log(c+x)}{4\,c^2}-\frac{b\log(c-x)}{4\,c^2}-\frac{b\log\left(-\frac{c+x}{c-x}\right)}{4\,x^2}-\frac{ac+bx}{2\,cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))/x^3,x, algorithm="giac")
```

```
[Out] 1/4*b*log(c + x)/c^2 - 1/4*b*log(c - x)/c^2 - 1/4*b*log(-(c + x)/(c - x))/x
^2 - 1/2*(a*c + b*x)/(c*x^2)
```

3.142 $\int \frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{x^4}\,dx$

Optimal. Leaf size=57

$$-\frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{3x^3}-\frac{b\log\left(c^2-x^2\right)}{6c^3}+\frac{b\log(x)}{3c^3}-\frac{b}{6cx^2}$$

```
[Out] -b/(6*c*x^2) - (a + b*ArcTanh[c/x])/(3*x^3) + (b*Log[x])/(3*c^3) - (b*Log[c
^2 - x^2])/(6*c^3)
```

Rubi [A] time = 0.0407439, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6097, 263, 266, 44}

$$-\frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{3x^3}-\frac{b\log\left(c^2-x^2\right)}{6c^3}+\frac{b\log(x)}{3c^3}-\frac{b}{6cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c/x])/x^4,x]
```

```
[Out] -b/(6*c*x^2) - (a + b*ArcTanh[c/x])/(3*x^3) + (b*Log[x])/(3*c^3) - (b*Log[c
^2 - x^2])/(6*c^3)
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{x^4}\,dx = -\frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{3x^3} - \frac{1}{3}(bc)\int \frac{1}{\left(1-\frac{c^2}{x^2}\right)x^5}\,dx$$
$$= -\frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{3x^3} - \frac{1}{3}(bc)\int \frac{1}{x^3\left(-c^2+x^2\right)}\,dx$$
$$= -\frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{3x^3} - \frac{1}{6}(bc)\,\text{Subst}\left(\int \frac{1}{x^2\left(-c^2+x\right)}\,dx, x, x^2\right)$$
$$= -\frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{3x^3} - \frac{1}{6}(bc)\,\text{Subst}\left(\int \left(-\frac{1}{c^4\left(c^2-x\right)} - \frac{1}{c^2x^2} - \frac{1}{c^4x}\right)dx, x, x^2\right)$$
$$= -\frac{b}{6cx^2} - \frac{a+b\tanh^{-1}\left(\frac{c}{x}\right)}{3x^3} + \frac{b\log(x)}{3c^3} - \frac{b\log\left(c^2-x^2\right)}{6c^3}$$

Mathematica [A] time = 0.0093336, size = 62, normalized size = 1.09

$$-\frac{a}{3x^3} - \frac{b\log\left(x^2-c^2\right)}{6c^3} + \frac{b\log(x)}{3c^3} - \frac{b}{6cx^2} - \frac{b\tanh^{-1}\left(\frac{c}{x}\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c/x])/x^4,x]
```

```
[Out] -a/(3*x^3) - b/(6*c*x^2) - (b*ArcTanh[c/x])/(3*x^3) + (b*Log[x])/(3*c^3) -
(b*Log[-c^2 + x^2])/(6*c^3)
```

Maple [A] time = 0.007, size = 57, normalized size = 1.

$$-\frac{a}{3\,x^3} - \frac{b}{3\,x^3}\text{Artanh}\left(\frac{c}{x}\right) - \frac{b}{6\,cx^2} - \frac{b}{6\,c^3}\ln\left(\frac{c}{x}-1\right) - \frac{b}{6\,c^3}\ln\left(1+\frac{c}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x))/x^4,x)
```

```
[Out] -1/3*a/x^3-1/3*b/x^3*arctanh(c/x)-1/6*b/c/x^2-1/6/c^3*b*ln(c/x-1)-1/6/c^3*b
*ln(1+c/x)
```

Maxima [A] time = 0.96182, size = 74, normalized size = 1.3

$$-\frac{1}{6}\left(c\left(\frac{\log\left(-c^2+x^2\right)}{c^4} - \frac{\log\left(x^2\right)}{c^4} + \frac{1}{c^2x^2}\right) + \frac{2\,\text{artanh}\left(\frac{c}{x}\right)}{x^3}\right)b - \frac{a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))/x^4,x, algorithm="maxima")
```

```
[Out] -1/6*(c*(log(-c^2 + x^2)/c^4 - log(x^2)/c^4 + 1/(c^2*x^2)) + 2*arctanh(c/x)
/x^3)*b - 1/3*a/x^3
```

Fricas [A] time = 1.67192, size = 144, normalized size = 2.53

$$-\frac{bx^3\log\left(-c^2+x^2\right)-2\,bx^3\log\left(x\right)+bc^3\log\left(-\frac{c+x}{c-x}\right)+2\,ac^3+bc^2x}{6\,c^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(b*x^3*log(-c^2 + x^2) - 2*b*x^3*log(x) + b*c^3*log(-(c + x)/(c - x))
+ 2*a*c^3 + b*c^2*x)/(c^3*x^3)
```

Sympy [A] time = 3.6407, size = 68, normalized size = 1.19

$$\begin{cases} -\frac{a}{3x^3}-\frac{b\operatorname{atanh}\left(\frac{c}{x}\right)}{3x^3}-\frac{b}{6cx^2}+\frac{b\log\left(x\right)}{3c^3}-\frac{b\log\left(-c+x\right)}{3c^3}-\frac{b\operatorname{atanh}\left(\frac{c}{x}\right)}{3c^3} & \text{for}\ c\neq 0 \\ -\frac{a}{3x^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x))/x**4,x)
```

```
[Out] Piecewise((-a/(3*x**3) - b*atanh(c/x)/(3*x**3) - b/(6*c*x**2) + b*log(x)/(3
*c**3) - b*log(-c + x)/(3*c**3) - b*atanh(c/x)/(3*c**3), Ne(c, 0)), (-a/(3*
x**3), True))
```

Giac [A] time = 1.12926, size = 85, normalized size = 1.49

$$-\frac{b\log\left(-c^2+x^2\right)}{6\,c^3}+\frac{b\log\left(x\right)}{3\,c^3}-\frac{b\log\left(-\frac{c+x}{c-x}\right)}{6\,x^3}-\frac{2\,ac^2+bcx}{6\,c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))/x^4,x, algorithm="giac")
```

```
[Out] -1/6*b*log(-c^2 + x^2)/c^3 + 1/3*b*log(x)/c^3 - 1/6*b*log(-(c + x)/(c - x))
/x^3 - 1/6*(2*a*c^2 + b*c*x)/(c^2*x^3)
```

3.143 $\int x^3 \left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^2 dx$

Optimal. Leaf size=123

$$-\frac{1}{4}c^4\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{1}{2}bc^3x\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)+\frac{1}{6}bcx^3\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)+\frac{1}{4}x^4\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{1}{12}b^2c^2x$$

```
[Out] (b^2*c^2*x^2)/12 + (b*c^3*x*(a + b*ArcCoth[x/c]))/2 + (b*c*x^3*(a + b*ArcCo
th[x/c]))/6 - (c^4*(a + b*ArcCoth[x/c])^2)/4 + (x^4*(a + b*ArcCoth[x/c])^2)
/4 + (b^2*c^4*Log[1 - c^2/x^2])/3 + (2*b^2*c^4*Log[x])/3
```

Rubi [C] time = 1.70313, antiderivative size = 812, normalized size of antiderivative = 6.6, number of steps used = 88, number of rules used = 34, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 2.125, Rules used = {6099, 2454, 2398, 2411, 2347, 2344, 2301, 2316, 2315, 2314, 31, 2319, 44, 2455, 263, 43, 6742, 30, 2557, 12, 2466, 2448, 2462, 260, 2416, 2394, 2393, 2391, 193, 2410, 2395, 36, 29, 2390}

$$-\frac{1}{16}\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2c^4-\frac{1}{16}b^2\log^2\left(\frac{c+x}{x}\right)c^4+\frac{5}{48}b^2\log\left(1-\frac{c}{x}\right)c^4+\frac{5}{48}b^2\log(c-x)c^4+\frac{1}{8}b^2\log\left(\frac{c}{x}+1\right)\log(c-$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^3*(a + b*ArcTanh[c/x])^2,x]
```

```
[Out] (a*b*c^3*x)/4 - (a*b*c^2*x^2)/8 + (b^2*c^2*x^2)/12 + (a*b*c*x^3)/12 + (5*b^
2*c^4*Log[1 - c/x])/48 - (b^2*c^3*x*Log[1 - c/x])/8 + (b^2*c^2*x^2*Log[1 -
c/x])/16 - (b^2*c*x^3*Log[1 - c/x])/24 + (b*c^3*(1 - c/x)*x*(2*a - b*Log[1
- c/x]))/8 + (b*c^2*x^2*(2*a - b*Log[1 - c/x]))/16 + (b*c*x^3*(2*a - b*Log[
1 - c/x]))/24 - (c^4*(2*a - b*Log[1 - c/x])^2)/16 + (x^4*(2*a - b*Log[1 - c
/x])^2)/16 + (b^2*c^3*x*Log[1 + c/x])/8 + (b^2*c^2*x^2*Log[1 + c/x])/16 + (
b^2*c*x^3*Log[1 + c/x])/24 + (a*b*x^4*Log[1 + c/x])/4 - (b^2*x^4*Log[1 - c/
x]*Log[1 + c/x])/8 + (5*b^2*c^4*Log[c - x])/48 + (b^2*c^4*Log[1 + c/x]*Log[
c - x])/8 + (a*b*c^4*Log[x])/4 + (11*b^2*c^4*Log[x])/24 + (b^2*c^4*Log[c -
x]*Log[x/c])/8 - (a*b*c^4*Log[c + x])/4 + (5*b^2*c^4*Log[c + x])/48 + (b^2*
c^4*Log[1 - c/x]*Log[c + x])/8 - (b^2*c^4*Log[(c - x)/(2*c)]*Log[c + x])/8
+ (b^2*c^4*Log[-(x/c)]*Log[c + x])/8 - (b^2*c^4*Log[c - x]*Log[(c + x)/(2*c
)])/8 + (11*b^2*c^4*Log[(c + x)/x])/48 + (b^2*c^3*x*Log[(c + x)/x])/8 - (b^
2*c^2*x^2*Log[(c + x)/x])/16 + (b^2*c*x^3*Log[(c + x)/x])/24 - (b^2*c^4*Log
[(c + x)/x]^2)/16 + (b^2*x^4*Log[(c + x)/x]^2)/16 - (b^2*c^4*PolyLog[2, (c
- x)/(2*c)])/8 - (b^2*c^4*PolyLog[2, -(c/x)])/8 - (b^2*c^4*PolyLog[2, c/x])
/8 - (b^2*c^4*PolyLog[2, (c + x)/(2*c)])/8 + (b^2*c^4*PolyLog[2, 1 - x/c])/
8 + (b^2*c^4*PolyLog[2, 1 + x/c])/8
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
```

```
!(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2316

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[
((a + b*Log[-((c*d)/e)])*Log[d + e*x])/e, x] + Dist[b, Int[Log[-((e*x)/d)]/
(d + e*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[-((c*d)/e), 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2319

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2557

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] :> With[{z = IntHide[u, x]}, Dist[Log[v
]*Log[w], z, x] + (-Int[SimplifyIntegrand[(z*Log[w]*D[v, x])/v, x], x] - In
t[SimplifyIntegrand[(z*Log[v]*D[w, x])/w, x], x]) /; InverseFunctionFreeQ[z
, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rubi steps

$$\int x^3 \left(a + b\tanh^{-1}\left(\frac{c}{x}\right)\right)^2 dx = \int \left(\frac{1}{4}x^3 \left(2a - b\log\left(1 - \frac{c}{x}\right)\right)^2 + \frac{1}{2}bx^3 \left(2a - b\log\left(1 - \frac{c}{x}\right)\right)\log\left(1 + \frac{c}{x}\right) + \frac{1}{4}b^2x^3\log^2\left(\right.\right.$$

$$= \frac{1}{4}\int x^3 \left(2a - b\log\left(1 - \frac{c}{x}\right)\right)^2 dx + \frac{1}{2}b\int x^3 \left(2a - b\log\left(1 - \frac{c}{x}\right)\right)\log\left(1 + \frac{c}{x}\right) dx + \frac{1}{4}b$$

$$= -\left(\frac{1}{4}\text{Subst}\left(\int \frac{(2a - b\log(1 - cx))^2}{x^5} dx, x, \frac{1}{x}\right)\right) + \frac{1}{2}b\int \left(2ax^3\log\left(1 + \frac{c}{x}\right) - bx^3\log\left(\right.\right.$$

$$= \frac{1}{16}x^4 \left(2a - b\log\left(1 - \frac{c}{x}\right)\right)^2 + \frac{1}{16}b^2x^4\log^2\left(\frac{c + x}{x}\right) + (ab)\int x^3\log\left(1 + \frac{c}{x}\right) dx - \frac{1}{2}b^2$$

$$= \frac{1}{16}x^4 \left(2a - b\log\left(1 - \frac{c}{x}\right)\right)^2 + \frac{1}{4}abx^4\log\left(1 + \frac{c}{x}\right) - \frac{1}{8}b^2x^4\log\left(1 - \frac{c}{x}\right)\log\left(1 + \frac{c}{x}\right) + \frac{1}{16}$$

$$= \frac{1}{16}x^4 \left(2a - b\log\left(1 - \frac{c}{x}\right)\right)^2 + \frac{1}{4}abx^4\log\left(1 + \frac{c}{x}\right) - \frac{1}{8}b^2x^4\log\left(1 - \frac{c}{x}\right)\log\left(1 + \frac{c}{x}\right) + \frac{1}{16}$$

$$= \frac{1}{24}bcx^3 \left(2a - b\log\left(1 - \frac{c}{x}\right)\right) + \frac{1}{16}x^4 \left(2a - b\log\left(1 - \frac{c}{x}\right)\right)^2 + \frac{1}{4}abx^4\log\left(1 + \frac{c}{x}\right) - \frac{1}{8}b^2$$

$$= \frac{1}{4}abc^3x - \frac{1}{8}abc^2x^2 + \frac{1}{12}abcx^3 + \frac{1}{16}bc^2x^2 \left(2a - b\log\left(1 - \frac{c}{x}\right)\right) + \frac{1}{24}bcx^3 \left(2a - b\log\left(1\right.\right.$$

$$= \frac{1}{4}abc^3x - \frac{1}{16}b^2c^3x - \frac{1}{8}abc^2x^2 + \frac{1}{24}b^2c^2x^2 + \frac{1}{12}abcx^3 + \frac{1}{24}b^2c^4\log\left(1 - \frac{c}{x}\right) - \frac{1}{8}b^2c^3x$$

$$= \frac{1}{4}abc^3x - \frac{1}{8}abc^2x^2 + \frac{1}{24}b^2c^2x^2 + \frac{1}{12}abcx^3 + \frac{5}{48}b^2c^4\log\left(1 - \frac{c}{x}\right) - \frac{1}{8}b^2c^3x\log\left(1 - \frac{c}{x}\right)$$

$$= \frac{1}{4}abc^3x - \frac{1}{8}abc^2x^2 + \frac{1}{24}b^2c^2x^2 + \frac{1}{12}abcx^3 + \frac{5}{48}b^2c^4\log\left(1 - \frac{c}{x}\right) - \frac{1}{8}b^2c^3x\log\left(1 - \frac{c}{x}\right)$$

$$= \frac{1}{4}abc^3x - \frac{1}{8}abc^2x^2 + \frac{1}{12}b^2c^2x^2 + \frac{1}{12}abcx^3 + \frac{5}{48}b^2c^4\log\left(1 - \frac{c}{x}\right) - \frac{1}{8}b^2c^3x\log\left(1 - \frac{c}{x}\right)$$

$$= \frac{1}{4}abc^3x - \frac{1}{8}abc^2x^2 + \frac{1}{12}b^2c^2x^2 + \frac{1}{12}abcx^3 + \frac{5}{48}b^2c^4\log\left(1 - \frac{c}{x}\right) - \frac{1}{8}b^2c^3x\log\left(1 - \frac{c}{x}\right)$$

$$= \frac{1}{4}abc^3x - \frac{1}{8}abc^2x^2 + \frac{1}{12}b^2c^2x^2 + \frac{1}{12}abcx^3 + \frac{5}{48}b^2c^4\log\left(1 - \frac{c}{x}\right) - \frac{1}{8}b^2c^3x\log\left(1 - \frac{c}{x}\right)$$

Mathematica [A] time = 0.0610989, size = 131, normalized size = 1.07

$$\frac{1}{12}\left(3a^2x^4 + 2bx\tanh^{-1}\left(\frac{c}{x}\right)\left(3ax^3 + bc\left(3c^2 + x^2\right)\right) + 6abc^3x + bc^4(3a + 4b)\log(x - c) - 3abc^4\log(c + x) + 2abcx^3 +\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcTanh[c/x])^2,x]
```

```
[Out] (6*a*b*c^3*x + b^2*c^2*x^2 + 2*a*b*c*x^3 + 3*a^2*x^4 + 2*b*x*(3*a*x^3 + b*c
*(3*c^2 + x^2))*ArcTanh[c/x] + 3*b^2*(-c^4 + x^4)*ArcTanh[c/x]^2 + b*(3*a +
 4*b)*c^4*Log[-c + x] - 3*a*b*c^4*Log[c + x] + 4*b^2*c^4*Log[c + x])/12
```

Maple [B] time = 0.007, size = 328, normalized size = 2.7

$$\frac{a^2x^4}{4}+\frac{x^4b^2}{4}\left(\mathrm{Artanh}\left(\frac{c}{x}\right)\right)^2+\frac{c^4b^2}{4}\mathrm{Artanh}\left(\frac{c}{x}\right)\ln\left(\frac{c}{x}-1\right)+\frac{b^2cx^3}{6}\mathrm{Artanh}\left(\frac{c}{x}\right)+\frac{c^3b^2x}{2}\mathrm{Artanh}\left(\frac{c}{x}\right)-\frac{c^4b^2}{4}\mathrm{Artanh}\left(\frac{c}{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c/x))^2,x)
```

```
[Out] 1/4*a^2*x^4+1/4*b^2*x^4*arctanh(c/x)^2+1/4*c^4*b^2*arctanh(c/x)*ln(c/x-1)+1
/6*c*b^2*x^3*arctanh(c/x)+1/2*c^3*b^2*x*arctanh(c/x)-1/4*c^4*b^2*arctanh(c/
x)*ln(1+c/x)+1/16*c^4*b^2*ln(c/x-1)^2-1/8*c^4*b^2*ln(c/x-1)*ln(1/2+1/2*c/x)
+1/8*c^4*b^2*ln(-1/2*c/x+1/2)*ln(1/2+1/2*c/x)-1/8*c^4*b^2*ln(-1/2*c/x+1/2)*
ln(1+c/x)+1/16*c^4*b^2*ln(1+c/x)^2+1/3*c^4*b^2*ln(c/x-1)+1/12*b^2*c^2*x^2-2
/3*c^4*b^2*ln(c/x)+1/3*c^4*b^2*ln(1+c/x)+1/2*a*b*x^4*arctanh(c/x)+1/4*c^4*a
*b*ln(c/x-1)+1/6*a*b*c*x^3+1/2*c^3*a*b*x-1/4*c^4*a*b*ln(1+c/x)
```

Maxima [A] time = 0.981755, size = 255, normalized size = 2.07

$$\frac{1}{4}b^2x^4\,\mathrm{artanh}\left(\frac{c}{x}\right)^2+\frac{1}{4}a^2x^4+\frac{1}{12}\left(6\,x^4\,\mathrm{artanh}\left(\frac{c}{x}\right)-\left(3\,c^3\log\left(c+x\right)-3\,c^3\log\left(-c+x\right)-6\,c^2x-2\,x^3\right)c\right)ab+\frac{1}{48}\left(\left(3\,c^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c/x))^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*x^4*arctanh(c/x)^2 + 1/4*a^2*x^4 + 1/12*(6*x^4*arctanh(c/x) - (3*c^
3*log(c + x) - 3*c^3*log(-c + x) - 6*c^2*x - 2*x^3)*c)*a*b + 1/48*((3*c^2*l
og(c + x)^2 + 3*c^2*log(-c + x)^2 + 16*c^2*log(c + x) + 4*x^2 - 2*(3*c^2*lo
g(c + x) - 8*c^2)*log(-c + x))*c^2 - 4*(3*c^3*log(c + x) - 3*c^3*log(-c + x
) - 6*c^2*x - 2*x^3)*c*arctanh(c/x))*b^2
```

Fricas [A] time = 1.67716, size = 350, normalized size = 2.85

$$\frac{1}{2}abc^3x+\frac{1}{12}b^2c^2x^2+\frac{1}{6}abcx^3+\frac{1}{4}a^2x^4-\frac{1}{12}\left(3\,ab-4\,b^2\right)c^4\log\left(c+x\right)+\frac{1}{12}\left(3\,ab+4\,b^2\right)c^4\log\left(-c+x\right)-\frac{1}{16}\left(b^2c^4-b^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c/x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*a*b*c^3*x + 1/12*b^2*c^2*x^2 + 1/6*a*b*c*x^3 + 1/4*a^2*x^4 - 1/12*(3*a*
b - 4*b^2)*c^4*log(c + x) + 1/12*(3*a*b + 4*b^2)*c^4*log(-c + x) - 1/16*(b^
2*c^4 - b^2*x^4)*log(-(c + x)/(c - x))^2 + 1/12*(3*b^2*c^3*x + b^2*c*x^3 +
3*a*b*x^4)*log(-(c + x)/(c - x))
```

Sympy [A] time = 3.44274, size = 158, normalized size = 1.28

$$\frac{a^2x^4}{4}-\frac{abc^4\,\mathrm{atanh}\left(\frac{c}{x}\right)}{2}+\frac{abc^3x}{2}+\frac{abcx^3}{6}+\frac{abx^4\,\mathrm{atanh}\left(\frac{c}{x}\right)}{2}+\frac{2b^2c^4\log\left(-c+x\right)}{3}-\frac{b^2c^4\,\mathrm{atanh}^2\left(\frac{c}{x}\right)}{4}+\frac{2b^2c^4\,\mathrm{atanh}\left(\frac{c}{x}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c/x))**2,x)
```

```
[Out] a**2*x**4/4 - a*b*c**4*atanh(c/x)/2 + a*b*c**3*x/2 + a*b*c*x**3/6 + a*b*x**
4*atanh(c/x)/2 + 2*b**2*c**4*log(-c + x)/3 - b**2*c**4*atanh(c/x)**2/4 + 2*
b**2*c**4*atanh(c/x)/3 + b**2*c**3*x*atanh(c/x)/2 + b**2*c**2*x**2/12 + b**
2*c*x**3*atanh(c/x)/6 + b**2*x**4*atanh(c/x)**2/4
```

Giac [A] time = 1.22244, size = 209, normalized size = 1.7

$$\frac{1}{2}abc^3x + \frac{1}{12}b^2c^2x^2 + \frac{1}{6}abcx^3 + \frac{1}{4}a^2x^4 - \frac{1}{16}\left(b^2c^4 - b^2x^4\right)\log\left(-\frac{c+x}{c-x}\right)^2 - \frac{1}{12}\left(3\,abc^4 - 4\,b^2c^4\right)\log\left(c+x\right) + \frac{1}{12}\left(3\,ab\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c/x))^2,x, algorithm="giac")
```

```
[Out] 1/2*a*b*c^3*x + 1/12*b^2*c^2*x^2 + 1/6*a*b*c*x^3 + 1/4*a^2*x^4 - 1/16*(b^2*
c^4 - b^2*x^4)*log(-(c + x)/(c - x))^2 - 1/12*(3*a*b*c^4 - 4*b^2*c^4)*log(c
 + x) + 1/12*(3*a*b*c^4 + 4*b^2*c^4)*log(c - x) + 1/12*(3*b^2*c^3*x + b^2*c
*x^3 + 3*a*b*x^4)*log(-(c + x)/(c - x))
```

3.144 $\int x^2 \left(a + b \tanh^{-1}\left(\frac{c}{x}\right)\right)^2 dx$

Optimal. Leaf size=142

$$\frac{1}{3}b^2c^3\text{PolyLog}\left(2, \frac{2}{\frac{c}{x}+1} - 1\right) - \frac{1}{3}c^3\left(a + b\coth^{-1}\left(\frac{x}{c}\right)\right)^2 - \frac{2}{3}bc^3\log\left(2 - \frac{2}{\frac{c}{x}+1}\right)\left(a + b\coth^{-1}\left(\frac{x}{c}\right)\right) + \frac{1}{3}bcx^2\left(a + b\coth\right.$$

```
[Out] (b^2*c^2*x)/3 - (b^2*c^3*ArcCoth[x/c])/3 + (b*c*x^2*(a + b*ArcCoth[x/c]))/3
 - (c^3*(a + b*ArcCoth[x/c])^2)/3 + (x^3*(a + b*ArcCoth[x/c])^2)/3 - (2*b*c
^3*(a + b*ArcCoth[x/c])*Log[2 - 2/(1 + c/x)])/3 + (b^2*c^3*PolyLog[2, -1 +
2/(1 + c/x)])/3
```

Rubi [B] time = 1.38924, antiderivative size = 695, normalized size of antiderivative = 4.89, number of steps used = 73, number of rules used = 34, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 2.125, Rules used = {6099, 2454, 2398, 2411, 2347, 2344, 2301, 2316, 2315, 2314, 31, 2319, 44, 2455, 263, 43, 6742, 30, 2557, 12, 2466, 2448, 2462, 260, 2416, 2394, 2393, 2391, 193, 2410, 2395, 36, 29, 2390}

$$-\frac{1}{6}b^2c^3\text{PolyLog}\left(2, \frac{c-x}{2c}\right) + \frac{1}{6}b^2c^3\text{PolyLog}\left(2, -\frac{c}{x}\right) - \frac{1}{6}b^2c^3\text{PolyLog}\left(2, \frac{c}{x}\right) + \frac{1}{6}b^2c^3\text{PolyLog}\left(2, \frac{c+x}{2c}\right) + \frac{1}{6}b^2c^3\text{PolyL}$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^2*(a + b*ArcTanh[c/x])^2,x]
```

```
[Out] -(a*b*c^2*x)/3 + (b^2*c^2*x)/3 + (a*b*c*x^2)/6 + (b^2*c^3*Log[1 - c/x])/12
+ (b^2*c^2*x*Log[1 - c/x])/6 - (b^2*c*x^2*Log[1 - c/x])/12 + (b*c^2*(1 - c/
x)*x*(2*a - b*Log[1 - c/x]))/6 + (b*c*x^2*(2*a - b*Log[1 - c/x]))/12 - (c^3
*(2*a - b*Log[1 - c/x])^2)/12 + (x^3*(2*a - b*Log[1 - c/x])^2)/12 + (b^2*c^
2*x*Log[1 + c/x])/6 + (b^2*c*x^2*Log[1 + c/x])/12 + (a*b*x^3*Log[1 + c/x])/
3 - (b^2*x^3*Log[1 - c/x]*Log[1 + c/x])/6 - (b^2*c^3*Log[c - x])/12 + (b^2*
c^3*Log[1 + c/x]*Log[c - x])/6 + (a*b*c^3*Log[x])/3 + (b^2*c^3*Log[c - x]*L
og[x/c])/6 + (a*b*c^3*Log[c + x])/3 + (b^2*c^3*Log[c + x])/12 - (b^2*c^3*Lo
g[1 - c/x]*Log[c + x])/6 + (b^2*c^3*Log[(c - x)/(2*c)]*Log[c + x])/6 - (b^2
*c^3*Log[-(x/c)]*Log[c + x])/6 - (b^2*c^3*Log[c - x]*Log[(c + x)/(2*c)])/6
 - (b^2*c^3*Log[(c + x)/x])/4 - (b^2*c^2*x*Log[(c + x)/x])/6 + (b^2*c*x^2*Lo
g[(c + x)/x])/12 + (b^2*c^3*Log[(c + x)/x]^2)/12 + (b^2*x^3*Log[(c + x)/x]^
2)/12 - (b^2*c^3*PolyLog[2, (c - x)/(2*c)])/6 + (b^2*c^3*PolyLog[2, -(c/x)]
)/6 - (b^2*c^3*PolyLog[2, c/x])/6 + (b^2*c^3*PolyLog[2, (c + x)/(2*c)])/6 +
 (b^2*c^3*PolyLog[2, 1 - x/c])/6 - (b^2*c^3*PolyLog[2, 1 + x/c])/6
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2316

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[
((a + b*Log[-((c*d)/e)])*Log[d + e*x])/e, x] + Dist[b, Int[Log[-((e*x)/d)]/
(d + e*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[-((c*d)/e), 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2319

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2557

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] :> With[{z = IntHide[u, x]}, Dist[Log[v
]*Log[w], z, x] + (-Int[SimplifyIntegrand[(z*Log[w]*D[v, x])/v, x], x] - In
t[SimplifyIntegrand[(z*Log[v]*D[w, x])/w, x], x]) /; InverseFunctionFreeQ[z
, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
```

```
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rubi steps

$$\int x^2\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^2dx = \int\left(\frac{1}{4}x^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2+\frac{1}{2}bx^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)\log\left(1+\frac{c}{x}\right)+\frac{1}{4}b^2x^2\log^2\left(\right.\right.$$

$$=\frac{1}{4}\int x^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2dx+\frac{1}{2}b\int x^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)\log\left(1+\frac{c}{x}\right)dx+\frac{1}{4}b$$

$$=-\left(\frac{1}{4}\text{Subst}\left(\int\frac{(2a-b\log(1-cx))^2}{x^4}dx,x,\frac{1}{x}\right)\right)+\frac{1}{2}b\int\left(2ax^2\log\left(1+\frac{c}{x}\right)-bx^2\log\left(\right.\right.$$

$$=\frac{1}{12}x^3\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2+\frac{1}{12}b^2x^3\log^2\left(\frac{c+x}{x}\right)+(ab)\int x^2\log\left(1+\frac{c}{x}\right)dx-\frac{1}{2}b^2$$

$$=\frac{1}{12}x^3\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2+\frac{1}{3}abx^3\log\left(1+\frac{c}{x}\right)-\frac{1}{6}b^2x^3\log\left(1-\frac{c}{x}\right)\log\left(1+\frac{c}{x}\right)+\frac{1}{12}$$

$$=\frac{1}{12}x^3\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2+\frac{1}{3}abx^3\log\left(1+\frac{c}{x}\right)-\frac{1}{6}b^2x^3\log\left(1-\frac{c}{x}\right)\log\left(1+\frac{c}{x}\right)+\frac{1}{12}$$

$$=\frac{1}{12}bcx^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)+\frac{1}{12}x^3\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2+\frac{1}{3}abx^3\log\left(1+\frac{c}{x}\right)-\frac{1}{6}b^2$$

$$=-\frac{1}{3}abc^2x+\frac{1}{6}abcx^2+\frac{1}{6}bc^2\left(1-\frac{c}{x}\right)x\left(2a-b\log\left(1-\frac{c}{x}\right)\right)+\frac{1}{12}bcx^2\left(2a-b\log\left(1-\frac{c}{x}\right.\right.$$

$$=-\frac{1}{3}abc^2x+\frac{1}{6}b^2c^2x+\frac{1}{6}abcx^2+\frac{1}{12}b^2c^3\log\left(1-\frac{c}{x}\right)+\frac{1}{6}b^2c^2x\log\left(1-\frac{c}{x}\right)-\frac{1}{12}b^2cx^2\log$$

$$=-\frac{1}{3}abc^2x+\frac{1}{6}b^2c^2x+\frac{1}{6}abcx^2+\frac{1}{12}b^2c^3\log\left(1-\frac{c}{x}\right)+\frac{1}{6}b^2c^2x\log\left(1-\frac{c}{x}\right)-\frac{1}{12}b^2cx^2\log$$

$$=-\frac{1}{3}abc^2x+\frac{1}{6}b^2c^2x+\frac{1}{6}abcx^2+\frac{1}{12}b^2c^3\log\left(1-\frac{c}{x}\right)+\frac{1}{6}b^2c^2x\log\left(1-\frac{c}{x}\right)-\frac{1}{12}b^2cx^2\log$$

$$=-\frac{1}{3}abc^2x+\frac{1}{3}b^2c^2x+\frac{1}{6}abcx^2+\frac{1}{12}b^2c^3\log\left(1-\frac{c}{x}\right)+\frac{1}{6}b^2c^2x\log\left(1-\frac{c}{x}\right)-\frac{1}{12}b^2cx^2\log$$

$$=-\frac{1}{3}abc^2x+\frac{1}{3}b^2c^2x+\frac{1}{6}abcx^2+\frac{1}{12}b^2c^3\log\left(1-\frac{c}{x}\right)+\frac{1}{6}b^2c^2x\log\left(1-\frac{c}{x}\right)-\frac{1}{12}b^2cx^2\log$$

$$=-\frac{1}{3}abc^2x+\frac{1}{3}b^2c^2x+\frac{1}{6}abcx^2+\frac{1}{12}b^2c^3\log\left(1-\frac{c}{x}\right)+\frac{1}{6}b^2c^2x\log\left(1-\frac{c}{x}\right)-\frac{1}{12}b^2cx^2\log$$

Mathematica [A] time = 0.303487, size = 145, normalized size = 1.02

$$\frac{1}{3}\left(b^2c^3\text{PolyLog}\left(2,e^{-2\tanh^{-1}\left(\frac{c}{x}\right)}\right)+a^2x^3+abc^3\log\left(1-\frac{c^2}{x^2}\right)+b\tanh^{-1}\left(\frac{c}{x}\right)\left(2ax^3-2bc^3\log\left(1-e^{-2\tanh^{-1}\left(\frac{c}{x}\right)}\right)-bc^3\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(a + b*ArcTanh[c/x])^2,x]
```

```
[Out] (b^2*c^2*x + a*b*c*x^2 + a^2*x^3 + b^2*(-c^3 + x^3)*ArcTanh[c/x]^2 + b*ArcT
anh[c/x]*(-(b*c^3) + b*c*x^2 + 2*a*x^3 - 2*b*c^3*Log[1 - E^(-2*ArcTanh[c/x]
)]) + a*b*c^3*Log[1 - c^2/x^2] - 2*a*b*c^3*Log[c/x] + b^2*c^3*PolyLog[2, E^
(-2*ArcTanh[c/x])])/3
```

Maple [B] time = 0.032, size = 391, normalized size = 2.8

$$\frac{x^3a^2}{3}+\frac{x^3b^2}{3}\left(\mathrm{Artanh}\left(\frac{c}{x}\right)\right)^2+\frac{c^3b^2}{3}\mathrm{Artanh}\left(\frac{c}{x}\right)\ln\left(\frac{c}{x}-1\right)+\frac{b^2cx^2}{3}\mathrm{Artanh}\left(\frac{c}{x}\right)-\frac{2\,c^3b^2}{3}\ln\left(\frac{c}{x}\right)\mathrm{Artanh}\left(\frac{c}{x}\right)+\frac{c^3b^2}{3}\mathrm{Art}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c/x))^2,x)
```

```
[Out] 1/3*x^3*a^2+1/3*b^2*x^3*arctanh(c/x)^2+1/3*c^3*b^2*arctanh(c/x)*ln(c/x-1)+1
/3*c*b^2*arctanh(c/x)*x^2-2/3*c^3*b^2*ln(c/x)*arctanh(c/x)+1/3*c^3*b^2*arct
anh(c/x)*ln(1+c/x)+1/3*b^2*c^2*x+1/6*c^3*b^2*ln(c/x-1)-1/6*c^3*b^2*ln(1+c/x
)-1/3*c^3*b^2*dilog(1/2+1/2*c/x)-1/6*c^3*b^2*ln(c/x-1)*ln(1/2+1/2*c/x)+1/12
*c^3*b^2*ln(c/x-1)^2-1/6*c^3*b^2*ln(-1/2*c/x+1/2)*ln(1/2+1/2*c/x)+1/6*c^3*b
^2*ln(-1/2*c/x+1/2)*ln(1+c/x)-1/12*c^3*b^2*ln(1+c/x)^2+1/3*c^3*b^2*dilog(c/
x)+1/3*c^3*b^2*dilog(1+c/x)+1/3*c^3*b^2*ln(c/x)*ln(1+c/x)+2/3*a*b*x^3*arcta
nh(c/x)+1/3*c^3*a*b*ln(c/x-1)+1/3*a*b*c*x^2-2/3*c^3*a*b*ln(c/x)+1/3*c^3*a*b
*ln(1+c/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{3}a^2x^3+\frac{1}{3}\left(2\,x^3\,\mathrm{artanh}\left(\frac{c}{x}\right)+\left(c^2\log\left(-c^2+x^2\right)+x^2\right)c\right)ab+\frac{1}{12}\left(6\,c^4\int-\frac{\log\left(c+x\right)}{3\left(c^2-x^2\right)}\,dx+x^3\log\left(c+x\right)^2+6\,c^3\int-\frac{x\,\mathrm{lo}}{3\,(}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c/x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*x^3 + 1/3*(2*x^3*arctanh(c/x) + (c^2*log(-c^2 + x^2) + x^2)*c)*a*b
+ 1/12*(6*c^4*integrate(-1/3*log(c + x)/(c^2 - x^2), x) + x^3*log(c + x)^2
+ 6*c^3*integrate(-1/3*x*log(c + x)/(c^2 - x^2), x) - (c*log(c + x) - c*log
(-c + x) - 2*x)*c^2 - (c^3 - x^3)*log(-c + x)^2 + (c^2*log(-c^2 + x^2) + x^
2)*c + 12*c*integrate(-1/3*x^3*log(c + x)/(c^2 - x^2), x) - 2*(c*x^2 + (c^3
 + x^3)*log(c + x))*log(-c + x))*b^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(b^2x^2\,\mathrm{artanh}\left(\frac{c}{x}\right)^2+2\,abx^2\,\mathrm{artanh}\left(\frac{c}{x}\right)+a^2x^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c/x))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^2*arctanh(c/x)^2 + 2*a*b*x^2*arctanh(c/x) + a^2*x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\left(a+b\,\mathrm{atanh}\left(\frac{c}{x}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c/x))**2,x)
```

```
[Out] Integral(x**2*(a + b*atanh(c/x))**2, x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \operatorname{artanh}\left(\frac{c}{x}\right) + a\right)^2 x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c/x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c/x) + a)^2*x^2, x)
```

3.145 $\int x\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^2\,dx$

Optimal. Leaf size=83

$$-\frac{1}{2}c^2\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{1}{2}x^2\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2+bcx\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)+\frac{1}{2}b^2c^2\log\left(1-\frac{c^2}{x^2}\right)+b^2c^2\log(x)$$

```
[Out] b*c*x*(a + b*ArcCoth[x/c]) - (c^2*(a + b*ArcCoth[x/c])^2)/2 + (x^2*(a + b*A
rcCoth[x/c])^2)/2 + (b^2*c^2*Log[1 - c^2/x^2])/2 + b^2*c^2*Log[x]
```

Rubi [C] time = 1.04099, antiderivative size = 574, normalized size of antiderivative = 6.92, number of steps used = 58, number of rules used = 32, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 2.286, Rules used = {6099, 2454, 2398, 2411, 2347, 2344, 2301, 2316, 2315, 2314, 31, 2455, 193, 43, 6742, 30, 2557, 12, 2466, 2448, 263, 2462, 260, 2416, 2394, 2393, 2391, 2410, 2395, 36, 29, 2390}

$$-\frac{1}{4}b^2c^2\text{PolyLog}\left(2,\frac{c-x}{2c}\right)-\frac{1}{4}b^2c^2\text{PolyLog}\left(2,-\frac{c}{x}\right)-\frac{1}{4}b^2c^2\text{PolyLog}\left(2,\frac{c}{x}\right)-\frac{1}{4}b^2c^2\text{PolyLog}\left(2,\frac{c+x}{2c}\right)+\frac{1}{4}b^2c^2\text{PolyL}$$

Warning: Unable to verify antiderivative.

```
[In] Int[x*(a + b*ArcTanh[c/x])^2,x]
```

```
[Out] (a*b*c*x)/2 - (b^2*c*x*Log[1 - c/x])/4 + (b*c*(1 - c/x)*x*(2*a - b*Log[1 -
c/x]))/4 - (c^2*(2*a - b*Log[1 - c/x])^2)/8 + (x^2*(2*a - b*Log[1 - c/x])^2
)/8 + (b^2*c*x*Log[1 + c/x])/4 + (a*b*x^2*Log[1 + c/x])/2 - (b^2*x^2*Log[1
- c/x]*Log[1 + c/x])/4 + (b^2*c^2*Log[c - x])/4 + (b^2*c^2*Log[1 + c/x]*Log
[c - x])/4 + (a*b*c^2*Log[x])/2 + (b^2*c^2*Log[x])/2 + (b^2*c^2*Log[c - x]*
Log[x/c])/4 - (a*b*c^2*Log[c + x])/2 + (b^2*c^2*Log[c + x])/4 + (b^2*c^2*Lo
g[1 - c/x]*Log[c + x])/4 - (b^2*c^2*Log[(c - x)/(2*c)]*Log[c + x])/4 + (b^2
*c^2*Log[-(x/c)]*Log[c + x])/4 - (b^2*c^2*Log[c - x]*Log[(c + x)/(2*c)])/4
+ (b^2*c^2*Log[(c + x)/x])/4 + (b^2*c*x*Log[(c + x)/x])/4 - (b^2*c^2*Log[(c
 + x)/x]^2)/8 + (b^2*x^2*Log[(c + x)/x]^2)/8 - (b^2*c^2*PolyLog[2, (c - x)/
(2*c)])/4 - (b^2*c^2*PolyLog[2, -(c/x)])/4 - (b^2*c^2*PolyLog[2, c/x])/4 -
(b^2*c^2*PolyLog[2, (c + x)/(2*c)])/4 + (b^2*c^2*PolyLog[2, 1 - x/c])/4 + (
b^2*c^2*PolyLog[2, 1 + x/c])/4
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
```

```
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2316

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[
((a + b*Log[-((c*d)/e)])*Log[d + e*x])/e, x] + Dist[b, Int[Log[-((e*x)/d)]/
(d + e*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[-((c*d)/e), 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
```

```
+ 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2557

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] :> With[{z = IntHide[u, x]}, Dist[Log[v
]*Log[w], z, x] + (-Int[SimplifyIntegrand[(z*Log[w]*D[v, x])/v, x], x] - In
t[SimplifyIntegrand[(z*Log[v]*D[w, x])/w, x], x]) /; InverseFunctionFreeQ[z
, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
```

```
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rubi steps

$$\int x\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^2\,dx=\int\left(\frac{1}{4}x\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2+\frac{1}{2}bx\left(2a-b\log\left(1-\frac{c}{x}\right)\right)\log\left(1+\frac{c}{x}\right)+\frac{1}{4}b^2x\log^2\left(1+\frac{c}{x}\right)\right.$$

$$=\frac{1}{4}\int x\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2\,dx+\frac{1}{2}b\int x\left(2a-b\log\left(1-\frac{c}{x}\right)\right)\log\left(1+\frac{c}{x}\right)\,dx+\frac{1}{4}b^2\int x\,l$$

$$=-\left(\frac{1}{4}\operatorname{Subst}\left(\int\frac{(2a-b\log(1-cx))^2}{x^3}\,dx,x,\frac{1}{x}\right)\right)+\frac{1}{2}b\int\left(2ax\log\left(1+\frac{c}{x}\right)-bx\log\left(1-\frac{c}{x}\right)\right.$$

$$=\frac{1}{8}x^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2+\frac{1}{8}b^2x^2\log^2\left(\frac{c+x}{x}\right)+(ab)\int x\log\left(1+\frac{c}{x}\right)\,dx-\frac{1}{2}b^2\int x\log$$

$$=\frac{1}{8}x^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2+\frac{1}{2}abx^2\log\left(1+\frac{c}{x}\right)-\frac{1}{4}b^2x^2\log\left(1-\frac{c}{x}\right)\log\left(1+\frac{c}{x}\right)+\frac{1}{8}b^2x^2\,l$$

$$=\frac{1}{8}x^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2+\frac{1}{2}abx^2\log\left(1+\frac{c}{x}\right)-\frac{1}{4}b^2x^2\log\left(1-\frac{c}{x}\right)\log\left(1+\frac{c}{x}\right)+\frac{1}{8}b^2x^2\,l$$

$$=\frac{1}{4}bc\left(1-\frac{c}{x}\right)x\left(2a-b\log\left(1-\frac{c}{x}\right)\right)+\frac{1}{8}x^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2+\frac{1}{2}abx^2\log\left(1+\frac{c}{x}\right)-\frac{1}{4}b$$

$$=\frac{1}{2}abcx+\frac{1}{4}bc\left(1-\frac{c}{x}\right)x\left(2a-b\log\left(1-\frac{c}{x}\right)\right)-\frac{1}{8}c^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2+\frac{1}{8}x^2\left(2a-b\log\right.$$

$$=\frac{1}{2}abcx-\frac{1}{4}b^2cx\log\left(1-\frac{c}{x}\right)+\frac{1}{4}bc\left(1-\frac{c}{x}\right)x\left(2a-b\log\left(1-\frac{c}{x}\right)\right)-\frac{1}{8}c^2\left(2a-b\log\left(1-\frac{c}{x}\right.\right.$$

$$=\frac{1}{2}abcx-\frac{1}{4}b^2cx\log\left(1-\frac{c}{x}\right)+\frac{1}{4}bc\left(1-\frac{c}{x}\right)x\left(2a-b\log\left(1-\frac{c}{x}\right)\right)-\frac{1}{8}c^2\left(2a-b\log\left(1-\frac{c}{x}\right.\right.$$

$$=\frac{1}{2}abcx-\frac{1}{4}b^2cx\log\left(1-\frac{c}{x}\right)+\frac{1}{4}bc\left(1-\frac{c}{x}\right)x\left(2a-b\log\left(1-\frac{c}{x}\right)\right)-\frac{1}{8}c^2\left(2a-b\log\left(1-\frac{c}{x}\right.\right.$$

$$=\frac{1}{2}abcx-\frac{1}{4}b^2cx\log\left(1-\frac{c}{x}\right)+\frac{1}{4}bc\left(1-\frac{c}{x}\right)x\left(2a-b\log\left(1-\frac{c}{x}\right)\right)-\frac{1}{8}c^2\left(2a-b\log\left(1-\frac{c}{x}\right.\right.$$

$$=\frac{1}{2}abcx-\frac{1}{4}b^2cx\log\left(1-\frac{c}{x}\right)+\frac{1}{4}bc\left(1-\frac{c}{x}\right)x\left(2a-b\log\left(1-\frac{c}{x}\right)\right)-\frac{1}{8}c^2\left(2a-b\log\left(1-\frac{c}{x}\right.\right.$$

$$=\frac{1}{2}abcx-\frac{1}{4}b^2cx\log\left(1-\frac{c}{x}\right)+\frac{1}{4}bc\left(1-\frac{c}{x}\right)x\left(2a-b\log\left(1-\frac{c}{x}\right)\right)-\frac{1}{8}c^2\left(2a-b\log\left(1-\frac{c}{x}\right.\right.$$

$$=\frac{1}{2}abcx-\frac{1}{4}b^2cx\log\left(1-\frac{c}{x}\right)+\frac{1}{4}bc\left(1-\frac{c}{x}\right)x\left(2a-b\log\left(1-\frac{c}{x}\right)\right)-\frac{1}{8}c^2\left(2a-b\log\left(1-\frac{c}{x}\right.\right.$$

Mathematica [A] time = 0.0472649, size = 92, normalized size = 1.11

$$\frac{1}{2}\left(a^2x^2+bc^2(a+b)\log(x-c)-abc^2\log(c+x)+2abcx+2bx\tanh^{-1}\left(\frac{c}{x}\right)(ax+bc)+b^2\left(x^2-c^2\right)\tanh^{-1}\left(\frac{c}{x}\right)^2+b^2c^2\,\mathrm{lo}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcTanh[c/x])^2,x]
```

```
[Out] (2*a*b*c*x + a^2*x^2 + 2*b*x*(b*c + a*x)*ArcTanh[c/x] + b^2*(-c^2 + x^2)*Ar
cTanh[c/x]^2 + b*(a + b)*c^2*Log[-c + x] - a*b*c^2*Log[c + x] + b^2*c^2*Log
```

```
[c + x])/2
```

Maple [B] time = 0.025, size = 287, normalized size = 3.5

$$\frac{a^2x^2}{2} + \frac{b^2x^2}{2}\left(\mathrm{Artanh}\left(\frac{c}{x}\right)\right)^2 + cb^2x\mathrm{Artanh}\left(\frac{c}{x}\right) + \frac{b^2c^2}{2}\mathrm{Artanh}\left(\frac{c}{x}\right)\ln\left(\frac{c}{x}-1\right) - \frac{b^2c^2}{2}\mathrm{Artanh}\left(\frac{c}{x}\right)\ln\left(1+\frac{c}{x}\right) - \frac{b^2c^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c/x))^2,x)
```

```
[Out] 1/2*a^2*x^2+1/2*b^2*x^2*arctanh(c/x)^2+c*b^2*x*arctanh(c/x)+1/2*c^2*b^2*arc
tanh(c/x)*ln(c/x-1)-1/2*c^2*b^2*arctanh(c/x)*ln(1+c/x)-1/4*c^2*b^2*ln(c/x-1
)*ln(1/2+1/2*c/x)+1/8*c^2*b^2*ln(c/x-1)^2+1/2*c^2*b^2*ln(c/x-1)-c^2*b^2*ln(
c/x)+1/2*c^2*b^2*ln(1+c/x)-1/4*c^2*b^2*ln(-1/2*c/x+1/2)*ln(1+c/x)+1/4*c^2*b
^2*ln(-1/2*c/x+1/2)*ln(1/2+1/2*c/x)+1/8*c^2*b^2*ln(1+c/x)^2+a*b*x^2*arctanh
(c/x)+a*b*c*x+1/2*c^2*a*b*ln(c/x-1)-1/2*c^2*a*b*ln(1+c/x)
```

Maxima [A] time = 0.975595, size = 184, normalized size = 2.22

$$\frac{1}{2}b^2x^2\,\mathrm{artanh}\left(\frac{c}{x}\right)^2 + \frac{1}{2}a^2x^2 + \frac{1}{2}\left(2x^2\,\mathrm{artanh}\left(\frac{c}{x}\right) - (c\log(c+x) - c\log(-c+x) - 2x)c\right)ab + \frac{1}{8}\left(\left(\log(c+x)^2 - 2(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c/x))^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^2*arctanh(c/x)^2 + 1/2*a^2*x^2 + 1/2*(2*x^2*arctanh(c/x) - (c*log
(c + x) - c*log(-c + x) - 2*x)*c)*a*b + 1/8*((log(c + x)^2 - 2*(log(c + x)
- 2)*log(-c + x) + log(-c + x)^2 + 4*log(c + x))*c^2 - 4*(c*log(c + x) - c*
log(-c + x) - 2*x)*c*arctanh(c/x))*b^2
```

Fricas [A] time = 1.76187, size = 254, normalized size = 3.06

$$abcx + \frac{1}{2}a^2x^2 - \frac{1}{2}\left(ab - b^2\right)c^2\log(c+x) + \frac{1}{2}\left(ab + b^2\right)c^2\log(-c+x) - \frac{1}{8}\left(b^2c^2 - b^2x^2\right)\log\left(-\frac{c+x}{c-x}\right)^2 + \frac{1}{2}\left(b^2cx + ab\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c/x))^2,x, algorithm="fricas")
```

```
[Out] a*b*c*x + 1/2*a^2*x^2 - 1/2*(a*b - b^2)*c^2*log(c + x) + 1/2*(a*b + b^2)*c^
2*log(-c + x) - 1/8*(b^2*c^2 - b^2*x^2)*log(-(c + x)/(c - x))^2 + 1/2*(b^2*
c*x + a*b*x^2)*log(-(c + x)/(c - x))
```

Sympy [A] time = 1.05, size = 104, normalized size = 1.25

$$\frac{a^2x^2}{2} - abc^2\,\mathrm{atanh}\left(\frac{c}{x}\right) + abcx + abx^2\,\mathrm{atanh}\left(\frac{c}{x}\right) + b^2c^2\log(-c+x) - \frac{b^2c^2\,\mathrm{atanh}^2\left(\frac{c}{x}\right)}{2} + b^2c^2\,\mathrm{atanh}\left(\frac{c}{x}\right) + b^2cx\,\mathrm{ata}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c/x))**2,x)
```

```
[Out] a**2*x**2/2 - a*b*c**2*atanh(c/x) + a*b*c*x + a*b*x**2*atanh(c/x) + b**2*c*
*2*log(-c + x) - b**2*c**2*atanh(c/x)**2/2 + b**2*c**2*atanh(c/x) + b**2*c*
x*atanh(c/x) + b**2*x**2*atanh(c/x)**2/2
```

Giac [A] time = 1.21089, size = 159, normalized size = 1.92

$$abcx + \frac{1}{2}a^2x^2 - \frac{1}{8}\left(b^2c^2 - b^2x^2\right)\log\left(-\frac{c+x}{c-x}\right)^2 - \frac{1}{2}\left(abc^2 - b^2c^2\right)\log(c+x) + \frac{1}{2}\left(abc^2 + b^2c^2\right)\log(c-x) + \frac{1}{2}\left(b^2cx + abx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c/x))^2,x, algorithm="giac")
```

```
[Out] a*b*c*x + 1/2*a^2*x^2 - 1/8*(b^2*c^2 - b^2*x^2)*log(-(c + x)/(c - x))^2 - 1
/2*(a*b*c^2 - b^2*c^2)*log(c + x) + 1/2*(a*b*c^2 + b^2*c^2)*log(c - x) + 1/
2*(b^2*c*x + a*b*x^2)*log(-(c + x)/(c - x))
```

3.146 $\int \left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^2 dx$

Optimal. Leaf size=74

$$b^2(-c)\text{PolyLog}\left(2,-\frac{c+x}{c-x}\right)+c\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2+x\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2-2bc\log\left(\frac{2c}{c-x}\right)\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)$$

```
[Out] c*(a + b*ArcCoth[x/c])^2 + x*(a + b*ArcCoth[x/c])^2 - 2*b*c*(a + b*ArcCoth[
x/c])*Log[(2*c)/(c - x)] - b^2*c*PolyLog[2, -((c + x)/(c - x))]
```

Rubi [B] time = 0.402079, antiderivative size = 370, normalized size of antiderivative = 5., number of steps used = 31, number of rules used = 14, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.167, Rules used = {6093, 2448, 263, 31, 2449, 2391, 2556, 12, 2462, 260, 2416, 2394, 2315, 2393}

$$-\frac{1}{2}b^2c\text{PolyLog}\left(2,\frac{c-x}{2c}\right)+\frac{1}{2}b^2c\text{PolyLog}\left(2,-\frac{c}{x}\right)-\frac{1}{2}b^2c\text{PolyLog}\left(2,\frac{c}{x}\right)+\frac{1}{2}b^2c\text{PolyLog}\left(2,\frac{c+x}{2c}\right)+\frac{1}{2}b^2c\text{PolyLo}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcTanh[c/x])^2,x]
```

```
[Out] a^2*x - a*b*x*Log[1 - c/x] - (b^2*(c - x)*Log[1 - c/x]^2)/4 + a*b*x*Log[1 +
 c/x] - (b^2*x*Log[1 - c/x]*Log[1 + c/x])/2 + (b^2*(c + x)*Log[1 + c/x]^2)/
4 - (b^2*c*Log[1 - c/x]*Log[-c - x])/2 + a*b*c*Log[c - x] + (b^2*c*Log[-c -
 x]*Log[(c - x)/(2*c)])/2 - (b^2*c*Log[-c - x]*Log[-(x/c)])/2 + (b^2*c*Log[
1 + c/x]*Log[-c + x])/2 + (b^2*c*Log[x/c]*Log[-c + x])/2 + a*b*c*Log[c + x]
 - (b^2*c*Log[-c + x]*Log[(c + x)/(2*c)])/2 - (b^2*c*PolyLog[2, (c - x)/(2*
c)])/2 + (b^2*c*PolyLog[2, -(c/x)])/2 - (b^2*c*PolyLog[2, c/x])/2 + (b^2*c*
PolyLog[2, (c + x)/(2*c)])/2 + (b^2*c*PolyLog[2, 1 - x/c])/2 - (b^2*c*PolyL
og[2, 1 + x/c])/2
```

Rule 6093

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.), x_Symbol] :> Int[Expand
Integrand[(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 - c*x^n])/2)^p, x], x] /; Fr
eeQ[{a, b, c, n}, x] && IGtQ[p, 0] && IntegerQ[n]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2449

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)/(x_))^(p_.)]*(b_.))^(q_), x_Symbol] :>
 Simp[((e + d*x)*(a + b*Log[c*(d + e/x)^p])^q)/d, x] + Dist[(b*e*p*q)/d, In
t[(a + b*Log[c*(d + e/x)^p])^(q - 1)/x, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && IGtQ[q, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2556

```
Int[Log[v_]*Log[w_], x_Symbol] :> Simp[x*Log[v]*Log[w], x] + (-Int[Simplify
Integrand[(x*Log[w]*D[v, x])/v, x], x] - Int[SimplifyIntegrand[(x*Log[v]*D[
w, x])/w, x], x]) /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w,
x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
```

```
(e*f - d*g), 0]
```

Rubi steps

$$\int \left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^2 dx = \int \left(a^2 - ab\log\left(1-\frac{c}{x}\right) + \frac{1}{4}b^2\log^2\left(1-\frac{c}{x}\right) + ab\log\left(1+\frac{c}{x}\right) - \frac{1}{2}b^2\log\left(1-\frac{c}{x}\right)\log\left(1+\ldots\right.\right.$$

$$= a^2x - (ab)\int \log\left(1-\frac{c}{x}\right) dx + (ab)\int \log\left(1+\frac{c}{x}\right) dx + \frac{1}{4}b^2\int \log^2\left(1-\frac{c}{x}\right) dx + \frac{1}{4}b^2\int \ldots$$

$$= a^2x - abx\log\left(1-\frac{c}{x}\right) - \frac{1}{4}b^2(c-x)\log^2\left(1-\frac{c}{x}\right) + abx\log\left(1+\frac{c}{x}\right) - \frac{1}{2}b^2x\log\left(1-\frac{c}{x}\right)\log\ldots$$

$$= a^2x - abx\log\left(1-\frac{c}{x}\right) - \frac{1}{4}b^2(c-x)\log^2\left(1-\frac{c}{x}\right) + abx\log\left(1+\frac{c}{x}\right) - \frac{1}{2}b^2x\log\left(1-\frac{c}{x}\right)\log\ldots$$

$$= a^2x - abx\log\left(1-\frac{c}{x}\right) - \frac{1}{4}b^2(c-x)\log^2\left(1-\frac{c}{x}\right) + abx\log\left(1+\frac{c}{x}\right) - \frac{1}{2}b^2x\log\left(1-\frac{c}{x}\right)\log\ldots$$

$$= a^2x - abx\log\left(1-\frac{c}{x}\right) - \frac{1}{4}b^2(c-x)\log^2\left(1-\frac{c}{x}\right) + abx\log\left(1+\frac{c}{x}\right) - \frac{1}{2}b^2x\log\left(1-\frac{c}{x}\right)\log\ldots$$

$$= a^2x - abx\log\left(1-\frac{c}{x}\right) - \frac{1}{4}b^2(c-x)\log^2\left(1-\frac{c}{x}\right) + abx\log\left(1+\frac{c}{x}\right) - \frac{1}{2}b^2x\log\left(1-\frac{c}{x}\right)\log\ldots$$

$$= a^2x - abx\log\left(1-\frac{c}{x}\right) - \frac{1}{4}b^2(c-x)\log^2\left(1-\frac{c}{x}\right) + abx\log\left(1+\frac{c}{x}\right) - \frac{1}{2}b^2x\log\left(1-\frac{c}{x}\right)\log\ldots$$

$$= a^2x - abx\log\left(1-\frac{c}{x}\right) - \frac{1}{4}b^2(c-x)\log^2\left(1-\frac{c}{x}\right) + abx\log\left(1+\frac{c}{x}\right) - \frac{1}{2}b^2x\log\left(1-\frac{c}{x}\right)\log\ldots$$

$$= a^2x - abx\log\left(1-\frac{c}{x}\right) - \frac{1}{4}b^2(c-x)\log^2\left(1-\frac{c}{x}\right) + abx\log\left(1+\frac{c}{x}\right) - \frac{1}{2}b^2x\log\left(1-\frac{c}{x}\right)\log\ldots$$

Mathematica [A] time = 0.121809, size = 97, normalized size = 1.31

$$b^2c\mathrm{PolyLog}\left(2, e^{-2\tanh^{-1}\left(\frac{c}{x}\right)}\right) + a\left(ax + bc\log\left(1-\frac{c^2}{x^2}\right) - 2bc\log\left(\frac{c}{x}\right)\right) + 2b\tanh^{-1}\left(\frac{c}{x}\right)\left(ax - bc\log\left(1-e^{-2\tanh^{-1}\left(\frac{c}{x}\right)}\right)\ldots\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c/x])^2,x]
```

```
[Out] b^2*(-c + x)*ArcTanh[c/x]^2 + 2*b*ArcTanh[c/x]*(a*x - b*c*Log[1 - E^(-2*Arc
Tanh[c/x])]) + a*(a*x + b*c*Log[1 - c^2/x^2] - 2*b*c*Log[c/x]) + b^2*c*Poly
Log[2, E^(-2*ArcTanh[c/x])]
```

Maple [B] time = 0.012, size = 282, normalized size = 3.8

$$a^2x + b^2x\left(\mathrm{Artanh}\left(\frac{c}{x}\right)\right)^2 + cb^2\mathrm{Artanh}\left(\frac{c}{x}\right)\ln\left(\frac{c}{x}-1\right) - 2\,cb^2\ln\left(\frac{c}{x}\right)\mathrm{Artanh}\left(\frac{c}{x}\right) + cb^2\mathrm{Artanh}\left(\frac{c}{x}\right)\ln\left(1+\frac{c}{x}\right) - cb^2\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x))^2,x)
```

```
[Out] a^2*x+b^2*x*arctanh(c/x)^2+c*b^2*arctanh(c/x)*ln(c/x-1)-2*c*b^2*ln(c/x)*arc
tanh(c/x)+c*b^2*arctanh(c/x)*ln(1+c/x)-c*b^2*dilog(1/2+1/2*c/x)-1/2*c*b^2*l
n(c/x-1)*ln(1/2+1/2*c/x)+1/4*c*b^2*ln(c/x-1)^2-1/2*c*b^2*ln(-1/2*c/x+1/2)*l
```

```
n(1/2+1/2*c/x)+1/2*c*b^2*ln(-1/2*c/x+1/2)*ln(1+c/x)-1/4*c*b^2*ln(1+c/x)^2+c
*b^2*dilog(c/x)+c*b^2*dilog(1+c/x)+c*b^2*ln(c/x)*ln(1+c/x)+2*a*b*x*arctanh(
c/x)+c*a*b*ln(c/x-1)-2*c*a*b*ln(c/x)+c*a*b*ln(1+c/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\left(2\,x\,\operatorname{artanh}\left(\frac{c}{x}\right)+c\log\left(-c^2+x^2\right)\right)ab+\frac{1}{4}\left(x\log\left(c+x\right)^2-2\left(c+x\right)\log\left(c+x\right)\log\left(-c+x\right)-\left(c-x\right)\log\left(-c+x\right)^2+\int -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^2,x, algorithm="maxima")
```

```
[Out] (2*x*arctanh(c/x) + c*log(-c^2 + x^2))*a*b + 1/4*(x*log(c + x)^2 - 2*(c + x
)*log(c + x)*log(-c + x) - (c - x)*log(-c + x)^2 + integrate(-2*(c^2 + 3*c*
x)*log(c + x)/(c^2 - x^2), x))*b^2 + a^2*x
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(b^2\operatorname{artanh}\left(\frac{c}{x}\right)^2+2\,ab\operatorname{artanh}\left(\frac{c}{x}\right)+a^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*arctanh(c/x)^2 + 2*a*b*arctanh(c/x) + a^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\operatorname{atanh}\left(\frac{c}{x}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x))**2,x)
```

```
[Out] Integral((a + b*atanh(c/x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\operatorname{artanh}\left(\frac{c}{x}\right)+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c/x) + a)^2, x)
```

3.147 $\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^2}{x}\,dx$

Optimal. Leaf size=133

$$b\text{PolyLog}\left(2,1-\frac{2}{1-\frac{c}{x}}\right)\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)-b\text{PolyLog}\left(2,\frac{2}{1-\frac{c}{x}}-1\right)\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)-\frac{1}{2}b^2\text{PolyLog}\left(3,1-\frac{2}{1-\ldots}\right.$$

```
[Out] -2*(a + b*ArcCoth[x/c])^2*ArcTanh[1 - 2/(1 - c/x)] + b*(a + b*ArcCoth[x/c])
*PolyLog[2, 1 - 2/(1 - c/x)] - b*(a + b*ArcCoth[x/c])*PolyLog[2, -1 + 2/(1
- c/x)] - (b^2*PolyLog[3, 1 - 2/(1 - c/x)])/2 + (b^2*PolyLog[3, -1 + 2/(1 -
 c/x)])/2
```

Rubi [A] time = 0.314473, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6095, 5914, 6052, 5948, 6058, 6610}

$$b\text{PolyLog}\left(2,1-\frac{2}{1-\frac{c}{x}}\right)\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)-b\text{PolyLog}\left(2,\frac{2}{1-\frac{c}{x}}-1\right)\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)-\frac{1}{2}b^2\text{PolyLog}\left(3,1-\frac{2}{1-\ldots}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c/x])^2/x,x]
```

```
[Out] -2*(a + b*ArcCoth[x/c])^2*ArcTanh[1 - 2/(1 - c/x)] + b*(a + b*ArcCoth[x/c])
*PolyLog[2, 1 - 2/(1 - c/x)] - b*(a + b*ArcCoth[x/c])*PolyLog[2, -1 + 2/(1
- c/x)] - (b^2*PolyLog[3, 1 - 2/(1 - c/x)])/2 + (b^2*PolyLog[3, -1 + 2/(1 -
 c/x)])/2
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[
1/n, Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
 n}, x] && IGtQ[p, 0]
```

Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
 e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^2}{x}\,dx = -\operatorname{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x}\,dx,x,\frac{1}{x}\right)$$

$$= -2\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1-\frac{c}{x}}\right)+(4bc)\operatorname{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(cx)\right)\tanh^{-1}\left(1\right.}{1-c^2x^2}\right.$$

$$= -2\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1-\frac{c}{x}}\right)-(2bc)\operatorname{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1-cx}\right)}{1-c^2x^2}\right.$$

$$= -2\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1-\frac{c}{x}}\right)+b\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)\operatorname{Li}_2\left(1-\frac{2}{1-\frac{c}{x}}\right)-b\left(a+\right.$$

$$= -2\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1-\frac{c}{x}}\right)+b\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)\operatorname{Li}_2\left(1-\frac{2}{1-\frac{c}{x}}\right)-b\left(a+\right.$$

Mathematica [A] time = 0.0921291, size = 114, normalized size = 0.86

$$\frac{1}{2}b\left(2\operatorname{PolyLog}\left(2,\frac{c+x}{c-x}\right)\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)-2\operatorname{PolyLog}\left(2,\frac{c+x}{x-c}\right)\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)+b\left(\operatorname{PolyLog}\left(3,\frac{c+x}{x-c}\right)-\operatorname{Poly}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c/x])^2/x,x]
```

```
[Out] -2*(a + b*ArcTanh[c/x])^2*ArcTanh[(c + x)/(c - x)] + (b*(2*(a + b*ArcTanh[c
/x])*PolyLog[2, (c + x)/(c - x)] - 2*(a + b*ArcTanh[c/x])*PolyLog[2, (c + x
)/(-c + x)] + b*(-PolyLog[3, (c + x)/(c - x)] + PolyLog[3, (c + x)/(-c + x)
])))/2
```

Maple [C] time = 0.162, size = 780, normalized size = 5.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x))^2/x,x)
```

```
[Out] -a^2*ln(c/x)-b^2*ln(c/x)*arctanh(c/x)^2+b^2*arctanh(c/x)*polylog(2,-(1+c/x)
^2/(1-c^2/x^2))-1/2*b^2*polylog(3,-(1+c/x)^2/(1-c^2/x^2))+b^2*arctanh(c/x)^
2*ln((1+c/x)^2/(1-c^2/x^2)-1)-b^2*arctanh(c/x)^2*ln(1-(1+c/x)/(1-c^2/x^2)^(
1/2))-2*b^2*arctanh(c/x)*polylog(2,(1+c/x)/(1-c^2/x^2)^(1/2))+2*b^2*polylog
(3,(1+c/x)/(1-c^2/x^2)^(1/2))-b^2*arctanh(c/x)^2*ln(1+(1+c/x)/(1-c^2/x^2)^(
1/2))-2*b^2*arctanh(c/x)*polylog(2,-(1+c/x)/(1-c^2/x^2)^(1/2))+2*b^2*polylo
g(3,-(1+c/x)/(1-c^2/x^2)^(1/2))+1/2*I*b^2*Pi*csgn(I*((1+c/x)^2/(1-c^2/x^2)-
1))*csgn(I*((1+c/x)^2/(1-c^2/x^2)-1)/((1+c/x)^2/(1-c^2/x^2)+1))^2*arctanh(c
/x)^2-1/2*I*b^2*Pi*csgn(I*((1+c/x)^2/(1-c^2/x^2)-1)/((1+c/x)^2/(1-c^2/x^2)+
1))^3*arctanh(c/x)^2+1/2*I*b^2*Pi*csgn(I/((1+c/x)^2/(1-c^2/x^2)+1))*csgn(I*
((1+c/x)^2/(1-c^2/x^2)-1)/((1+c/x)^2/(1-c^2/x^2)+1))^2*arctanh(c/x)^2-1/2*I
*b^2*Pi*csgn(I*((1+c/x)^2/(1-c^2/x^2)-1))*csgn(I/((1+c/x)^2/(1-c^2/x^2)+1))
*csgn(I*((1+c/x)^2/(1-c^2/x^2)-1)/((1+c/x)^2/(1-c^2/x^2)+1))*arctanh(c/x)^2
-2*a*b*ln(c/x)*arctanh(c/x)+a*b*ln(c/x)*ln(1+c/x)+a*b*dilog(c/x)+a*b*dilog(
1+c/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$a^2 \log(x) + \int \frac{b^2\left(\log\left(\frac{c}{x}+1\right) - \log\left(-\frac{c}{x}+1\right)\right)^2}{4x} + \frac{ab\left(\log\left(\frac{c}{x}+1\right) - \log\left(-\frac{c}{x}+1\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^2/x,x, algorithm="maxima")
```

```
[Out] a^2*log(x) + integrate(1/4*b^2*(log(c/x + 1) - log(-c/x + 1))^2/x + a*b*(lo
g(c/x + 1) - log(-c/x + 1))/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2 \operatorname{artanh}\left(\frac{c}{x}\right)^2 + 2\,ab \operatorname{artanh}\left(\frac{c}{x}\right) + a^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^2/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c/x)^2 + 2*a*b*arctanh(c/x) + a^2)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + b \operatorname{atanh}\left(\frac{c}{x}\right)\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x))**2/x,x)
```

```
[Out] Integral((a + b*atanh(c/x))**2/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \operatorname{artanh}\left(\frac{c}{x}\right) + a\right)^2}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^2/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c/x) + a)^2/x, x)
```

3.148 $\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^2}{x^2} dx$

Optimal. Leaf size=87

$$\frac{b^2\text{PolyLog}\left(2,1-\frac{2}{1-\frac{c}{x}}\right)}{c}-\frac{\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2}{c}-\frac{\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2}{x}+\frac{2b\log\left(\frac{2}{1-\frac{c}{x}}\right)\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)}{c}$$

```
[Out] -((a + b*ArcCoth[x/c])^2/c) - (a + b*ArcCoth[x/c])^2/x + (2*b*(a + b*ArcCot
h[x/c])*Log[2/(1 - c/x)])/c + (b^2*PolyLog[2, 1 - 2/(1 - c/x)])/c
```

Rubi [B] time = 0.506839, antiderivative size = 205, normalized size of antiderivative = 2.36, number of steps used = 28, number of rules used = 12, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {6099, 2454, 2389, 2296, 2295, 6715, 2430, 43, 2416, 2394, 2393, 2391}

$$\frac{b^2\text{PolyLog}\left(2,-\frac{c-x}{2x}\right)}{2c}-\frac{b^2\text{PolyLog}\left(2,\frac{c+x}{2x}\right)}{2c}-\frac{b\log\left(\frac{c+x}{2x}\right)\left(2a-b\log\left(1-\frac{c}{x}\right)\right)}{2c}-\frac{b\log\left(\frac{c+x}{x}\right)\left(2a-b\log\left(1-\frac{c}{x}\right)\right)}{2x}+\frac{(1}{}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcTanh[c/x])^2/x^2,x]
```

```
[Out] ((1 - c/x)*(2*a - b*Log[1 - c/x])^2)/(4*c) - (b*(2*a - b*Log[1 - c/x])*Log[
(c + x)/(2*x)])/(2*c) - (b*(2*a - b*Log[1 - c/x])*Log[(c + x)/x])/(2*x) - (
b^2*Log[-(c - x)/(2*x)]*Log[(c + x)/x])/(2*c) - (b^2*(1 + c/x)*Log[(c + x)/
x]^2)/(4*c) + (b^2*PolyLog[2, -(c - x)/(2*x)])/(2*c) - (b^2*PolyLog[2, (c +
 x)/(2*x)])/(2*c)
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 2430

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.)), x_Symbol] :> Simp[x*(a + b*Log[c
*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]), x] + (-Dist[g*j*m, Int[(x*(a +
 b*Log[c*(d + e*x)^n])^p)/(i + j*x), x], x] - Dist[b*e*n*p, Int[(x*(a + b*L
og[c*(d + e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /
; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^2}{x^2}\,dx = \int \left(\frac{\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2}{4x^2} + \frac{b\left(2a-b\log\left(1-\frac{c}{x}\right)\right)\log\left(1+\frac{c}{x}\right)}{2x^2} + \frac{b^2\log^2\left(1+\frac{c}{x}\right)}{4x^2}\right)dx$$

$$= \frac{1}{4}\int \frac{\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2}{x^2}\,dx + \frac{1}{2}b\int \frac{\left(2a-b\log\left(1-\frac{c}{x}\right)\right)\log\left(1+\frac{c}{x}\right)}{x^2}\,dx + \frac{1}{4}b^2\int \frac{\log^2\left(\right.}{x}$$

$$= -\left(\frac{1}{4}\text{Subst}\left(\int (2a-b\log(1-cx))^2\,dx, x, \frac{1}{x}\right)\right) - \frac{1}{2}b\,\text{Subst}\left(\int (2a-b\log(1-cx))\log(1\right.$$

$$= -\frac{b\left(2a-b\log\left(1-\frac{c}{x}\right)\right)\log\left(\frac{c+x}{x}\right)}{2x} + \frac{\text{Subst}\left(\int (2a-b\log(x))^2\,dx, x, 1-\frac{c}{x}\right)}{4c} - \frac{b^2\,\text{Subst}\left(\int\right.}{}$$

$$= \frac{\left(1-\frac{c}{x}\right)\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2}{4c} - \frac{b\left(2a-b\log\left(1-\frac{c}{x}\right)\right)\log\left(\frac{c+x}{x}\right)}{2x} - \frac{b^2\left(1+\frac{c}{x}\right)\log^2\left(\frac{c+x}{x}\right)}{4c} +$$

$$= -\frac{ab}{x} - \frac{b^2}{2x} + \frac{\left(1-\frac{c}{x}\right)\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2}{4c} + \frac{b^2\left(1+\frac{c}{x}\right)\log\left(\frac{c+x}{x}\right)}{2c} - \frac{b\left(2a-b\log\left(1-\frac{c}{x}\right)\right)}{2x}$$

$$= -\frac{b^2}{x} - \frac{b^2\left(1-\frac{c}{x}\right)\log\left(1-\frac{c}{x}\right)}{2c} + \frac{\left(1-\frac{c}{x}\right)\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2}{4c} - \frac{b\left(2a-b\log\left(1-\frac{c}{x}\right)\right)\log\left(\right.}{2c}$$

$$= -\frac{b^2}{2x} - \frac{b^2\left(1-\frac{c}{x}\right)\log\left(1-\frac{c}{x}\right)}{2c} + \frac{\left(1-\frac{c}{x}\right)\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2}{4c} - \frac{b\left(2a-b\log\left(1-\frac{c}{x}\right)\right)\log}{2c}$$

$$= \frac{\left(1-\frac{c}{x}\right)\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2}{4c} - \frac{b\left(2a-b\log\left(1-\frac{c}{x}\right)\right)\log\left(\frac{c+x}{2x}\right)}{2c} - \frac{b\left(2a-b\log\left(1-\frac{c}{x}\right)\right)\text{lo}}{2x}$$

Mathematica [A] time = 0.0965751, size = 101, normalized size = 1.16

$$\frac{-b^2x\text{PolyLog}\left(2,-e^{-2\tanh^{-1}\left(\frac{c}{x}\right)}\right) + a\left(2bx\log\left(\frac{1}{\sqrt{1-\frac{c^2}{x^2}}}\right) - ac\right) + 2b\tanh^{-1}\left(\frac{c}{x}\right)\left(bx\log\left(e^{-2\tanh^{-1}\left(\frac{c}{x}\right)}+1\right) - ac\right) + b^2(x-}{cx}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c/x])^2/x^2,x]
```

```
[Out] (b^2*(-c + x)*ArcTanh[c/x]^2 + 2*b*ArcTanh[c/x]*(-(a*c) + b*x*Log[1 + E^(-2
*ArcTanh[c/x])]) + a*(-(a*c) + 2*b*x*Log[1/Sqrt[1 - c^2/x^2]]) - b^2*x*Poly
Log[2, -E^(-2*ArcTanh[c/x])])/(c*x)
```

Maple [A] time = 0.004, size = 144, normalized size = 1.7

$$-\frac{a^2}{x} - \frac{b^2}{x}\left(\text{Artanh}\left(\frac{c}{x}\right)\right)^2 - \frac{b^2}{c}\left(\text{Artanh}\left(\frac{c}{x}\right)\right)^2 + 2\frac{b^2}{c}\text{Artanh}\left(\frac{c}{x}\right)\ln\left(\left(1+\frac{c}{x}\right)^2\left(1-\frac{c^2}{x^2}\right)^{-1}+1\right) + \frac{b^2}{c}\text{polylog}\left(2,-\left(1\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x))^2/x^2,x)
```

```
[Out] -a^2/x-arctanh(c/x)^2/x*b^2-1/c*b^2*arctanh(c/x)^2+2/c*arctanh(c/x)*ln((1+c
/x)^2/(1-c^2/x^2)+1)*b^2+1/c*polylog(2,-(1+c/x)^2/(1-c^2/x^2))*b^2-2*a*b/x*
```

```
arctanh(c/x)-1/c*a*b*ln(1-c^2/x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{4}\left(c^3\int -\frac{\log(x)^2}{c^3x^2-cx^4}\,dx+c^2\left(\frac{\log\left(-c^2+x^2\right)}{c^3}-\frac{\log\left(x^2\right)}{c^3}\right)-4\,c^2\int -\frac{x\log(c+x)}{c^3x^2-cx^4}\,dx+2\,c^2\int -\frac{x\log(x)}{c^3x^2-cx^4}\,dx+2\,c\left(\frac{\log(-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*(c^3*integrate(-log(x)^2/(c^3*x^2 - c*x^4), x) + c^2*(log(-c^2 + x^2)/c
^3 - log(x^2)/c^3) - 4*c^2*integrate(-x*log(c + x)/(c^3*x^2 - c*x^4), x) +
2*c^2*integrate(-x*log(x)/(c^3*x^2 - c*x^4), x) + 2*c*(log(-c + x)/c^2 - lo
g(x)/c^2 + 1/(c*x))*log(-c/x + 1) - c*(log(c + x)/c^2 - log(-c + x)/c^2) -
c*integrate(-x^2*log(x)^2/(c^3*x^2 - c*x^4), x) - 2*c*integrate(-x^2*log(c
+ x)/(c^3*x^2 - c*x^4), x) + 4*c*integrate(-x^2*log(x)/(c^3*x^2 - c*x^4), x
) - log(-c/x + 1)^2/x - (c*log(c + x)^2 - 2*((c + x)*log(c + x) - (c + x)*l
og(x) - c)*log(-c + x))/(c*x) - (x*log(-c + x)^2 + x*log(x)^2 - 2*(x*log(x)
 - x)*log(-c + x) - 2*x*log(x) + 2*c)/(c*x) - 2*integrate(-x^3*log(c + x)/(
c^3*x^2 - c*x^4), x) + 2*integrate(-x^3*log(x)/(c^3*x^2 - c*x^4), x))*b^2 -
 a*b*(2*c*arctanh(c/x)/x + log(-c^2/x^2 + 1))/c - a^2/x
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2\,\text{artanh}\left(\frac{c}{x}\right)^2+2\,ab\,\text{artanh}\left(\frac{c}{x}\right)+a^2}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^2/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c/x)^2 + 2*a*b*arctanh(c/x) + a^2)/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+b\,\text{atanh}\left(\frac{c}{x}\right)\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x))**2/x**2,x)
```

```
[Out] Integral((a + b*atanh(c/x))**2/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(b\,\text{artanh}\left(\frac{c}{x}\right)+a\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c/x) + a)^2/x^2, x)
```

3.149 $\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^2}{x^3}\,dx$

Optimal. Leaf size=87

$$\frac{\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2}{2c^2}-\frac{\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2}{2x^2}-\frac{ab}{cx}-\frac{b^2\log\left(1-\frac{c^2}{x^2}\right)}{2c^2}-\frac{b^2\coth^{-1}\left(\frac{x}{c}\right)}{cx}$$

```
[Out] -((a*b)/(c*x)) - (b^2*ArcCoth[x/c])/(c*x) + (a + b*ArcCoth[x/c])^2/(2*c^2)
- (a + b*ArcCoth[x/c])^2/(2*x^2) - (b^2*Log[1 - c^2/x^2])/(2*c^2)
```

Rubi [C] time = 1.24698, antiderivative size = 707, normalized size of antiderivative = 8.13, number of steps used = 66, number of rules used = 23, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.438, Rules used = {6099, 2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304, 2395, 43, 6742, 30, 2557, 12, 2466, 2462, 260, 2416, 2394, 2315, 2393, 2391}

$$\frac{b^2\text{PolyLog}\left(2,\frac{c-x}{2c}\right)}{4c^2}+\frac{b^2\text{PolyLog}\left(2,-\frac{c}{x}\right)}{4c^2}+\frac{b^2\text{PolyLog}\left(2,\frac{c}{x}\right)}{4c^2}+\frac{b^2\text{PolyLog}\left(2,\frac{c+x}{2c}\right)}{4c^2}-\frac{b^2\text{PolyLog}\left(2,1-\frac{x}{c}\right)}{4c^2}-b^2\text{Poly}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcTanh[c/x])^2/x^3,x]
```

```
[Out] -(b^2*(1 - c/x)^2)/(16*c^2) - (b^2*(1 + c/x)^2)/(16*c^2) + (a*b)/(4*x^2) +
b^2/(8*x^2) - (3*a*b)/(2*c*x) + (b^2*Log[1 - c/x])/(8*c^2) - (3*b^2*(1 - c/
x)*Log[1 - c/x])/(4*c^2) - (b^2*Log[1 - c/x])/(8*x^2) - (b*(1 - c/x)^2*(2*a
 - b*Log[1 - c/x]))/(8*c^2) + ((1 - c/x)*(2*a - b*Log[1 - c/x])^2)/(4*c^2)
- ((1 - c/x)^2*(2*a - b*Log[1 - c/x])^2)/(8*c^2) + (b^2*Log[1 - c/x]*Log[1
+ c/x])/(4*x^2) - (b^2*Log[1 + c/x]*Log[c - x])/(4*c^2) - (b^2*Log[c - x]*L
og[x/c])/(4*c^2) - (b^2*Log[1 - c/x]*Log[c + x])/(4*c^2) + (b^2*Log[(c - x)
/(2*c)]*Log[c + x])/(4*c^2) - (b^2*Log[-(x/c)]*Log[c + x])/(4*c^2) + (b^2*L
og[c - x]*Log[(c + x)/(2*c)])/(4*c^2) + (a*b*Log[(c + x)/x])/(2*c^2) + (b^2
*Log[(c + x)/x])/(8*c^2) - (3*b^2*(1 + c/x)*Log[(c + x)/x])/(4*c^2) + (b^2*
(1 + c/x)^2*Log[(c + x)/x])/(8*c^2) - (a*b*Log[(c + x)/x])/(2*x^2) - (b^2*L
og[(c + x)/x])/(8*x^2) + (b^2*(1 + c/x)*Log[(c + x)/x]^2)/(4*c^2) - (b^2*(1
 + c/x)^2*Log[(c + x)/x]^2)/(8*c^2) + (b^2*PolyLog[2, (c - x)/(2*c)])/(4*c^
2) + (b^2*PolyLog[2, -(c/x)])/(4*c^2) + (b^2*PolyLog[2, c/x])/(4*c^2) + (b^
2*PolyLog[2, (c + x)/(2*c)])/(4*c^2) - (b^2*PolyLog[2, 1 - x/c])/(4*c^2) -
(b^2*PolyLog[2, 1 + x/c])/(4*c^2)
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2557

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] :> With[{z = IntHide[u, x]}, Dist[Log[v
]*Log[w], z, x] + (-Int[SimplifyIntegrand[(z*Log[w]*D[v, x])/v, x], x] - In
t[SimplifyIntegrand[(z*Log[v]*D[w, x])/w, x], x]) /; InverseFunctionFreeQ[z
, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^2}{x^3}\,dx = \int \left(\frac{\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2}{4x^3}+\frac{b\left(2a-b\log\left(1-\frac{c}{x}\right)\right)\log\left(1+\frac{c}{x}\right)}{2x^3}+\frac{b^2\log^2\left(1+\frac{c}{x}\right)}{4x^3}\right)dx$$

$$=\frac{1}{4}\int \frac{\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2}{x^3}\,dx+\frac{1}{2}b\int \frac{\left(2a-b\log\left(1-\frac{c}{x}\right)\right)\log\left(1+\frac{c}{x}\right)}{x^3}\,dx+\frac{1}{4}b^2\int \frac{\log^2\left(\right.}{x}$$

$$=-\left(\frac{1}{4}\,\mathrm{Subst}\left(\int x(2a-b\log(1-cx))^2\,dx,x,\frac{1}{x}\right)\right)+\frac{1}{2}b\int\left(\frac{2a\log\left(1+\frac{c}{x}\right)}{x^3}-\frac{b\log\left(1-\frac{c}{x}\right)}{x^3}\right.$$

$$=-\left(\frac{1}{4}\,\mathrm{Subst}\left(\int\left(\frac{(2a-b\log(1-cx))^2}{c}-\frac{(1-cx)(2a-b\log(1-cx))^2}{c}\right)dx,x,\frac{1}{x}\right)\right)+(ab)$$

$$=\frac{b^2\log\left(1-\frac{c}{x}\right)\log\left(1+\frac{c}{x}\right)}{4x^2}-(ab)\,\mathrm{Subst}\left(\int x\log(1+cx)\,dx,x,\frac{1}{x}\right)+\frac{1}{2}b^2\int\frac{c\log\left(1-\frac{c}{x}\right)}{2x^3(c+x)}$$

$$=\frac{b^2\log\left(1-\frac{c}{x}\right)\log\left(1+\frac{c}{x}\right)}{4x^2}-\frac{ab\log\left(\frac{c+x}{x}\right)}{2x^2}+\frac{\mathrm{Subst}\left(\int(2a-b\log(x))^2\,dx,x,1-\frac{c}{x}\right)}{4c^2}-\mathrm{Sub}$$

$$=\frac{\left(1-\frac{c}{x}\right)\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2}{4c^2}-\frac{\left(1-\frac{c}{x}\right)^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2}{8c^2}+\frac{b^2\log\left(1-\frac{c}{x}\right)\log\left(1+\frac{c}{x}\right)}{4x^2}$$

$$=-\frac{b^2\left(1-\frac{c}{x}\right)^2}{16c^2}-\frac{b^2\left(1+\frac{c}{x}\right)^2}{16c^2}+\frac{ab}{4x^2}-\frac{3ab}{2cx}+\frac{b^2}{2cx}-\frac{b\left(1-\frac{c}{x}\right)^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)}{8c^2}+\frac{\left(1-\right.}{}$$

$$=-\frac{b^2\left(1-\frac{c}{x}\right)^2}{16c^2}-\frac{b^2\left(1+\frac{c}{x}\right)^2}{16c^2}+\frac{ab}{4x^2}-\frac{3ab}{2cx}-\frac{b^2\left(1-\frac{c}{x}\right)\log\left(1-\frac{c}{x}\right)}{2c^2}-\frac{b\left(1-\frac{c}{x}\right)^2\left(2a-b\log\right.}{8c^2}$$

$$=-\frac{b^2\left(1-\frac{c}{x}\right)^2}{16c^2}-\frac{b^2\left(1+\frac{c}{x}\right)^2}{16c^2}+\frac{ab}{4x^2}-\frac{3ab}{2cx}-\frac{b^2\left(1-\frac{c}{x}\right)\log\left(1-\frac{c}{x}\right)}{2c^2}-\frac{b^2\log\left(1-\frac{c}{x}\right)}{8x^2}-\frac{b\left(1\right.}{}$$

$$=-\frac{b^2\left(1-\frac{c}{x}\right)^2}{16c^2}-\frac{b^2\left(1+\frac{c}{x}\right)^2}{16c^2}+\frac{ab}{4x^2}-\frac{3ab}{2cx}-\frac{3b^2\left(1-\frac{c}{x}\right)\log\left(1-\frac{c}{x}\right)}{4c^2}-\frac{b^2\log\left(1-\frac{c}{x}\right)}{8x^2}-\frac{b\left(\right.}{}$$

$$=-\frac{b^2\left(1-\frac{c}{x}\right)^2}{16c^2}-\frac{b^2\left(1+\frac{c}{x}\right)^2}{16c^2}+\frac{ab}{4x^2}+\frac{b^2}{8x^2}-\frac{3ab}{2cx}+\frac{b^2\log\left(1-\frac{c}{x}\right)}{8c^2}-\frac{3b^2\left(1-\frac{c}{x}\right)\log\left(1-\frac{c}{x}\right.}{4c^2}$$

$$=-\frac{b^2\left(1-\frac{c}{x}\right)^2}{16c^2}-\frac{b^2\left(1+\frac{c}{x}\right)^2}{16c^2}+\frac{ab}{4x^2}+\frac{b^2}{8x^2}-\frac{3ab}{2cx}+\frac{b^2\log\left(1-\frac{c}{x}\right)}{8c^2}-\frac{3b^2\left(1-\frac{c}{x}\right)\log\left(1-\frac{c}{x}\right.}{4c^2}$$

$$=-\frac{b^2\left(1-\frac{c}{x}\right)^2}{16c^2}-\frac{b^2\left(1+\frac{c}{x}\right)^2}{16c^2}+\frac{ab}{4x^2}+\frac{b^2}{8x^2}-\frac{3ab}{2cx}+\frac{b^2\log\left(1-\frac{c}{x}\right)}{8c^2}-\frac{3b^2\left(1-\frac{c}{x}\right)\log\left(1-\frac{c}{x}\right.}{4c^2}$$

Mathematica [A] time = 0.067157, size = 119, normalized size = 1.37

$$-\frac{a^2c^2+abx^2\log(x-c)-abx^2\log(c+x)+2abcx+2bc\tanh^{-1}\left(\frac{c}{x}\right)(ac+bx)+b^2\left(c^2-x^2\right)\tanh^{-1}\left(\frac{c}{x}\right)^2+b^2x^2\log(x}{2c^2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c/x])^2/x^3,x]
```

```
[Out] -(a^2*c^2 + 2*a*b*c*x + 2*b*c*(a*c + b*x)*ArcTanh[c/x] + b^2*(c^2 - x^2)*Ar
cTanh[c/x]^2 - 2*b^2*x^2*Log[x] + a*b*x^2*Log[-c + x] + b^2*x^2*Log[-c + x]
 - a*b*x^2*Log[c + x] + b^2*x^2*Log[c + x])/(2*c^2*x^2)
```

Maple [B] time = 0.016, size = 284, normalized size = 3.3

$$-\frac{a^2}{2\,x^2}-\frac{b^2}{2\,x^2}\left(\mathrm{Artanh}\left(\frac{c}{x}\right)\right)^2-\frac{b^2}{cx}\mathrm{Artanh}\left(\frac{c}{x}\right)-\frac{b^2}{2\,c^2}\mathrm{Artanh}\left(\frac{c}{x}\right)\ln\left(\frac{c}{x}-1\right)+\frac{b^2}{2\,c^2}\mathrm{Artanh}\left(\frac{c}{x}\right)\ln\left(1+\frac{c}{x}\right)+\frac{b^2}{4\,c^2}\ln\left(\frac{c}{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x))^2/x^3,x)
```

```
[Out] -1/2*a^2/x^2-1/2*b^2/x^2*arctanh(c/x)^2-1/c*b^2*arctanh(c/x)/x-1/2/c^2*b^2*
arctanh(c/x)*ln(c/x-1)+1/2/c^2*b^2*arctanh(c/x)*ln(1+c/x)+1/4/c^2*b^2*ln(c/
x-1)*ln(1/2+1/2*c/x)-1/8/c^2*b^2*ln(c/x-1)^2-1/2/c^2*b^2*ln(c/x-1)-1/2/c^2*
b^2*ln(1+c/x)+1/4/c^2*b^2*ln(-1/2*c/x+1/2)*ln(1+c/x)-1/4/c^2*b^2*ln(-1/2*c/
x+1/2)*ln(1/2+1/2*c/x)-1/8/c^2*b^2*ln(1+c/x)^2-a*b/x^2*arctanh(c/x)-a*b/c/x
-1/2/c^2*a*b*ln(c/x-1)+1/2/c^2*a*b*ln(1+c/x)
```

Maxima [B] time = 0.989422, size = 223, normalized size = 2.56

$$\frac{1}{2}\left(c\left(\frac{\log(c+x)}{c^3}-\frac{\log(-c+x)}{c^3}-\frac{2}{c^2x}\right)-\frac{2\,\mathrm{artanh}\left(\frac{c}{x}\right)}{x^2}\right)ab-\frac{1}{8}\left(c^2\left(\frac{\log(c+x)^2-2\,(\log(c+x)-2)\log(-c+x)+\log(-\right.}{c^4}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*(c*(log(c + x)/c^3 - log(-c + x)/c^3 - 2/(c^2*x)) - 2*arctanh(c/x)/x^2)
*a*b - 1/8*(c^2*((log(c + x)^2 - 2*(log(c + x) - 2)*log(-c + x) + log(-c +
x)^2 + 4*log(c + x))/c^4 - 8*log(x)/c^4) - 4*c*(log(c + x)/c^3 - log(-c + x
)/c^3 - 2/(c^2*x))*arctanh(c/x))*b^2 - 1/2*b^2*arctanh(c/x)^2/x^2 - 1/2*a^2
/x^2
```

Fricas [A] time = 1.86745, size = 288, normalized size = 3.31

$$\frac{8\,b^2x^2\log(x)-4\,a^2c^2-8\,abcx+4\left(ab-b^2\right)x^2\log(c+x)-4\left(ab+b^2\right)x^2\log(-c+x)-\left(b^2c^2-b^2x^2\right)\log\left(-\frac{c+x}{c-x}\right)^2-4\,(al}{8\,c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/8*(8*b^2*x^2*log(x) - 4*a^2*c^2 - 8*a*b*c*x + 4*(a*b - b^2)*x^2*log(c + x
) - 4*(a*b + b^2)*x^2*log(-c + x) - (b^2*c^2 - b^2*x^2)*log(-(c + x)/(c - x
```

```
))^2 - 4*(a*b*c^2 + b^2*c*x)*log(-(c + x)/(c - x)))/(c^2*x^2)
```

Sympy [A] time = 1.96205, size = 124, normalized size = 1.43

$$\begin{cases} -\frac{a^2}{2x^2} - \frac{ab\,\mathrm{atanh}\left(\frac{c}{x}\right)}{x^2} - \frac{ab}{cx} + \frac{ab\,\mathrm{atanh}\left(\frac{c}{x}\right)}{c^2} - \frac{b^2\,\mathrm{atanh}^2\left(\frac{c}{x}\right)}{2x^2} - \frac{b^2\,\mathrm{atanh}\left(\frac{c}{x}\right)}{cx} + \frac{b^2\log(x)}{c^2} - \frac{b^2\log(-c+x)}{c^2} + \frac{b^2\,\mathrm{atanh}^2\left(\frac{c}{x}\right)}{2c^2} - \frac{b^2\,\mathrm{atanh}\left(\frac{c}{x}\right)}{c^2} \\ -\frac{a^2}{2x^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x))**2/x**3,x)
```

```
[Out] Piecewise((-a**2/(2*x**2) - a*b*atanh(c/x)/x**2 - a*b/(c*x) + a*b*atanh(c/x
)/c**2 - b**2*atanh(c/x)**2/(2*x**2) - b**2*atanh(c/x)/(c*x) + b**2*log(x)/
c**2 - b**2*log(-c + x)/c**2 + b**2*atanh(c/x)**2/(2*c**2) - b**2*atanh(c/x
)/c**2, Ne(c, 0)), (-a**2/(2*x**2), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\,\mathrm{artanh}\left(\frac{c}{x}\right) + a\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^2/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c/x) + a)^2/x^3, x)
```

3.150 $\int x^3 \left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^3 dx$

Optimal. Leaf size=203

$$b^3c^4\text{PolyLog}\left(2,\frac{2}{\frac{c}{x}+1}-1\right)+\frac{1}{4}b^2c^2x^2\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)-2b^2c^4\log\left(2-\frac{2}{\frac{c}{x}+1}\right)\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)-\frac{1}{4}c^4\left(a+b\coth^{-1}\right.$$

```
[Out] (b^3*c^3*x)/4 - (b^3*c^4*ArcCoth[x/c])/4 + (b^2*c^2*x^2*(a + b*ArcCoth[x/c]
))/4 - b*c^4*(a + b*ArcCoth[x/c])^2 + (3*b*c^3*x*(a + b*ArcCoth[x/c])^2)/4
+ (b*c*x^3*(a + b*ArcCoth[x/c])^2)/4 - (c^4*(a + b*ArcCoth[x/c])^3)/4 + (x^
4*(a + b*ArcCoth[x/c])^3)/4 - 2*b^2*c^4*(a + b*ArcCoth[x/c])*Log[2 - 2/(1 +
 c/x)] + b^3*c^4*PolyLog[2, -1 + 2/(1 + c/x)]
```

Rubi [F] time = 4.39036, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^3 \left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^3 dx$$

Verification is Not applicable to the result.

```
[In] Int[x^3*(a + b*ArcTanh[c/x])^3,x]
```

```
[Out] (3*a^2*b*c^3*x)/8 - (5*a*b^2*c^3*x)/16 + (b^3*c^3*x)/16 - (3*a^2*b*c^2*x^2)
/16 + (3*a*b^2*c^2*x^2)/16 + (a^2*b*c*x^3)/8 + (b^3*c^4*Log[1 - c/x])/32 -
(3*a*b^2*c^3*x*Log[1 - c/x])/8 + (3*a*b^2*c^2*x^2*Log[1 - c/x])/16 - (a*b^2
*c*x^3*Log[1 - c/x])/8 + (5*b^2*c^3*(1 - c/x)*x*(2*a - b*Log[1 - c/x]))/32
+ (b^2*c^2*x^2*(2*a - b*Log[1 - c/x]))/32 - (5*b*c^4*(2*a - b*Log[1 - c/x])
^2)/64 + (3*b*c^3*(1 - c/x)*x*(2*a - b*Log[1 - c/x])^2)/32 + (3*b*c^2*x^2*(
2*a - b*Log[1 - c/x])^2)/64 + (b*c*x^3*(2*a - b*Log[1 - c/x])^2)/32 - (c^4*
(2*a - b*Log[1 - c/x])^3)/32 + (x^4*(2*a - b*Log[1 - c/x])^3)/32 + (3*a*b^2
*c^3*x*Log[1 + c/x])/8 + (3*a*b^2*c^2*x^2*Log[1 + c/x])/16 + (a*b^2*c*x^3*L
og[1 + c/x])/8 + (3*a^2*b*x^4*Log[1 + c/x])/8 - (3*a*b^2*x^4*Log[1 - c/x]*L
og[1 + c/x])/8 + (5*a*b^2*c^4*Log[c - x])/16 + (3*a*b^2*c^4*Log[1 + c/x]*Lo
g[c - x])/8 - (3*b*c^4*(2*a - b*Log[1 - c/x])^2*Log[c/x])/32 + (11*a*b^2*c^
4*Log[x])/8 + (3*a*b^2*c^4*Log[c - x]*Log[x/c])/8 - (3*a^2*b*c^4*Log[c + x]
)/8 + (5*a*b^2*c^4*Log[c + x])/16 + (3*a*b^2*c^4*Log[1 - c/x]*Log[c + x])/8
 - (3*a*b^2*c^4*Log[(c - x)/(2*c)]*Log[c + x])/8 + (3*a*b^2*c^4*Log[-(x/c)]
*Log[c + x])/8 - (3*a*b^2*c^4*Log[c - x]*Log[(c + x)/(2*c)])/8 + (11*a*b^2*
c^4*Log[(c + x)/x])/16 - (b^3*c^4*Log[(c + x)/x])/32 + (3*a*b^2*c^3*x*Log[(
c + x)/x])/8 - (5*b^3*c^3*(1 + c/x)*x*Log[(c + x)/x])/32 - (3*a*b^2*c^2*x^2
*Log[(c + x)/x])/16 + (b^3*c^2*x^2*Log[(c + x)/x])/32 + (a*b^2*c*x^3*Log[(c
 + x)/x])/8 - (3*a*b^2*c^4*Log[(c + x)/x]^2)/16 + (5*b^3*c^4*Log[(c + x)/x]
^2)/64 + (3*b^3*c^3*(1 + c/x)*x*Log[(c + x)/x]^2)/32 - (3*b^3*c^2*x^2*Log[(
c + x)/x]^2)/64 + (b^3*c*x^3*Log[(c + x)/x]^2)/32 + (3*a*b^2*x^4*Log[(c + x
)/x]^2)/16 + (3*b^3*c^4*Log[-(c/x)]*Log[(c + x)/x]^2)/32 - (b^3*c^4*Log[(c
+ x)/x]^3)/32 + (b^3*x^4*Log[(c + x)/x]^3)/32 + (3*b^2*c^4*(2*a - b*Log[1 -
 c/x])*PolyLog[2, 1 - c/x])/16 - (3*a*b^2*c^4*PolyLog[2, (c - x)/(2*c)])/8
- (3*a*b^2*c^4*PolyLog[2, -(c/x)])/8 + (11*b^3*c^4*PolyLog[2, -(c/x)])/32 -
 (11*b^3*c^4*PolyLog[2, c/x])/32 - (3*a*b^2*c^4*PolyLog[2, (c + x)/(2*c)])/
8 + (3*b^3*c^4*Log[(c + x)/x]*PolyLog[2, (c + x)/x])/16 + (3*a*b^2*c^4*Poly
Log[2, 1 - x/c])/8 + (3*a*b^2*c^4*PolyLog[2, 1 + x/c])/8 + (3*b^3*c^4*PolyL
og[3, 1 - c/x])/16 - (3*b^3*c^4*PolyLog[3, (c + x)/x])/16 + (3*b^3*Defer[In
t][x^3*Log[1 - c/x]^2*Log[1 + c/x], x])/8 - (3*b^3*Defer[Int][x^3*Log[1 - c
/x]*Log[1 + c/x]^2, x])/8
```

Rubi steps

$$\int x^3 \left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^3 dx = \int \left(\frac{1}{8}x^3\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3 + \frac{3}{8}bx^3\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2\log\left(1+\frac{c}{x}\right) + \frac{3}{8}b^2x^3 \left(2a -\right.\right.$$

$$= \frac{1}{8}\int x^3 \left(2a - b\log\left(1-\frac{c}{x}\right)\right)^3 dx + \frac{1}{8}(3b)\int x^3 \left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2 \log\left(1+\frac{c}{x}\right) dx +$$

$$= -\left(\frac{1}{8}\text{Subst}\left(\int \frac{(2a-b\log(1-cx))^3}{x^5}dx, x, \frac{1}{x}\right)\right) + \frac{1}{8}(3b)\int\left(4a^2x^3\log\left(1+\frac{c}{x}\right) - 4abx\right.$$

$$= \frac{1}{32}x^4\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3 + \frac{1}{32}b^3x^4\log^3\left(\frac{c+x}{x}\right) + \frac{1}{2}\left(3a^2b\right)\int x^3\log\left(1+\frac{c}{x}\right)dx +$$

$$= \frac{1}{32}x^4\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3 + \frac{3}{8}a^2bx^4\log\left(1+\frac{c}{x}\right) - \frac{3}{8}ab^2x^4\log\left(1-\frac{c}{x}\right)\log\left(1+\frac{c}{x}\right) +$$

$$= \frac{1}{32}x^4\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3 + \frac{3}{8}a^2bx^4\log\left(1+\frac{c}{x}\right) - \frac{3}{8}ab^2x^4\log\left(1-\frac{c}{x}\right)\log\left(1+\frac{c}{x}\right) +$$

$$= \frac{1}{32}bcx^3\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2 + \frac{1}{32}x^4\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3 + \frac{3}{8}a^2bx^4\log\left(1+\frac{c}{x}\right) - \frac{3}{8}$$

$$= \frac{3}{8}a^2bc^3x - \frac{3}{16}a^2bc^2x^2 + \frac{1}{8}a^2bcx^3 + \frac{3}{64}bc^2x^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2 + \frac{1}{32}bcx^3\left(2a - b\,\mathrm{lo}\right.$$

$$= \frac{3}{8}a^2bc^3x - \frac{3}{16}a^2bc^2x^2 + \frac{1}{8}a^2bcx^3 - \frac{3}{8}ab^2c^3x\log\left(1-\frac{c}{x}\right) + \frac{3}{16}ab^2c^2x^2\log\left(1-\frac{c}{x}\right) - \frac{1}{8}$$

$$= \frac{3}{8}a^2bc^3x - \frac{3}{16}a^2bc^2x^2 + \frac{1}{8}a^2bcx^3 - \frac{3}{8}ab^2c^3x\log\left(1-\frac{c}{x}\right) + \frac{3}{16}ab^2c^2x^2\log\left(1-\frac{c}{x}\right) - \frac{1}{8}$$

$$= \frac{3}{8}a^2bc^3x - \frac{5}{16}ab^2c^3x + \frac{1}{16}b^3c^3x - \frac{3}{16}a^2bc^2x^2 + \frac{1}{16}ab^2c^2x^2 + \frac{1}{8}a^2bcx^3 + \frac{1}{32}b^3c^4\log\left(1\right.$$

$$= \frac{3}{8}a^2bc^3x - \frac{5}{16}ab^2c^3x + \frac{1}{16}b^3c^3x - \frac{3}{16}a^2bc^2x^2 + \frac{3}{16}ab^2c^2x^2 + \frac{1}{8}a^2bcx^3 + \frac{1}{32}b^3c^4\log\left(1\right.$$

$$= \frac{3}{8}a^2bc^3x - \frac{5}{16}ab^2c^3x + \frac{1}{16}b^3c^3x - \frac{3}{16}a^2bc^2x^2 + \frac{3}{16}ab^2c^2x^2 + \frac{1}{8}a^2bcx^3 + \frac{1}{32}b^3c^4\log\left(1\right.$$

$$= \frac{3}{8}a^2bc^3x - \frac{5}{16}ab^2c^3x + \frac{1}{16}b^3c^3x - \frac{3}{16}a^2bc^2x^2 + \frac{3}{16}ab^2c^2x^2 + \frac{1}{8}a^2bcx^3 + \frac{1}{32}b^3c^4\log\left(1\right.$$

Mathematica [A] time = 0.603465, size = 286, normalized size = 1.41

$$\frac{1}{8}\left(8b^3c^4\text{PolyLog}\left(2,e^{-2\tanh^{-1}\left(\frac{c}{x}\right)}\right) + 2b\tanh^{-1}\left(\frac{c}{x}\right)\left(3a^2x^4 + 2abcx\left(3c^2+x^2\right) + b^2\left(c^2x^2-c^4\right)\right) - 8b^2c^4\log\left(1-e^{-2\tan}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*(a + b*ArcTanh[c/x])^3,x]
```

```
[Out] (-2*a*b^2*c^4 + 6*a^2*b*c^3*x + 2*b^3*c^3*x + 2*a*b^2*c^2*x^2 + 2*a^2*b*c*x
^3 + 2*a^3*x^4 + 2*b^2*(b*c*(-4*c^3 + 3*c^2*x + x^3) + 3*a*(-c^4 + x^4))*Ar
```

```
cTanh[c/x]^2 + 2*b^3*(-c^4 + x^4)*ArcTanh[c/x]^3 + 2*b*ArcTanh[c/x]*(3*a^2*
x^4 + 2*a*b*c*x*(3*c^2 + x^2) + b^2*(-c^4 + c^2*x^2) - 8*b^2*c^4*Log[1 - E^
(-2*ArcTanh[c/x])]) + 3*a^2*b*c^4*Log[1 - c/x] - 16*a*b^2*c^4*Log[c/(Sqrt[1
 - c^2/x^2]*x)] - 3*a^2*b*c^4*Log[(c + x)/x] + 8*b^3*c^4*PolyLog[2, E^(-2*A
rcTanh[c/x])])/8
```

Maple [C] time = 0.332, size = 1410, normalized size = 7.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c/x))^3,x)
```

```
[Out] 3/4*a^2*b*x^4*arctanh(c/x)+3/8*c^4*b^3*arctanh(c/x)^2*ln(c/x-1)+1/4*c^4*b^3
/(c/x+1-(1-c^2/x^2)^(1/2))*(1-c^2/x^2)^(1/2)+c^4*a*b^2*ln(c/x-1)+c^4*a*b^2*
ln(1+c/x)-3/16*I*c^4*b^3*Pi*csgn(I*(1+c/x)^2/(-1+c^2/x^2))*csgn(I*(1+c/x)^2
/(-1+c^2/x^2)/((1+c/x)^2/(1-c^2/x^2)+1))^2*arctanh(c/x)^2-2*c^4*b^3*dilog(1
+(1+c/x)/(1-c^2/x^2)^(1/2))+2*c^4*b^3*dilog((1+c/x)/(1-c^2/x^2)^(1/2))+3/16
*I*c^4*b^3*Pi*csgn(I*(1+c/x)/(1-c^2/x^2)^(1/2))^2*csgn(I*(1+c/x)^2/(-1+c^2/
x^2))*arctanh(c/x)^2+3/8*I*c^4*b^3*Pi*csgn(I*(1+c/x)/(1-c^2/x^2)^(1/2))*csg
n(I*(1+c/x)^2/(-1+c^2/x^2))^2*arctanh(c/x)^2+3/16*I*c^4*b^3*Pi*csgn(I/((1+c
/x)^2/(1-c^2/x^2)+1))*csgn(I*(1+c/x)^2/(-1+c^2/x^2)/((1+c/x)^2/(1-c^2/x^2)+
1))^2*arctanh(c/x)^2-2*c^4*b^3*arctanh(c/x)*ln(1+(1+c/x)/(1-c^2/x^2)^(1/2))
+3/4*c^4*b^3*arctanh(c/x)^2*ln((1+c/x)/(1-c^2/x^2)^(1/2))+3/4*c^3*b^3*arcta
nh(c/x)^2*x+1/4*c*b^3*arctanh(c/x)^2*x^3-3/16*I*c^4*b^3*Pi*csgn(I/((1+c/x)^
2/(1-c^2/x^2)+1))*csgn(I*(1+c/x)^2/(-1+c^2/x^2))*csgn(I*(1+c/x)^2/(-1+c^2/x
^2)/((1+c/x)^2/(1-c^2/x^2)+1))*arctanh(c/x)^2+3/16*c^4*a*b^2*ln(1+c/x)^2+1/
4*b^3*x^4*arctanh(c/x)^3-1/4*c^4*b^3*arctanh(c/x)+3/4*c^3*a^2*b*x-1/4*c^4*b
^3/((1-c^2/x^2)^(1/2)+c/x+1)*(1-c^2/x^2)^(1/2)+1/4*x^4*a^3+3/4*a*b^2*x^4*ar
ctanh(c/x)^2+3/8*I*c^4*b^3*Pi*csgn(I/((1+c/x)^2/(1-c^2/x^2)+1))^2*arctanh(c
/x)^2-3/8*I*c^4*b^3*Pi*csgn(I/((1+c/x)^2/(1-c^2/x^2)+1))^3*arctanh(c/x)^2+c
^4*b^3*arctanh(c/x)^2-1/4*c^4*b^3*arctanh(c/x)^3+3/8*c^4*a^2*b*ln(c/x-1)+1/
4*c*a^2*b*x^3+1/4*c^2*b^2*x^2*a-3/8*c^4*a^2*b*ln(1+c/x)-2*c^4*a*b^2*ln(c/x)
+1/4*c^2*b^3*arctanh(c/x)*x^2-3/8*c^4*b^3*arctanh(c/x)^2*ln(1+c/x)+3/16*c^4
*a*b^2*ln(c/x-1)^2+1/2*c*a*b^2*arctanh(c/x)*x^3+3/2*c^3*a*b^2*x*arctanh(c/x
)+3/16*I*c^4*b^3*Pi*csgn(I*(1+c/x)^2/(-1+c^2/x^2)/((1+c/x)^2/(1-c^2/x^2)+1)
)^3*arctanh(c/x)^2+3/16*I*c^4*b^3*Pi*csgn(I*(1+c/x)^2/(-1+c^2/x^2))^3*arcta
nh(c/x)^2-3/8*c^4*a*b^2*ln(-1/2*c/x+1/2)*ln(1+c/x)+3/8*c^4*a*b^2*ln(-1/2*c/
x+1/2)*ln(1/2+1/2*c/x)+3/4*c^4*a*b^2*arctanh(c/x)*ln(c/x-1)-3/4*c^4*a*b^2*a
rctanh(c/x)*ln(1+c/x)-3/8*c^4*a*b^2*ln(c/x-1)*ln(1/2+1/2*c/x)-3/8*I*c^4*b^3
*Pi*arctanh(c/x)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{3}{4}\,ab^2x^4\,\operatorname{artanh}\left(\frac{c}{x}\right)^2+\frac{1}{4}\,a^3x^4+\frac{1}{8}\left(6\,x^4\,\operatorname{artanh}\left(\frac{c}{x}\right)-\left(3\,c^3\log\left(c+x\right)-3\,c^3\log\left(-c+x\right)-6\,c^2x-2\,x^3\right)c\right)a^2b+\frac{1}{16}\left(\left(3\,c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c/x))^3,x, algorithm="maxima")
```

```
[Out] 3/4*a*b^2*x^4*arctanh(c/x)^2 + 1/4*a^3*x^4 + 1/8*(6*x^4*arctanh(c/x) - (3*c
^3*log(c + x) - 3*c^3*log(-c + x) - 6*c^2*x - 2*x^3)*c)*a^2*b + 1/16*((3*c^
```

```
2*log(c + x)^2 + 3*c^2*log(-c + x)^2 + 16*c^2*log(c + x) + 4*x^2 - 2*(3*c^2
*log(c + x) - 8*c^2)*log(-c + x))*c^2 - 4*(3*c^3*log(c + x) - 3*c^3*log(-c
+ x) - 6*c^2*x - 2*x^3)*c*arctanh(c/x))*a*b^2 + 1/32*(16*c^5*integrate(-log
(c + x)/(c^2 - x^2), x) + 40*c^4*integrate(-x*log(c + x)/(c^2 - x^2), x) -
2*(c*log(c + x) - c*log(-c + x) - 2*x)*c^3 - (c^4 - x^4)*log(c + x)^3 + (c^
4 - x^4)*log(-c + x)^3 + 2*(c^2*log(-c^2 + x^2) + x^2)*c^2 + 8*c^2*integrat
e(-x^3*log(c + x)/(c^2 - x^2), x) + 2*(3*c^3*x + c*x^3)*log(c + x)^2 - (8*c
^4 - 6*c^3*x - 2*c*x^3 + 3*(c^4 - x^4)*log(c + x))*log(-c + x)^2 - (4*c^2*x
^2 - 3*(c^4 - x^4)*log(c + x)^2 + 4*(4*c^4 + 3*c^3*x + c*x^3)*log(c + x))*l
og(-c + x))*b^3
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(b^3x^3\,\mathrm{artanh}\left(\frac{c}{x}\right)^3+3\,ab^2x^3\,\mathrm{artanh}\left(\frac{c}{x}\right)^2+3\,a^2bx^3\,\mathrm{artanh}\left(\frac{c}{x}\right)+a^3x^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c/x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^3*arctanh(c/x)^3 + 3*a*b^2*x^3*arctanh(c/x)^2 + 3*a^2*b*x^3*
arctanh(c/x) + a^3*x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\left(a+b\,\mathrm{atanh}\left(\frac{c}{x}\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c/x))**3,x)
```

```
[Out] Integral(x**3*(a + b*atanh(c/x))**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\,\mathrm{artanh}\left(\frac{c}{x}\right)+a\right)^3x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c/x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c/x) + a)^3*x^3, x)
```

3.151 $\int x^2 \left(a+b \tanh^{-1}\left(\frac{c}{x}\right)\right)^3 \, dx$

Optimal. Leaf size=217

$$b^2c^3\text{PolyLog}\left(2,\frac{2}{\frac{c}{x}+1}-1\right)\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)+\frac{1}{2}b^3c^3\text{PolyLog}\left(3,\frac{2}{\frac{c}{x}+1}-1\right)+b^2c^2x\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)-\frac{1}{3}c^3\left(a+b\right.$$

```
[Out] b^2*c^2*x*(a + b*ArcCoth[x/c]) - (b*c^3*(a + b*ArcCoth[x/c])^2)/2 + (b*c*x^
2*(a + b*ArcCoth[x/c])^2)/2 - (c^3*(a + b*ArcCoth[x/c])^3)/3 + (x^3*(a + b*
ArcCoth[x/c])^3)/3 - b*c^3*(a + b*ArcCoth[x/c])^2*Log[2 - 2/(1 + c/x)] + (b
^3*c^3*Log[1 - c^2/x^2])/2 + b^3*c^3*Log[x] + b^2*c^3*(a + b*ArcCoth[x/c])*
PolyLog[2, -1 + 2/(1 + c/x)] + (b^3*c^3*PolyLog[3, -1 + 2/(1 + c/x)])/2
```

Rubi [F] time = 3.30156, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int x^2 \left(a+b \tanh^{-1}\left(\frac{c}{x}\right)\right)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2*(a + b*ArcTanh[c/x])^3,x]
```

```
[Out] -(a^2*b*c^2*x)/2 + (3*a*b^2*c^2*x)/4 + (a^2*b*c*x^2)/4 + (a*b^2*c^2*x*Log[1
 - c/x])/2 - (a*b^2*c*x^2*Log[1 - c/x])/4 + (b^2*c^2*(1 - c/x)*x*(2*a - b*L
og[1 - c/x]))/8 - (b*c^3*(2*a - b*Log[1 - c/x])^2)/16 + (b*c^2*(1 - c/x)*x*
(2*a - b*Log[1 - c/x])^2)/8 + (b*c*x^2*(2*a - b*Log[1 - c/x])^2)/16 - (c^3*
(2*a - b*Log[1 - c/x])^3)/24 + (x^3*(2*a - b*Log[1 - c/x])^3)/24 + (a*b^2*c
^2*x*Log[1 + c/x])/2 + (a*b^2*c*x^2*Log[1 + c/x])/4 + (a^2*b*x^3*Log[1 + c/
x])/2 - (a*b^2*x^3*Log[1 - c/x]*Log[1 + c/x])/2 - (a*b^2*c^3*Log[c - x])/4
+ (a*b^2*c^3*Log[1 + c/x]*Log[c - x])/2 - (b*c^3*(2*a - b*Log[1 - c/x])^2*L
og[c/x])/8 + (b^3*c^3*Log[x])/4 + (a*b^2*c^3*Log[c - x]*Log[x/c])/2 + (a^2*
b*c^3*Log[c + x])/2 + (a*b^2*c^3*Log[c + x])/4 - (a*b^2*c^3*Log[1 - c/x]*Lo
g[c + x])/2 + (a*b^2*c^3*Log[(c - x)/(2*c)]*Log[c + x])/2 - (a*b^2*c^3*Log[
-(x/c)]*Log[c + x])/2 - (a*b^2*c^3*Log[c - x]*Log[(c + x)/(2*c)])/2 - (3*a*
b^2*c^3*Log[(c + x)/x])/4 - (a*b^2*c^2*x*Log[(c + x)/x])/2 + (b^3*c^2*(1 +
c/x)*x*Log[(c + x)/x])/8 + (a*b^2*c*x^2*Log[(c + x)/x])/4 + (a*b^2*c^3*Log[
(c + x)/x]^2)/4 - (b^3*c^3*Log[(c + x)/x]^2)/16 - (b^3*c^2*(1 + c/x)*x*Log[
(c + x)/x]^2)/8 + (b^3*c*x^2*Log[(c + x)/x]^2)/16 + (a*b^2*x^3*Log[(c + x)/
x]^2)/4 - (b^3*c^3*Log[-(c/x)]*Log[(c + x)/x]^2)/8 + (b^3*c^3*Log[(c + x)/x
]^3)/24 + (b^3*x^3*Log[(c + x)/x]^3)/24 + (b^2*c^3*(2*a - b*Log[1 - c/x])*P
olyLog[2, 1 - c/x])/4 - (a*b^2*c^3*PolyLog[2, (c - x)/(2*c)])/2 + (a*b^2*c^
3*PolyLog[2, -(c/x)])/2 - (3*b^3*c^3*PolyLog[2, -(c/x)])/8 - (3*b^3*c^3*Pol
yLog[2, c/x])/8 + (a*b^2*c^3*PolyLog[2, (c + x)/(2*c)])/2 - (b^3*c^3*Log[(c
 + x)/x]*PolyLog[2, (c + x)/x])/4 + (a*b^2*c^3*PolyLog[2, 1 - x/c])/2 - (a*
b^2*c^3*PolyLog[2, 1 + x/c])/2 + (b^3*c^3*PolyLog[3, 1 - c/x])/4 + (b^3*c^3
*PolyLog[3, (c + x)/x])/4 + (3*b^3*Defer[Int][x^2*Log[1 - c/x]^2*Log[1 + c/
x], x])/8 - (3*b^3*Defer[Int][x^2*Log[1 - c/x]*Log[1 + c/x]^2, x])/8
```

Rubi steps

$$\int x^2\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^3dx=\int\left(\frac{1}{8}x^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3+\frac{3}{8}bx^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2\log\left(1+\frac{c}{x}\right)+\frac{3}{8}b^2x^2\left(2a\right.\right.$$

$$=\frac{1}{8}\int x^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3dx+\frac{1}{8}(3b)\int x^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2\log\left(1+\frac{c}{x}\right)dx+$$

$$=-\left(\frac{1}{8}\text{Subst}\left(\int\frac{(2a-b\log(1-cx))^3}{x^4}dx,x,\frac{1}{x}\right)\right)+\frac{1}{8}(3b)\int\left(4a^2x^2\log\left(1+\frac{c}{x}\right)-4abx\right.$$

$$=\frac{1}{24}x^3\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3+\frac{1}{24}b^3x^3\log^3\left(\frac{c+x}{x}\right)+\frac{1}{2}\left(3a^2b\right)\int x^2\log\left(1+\frac{c}{x}\right)dx+$$

$$=\frac{1}{24}x^3\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3+\frac{1}{2}a^2bx^3\log\left(1+\frac{c}{x}\right)-\frac{1}{2}ab^2x^3\log\left(1-\frac{c}{x}\right)\log\left(1+\frac{c}{x}\right)+$$

$$=\frac{1}{24}x^3\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3+\frac{1}{2}a^2bx^3\log\left(1+\frac{c}{x}\right)-\frac{1}{2}ab^2x^3\log\left(1-\frac{c}{x}\right)\log\left(1+\frac{c}{x}\right)+$$

$$=\frac{1}{16}bcx^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2+\frac{1}{24}x^3\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3+\frac{1}{2}a^2bx^3\log\left(1+\frac{c}{x}\right)-\frac{1}{2}$$

$$=-\frac{1}{2}a^2bc^2x+\frac{1}{4}a^2bcx^2+\frac{1}{8}bc^2\left(1-\frac{c}{x}\right)x\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2+\frac{1}{16}bcx^2\left(2a-b\log\left(1\right.\right.$$

$$=-\frac{1}{2}a^2bc^2x+\frac{1}{4}a^2bcx^2+\frac{1}{2}ab^2c^2x\log\left(1-\frac{c}{x}\right)-\frac{1}{4}ab^2cx^2\log\left(1-\frac{c}{x}\right)+\frac{1}{8}b^2c^2\left(1-\frac{c}{x}\right)x$$

$$=-\frac{1}{2}a^2bc^2x+\frac{1}{4}a^2bcx^2+\frac{1}{2}ab^2c^2x\log\left(1-\frac{c}{x}\right)-\frac{1}{4}ab^2cx^2\log\left(1-\frac{c}{x}\right)+\frac{1}{8}b^2c^2\left(1-\frac{c}{x}\right)x$$

$$=-\frac{1}{2}a^2bc^2x+\frac{1}{4}ab^2c^2x+\frac{1}{4}a^2bcx^2+\frac{1}{2}ab^2c^2x\log\left(1-\frac{c}{x}\right)-\frac{1}{4}ab^2cx^2\log\left(1-\frac{c}{x}\right)+\frac{1}{8}b^2c$$

$$=-\frac{1}{2}a^2bc^2x+\frac{3}{4}ab^2c^2x+\frac{1}{4}a^2bcx^2+\frac{1}{2}ab^2c^2x\log\left(1-\frac{c}{x}\right)-\frac{1}{4}ab^2cx^2\log\left(1-\frac{c}{x}\right)+\frac{1}{8}b^2c$$

$$=-\frac{1}{2}a^2bc^2x+\frac{3}{4}ab^2c^2x+\frac{1}{4}a^2bcx^2+\frac{1}{2}ab^2c^2x\log\left(1-\frac{c}{x}\right)-\frac{1}{4}ab^2cx^2\log\left(1-\frac{c}{x}\right)+\frac{1}{8}b^2c$$

$$=-\frac{1}{2}a^2bc^2x+\frac{3}{4}ab^2c^2x+\frac{1}{4}a^2bcx^2+\frac{1}{2}ab^2c^2x\log\left(1-\frac{c}{x}\right)-\frac{1}{4}ab^2cx^2\log\left(1-\frac{c}{x}\right)+\frac{1}{8}b^2c$$

Mathematica [C] time = 0.744349, size = 316, normalized size = 1.46

$$\frac{1}{6}\left(6ab^2\left(c^3\text{PolyLog}\left(2,e^{-2\tanh^{-1}\left(\frac{c}{x}\right)}\right)+\left(x^3-c^3\right)\tanh^{-1}\left(\frac{c}{x}\right)^2+c\tanh^{-1}\left(\frac{c}{x}\right)\left(-2c^2\log\left(1-e^{-2\tanh^{-1}\left(\frac{c}{x}\right)}\right)-c^2+x^2\right)\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(a + b*ArcTanh[c/x])^3,x]
```

```
[Out] (3*a^2*b*c*x^2 + 2*a^3*x^3 + 6*a^2*b*x^3*ArcTanh[c/x] + 3*a^2*b*c^3*Log[-c^
2 + x^2] + 6*a*b^2*(c^2*x + (-c^3 + x^3)*ArcTanh[c/x]^2 + c*ArcTanh[c/x]*(-
c^2 + x^2 - 2*c^2*Log[1 - E^(-2*ArcTanh[c/x])]) + c^3*PolyLog[2, E^(-2*ArcT
anh[c/x])]) + (b^3*((-I)*c^3*Pi^3 + 24*c^2*x*ArcTanh[c/x] - 12*c^3*ArcTanh[
c/x]^2 + 12*c*x^2*ArcTanh[c/x]^2 + 8*c^3*ArcTanh[c/x]^3 + 8*x^3*ArcTanh[c/x
```

```
]^3 - 24*c^3*ArcTanh[c/x]^2*Log[1 - E^(2*ArcTanh[c/x])] - 24*c^3*Log[c/(Sqr
t[1 - c^2/x^2]*x)] - 24*c^3*ArcTanh[c/x]*PolyLog[2, E^(2*ArcTanh[c/x])] + 1
2*c^3*PolyLog[3, E^(2*ArcTanh[c/x])]))/4)/6
```

Maple [C] time = 0.02, size = 2033, normalized size = 9.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c/x))^3,x)
```

```
[Out] 1/4*I*c^3*b^3*arctanh(c/x)^2*csgn(I*(1+c/x)^2/(-1+c^2/x^2))*csgn(I/((1+c/x)
^2/(1-c^2/x^2)+1))*csgn(I*(1+c/x)^2/(-1+c^2/x^2)/((1+c/x)^2/(1-c^2/x^2)+1))
*Pi+1/4*I*c^3*b^3*arctanh(c/x)^2*csgn(I*(1+c/x)^2/(-1+c^2/x^2))*csgn(I*(1+c
/x)^2/(-1+c^2/x^2)/((1+c/x)^2/(1-c^2/x^2)+1))^2*Pi+1/2*I*c^3*b^3*arctanh(c/
x)^2*csgn(I/((1+c/x)^2/(1-c^2/x^2)+1))*csgn(I*((1+c/x)^2/(1-c^2/x^2)-1)/((1
+c/x)^2/(1-c^2/x^2)+1))^2*Pi-1/2*I*c^3*b^3*arctanh(c/x)^2*csgn(I*(1+c/x)/(1
-c^2/x^2)^(1/2))*csgn(I*(1+c/x)^2/(-1+c^2/x^2))^2*Pi-1/2*I*c^3*b^3*arctanh(
c/x)^2*csgn(I/((1+c/x)^2/(1-c^2/x^2)+1))*csgn(I*((1+c/x)^2/(1-c^2/x^2)-1))*
csgn(I*((1+c/x)^2/(1-c^2/x^2)-1)/((1+c/x)^2/(1-c^2/x^2)+1))*Pi-1/4*I*c^3*b^
3*arctanh(c/x)^2*csgn(I*(1+c/x)/(1-c^2/x^2)^(1/2))^2*csgn(I*(1+c/x)^2/(-1+c
^2/x^2))*Pi-1/4*I*c^3*b^3*arctanh(c/x)^2*csgn(I/((1+c/x)^2/(1-c^2/x^2)+1))*
csgn(I*(1+c/x)^2/(-1+c^2/x^2)/((1+c/x)^2/(1-c^2/x^2)+1))^2*Pi+1/3*x^3*a^3+1
/2*I*c^3*b^3*arctanh(c/x)^2*csgn(I/((1+c/x)^2/(1-c^2/x^2)+1))^2*Pi+1/2*c*b^
3*arctanh(c/x)^2*x^2+1/4*c^3*a*b^2*ln(c/x-1)^2-1/2*c^3*a*b^2*ln(1+c/x)+1/2*
c^3*a^2*b*ln(c/x-1)+1/2*c^3*b^3*arctanh(c/x)^2*ln(1+c/x)+a^2*b*x^3*arctanh(
c/x)+a*b^2*x^3*arctanh(c/x)^2-1/2*I*c^3*b^3*arctanh(c/x)^2*csgn(I/((1+c/x)^
2/(1-c^2/x^2)+1))^3*Pi-1/2*I*c^3*b^3*arctanh(c/x)^2*csgn(I*((1+c/x)^2/(1-c^
2/x^2)-1)/((1+c/x)^2/(1-c^2/x^2)+1))^3*Pi+c^2*b^3*arctanh(c/x)*x+c^2*a*b^2*
x+1/2*c*a^2*b*x^2-c^3*b^3*arctanh(c/x)^2*ln(2)-2*c^3*b^3*arctanh(c/x)*polyl
og(2,-(1+c/x)/(1-c^2/x^2)^(1/2))-2*c^3*b^3*arctanh(c/x)*polylog(2,(1+c/x)/(
1-c^2/x^2)^(1/2))-c^3*b^3*arctanh(c/x)^2*ln(1-(1+c/x)/(1-c^2/x^2)^(1/2))-c^
3*a*b^2*dilog(1/2+1/2*c/x)-c^3*a^2*b*ln(c/x)+c^3*a*b^2*dilog(1+c/x)-c^3*b^3
*arctanh(c/x)^2*ln((1+c/x)/(1-c^2/x^2)^(1/2))-c^3*b^3*arctanh(c/x)^2*ln(1+(
1+c/x)/(1-c^2/x^2)^(1/2))-c^3*b^3*ln(c/x)*arctanh(c/x)^2+c^3*b^3*arctanh(c/
x)^2*ln((1+c/x)^2/(1-c^2/x^2)-1)+1/2*c^3*b^3*arctanh(c/x)^2*ln(c/x-1)+1/2*c
^3*a^2*b*ln(1+c/x)+c^3*a*b^2*dilog(c/x)-1/4*c^3*a*b^2*ln(1+c/x)^2+1/2*c^3*a
*b^2*ln(c/x-1)+1/3*c^3*b^3*arctanh(c/x)^3+2*c^3*b^3*polylog(3,-(1+c/x)/(1-c
^2/x^2)^(1/2))+2*c^3*b^3*polylog(3,(1+c/x)/(1-c^2/x^2)^(1/2))-1/2*c^3*b^3*a
rctanh(c/x)^2+c^3*b^3*arctanh(c/x)+1/3*b^3*x^3*arctanh(c/x)^3-c^3*b^3*ln(1+
(1+c/x)/(1-c^2/x^2)^(1/2))-c^3*b^3*ln((1+c/x)/(1-c^2/x^2)^(1/2)-1)-2*c^3*a*
b^2*ln(c/x)*arctanh(c/x)-1/4*I*c^3*b^3*arctanh(c/x)^2*csgn(I*(1+c/x)^2/(-1+
c^2/x^2))^3*Pi+c^3*a*b^2*arctanh(c/x)*ln(1+c/x)-1/2*c^3*a*b^2*ln(c/x-1)*ln(
1/2+1/2*c/x)-1/2*c^3*a*b^2*ln(-1/2*c/x+1/2)*ln(1/2+1/2*c/x)+c^3*a*b^2*arcta
nh(c/x)*ln(c/x-1)+1/2*c^3*a*b^2*ln(-1/2*c/x+1/2)*ln(1+c/x)-1/2*I*c^3*b^3*ar
ctanh(c/x)^2*Pi+c^3*a*b^2*ln(c/x)*ln(1+c/x)+c*a*b^2*x^2*arctanh(c/x)-1/4*I*
c^3*b^3*arctanh(c/x)^2*csgn(I*(1+c/x)^2/(-1+c^2/x^2)/((1+c/x)^2/(1-c^2/x^2)
+1))^3*Pi+1/2*I*c^3*b^3*arctanh(c/x)^2*csgn(I*((1+c/x)^2/(1-c^2/x^2)-1))*cs
gn(I*((1+c/x)^2/(1-c^2/x^2)-1)/((1+c/x)^2/(1-c^2/x^2)+1))^2*Pi
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{3}a^3x^3 + \frac{1}{2}\left(2x^3\operatorname{artanh}\left(\frac{c}{x}\right) + \left(c^2\log\left(-c^2 + x^2\right) + x^2\right)c\right)a^2b + \frac{1}{24}\left(b^3c^3 - b^3x^3\right)\log\left(-c + x\right)^3 + \frac{1}{8}\left(b^3cx^2 + 2ab^2x^3 + \left(b^3\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c/x))^3,x, algorithm="maxima")
```

```
[Out] 1/3*a^3*x^3 + 1/2*(2*x^3*arctanh(c/x) + (c^2*log(-c^2 + x^2) + x^2)*c)*a^2*
b + 1/24*(b^3*c^3 - b^3*x^3)*log(-c + x)^3 + 1/8*(b^3*c*x^2 + 2*a*b^2*x^3 +
 (b^3*c^3 + b^3*x^3)*log(c + x))*log(-c + x)^2 - integrate(-1/8*((b^3*c*x^2
 - b^3*x^3)*log(c + x)^3 + 6*(a*b^2*c*x^2 - a*b^2*x^3)*log(c + x)^2 + (2*b^
3*c*x^2 + 4*a*b^2*x^3 - 3*(b^3*c*x^2 - b^3*x^3)*log(c + x)^2 + 2*(b^3*c^3 -
 6*a*b^2*c*x^2 + (6*a*b^2 + b^3)*x^3)*log(c + x))*log(-c + x))/(c - x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(b^3x^2\,\text{artanh}\left(\frac{c}{x}\right)^3 + 3\,ab^2x^2\,\text{artanh}\left(\frac{c}{x}\right)^2 + 3\,a^2bx^2\,\text{artanh}\left(\frac{c}{x}\right) + a^3x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c/x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^2*arctanh(c/x)^3 + 3*a*b^2*x^2*arctanh(c/x)^2 + 3*a^2*b*x^2*
arctanh(c/x) + a^3*x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\left(a + b\,\text{atanh}\left(\frac{c}{x}\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c/x))**3,x)
```

```
[Out] Integral(x**2*(a + b*atanh(c/x))**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\,\text{artanh}\left(\frac{c}{x}\right) + a\right)^3 x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c/x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c/x) + a)^3*x^2, x)
```

3.152 $\int x\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^3\,dx$

Optimal. Leaf size=135

$$\frac{3}{2}b^3c^2\text{PolyLog}\left(2,\frac{2}{\frac{c}{x}+1}-1\right)-3b^2c^2\log\left(2-\frac{2}{\frac{c}{x}+1}\right)\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)-\frac{3}{2}bc^2\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2-\frac{1}{2}c^2\left(a+b\coth^{-}\right.$$

```
[Out] (-3*b*c^2*(a + b*ArcCoth[x/c])^2)/2 + (3*b*c*x*(a + b*ArcCoth[x/c])^2)/2 -
(c^2*(a + b*ArcCoth[x/c])^3)/2 + (x^2*(a + b*ArcCoth[x/c])^3)/2 - 3*b^2*c^2
*(a + b*ArcCoth[x/c])*Log[2 - 2/(1 + c/x)] + (3*b^3*c^2*PolyLog[2, -1 + 2/(
1 + c/x)])/2
```

Rubi [F] time = 2.24482, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^3\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x*(a + b*ArcTanh[c/x])^3,x]
```

```
[Out] (3*a^2*b*c*x)/4 - (3*a*b^2*c*x*Log[1 - c/x])/4 + (3*b*c*(1 - c/x)*x*(2*a -
b*Log[1 - c/x])^2)/16 - (c^2*(2*a - b*Log[1 - c/x])^3)/16 + (x^2*(2*a - b*L
og[1 - c/x])^3)/16 + (3*a*b^2*c*x*Log[1 + c/x])/4 + (3*a^2*b*x^2*Log[1 + c/
x])/4 - (3*a*b^2*x^2*Log[1 - c/x]*Log[1 + c/x])/4 + (3*a*b^2*c^2*Log[c - x]
)/4 + (3*a*b^2*c^2*Log[1 + c/x]*Log[c - x])/4 - (3*b*c^2*(2*a - b*Log[1 - c
/x])^2*Log[c/x])/16 + (3*a*b^2*c^2*Log[x])/2 + (3*a*b^2*c^2*Log[c - x]*Log[
x/c])/4 - (3*a^2*b*c^2*Log[c + x])/4 + (3*a*b^2*c^2*Log[c + x])/4 + (3*a*b^
2*c^2*Log[1 - c/x]*Log[c + x])/4 - (3*a*b^2*c^2*Log[(c - x)/(2*c)]*Log[c +
x])/4 + (3*a*b^2*c^2*Log[-(x/c)]*Log[c + x])/4 - (3*a*b^2*c^2*Log[c - x]*Lo
g[(c + x)/(2*c)])/4 + (3*a*b^2*c^2*Log[(c + x)/x])/4 + (3*a*b^2*c*x*Log[(c
+ x)/x])/4 - (3*a*b^2*c^2*Log[(c + x)/x]^2)/8 + (3*b^3*c*(1 + c/x)*x*Log[(c
 + x)/x]^2)/16 + (3*a*b^2*x^2*Log[(c + x)/x]^2)/8 + (3*b^3*c^2*Log[-(c/x)]*
Log[(c + x)/x]^2)/16 - (b^3*c^2*Log[(c + x)/x]^3)/16 + (b^3*x^2*Log[(c + x)
/x]^3)/16 + (3*b^2*c^2*(2*a - b*Log[1 - c/x])*PolyLog[2, 1 - c/x])/8 - (3*a
*b^2*c^2*PolyLog[2, (c - x)/(2*c)])/4 - (3*a*b^2*c^2*PolyLog[2, -(c/x)])/4
+ (3*b^3*c^2*PolyLog[2, -(c/x)])/8 - (3*b^3*c^2*PolyLog[2, c/x])/8 - (3*a*b
^2*c^2*PolyLog[2, (c + x)/(2*c)])/4 + (3*b^3*c^2*Log[(c + x)/x]*PolyLog[2,
(c + x)/x])/8 + (3*a*b^2*c^2*PolyLog[2, 1 - x/c])/4 + (3*a*b^2*c^2*PolyLog[
2, 1 + x/c])/4 + (3*b^3*c^2*PolyLog[3, 1 - c/x])/8 - (3*b^3*c^2*PolyLog[3,
(c + x)/x])/8 + (3*b^3*Defer[Int][x*Log[1 - c/x]^2*Log[1 + c/x], x])/8 - (3
*b^3*Defer[Int][x*Log[1 - c/x]*Log[1 + c/x]^2, x])/8
```

Rubi steps

$$\int x\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^3 dx = \int \left(\frac{1}{8}x\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3+\frac{3}{8}bx\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2\log\left(1+\frac{c}{x}\right)+\frac{3}{8}b^2x\left(2a-b\log\right.\right.$$

$$=\frac{1}{8}\int x\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3 dx+\frac{1}{8}(3b)\int x\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2\log\left(1+\frac{c}{x}\right)dx+\frac{1}{8}$$

$$=-\left(\frac{1}{8}\text{Subst}\left(\int\frac{(2a-b\log(1-cx))^3}{x^3}dx,x,\frac{1}{x}\right)\right)+\frac{1}{8}(3b)\int\left(4a^2x\log\left(1+\frac{c}{x}\right)-4abx\log\right.$$

$$=\frac{1}{16}x^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3+\frac{1}{16}b^3x^2\log^3\left(\frac{c+x}{x}\right)+\frac{1}{2}\left(3a^2b\right)\int x\log\left(1+\frac{c}{x}\right)dx+\frac{1}{4}$$

$$=\frac{1}{16}x^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3+\frac{3}{4}a^2bx^2\log\left(1+\frac{c}{x}\right)-\frac{3}{4}ab^2x^2\log\left(1-\frac{c}{x}\right)\log\left(1+\frac{c}{x}\right)+$$

$$=\frac{1}{16}x^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3+\frac{3}{4}a^2bx^2\log\left(1+\frac{c}{x}\right)-\frac{3}{4}ab^2x^2\log\left(1-\frac{c}{x}\right)\log\left(1+\frac{c}{x}\right)+$$

$$=\frac{3}{16}bc\left(1-\frac{c}{x}\right)x\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2+\frac{1}{16}x^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3+\frac{3}{4}a^2bx^2\log\left(1+\right.$$

$$=\frac{3}{4}a^2bcx+\frac{3}{16}bc\left(1-\frac{c}{x}\right)x\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2+\frac{1}{16}x^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3+\frac{3}{4}a^2bx^2$$

$$=\frac{3}{4}a^2bcx-\frac{3}{4}ab^2cx\log\left(1-\frac{c}{x}\right)+\frac{3}{16}bc\left(1-\frac{c}{x}\right)x\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2-\frac{1}{16}c^2\left(2a-b\log\right.$$

$$=\frac{3}{4}a^2bcx-\frac{3}{4}ab^2cx\log\left(1-\frac{c}{x}\right)+\frac{3}{16}bc\left(1-\frac{c}{x}\right)x\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2-\frac{1}{16}c^2\left(2a-b\log\right.$$

$$=\frac{3}{4}a^2bcx-\frac{3}{4}ab^2cx\log\left(1-\frac{c}{x}\right)+\frac{3}{16}bc\left(1-\frac{c}{x}\right)x\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2-\frac{1}{16}c^2\left(2a-b\log\right.$$

$$=\frac{3}{4}a^2bcx-\frac{3}{4}ab^2cx\log\left(1-\frac{c}{x}\right)+\frac{3}{16}bc\left(1-\frac{c}{x}\right)x\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2-\frac{1}{16}c^2\left(2a-b\log\right.$$

$$=\frac{3}{4}a^2bcx-\frac{3}{4}ab^2cx\log\left(1-\frac{c}{x}\right)+\frac{3}{16}bc\left(1-\frac{c}{x}\right)x\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2-\frac{1}{16}c^2\left(2a-b\log\right.$$

$$=\frac{3}{4}a^2bcx-\frac{3}{4}ab^2cx\log\left(1-\frac{c}{x}\right)+\frac{3}{16}bc\left(1-\frac{c}{x}\right)x\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2-\frac{1}{16}c^2\left(2a-b\log\right.$$

Mathematica [A] time = 0.317704, size = 193, normalized size = 1.43

$$\frac{1}{4}\left(6b^3c^2\text{PolyLog}\left(2,e^{-2\tanh^{-1}\left(\frac{c}{x}\right)}\right)+a\left(a\left(2ax^2-3bc^2\log\left(\frac{c+x}{x}\right)+6bcx\right)+3abc^2\log\left(1-\frac{c}{x}\right)-12b^2c^2\log\left(\frac{c}{x\sqrt{1-\ldots}}\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(a + b*ArcTanh[c/x])^3,x]
```

```
[Out] (6*b^2*(-c + x)*(b*c + a*(c + x))*ArcTanh[c/x]^2 + 2*b^3*(-c^2 + x^2)*ArcTa
nh[c/x]^3 + 6*b*ArcTanh[c/x]*(a*x*(2*b*c + a*x) - 2*b^2*c^2*Log[1 - E^(-2*A
rcTanh[c/x])]) + a*(3*a*b*c^2*Log[1 - c/x] - 12*b^2*c^2*Log[c/(Sqrt[1 - c^2
/x^2]*x)] + a*(6*b*c*x + 2*a*x^2 - 3*b*c^2*Log[(c + x)/x])) + 6*b^3*c^2*Pol
yLog[2, E^(-2*ArcTanh[c/x])])/4
```

Maple [C] time = 0.278, size = 5536, normalized size = 41.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c/x))^3,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{3}{2}ab^2x^2\operatorname{artanh}\left(\frac{c}{x}\right)^2+\frac{1}{2}a^3x^2+\frac{3}{4}\left(2x^2\operatorname{artanh}\left(\frac{c}{x}\right)-(c\log(c+x)-c\log(-c+x)-2x)c\right)a^2b+\frac{3}{8}\left(\left(\log(c+x)^2-2(\mathrm{lo}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c/x))^3,x, algorithm="maxima")
```

```
[Out] 3/2*a*b^2*x^2*arctanh(c/x)^2 + 1/2*a^3*x^2 + 3/4*(2*x^2*arctanh(c/x) - (c*l
og(c + x) - c*log(-c + x) - 2*x)*c)*a^2*b + 3/8*((log(c + x)^2 - 2*(log(c +
 x) - 2)*log(-c + x) + log(-c + x)^2 + 4*log(c + x))*c^2 - 4*(c*log(c + x)
- c*log(-c + x) - 2*x)*c*arctanh(c/x))*a*b^2 + 1/16*(6*c*x*log(c + x)^2 - (
c^2 - x^2)*log(c + x)^3 + (c^2 - x^2)*log(-c + x)^3 - 3*(2*c^2 - 2*c*x + (c
^2 - x^2)*log(c + x))*log(-c + x)^2 + 3*((c^2 - x^2)*log(c + x)^2 - 4*(c^2
+ c*x)*log(c + x))*log(-c + x) + 2*integrate(-6*(c^3 + 3*c^2*x)*log(c + x)/
(c^2 - x^2), x))*b^3
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(b^3x\operatorname{artanh}\left(\frac{c}{x}\right)^3+3\,ab^2x\operatorname{artanh}\left(\frac{c}{x}\right)^2+3\,a^2bx\operatorname{artanh}\left(\frac{c}{x}\right)+a^3x,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c/x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x*arctanh(c/x)^3 + 3*a*b^2*x*arctanh(c/x)^2 + 3*a^2*b*x*arctan
h(c/x) + a^3*x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(a+b\operatorname{atanh}\left(\frac{c}{x}\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c/x))**3,x)
```

```
[Out] Integral(x*(a + b*atanh(c/x))**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \operatorname{artanh}\left(\frac{c}{x}\right)+a\right)^3 x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c/x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c/x) + a)^3*x, x)
```

3.153 $\int \left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^3 dx$

Optimal. Leaf size=108

$$-3b^2c\text{PolyLog}\left(2,1-\frac{2c}{c-x}\right)\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)+\frac{3}{2}b^3c\text{PolyLog}\left(3,1-\frac{2c}{c-x}\right)+c\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3+x\left(a+b\coth^{-1}\right.$$

```
[Out] c*(a + b*ArcCoth[x/c])^3 + x*(a + b*ArcCoth[x/c])^3 - 3*b*c*(a + b*ArcCoth[
x/c])^2*Log[(2*c)/(c - x)] - 3*b^2*c*(a + b*ArcCoth[x/c])*PolyLog[2, 1 - (2
*c)/(c - x)] + (3*b^3*c*PolyLog[3, 1 - (2*c)/(c - x)])/2
```

Rubi [F] time = 0.700751, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^3 dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcTanh[c/x])^3,x]
```

```
[Out] a^3*x - (3*a^2*b*x*Log[1 - c/x])/2 - (3*a*b^2*(c - x)*Log[1 - c/x]^2)/4 + (
b^3*(c - x)*Log[1 - c/x]^3)/8 + (3*a^2*b*x*Log[1 + c/x])/2 - (3*a*b^2*x*Log
[1 - c/x]*Log[1 + c/x])/2 + (3*a*b^2*(c + x)*Log[1 + c/x]^2)/4 + (b^3*(c +
x)*Log[1 + c/x]^3)/8 - (3*a*b^2*c*Log[1 - c/x]*Log[-c - x])/2 + (3*a^2*b*c*
Log[c - x])/2 + (3*a*b^2*c*Log[-c - x]*Log[(c - x)/(2*c)])/2 - (3*b^3*c*Log
[1 - c/x]^2*Log[c/x])/8 - (3*a*b^2*c*Log[-c - x]*Log[-(x/c)])/2 + (3*a*b^2*
c*Log[1 + c/x]*Log[-c + x])/2 + (3*a*b^2*c*Log[x/c]*Log[-c + x])/2 + (3*a^2
*b*c*Log[c + x])/2 - (3*a*b^2*c*Log[-c + x]*Log[(c + x)/(2*c)])/2 - (3*b^3*
c*Log[-(c/x)]*Log[(c + x)/x]^2)/8 - (3*b^3*c*Log[1 - c/x]*PolyLog[2, 1 - c/
x])/4 - (3*a*b^2*c*PolyLog[2, (c - x)/(2*c)])/2 + (3*a*b^2*c*PolyLog[2, -(c
/x)])/2 - (3*a*b^2*c*PolyLog[2, c/x])/2 + (3*a*b^2*c*PolyLog[2, (c + x)/(2*
c)])/2 - (3*b^3*c*Log[(c + x)/x]*PolyLog[2, (c + x)/x])/4 + (3*a*b^2*c*Poly
Log[2, 1 - x/c])/2 - (3*a*b^2*c*PolyLog[2, 1 + x/c])/2 + (3*b^3*c*PolyLog[3
, 1 - c/x])/4 + (3*b^3*c*PolyLog[3, (c + x)/x])/4 + (3*b^3*Defer[Int][Log[1
 - c/x]^2*Log[1 + c/x], x])/8 - (3*b^3*Defer[Int][Log[1 - c/x]*Log[1 + c/x]
^2, x])/8
```

Rubi steps

$$\int \left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^3 dx = \int \left(a^3-\frac{3}{2}a^2b\log\left(1-\frac{c}{x}\right)+\frac{3}{4}ab^2\log^2\left(1-\frac{c}{x}\right)-\frac{1}{8}b^3\log^3\left(1-\frac{c}{x}\right)+\frac{3}{2}a^2b\log\left(1+\frac{c}{x}\right)-$$

$$= a^3x-\frac{1}{2}\left(3a^2b\right)\int \log\left(1-\frac{c}{x}\right)dx+\frac{1}{2}\left(3a^2b\right)\int \log\left(1+\frac{c}{x}\right)dx+\frac{1}{4}\left(3ab^2\right)\int \log^2\left(1-\frac{c}{x}\right)$$

$$= a^3x-\frac{3}{2}a^2bx\log\left(1-\frac{c}{x}\right)-\frac{3}{4}ab^2(c-x)\log^2\left(1-\frac{c}{x}\right)+\frac{1}{8}b^3(c-x)\log^3\left(1-\frac{c}{x}\right)+\frac{3}{2}a^2bx\,l$$

$$= a^3x-\frac{3}{2}a^2bx\log\left(1-\frac{c}{x}\right)-\frac{3}{4}ab^2(c-x)\log^2\left(1-\frac{c}{x}\right)+\frac{1}{8}b^3(c-x)\log^3\left(1-\frac{c}{x}\right)+\frac{3}{2}a^2bx\,l$$

$$= a^3x-\frac{3}{2}a^2bx\log\left(1-\frac{c}{x}\right)-\frac{3}{4}ab^2(c-x)\log^2\left(1-\frac{c}{x}\right)+\frac{1}{8}b^3(c-x)\log^3\left(1-\frac{c}{x}\right)+\frac{3}{2}a^2bx\,l$$

$$= a^3x-\frac{3}{2}a^2bx\log\left(1-\frac{c}{x}\right)-\frac{3}{4}ab^2(c-x)\log^2\left(1-\frac{c}{x}\right)+\frac{1}{8}b^3(c-x)\log^3\left(1-\frac{c}{x}\right)+\frac{3}{2}a^2bx\,l$$

$$= a^3x-\frac{3}{2}a^2bx\log\left(1-\frac{c}{x}\right)-\frac{3}{4}ab^2(c-x)\log^2\left(1-\frac{c}{x}\right)+\frac{1}{8}b^3(c-x)\log^3\left(1-\frac{c}{x}\right)+\frac{3}{2}a^2bx\,l$$

$$= a^3x-\frac{3}{2}a^2bx\log\left(1-\frac{c}{x}\right)-\frac{3}{4}ab^2(c-x)\log^2\left(1-\frac{c}{x}\right)+\frac{1}{8}b^3(c-x)\log^3\left(1-\frac{c}{x}\right)+\frac{3}{2}a^2bx\,l$$

$$= a^3x-\frac{3}{2}a^2bx\log\left(1-\frac{c}{x}\right)-\frac{3}{4}ab^2(c-x)\log^2\left(1-\frac{c}{x}\right)+\frac{1}{8}b^3(c-x)\log^3\left(1-\frac{c}{x}\right)+\frac{3}{2}a^2bx\,l$$

$$= a^3x-\frac{3}{2}a^2bx\log\left(1-\frac{c}{x}\right)-\frac{3}{4}ab^2(c-x)\log^2\left(1-\frac{c}{x}\right)+\frac{1}{8}b^3(c-x)\log^3\left(1-\frac{c}{x}\right)+\frac{3}{2}a^2bx\,l$$

Mathematica [C] time = 0.263006, size = 198, normalized size = 1.83

$$-3ab^2\left(\tanh^{-1}\left(\frac{c}{x}\right)\left((c-x)\tanh^{-1}\left(\frac{c}{x}\right)+2c\log\left(1-e^{-2\tanh^{-1}\left(\frac{c}{x}\right)}\right)\right)-c\text{PolyLog}\left(2,e^{-2\tanh^{-1}\left(\frac{c}{x}\right)}\right)\right)+\frac{1}{8}b^3\left(-24c\tanh\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c/x])^3,x]
```

```
[Out] a^3*x + 3*a^2*b*x*ArcTanh[c/x] + (3*a^2*b*c*Log[-c^2 + x^2])/2 - 3*a*b^2*(A
rcTanh[c/x]*((c - x)*ArcTanh[c/x] + 2*c*Log[1 - E^(-2*ArcTanh[c/x])]) - c*P
olyLog[2, E^(-2*ArcTanh[c/x])]) + (b^3*((-I)*c*Pi^3 + 8*c*ArcTanh[c/x]^3 +
8*x*ArcTanh[c/x]^3 - 24*c*ArcTanh[c/x]^2*Log[1 - E^(2*ArcTanh[c/x])] - 24*c
*ArcTanh[c/x]*PolyLog[2, E^(2*ArcTanh[c/x])] + 12*c*PolyLog[3, E^(2*ArcTanh
[c/x])]))/8
```

Maple [C] time = 0.121, size = 1756, normalized size = 16.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x))^3,x)
```

```
[Out] -6*c*a*b^2*ln(c/x)*arctanh(c/x)+3*c*a*b^2*ln(c/x)*ln(1+c/x)+3/2*c*a*b^2*ln(
-1/2*c/x+1/2)*ln(1+c/x)-3/2*c*a*b^2*ln(-1/2*c/x+1/2)*ln(1/2+1/2*c/x)+3*c*a*
b^2*arctanh(c/x)*ln(c/x-1)+3*c*a*b^2*arctanh(c/x)*ln(1+c/x)-3/2*c*a*b^2*ln(
```

```
c/x-1)*ln(1/2+1/2*c/x)-3/2*I*c*b^3*Pi*csgn(I*((1+c/x)^2/(1-c^2/x^2)-1))*csg
n(I/((1+c/x)^2/(1-c^2/x^2)+1))*csgn(I*((1+c/x)^2/(1-c^2/x^2)-1)/((1+c/x)^2/
(1-c^2/x^2)+1))*arctanh(c/x)^2+6*c*b^3*polylog(3,-(1+c/x)/(1-c^2/x^2)^(1/2)
)+6*c*b^3*polylog(3,(1+c/x)/(1-c^2/x^2)^(1/2))+c*b^3*arctanh(c/x)^3+b^3*x*a
rctanh(c/x)^3+3/4*I*c*b^3*Pi*csgn(I*(1+c/x)^2/(-1+c^2/x^2))*csgn(I*(1+c/x)^
2/(-1+c^2/x^2)/((1+c/x)^2/(1-c^2/x^2)+1))^2*arctanh(c/x)^2+3/2*I*c*b^3*Pi*c
sgn(I/((1+c/x)^2/(1-c^2/x^2)+1))^2*arctanh(c/x)^2-3/2*I*c*b^3*Pi*csgn(I*((1
+c/x)^2/(1-c^2/x^2)-1)/((1+c/x)^2/(1-c^2/x^2)+1))^3*arctanh(c/x)^2-3/2*I*c*
b^3*Pi*csgn(I/((1+c/x)^2/(1-c^2/x^2)+1))^3*arctanh(c/x)^2-3/4*I*c*b^3*Pi*cs
gn(I*(1+c/x)/(1-c^2/x^2)^(1/2))^2*csgn(I*(1+c/x)^2/(-1+c^2/x^2))*arctanh(c/
x)^2-3/4*I*c*b^3*Pi*csgn(I/((1+c/x)^2/(1-c^2/x^2)+1))*csgn(I*(1+c/x)^2/(-1+
c^2/x^2)/((1+c/x)^2/(1-c^2/x^2)+1))^2*arctanh(c/x)^2-3/2*I*c*b^3*Pi*csgn(I*
(1+c/x)/(1-c^2/x^2)^(1/2))*csgn(I*(1+c/x)^2/(-1+c^2/x^2))^2*arctanh(c/x)^2+
3/4*I*c*b^3*Pi*csgn(I/((1+c/x)^2/(1-c^2/x^2)+1))*csgn(I*(1+c/x)^2/(-1+c^2/x
^2))*csgn(I*(1+c/x)^2/(-1+c^2/x^2)/((1+c/x)^2/(1-c^2/x^2)+1))*arctanh(c/x)^
2+x*a^3-6*c*b^3*arctanh(c/x)*polylog(2,(1+c/x)/(1-c^2/x^2)^(1/2))-6*c*b^3*a
rctanh(c/x)*polylog(2,-(1+c/x)/(1-c^2/x^2)^(1/2))-3*c*b^3*arctanh(c/x)^2*ln
(1-(1+c/x)/(1-c^2/x^2)^(1/2))+3*c*a*b^2*dilog(c/x)+3/2*c*a^2*b*ln(c/x-1)+3/
2*c*a^2*b*ln(1+c/x)+3/4*c*a*b^2*ln(c/x-1)^2-3/4*c*a*b^2*ln(1+c/x)^2-3*c*b^3
*arctanh(c/x)^2*ln(2)-3*c*a^2*b*ln(c/x)+3*c*a*b^2*dilog(1+c/x)-3*c*a*b^2*di
log(1/2+1/2*c/x)+3*a*b^2*x*arctanh(c/x)^2+3*a^2*b*x*arctanh(c/x)+3/2*c*b^3*
arctanh(c/x)^2*ln(c/x-1)+3/2*c*b^3*arctanh(c/x)^2*ln(1+c/x)-3*c*b^3*arctanh
(c/x)^2*ln((1+c/x)/(1-c^2/x^2)^(1/2))-3*c*b^3*arctanh(c/x)^2*ln(1+(1+c/x)/(
1-c^2/x^2)^(1/2))-3*c*b^3*ln(c/x)*arctanh(c/x)^2+3*c*b^3*arctanh(c/x)^2*ln(
(1+c/x)^2/(1-c^2/x^2)-1)-3/4*I*c*b^3*Pi*csgn(I*(1+c/x)^2/(-1+c^2/x^2)/((1+c
/x)^2/(1-c^2/x^2)+1))^3*arctanh(c/x)^2-3/4*I*c*b^3*Pi*csgn(I*(1+c/x)^2/(-1+
c^2/x^2))^3*arctanh(c/x)^2-3/2*I*c*b^3*Pi*arctanh(c/x)^2+3/2*I*c*b^3*Pi*csg
n(I*((1+c/x)^2/(1-c^2/x^2)-1))*csgn(I*((1+c/x)^2/(1-c^2/x^2)-1)/((1+c/x)^2/
(1-c^2/x^2)+1))^2*arctanh(c/x)^2+3/2*I*c*b^3*Pi*csgn(I/((1+c/x)^2/(1-c^2/x^
2)+1))*csgn(I*((1+c/x)^2/(1-c^2/x^2)-1)/((1+c/x)^2/(1-c^2/x^2)+1))^2*arctan
h(c/x)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{3}{2}\left(2\,x\,\operatorname{artanh}\left(\frac{c}{x}\right)+c\log\left(-c^2+x^2\right)\right)a^2b+a^3x+\frac{1}{8}\left(b^3c-b^3x\right)\log\left(-c+x\right)^3+\frac{3}{8}\left(2\,ab^2x+\left(b^3c+b^3x\right)\log\left(c+x\right)\right)\log\left(\cdot\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^3,x, algorithm="maxima")
```

```
[Out] 3/2*(2*x*arctanh(c/x) + c*log(-c^2 + x^2))*a^2*b + a^3*x + 1/8*(b^3*c - b^3
*x)*log(-c + x)^3 + 3/8*(2*a*b^2*x + (b^3*c + b^3*x)*log(c + x))*log(-c + x
)^2 - integrate(-1/8*((b^3*c - b^3*x)*log(c + x)^3 + 6*(a*b^2*c - a*b^2*x)*
log(c + x)^2 + 3*(4*a*b^2*x - (b^3*c - b^3*x)*log(c + x)^2 - 2*(2*a*b^2*c -
 b^3*c - (2*a*b^2 + b^3)*x)*log(c + x))*log(-c + x))/(c - x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(b^3\operatorname{artanh}\left(\frac{c}{x}\right)^3+3\,ab^2\operatorname{artanh}\left(\frac{c}{x}\right)^2+3\,a^2b\operatorname{artanh}\left(\frac{c}{x}\right)+a^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*arctanh(c/x)^3 + 3*a*b^2*arctanh(c/x)^2 + 3*a^2*b*arctanh(c/x)
 + a^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+b\operatorname{atanh}\left(\frac{c}{x}\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x))**3,x)
```

```
[Out] Integral((a + b*atanh(c/x))**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\operatorname{artanh}\left(\frac{c}{x}\right)+a\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c/x) + a)^3, x)
```

3.154 $\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^3}{x}\,dx$

Optimal. Leaf size=208

$$-\frac{3}{2}b^2\text{PolyLog}\left(3,1-\frac{2}{1-\frac{c}{x}}\right)\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)+\frac{3}{2}b^2\text{PolyLog}\left(3,\frac{2}{1-\frac{c}{x}}-1\right)\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)+\frac{3}{2}b\text{PolyLog}\left(2,1-\right.$$

```
[Out] -2*(a + b*ArcCoth[x/c])^3*ArcTanh[1 - 2/(1 - c/x)] + (3*b*(a + b*ArcCoth[x/
c])^2*PolyLog[2, 1 - 2/(1 - c/x)])/2 - (3*b*(a + b*ArcCoth[x/c])^2*PolyLog[
2, -1 + 2/(1 - c/x)])/2 - (3*b^2*(a + b*ArcCoth[x/c])*PolyLog[3, 1 - 2/(1 -
 c/x)])/2 + (3*b^2*(a + b*ArcCoth[x/c])*PolyLog[3, -1 + 2/(1 - c/x)])/2 + (
3*b^3*PolyLog[4, 1 - 2/(1 - c/x)])/4 - (3*b^3*PolyLog[4, -1 + 2/(1 - c/x)])
/4
```

Rubi [A] time = 0.508605, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {6095, 5914, 6052, 5948, 6058, 6062, 6610}

$$-\frac{3}{2}b^2\text{PolyLog}\left(3,1-\frac{2}{1-\frac{c}{x}}\right)\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)+\frac{3}{2}b^2\text{PolyLog}\left(3,\frac{2}{1-\frac{c}{x}}-1\right)\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)+\frac{3}{2}b\text{PolyLog}\left(2,1-\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c/x])^3/x,x]
```


```
[Out] -2*(a + b*ArcCoth[x/c])^3*ArcTanh[1 - 2/(1 - c/x)] + (3*b*(a + b*ArcCoth[x/
c])^2*PolyLog[2, 1 - 2/(1 - c/x)])/2 - (3*b*(a + b*ArcCoth[x/c])^2*PolyLog[
2, -1 + 2/(1 - c/x)])/2 - (3*b^2*(a + b*ArcCoth[x/c])*PolyLog[3, 1 - 2/(1 -
 c/x)])/2 + (3*b^2*(a + b*ArcCoth[x/c])*PolyLog[3, -1 + 2/(1 - c/x)])/2 + (
3*b^3*PolyLog[4, 1 - 2/(1 - c/x)])/4 - (3*b^3*PolyLog[4, -1 + 2/(1 - c/x)])
/4
```


Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[
1/n, Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
 n}, x] && IGtQ[p, 0]
```

Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
 e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6062

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[k + 1, u])/(2
*c*d), x] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[k + 1,
u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && Eq
Q[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^3}{x}\,dx = -\operatorname{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{x}\,dx, x, \frac{1}{x}\right)$$

$$= -2\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1-\frac{c}{x}}\right)+(6bc)\operatorname{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2\tanh^{-1}}{1-c^2x^2}\right.$$

$$= -2\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1-\frac{c}{x}}\right)-(3bc)\operatorname{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{}{1}\right.}{1-c^2x^2}\right.$$

$$= -2\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1-\frac{c}{x}}\right)+\frac{3}{2}b\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2\operatorname{Li}_2\left(1-\frac{2}{1-\frac{c}{x}}\right)-\frac{3}{2}$$

$$= -2\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1-\frac{c}{x}}\right)+\frac{3}{2}b\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2\operatorname{Li}_2\left(1-\frac{2}{1-\frac{c}{x}}\right)-\frac{3}{2}$$

$$= -2\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1-\frac{c}{x}}\right)+\frac{3}{2}b\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2\operatorname{Li}_2\left(1-\frac{2}{1-\frac{c}{x}}\right)-\frac{3}{2}$$

Mathematica [A] time = 0.191137, size = 171, normalized size = 0.82

$$\frac{3}{4}b\left(2\operatorname{PolyLog}\left(2,\frac{c+x}{c-x}\right)\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^2-2\operatorname{PolyLog}\left(2,\frac{c+x}{x-c}\right)\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^2+b\left(-2\operatorname{PolyLog}\left(3,\frac{c+x}{c-x}\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c/x])^3/x,x]
```

```
[Out] -2*(a + b*ArcTanh[c/x])^3*ArcTanh[(c + x)/(c - x)] + (3*b*(2*(a + b*ArcTanh
[c/x])^2*PolyLog[2, (c + x)/(c - x)] - 2*(a + b*ArcTanh[c/x])^2*PolyLog[2,
(c + x)/(-c + x)] + b*(-2*(a + b*ArcTanh[c/x])*PolyLog[3, (c + x)/(c - x)]
+ 2*(a + b*ArcTanh[c/x])*PolyLog[3, (c + x)/(-c + x)] + b*(PolyLog[4, (c +
x)/(c - x)] - PolyLog[4, (c + x)/(-c + x)]))))/4
```

Maple [C] time = 0.07, size = 1631, normalized size = 7.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x))^3/x,x)
```

```
[Out] 3/2*a^2*b*dilog(1+c/x)-3/2*a*b^2*polylog(3,-(1+c/x)^2/(1-c^2/x^2))+6*a*b^2*
polylog(3,(1+c/x)/(1-c^2/x^2)^(1/2))+6*a*b^2*polylog(3,-(1+c/x)/(1-c^2/x^2)
^(1/2))-b^3*arctanh(c/x)^3*ln(1-(1+c/x)/(1-c^2/x^2)^(1/2))-3*b^3*arctanh(c/
x)^2*polylog(2,(1+c/x)/(1-c^2/x^2)^(1/2))+6*b^3*arctanh(c/x)*polylog(3,(1+c
/x)/(1-c^2/x^2)^(1/2))-b^3*arctanh(c/x)^3*ln(1+(1+c/x)/(1-c^2/x^2)^(1/2))-3
*b^3*arctanh(c/x)^2*polylog(2,-(1+c/x)/(1-c^2/x^2)^(1/2))+6*b^3*arctanh(c/x
)*polylog(3,-(1+c/x)/(1-c^2/x^2)^(1/2))-b^3*ln(c/x)*arctanh(c/x)^3+b^3*arct
anh(c/x)^3*ln((1+c/x)^2/(1-c^2/x^2)-1)+3/2*b^3*arctanh(c/x)^2*polylog(2,-(1
+c/x)^2/(1-c^2/x^2))-3/2*b^3*arctanh(c/x)*polylog(3,-(1+c/x)^2/(1-c^2/x^2))
+3/2*a^2*b*dilog(c/x)-3/2*I*a*b^2*Pi*csgn(I*((1+c/x)^2/(1-c^2/x^2)-1))*csgn
(I/((1+c/x)^2/(1-c^2/x^2)+1))*csgn(I*((1+c/x)^2/(1-c^2/x^2)-1)/((1+c/x)^2/(
1-c^2/x^2)+1))*arctanh(c/x)^2-3*a*b^2*arctanh(c/x)^2*ln(1+(1+c/x)/(1-c^2/x^
2)^(1/2))-6*a*b^2*arctanh(c/x)*polylog(2,-(1+c/x)/(1-c^2/x^2)^(1/2))-3*a*b^
2*ln(c/x)*arctanh(c/x)^2+3*a*b^2*arctanh(c/x)*polylog(2,-(1+c/x)^2/(1-c^2/x
^2))-3*a^2*b*ln(c/x)*arctanh(c/x)+3/2*a^2*b*ln(c/x)*ln(1+c/x)+3*a*b^2*arcta
nh(c/x)^2*ln((1+c/x)^2/(1-c^2/x^2)-1)-3*a*b^2*arctanh(c/x)^2*ln(1-(1+c/x)/(
1-c^2/x^2)^(1/2))-6*a*b^2*arctanh(c/x)*polylog(2,(1+c/x)/(1-c^2/x^2)^(1/2))
+1/2*I*b^3*Pi*csgn(I*((1+c/x)^2/(1-c^2/x^2)-1))*csgn(I*((1+c/x)^2/(1-c^2/x^
2)-1)/((1+c/x)^2/(1-c^2/x^2)+1))^2*arctanh(c/x)^3+1/2*I*b^3*Pi*csgn(I/((1+c
/x)^2/(1-c^2/x^2)+1))*csgn(I*((1+c/x)^2/(1-c^2/x^2)-1)/((1+c/x)^2/(1-c^2/x^
2)+1))^2*arctanh(c/x)^3-3/2*I*a*b^2*Pi*csgn(I*((1+c/x)^2/(1-c^2/x^2)-1)/((1
+c/x)^2/(1-c^2/x^2)+1))^3*arctanh(c/x)^2+3/2*I*a*b^2*Pi*csgn(I*((1+c/x)^2/(
1-c^2/x^2)-1))*csgn(I*((1+c/x)^2/(1-c^2/x^2)-1)/((1+c/x)^2/(1-c^2/x^2)+1))^
2*arctanh(c/x)^2+3/2*I*a*b^2*Pi*csgn(I/((1+c/x)^2/(1-c^2/x^2)+1))*csgn(I*((
1+c/x)^2/(1-c^2/x^2)-1)/((1+c/x)^2/(1-c^2/x^2)+1))^2*arctanh(c/x)^2-1/2*I*b
^3*Pi*csgn(I*((1+c/x)^2/(1-c^2/x^2)-1))*csgn(I/((1+c/x)^2/(1-c^2/x^2)+1))*c
sgn(I*((1+c/x)^2/(1-c^2/x^2)-1)/((1+c/x)^2/(1-c^2/x^2)+1))*arctanh(c/x)^3-a
^3*ln(c/x)+3/4*b^3*polylog(4,-(1+c/x)^2/(1-c^2/x^2))-6*b^3*polylog(4,(1+c/x
)/(1-c^2/x^2)^(1/2))-6*b^3*polylog(4,-(1+c/x)/(1-c^2/x^2)^(1/2))-1/2*I*b^3*
Pi*csgn(I*((1+c/x)^2/(1-c^2/x^2)-1)/((1+c/x)^2/(1-c^2/x^2)+1))^3*arctanh(c/
x)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$a^3 \log(x) + \int \frac{b^3\left(\log\left(\frac{c}{x}+1\right) - \log\left(-\frac{c}{x}+1\right)\right)^3}{8\,x} + \frac{3\,ab^2\left(\log\left(\frac{c}{x}+1\right) - \log\left(-\frac{c}{x}+1\right)\right)^2}{4\,x} + \frac{3\,a^2b\left(\log\left(\frac{c}{x}+1\right) - \log\left(-\frac{c}{x}+1\right)\right)}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^3/x,x, algorithm="maxima")
```

```
[Out] a^3*log(x) + integrate(1/8*b^3*(log(c/x + 1) - log(-c/x + 1))^3/x + 3/4*a*b
^2*(log(c/x + 1) - log(-c/x + 1))^2/x + 3/2*a^2*b*(log(c/x + 1) - log(-c/x
+ 1))/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3\,\text{artanh}\left(\frac{c}{x}\right)^3 + 3\,ab^2\,\text{artanh}\left(\frac{c}{x}\right)^2 + 3\,a^2b\,\text{artanh}\left(\frac{c}{x}\right) + a^3}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^3/x,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctanh(c/x)^3 + 3*a*b^2*arctanh(c/x)^2 + 3*a^2*b*arctanh(c/x
) + a^3)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + b\,\text{atanh}\left(\frac{c}{x}\right)\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x))**3/x,x)
```

```
[Out] Integral((a + b*atanh(c/x))**3/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\,\text{artanh}\left(\frac{c}{x}\right) + a\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^3/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c/x) + a)^3/x, x)
```

3.155 $\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^3}{x^2}\,dx$

Optimal. Leaf size=126

$$\frac{3b^2\text{PolyLog}\left(2,1-\frac{2}{1-\frac{c}{x}}\right)\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)}{c}-\frac{3b^3\text{PolyLog}\left(3,1-\frac{2}{1-\frac{c}{x}}\right)}{2c}-\frac{\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3}{c}-\frac{\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3}{x}+$$

```
[Out] -((a + b*ArcCoth[x/c])^3/c) - (a + b*ArcCoth[x/c])^3/x + (3*b*(a + b*ArcCot
h[x/c])^2*Log[2/(1 - c/x)])/c + (3*b^2*(a + b*ArcCoth[x/c])*PolyLog[2, 1 -
2/(1 - c/x)])/c - (3*b^3*PolyLog[3, 1 - 2/(1 - c/x)])/(2*c)
```

Rubi [B] time = 2.23691, antiderivative size = 387, normalized size of antiderivative = 3.07, number of steps used = 82, number of rules used = 23, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.438, Rules used = {6099, 2454, 2389, 2296, 2295, 6715, 2430, 2416, 2396, 2433, 2374, 6589, 2411, 2346, 2301, 6742, 43, 2394, 2393, 2391, 2375, 2317, 2425}

$$\frac{3b^2\text{PolyLog}\left(2,-\frac{c-x}{2x}\right)\left(2a-b\log\left(1-\frac{c}{x}\right)\right)}{2c}+\frac{3b^3\text{PolyLog}\left(3,-\frac{c-x}{2x}\right)}{2c}+\frac{3b^3\text{PolyLog}\left(3,\frac{c+x}{2x}\right)}{2c}-\frac{3b^3\log\left(\frac{c+x}{x}\right)\text{PolyLog}\left(2,\right.}{2c}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcTanh[c/x])^3/x^2,x]
```

```
[Out] ((1 - c/x)*(2*a - b*Log[1 - c/x])^3)/(8*c) - (3*b*(2*a - b*Log[1 - c/x])^2*
Log[(c + x)/(2*x)])/(4*c) + (3*b*(2*a - b*Log[1 - c/x])^2*Log[(c + x)/x])/(
8*c) - (3*b*(2*a - b*Log[1 - c/x])^2*Log[(c + x)/x])/(8*x) - (3*b^2*(2*a -
b*Log[1 - c/x])*Log[(c + x)/x]^2)/(8*c) - (3*b^2*(2*a - b*Log[1 - c/x])*Log
[(c + x)/x]^2)/(8*x) - (3*b^3*Log[-(c - x)/(2*x)]*Log[(c + x)/x]^2)/(4*c) -
 (b^3*(1 + c/x)*Log[(c + x)/x]^3)/(8*c) + (3*b^2*(2*a - b*Log[1 - c/x])*Pol
yLog[2, -(c - x)/(2*x)])/(2*c) - (3*b^3*Log[(c + x)/x]*PolyLog[2, (c + x)/(
2*x)])/(2*c) + (3*b^3*PolyLog[3, -(c - x)/(2*x)])/(2*c) + (3*b^3*PolyLog[3,
 (c + x)/(2*x)])/(2*c)
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 2430

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.)), x_Symbol] :> Simp[x*(a + b*Log[c
*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]), x] + (-Dist[g*j*m, Int[(x*(a +
 b*Log[c*(d + e*x)^n])^p)/(i + j*x), x], x] - Dist[b*e*n*p, Int[(x*(a + b*L
og[c*(d + e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /
; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2346

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Dist[d, Int[((d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p)/x,
 x], x] + Dist[e, Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; Fre
eQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2375

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[(Log[d*(e + f*x^m)^r]*(a + b*Log[
```

```
c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[(f*m*r)/(b*n*(p + 1)), Int[(x^(m
- 1)*(a + b*Log[c*x^n])^(p + 1))/(e + f*x^m), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2425

```
Int[(Log[(f_.)*(x_)^(m_.)]*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b
_.)))/(x_), x_Symbol] :> Simp[(Log[f*x^m]^2*(a + b*Log[c*(d + e*x)^n]))/(2*
m), x] - Dist[(b*e*n)/(2*m), Int[Log[f*x^m]^2/(d + e*x), x], x] /; FreeQ[{a
, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^3}{x^2}\,dx = \int \left(\frac{\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3}{8x^2}+\frac{3b\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2\log\left(1+\frac{c}{x}\right)}{8x^2}+\frac{3b^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right.}{8x^2}\right.$$

$$=\frac{1}{8}\int \frac{\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3}{x^2}\,dx+\frac{1}{8}(3b)\int \frac{\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2\log\left(1+\frac{c}{x}\right)}{x^2}\,dx+\frac{1}{8}\left(3b^2\right)\int$$

$$=-\left(\frac{1}{8}\operatorname{Subst}\left(\int(2a-b\log(1-cx))^3\,dx,x,\frac{1}{x}\right)\right)-\frac{1}{8}(3b)\operatorname{Subst}\left(\int(2a-b\log(1-cx))^2\log\right.$$

$$=-\frac{3b\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2\log\left(\frac{c+x}{x}\right)}{8x}-\frac{3b^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)\log^2\left(\frac{c+x}{x}\right)}{8x}+\frac{\operatorname{Subst}\left(\int(2a\right.}{}$$

$$=\frac{\left(1-\frac{c}{x}\right)\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3}{8c}-\frac{3b\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2\log\left(\frac{c+x}{x}\right)}{8x}-\frac{3b^2\left(2a-b\log\left(1-\frac{c}{x}\right.\right.}{8x}$$

$$=\frac{3b\left(1-\frac{c}{x}\right)\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2}{8c}+\frac{\left(1-\frac{c}{x}\right)\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3}{8c}-\frac{3b\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2}{8x}$$

$$=-\frac{3ab^2}{2x}+\frac{3b^3}{4x}+\frac{3b\left(1-\frac{c}{x}\right)\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2}{8c}+\frac{\left(1-\frac{c}{x}\right)\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3}{8c}-\frac{3b\left(2a\right.}{}$$

$$=-\frac{3ab^2}{2x}-\frac{3b^3\left(1-\frac{c}{x}\right)\log\left(1-\frac{c}{x}\right)}{4c}+\frac{\left(1-\frac{c}{x}\right)\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3}{8c}-\frac{3b\left(2a-b\log\left(1-\frac{c}{x}\right)\right)}{8c}$$

$$=-\frac{3b^3}{4x}-\frac{3b^3\left(1-\frac{c}{x}\right)\log\left(1-\frac{c}{x}\right)}{4c}+\frac{\left(1-\frac{c}{x}\right)\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3}{8c}-\frac{3b\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2}{4c}$$

$$=\frac{\left(1-\frac{c}{x}\right)\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3}{8c}-\frac{3b\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2\log\left(\frac{c+x}{2x}\right)}{4c}+\frac{3b\left(2a-b\log\left(1-\frac{c}{x}\right.\right.}{8c}$$

$$=\frac{\left(1-\frac{c}{x}\right)\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3}{8c}-\frac{3b\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2\log\left(\frac{c+x}{2x}\right)}{4c}+\frac{3b\left(2a-b\log\left(1-\frac{c}{x}\right.\right.}{8c}$$

Mathematica [A] time = 0.122466, size = 205, normalized size = 1.63

$$-\frac{3ab^2\left(\operatorname{PolyLog}\left(2,-e^{-2\tanh^{-1}\left(\frac{c}{x}\right)}\right)+\tanh^{-1}\left(\frac{c}{x}\right)\left(\frac{c\tanh^{-1}\left(\frac{c}{x}\right)}{x}-\tanh^{-1}\left(\frac{c}{x}\right)-2\log\left(e^{-2\tanh^{-1}\left(\frac{c}{x}\right)}+1\right)\right)\right)}{c}-\frac{b^3\left(3\tanh^{-1}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c/x])^3/x^2,x]
```

```
[Out] -(a^3/x) - (3*a^2*b*ArcTanh[c/x])/x - (3*a^2*b*Log[1 - c^2/x^2])/(2*c) - (3
*a*b^2*(ArcTanh[c/x]*(-ArcTanh[c/x] + (c*ArcTanh[c/x])/x - 2*Log[1 + E^(-2*
ArcTanh[c/x])]) + PolyLog[2, -E^(-2*ArcTanh[c/x])]))/c - (b^3*(ArcTanh[c/x]
^2*(-ArcTanh[c/x] + (c*ArcTanh[c/x])/x - 3*Log[1 + E^(-2*ArcTanh[c/x])]) +
3*ArcTanh[c/x]*PolyLog[2, -E^(-2*ArcTanh[c/x])] + (3*PolyLog[3, -E^(-2*ArcT
anh[c/x])])/2))/c
```

Maple [B] time = 0.004, size = 298, normalized size = 2.4

$$-\frac{a^3}{x}-\frac{b^3}{x}\left(\operatorname{Artanh}\left(\frac{c}{x}\right)\right)^3-\frac{b^3}{c}\left(\operatorname{Artanh}\left(\frac{c}{x}\right)\right)^3+3\frac{b^3}{c}\left(\operatorname{Artanh}\left(\frac{c}{x}\right)\right)^2\ln\left(\left(1+\frac{c}{x}\right)^2\left(1-\frac{c^2}{x^2}\right)^{-1}+1\right)+3\frac{b^3}{c}\operatorname{Artanh}\left(\frac{c}{x}\right)\mathrm{p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x))^3/x^2,x)
```

```
[Out] -a^3/x-b^3/x*arctanh(c/x)^3-1/c*b^3*arctanh(c/x)^3+3/c*b^3*arctanh(c/x)^2*l
n((1+c/x)^2/(1-c^2/x^2)+1)+3/c*b^3*arctanh(c/x)*polylog(2,-(1+c/x)^2/(1-c^2
/x^2))-3/2/c*b^3*polylog(3,-(1+c/x)^2/(1-c^2/x^2))-3*arctanh(c/x)^2/x*a*b^2
+6/c*ln((1+c/x)^2/(1-c^2/x^2)+1)*arctanh(c/x)*a*b^2-3/c*a*b^2*arctanh(c/x)^
2+3/c*polylog(2,-(1+c/x)^2/(1-c^2/x^2))*a*b^2-3/x*a^2*b*arctanh(c/x)-3/2/c*
a^2*b*ln(1-c^2/x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{3\,a^2b\left(\frac{2\,c\,\operatorname{artanh}\left(\frac{c}{x}\right)}{x}+\log\left(-\frac{c^2}{x^2}+1\right)\right)}{2\,c}-\frac{a^3}{x}+\frac{\left(b^3c-b^3x\right)\log\left(-c+x\right)^3-3\left(2\,ab^2c+\left(b^3c+b^3x\right)\log\left(c+x\right)\right)\log\left(-c+x\right)^2}{8\,cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^3/x^2,x, algorithm="maxima")
```

```
[Out] -3/2*a^2*b*(2*c*arctanh(c/x)/x + log(-c^2/x^2 + 1))/c - a^3/x + 1/8*((b^3*c
 - b^3*x)*log(-c + x)^3 - 3*(2*a*b^2*c + (b^3*c + b^3*x)*log(c + x))*log(-c
 + x)^2)/(c*x) - integrate(-1/8*((b^3*c^2 - b^3*c*x)*log(c + x)^3 + 6*(a*b^
2*c^2 - a*b^2*c*x)*log(c + x)^2 - 3*(4*a*b^2*c*x + (b^3*c^2 - b^3*c*x)*log(
c + x)^2 + 2*(2*a*b^2*c^2 + b^3*x^2 - (2*a*b^2*c - b^3*c)*x)*log(c + x))*lo
g(-c + x))/(c^2*x^2 - c*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{b^3\operatorname{artanh}\left(\frac{c}{x}\right)^3+3\,ab^2\operatorname{artanh}\left(\frac{c}{x}\right)^2+3\,a^2b\operatorname{artanh}\left(\frac{c}{x}\right)+a^3}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^3/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctanh(c/x)^3 + 3*a*b^2*arctanh(c/x)^2 + 3*a^2*b*arctanh(c/x
) + a^3)/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + b\,\text{atanh}\left(\frac{c}{x}\right)\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x))**3/x**2,x)
```

```
[Out] Integral((a + b*atanh(c/x))**3/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\,\text{artanh}\left(\frac{c}{x}\right) + a\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^3/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c/x) + a)^3/x^2, x)
```

3.156 $\int \frac{\left(a+b \tanh^{-1}\left(\frac{c}{x}\right)\right)^3}{x^3} dx$

Optimal. Leaf size=139

$$\frac{3b^3\text{PolyLog}\left(2,1-\frac{2}{1-\frac{c}{x}}\right)}{2c^2}+\frac{3b^2\log\left(\frac{2}{1-\frac{c}{x}}\right)\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)}{c^2}-\frac{3b\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2}{2c^2}+\frac{\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3}{2c^2}-\frac{\left(a+b\,\text{co}\right.}{}$$

```
[Out] (-3*b*(a + b*ArcCoth[x/c])^2)/(2*c^2) - (3*b*(a + b*ArcCoth[x/c])^2)/(2*c*x
) + (a + b*ArcCoth[x/c])^3/(2*c^2) - (a + b*ArcCoth[x/c])^3/(2*x^2) + (3*b^
2*(a + b*ArcCoth[x/c])*Log[2/(1 - c/x)])/c^2 + (3*b^3*PolyLog[2, 1 - 2/(1 -
 c/x)])/(2*c^2)
```

Rubi [F] time = 2.10214, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^3}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcTanh[c/x])^3/x^3,x]
```

```
[Out] (-3*b^3*(1 - c/x)^2)/(64*c^2) - (3*a*b^2*(1 + c/x)^2)/(16*c^2) + (3*b^3*(1
+ c/x)^2)/(64*c^2) + (3*a^2*b)/(8*x^2) + (3*a*b^2)/(8*x^2) - (3*a^2*b)/(4*c
*x) - (3*b^3)/(2*c*x) + (3*a*b^2*Log[1 - c/x])/(8*c^2) - (3*a*b^2*(1 - c/x)
*Log[1 - c/x])/(4*c^2) - (3*b^3*(1 - c/x)*Log[1 - c/x])/(4*c^2) - (3*a*b^2*
Log[1 - c/x])/(8*x^2) - (3*b^2*(1 - c/x)^2*(2*a - b*Log[1 - c/x]))/(32*c^2)
 + (3*b*(1 - c/x)*(2*a - b*Log[1 - c/x])^2)/(8*c^2) - (3*b*(1 - c/x)^2*(2*a
 - b*Log[1 - c/x])^2)/(32*c^2) + ((1 - c/x)*(2*a - b*Log[1 - c/x])^3)/(8*c^
2) - ((1 - c/x)^2*(2*a - b*Log[1 - c/x])^3)/(16*c^2) + (3*a*b^2*Log[1 - c/x
]*Log[1 + c/x])/(4*x^2) - (3*a*b^2*Log[1 + c/x]*Log[c - x])/(4*c^2) - (3*a*
b^2*Log[c - x]*Log[x/c])/(4*c^2) - (3*a*b^2*Log[1 - c/x]*Log[c + x])/(4*c^2
) + (3*a*b^2*Log[(c - x)/(2*c)]*Log[c + x])/(4*c^2) - (3*a*b^2*Log[-(x/c)]*
Log[c + x])/(4*c^2) + (3*a*b^2*Log[c - x]*Log[(c + x)/(2*c)])/(4*c^2) + (3*
a^2*b*Log[(c + x)/x])/(4*c^2) + (3*a*b^2*Log[(c + x)/x])/(8*c^2) - (9*a*b^2
*(1 + c/x)*Log[(c + x)/x])/(4*c^2) + (3*b^3*(1 + c/x)*Log[(c + x)/x])/(4*c^
2) + (3*a*b^2*(1 + c/x)^2*Log[(c + x)/x])/(8*c^2) - (3*b^3*(1 + c/x)^2*Log[
(c + x)/x])/(32*c^2) - (3*a^2*b*Log[(c + x)/x])/(4*x^2) - (3*a*b^2*Log[(c +
 x)/x])/(8*x^2) + (3*a*b^2*(1 + c/x)*Log[(c + x)/x]^2)/(4*c^2) - (3*b^3*(1
+ c/x)*Log[(c + x)/x]^2)/(8*c^2) - (3*a*b^2*(1 + c/x)^2*Log[(c + x)/x]^2)/(
8*c^2) + (3*b^3*(1 + c/x)^2*Log[(c + x)/x]^2)/(32*c^2) + (b^3*(1 + c/x)*Log
[(c + x)/x]^3)/(8*c^2) - (b^3*(1 + c/x)^2*Log[(c + x)/x]^3)/(16*c^2) + (3*a
*b^2*PolyLog[2, (c - x)/(2*c)])/(4*c^2) + (3*a*b^2*PolyLog[2, -(c/x)])/(4*c
^2) + (3*a*b^2*PolyLog[2, c/x])/(4*c^2) + (3*a*b^2*PolyLog[2, (c + x)/(2*c)
])/(4*c^2) - (3*a*b^2*PolyLog[2, 1 - x/c])/(4*c^2) - (3*a*b^2*PolyLog[2, 1
+ x/c])/(4*c^2) + (3*b^3*Defer[Int][(Log[1 - c/x]^2*Log[1 + c/x])/x^3, x])/
8 - (3*b^3*Defer[Int][(Log[1 - c/x]*Log[1 + c/x]^2)/x^3, x])/8
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x}\right)\right)^3}{x^3}\,dx = \int\left(\frac{\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3}{8x^3}+\frac{3b\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2\log\left(1+\frac{c}{x}\right)}{8x^3}+\frac{3b^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right.}{8x^3}\right.$$

$$=\frac{1}{8}\int\frac{\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3}{x^3}dx+\frac{1}{8}(3b)\int\frac{\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2\log\left(1+\frac{c}{x}\right)}{x^3}dx+\frac{1}{8}\left(3b^2\right)\int$$

$$=-\left(\frac{1}{8}\operatorname{Subst}\left(\int x(2a-b\log(1-cx))^3\,dx,x,\frac{1}{x}\right)\right)+\frac{1}{8}(3b)\int\left(\frac{4a^2\log\left(1+\frac{c}{x}\right)}{x^3}-\frac{4ab\log(1}{}\right.$$

$$=-\left(\frac{1}{8}\operatorname{Subst}\left(\int\left(\frac{(2a-b\log(1-cx))^3}{c}-\frac{(1-cx)(2a-b\log(1-cx))^3}{c}\right)dx,x,\frac{1}{x}\right)\right)+\frac{1}{2}(3a$$

$$=\frac{3ab^2\log\left(1-\frac{c}{x}\right)\log\left(1+\frac{c}{x}\right)}{4x^2}-\frac{1}{2}\left(3a^2b\right)\operatorname{Subst}\left(\int x\log(1+cx)\,dx,x,\frac{1}{x}\right)-\frac{1}{4}\left(3ab^2\right)\operatorname{Sub}$$

$$=\frac{3ab^2\log\left(1-\frac{c}{x}\right)\log\left(1+\frac{c}{x}\right)}{4x^2}-\frac{3a^2b\log\left(\frac{c+x}{x}\right)}{4x^2}-\frac{1}{4}\left(3ab^2\right)\operatorname{Subst}\left(\int\left(-\frac{\log^2(1+cx)}{c}+\frac{(1}{}\right.\right.$$

$$=\frac{\left(1-\frac{c}{x}\right)\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3}{8c^2}-\frac{\left(1-\frac{c}{x}\right)^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^3}{16c^2}+\frac{3ab^2\log\left(1-\frac{c}{x}\right)\log\left(1+\right.}{4x^2}$$

$$=\frac{3a^2b}{8x^2}-\frac{3a^2b}{4cx}+\frac{3b\left(1-\frac{c}{x}\right)\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2}{8c^2}-\frac{3b\left(1-\frac{c}{x}\right)^2\left(2a-b\log\left(1-\frac{c}{x}\right)\right)^2}{32c^2}+\frac{(1}{}$$

$$=-\frac{3b^3\left(1-\frac{c}{x}\right)^2}{64c^2}+\frac{3b^3\left(1+\frac{c}{x}\right)^2}{64c^2}+\frac{3a^2b}{8x^2}-\frac{3a^2b}{4cx}-\frac{3ab^2}{2cx}-\frac{3b^3}{4cx}-\frac{3b^2\left(1-\frac{c}{x}\right)^2\left(2a-b\log(1\right.}{32c^2}$$

$$=-\frac{3b^3\left(1-\frac{c}{x}\right)^2}{64c^2}-\frac{3ab^2\left(1+\frac{c}{x}\right)^2}{16c^2}+\frac{3b^3\left(1+\frac{c}{x}\right)^2}{64c^2}+\frac{3a^2b}{8x^2}-\frac{3a^2b}{4cx}-\frac{3b^3}{2cx}-\frac{3b^3\left(1-\frac{c}{x}\right)\log(1}{4c^2}$$

$$=-\frac{3b^3\left(1-\frac{c}{x}\right)^2}{64c^2}-\frac{3ab^2\left(1+\frac{c}{x}\right)^2}{16c^2}+\frac{3b^3\left(1+\frac{c}{x}\right)^2}{64c^2}+\frac{3a^2b}{8x^2}-\frac{3a^2b}{4cx}-\frac{3b^3}{2cx}-\frac{3ab^2\left(1-\frac{c}{x}\right)\log(}{4c^2}$$

$$=-\frac{3b^3\left(1-\frac{c}{x}\right)^2}{64c^2}-\frac{3ab^2\left(1+\frac{c}{x}\right)^2}{16c^2}+\frac{3b^3\left(1+\frac{c}{x}\right)^2}{64c^2}+\frac{3a^2b}{8x^2}+\frac{3ab^2}{8x^2}-\frac{3a^2b}{4cx}-\frac{3b^3}{2cx}+\frac{3ab^2\log(}{8c^2}$$

$$=-\frac{3b^3\left(1-\frac{c}{x}\right)^2}{64c^2}-\frac{3ab^2\left(1+\frac{c}{x}\right)^2}{16c^2}+\frac{3b^3\left(1+\frac{c}{x}\right)^2}{64c^2}+\frac{3a^2b}{8x^2}+\frac{3ab^2}{8x^2}-\frac{3a^2b}{4cx}-\frac{3b^3}{2cx}+\frac{3ab^2\log(}{8c^2}$$

$$=-\frac{3b^3\left(1-\frac{c}{x}\right)^2}{64c^2}-\frac{3ab^2\left(1+\frac{c}{x}\right)^2}{16c^2}+\frac{3b^3\left(1+\frac{c}{x}\right)^2}{64c^2}+\frac{3a^2b}{8x^2}+\frac{3ab^2}{8x^2}-\frac{3a^2b}{4cx}-\frac{3b^3}{2cx}+\frac{3ab^2\log(}{8c^2}$$

Mathematica [A] time = 0.334353, size = 195, normalized size = 1.4

$$-6b^3x^2\operatorname{PolyLog}\left(2,-e^{-2\tanh^{-1}\left(\frac{c}{x}\right)}\right)+a\left(12b^2x^2\log\left(\frac{1}{\sqrt{1-\frac{c^2}{x^2}}}\right)-a\left(2ac^2+3bx^2\log\left(1-\frac{c}{x}\right)-3bx^2\log\left(\frac{c+x}{x}\right)+6bcx\right)\right)+$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c/x])^3/x^3,x]
```

```
[Out] (6*b^2*(-c + x)*(b*x + a*(c + x))*ArcTanh[c/x]^2 + 2*b^3*(-c^2 + x^2)*ArcTa
nh[c/x]^3 + 6*b*ArcTanh[c/x]*(-(a*c*(a*c + 2*b*x)) + 2*b^2*x^2*Log[1 + E^(-
2*ArcTanh[c/x])]) + a*(12*b^2*x^2*Log[1/Sqrt[1 - c^2/x^2]] - a*(2*a*c^2 + 6
*b*c*x + 3*b*x^2*Log[1 - c/x] - 3*b*x^2*Log[(c + x)/x])) - 6*b^3*x^2*PolyLo
```

```
g[2, -E^(-2*ArcTanh[c/x])])/(4*c^2*x^2)
```

Maple [C] time = 0.266, size = 6645, normalized size = 47.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x))^3/x^3,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^3/x^3,x, algorithm="maxima")
```

```
[Out] 3/4*(c*(log(c + x)/c^3 - log(-c + x)/c^3 - 2/(c^2*x)) - 2*arctanh(c/x)/x^2)
*a^2*b - 3/8*(c^2*((log(c + x)^2 - 2*(log(c + x) - 2)*log(-c + x) + log(-c
+ x)^2 + 4*log(c + x))/c^4 - 8*log(x)/c^4) - 4*c*(log(c + x)/c^3 - log(-c +
 x)/c^3 - 2/(c^2*x))*arctanh(c/x))*a*b^2 + 1/64*(32*c^4*integrate(-1/4*log(
x)^3/(c^4*x^3 - c^2*x^5), x) - 3*c^3*(log(c + x)/c^5 - log(-c + x)/c^5 - 2/
(c^4*x)) + 48*c^3*integrate(-1/4*x*log(x)^2/(c^4*x^3 - c^2*x^5), x) + 48*c^
3*integrate(-1/4*x*log(x)/(c^4*x^3 - c^2*x^5), x) - 6*c*(2*log(-c + x)/c^3
- 2*log(x)/c^3 + (c + 2*x)/(c^2*x^2))*log(-c/x + 1)^2 + 21*c^2*(log(c + x)/
c^4 + log(-c + x)/c^4 - 2*log(x)/c^4) - 32*c^2*integrate(-1/4*x^2*log(x)^3/
(c^4*x^3 - c^2*x^5), x) + 48*c^2*integrate(-1/4*x^2*log(x)^2/(c^4*x^3 - c^2
*x^5), x) - 384*c^2*integrate(-1/4*x^2*log(c + x)/(c^4*x^3 - c^2*x^5), x) +
 144*c^2*integrate(-1/4*x^2*log(x)/(c^4*x^3 - c^2*x^5), x) - 18*c*(log(c +
x)/c^3 - log(-c + x)/c^3) + c*(6*(2*x^2*log(-c + x)^2 + 2*x^2*log(x)^2 - 6*
x^2*log(x) + c^2 + 6*c*x - 2*(2*x^2*log(x) - 3*x^2)*log(-c + x))*log(-c/x +
 1)/(c^3*x^2) - (4*x^2*log(-c + x)^3 - 4*x^2*log(x)^3 + 18*x^2*log(x)^2 - 6
*(2*x^2*log(x) - 3*x^2)*log(-c + x)^2 - 42*x^2*log(x) + 3*c^2 + 42*c*x + 6*
(2*x^2*log(x)^2 - 6*x^2*log(x) + 7*x^2)*log(-c + x))/(c^3*x^2)) - 48*c*inte
grate(-1/4*x^3*log(x)^2/(c^4*x^3 - c^2*x^5), x) - 192*c*integrate(-1/4*x^3*
log(c + x)/(c^4*x^3 - c^2*x^5), x) + 336*c*integrate(-1/4*x^3*log(x)/(c^4*x
^3 - c^2*x^5), x) + 4*log(-c/x + 1)^3/x^2 - 2*(12*c*x*log(c + x)^2 + 2*(c^2
 - x^2)*log(c + x)^3 - 3*(c^2 - 2*c*x + x^2 - 2*(c^2 - x^2)*log(c + x) + 2*
(c^2 - x^2)*log(x))*log(-c + x)^2 - 3*(2*(c^2 - x^2)*log(c + x)^2 - 2*(c^2
- x^2)*log(x)^2 - c^2 - 6*c*x + 8*(c*x + x^2)*log(c + x) - 2*(c^2 + 2*c*x +
 5*x^2)*log(x))*log(-c + x))/(c^2*x^2) - 48*integrate(-1/4*x^4*log(x)^2/(c^
4*x^3 - c^2*x^5), x) - 192*integrate(-1/4*x^4*log(c + x)/(c^4*x^3 - c^2*x^5
), x) + 240*integrate(-1/4*x^4*log(x)/(c^4*x^3 - c^2*x^5), x))*b^3 - 3/2*a*
b^2*arctanh(c/x)^2/x^2 - 1/2*a^3/x^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3 \operatorname{artanh}\left(\frac{c}{x}\right)^3 + 3\,ab^2 \operatorname{artanh}\left(\frac{c}{x}\right)^2 + 3\,a^2 b \operatorname{artanh}\left(\frac{c}{x}\right) + a^3}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^3/x^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctanh(c/x)^3 + 3*a*b^2*arctanh(c/x)^2 + 3*a^2*b*arctanh(c/x
) + a^3)/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + b\operatorname{atanh}\left(\frac{c}{x}\right)\right)^3}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x))**3/x**3,x)
```

```
[Out] Integral((a + b*atanh(c/x))**3/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\operatorname{artanh}\left(\frac{c}{x}\right) + a\right)^3}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^3/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c/x) + a)^3/x^3, x)
```

3.157 $\int x^7 \left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right) dx$

Optimal. Leaf size=54

$$\frac{1}{8}x^8\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{8}bc^3x^2-\frac{1}{8}bc^4\tanh^{-1}\left(\frac{x^2}{c}\right)+\frac{1}{24}bcx^6$$

```
[Out] (b*c^3*x^2)/8 + (b*c*x^6)/24 + (x^8*(a + b*ArcTanh[c/x^2]))/8 - (b*c^4*ArcT
anh[x^2/c])/8
```

Rubi [A] time = 0.0417084, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6097, 263, 275, 302, 207}

$$\frac{1}{8}x^8\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{8}bc^3x^2-\frac{1}{8}bc^4\tanh^{-1}\left(\frac{x^2}{c}\right)+\frac{1}{24}bcx^6$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*ArcTanh[c/x^2]),x]
```

```
[Out] (b*c^3*x^2)/8 + (b*c*x^6)/24 + (x^8*(a + b*ArcTanh[c/x^2]))/8 - (b*c^4*ArcT
anh[x^2/c])/8
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int x^7\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)dx &= \frac{1}{8}x^8\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{4}(bc)\int\frac{x^5}{1-\frac{c^2}{x^4}}dx\\ &= \frac{1}{8}x^8\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{4}(bc)\int\frac{x^9}{-c^2+x^4}dx\\ &= \frac{1}{8}x^8\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{8}(bc)\,\text{Subst}\left(\int\frac{x^4}{-c^2+x^2}dx,x,x^2\right)\\ &= \frac{1}{8}x^8\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{8}(bc)\,\text{Subst}\left(\int\left(c^2+x^2+\frac{c^4}{-c^2+x^2}\right)dx,x,x^2\right)\\ &= \frac{1}{8}bc^3x^2+\frac{1}{24}bcx^6+\frac{1}{8}x^8\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{8}\left(bc^5\right)\text{Subst}\left(\int\frac{1}{-c^2+x^2}dx,x,x^2\right)\\ &= \frac{1}{8}bc^3x^2+\frac{1}{24}bcx^6+\frac{1}{8}x^8\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)-\frac{1}{8}bc^4\tanh^{-1}\left(\frac{x^2}{c}\right)\end{aligned}$$

Mathematica [A] time = 0.0140286, size = 73, normalized size = 1.35

$$\frac{ax^8}{8}+\frac{1}{8}bc^3x^2+\frac{1}{16}bc^4\log\left(x^2-c\right)-\frac{1}{16}bc^4\log\left(c+x^2\right)+\frac{1}{24}bcx^6+\frac{1}{8}bx^8\tanh^{-1}\left(\frac{c}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*ArcTanh[c/x^2]),x]
```

```
[Out] (b*c^3*x^2)/8 + (b*c*x^6)/24 + (a*x^8)/8 + (b*x^8*ArcTanh[c/x^2])/8 + (b*c^
4*Log[-c + x^2])/16 - (b*c^4*Log[c + x^2])/16
```

Maple [A] time = 0.016, size = 64, normalized size = 1.2

$$\frac{x^8a}{8}+\frac{bx^8}{8}\text{Artanh}\left(\frac{c}{x^2}\right)-\frac{bc^4}{16}\ln\left(1+\frac{c}{x^2}\right)+\frac{bcx^6}{24}+\frac{bc^3x^2}{8}+\frac{bc^4}{16}\ln\left(\frac{c}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a+b*arctanh(c/x^2)),x)
```

```
[Out] 1/8*x^8*a+1/8*b*x^8*arctanh(c/x^2)-1/16*b*c^4*ln(1+c/x^2)+1/24*b*c*x^6+1/8*
b*c^3*x^2+1/16*b*c^4*ln(c/x^2-1)
```

Maxima [A] time = 0.989646, size = 84, normalized size = 1.56

$$\frac{1}{8}ax^8+\frac{1}{48}\left(6x^8\,\text{artanh}\left(\frac{c}{x^2}\right)+\left(2x^6+6c^2x^2-3c^3\log\left(x^2+c\right)+3c^3\log\left(x^2-c\right)\right)c\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arctanh(c/x^2)),x, algorithm="maxima")
```

```
[Out] 1/8*a*x^8 + 1/48*(6*x^8*arctanh(c/x^2) + (2*x^6 + 6*c^2*x^2 - 3*c^3*log(x^2
 + c) + 3*c^3*log(x^2 - c))*c)*b
```

Fricas [A] time = 1.74963, size = 122, normalized size = 2.26

$$\frac{1}{8}ax^8 + \frac{1}{24}bcx^6 + \frac{1}{8}bc^3x^2 + \frac{1}{16}\left(bx^8 - bc^4\right)\log\left(\frac{x^2 + c}{x^2 - c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arctanh(c/x^2)),x, algorithm="fricas")
```

```
[Out] 1/8*a*x^8 + 1/24*b*c*x^6 + 1/8*b*c^3*x^2 + 1/16*(b*x^8 - b*c^4)*log((x^2 +
c)/(x^2 - c))
```

Sympy [A] time = 18.4974, size = 51, normalized size = 0.94

$$\frac{ax^8}{8} - \frac{bc^4 \operatorname{atanh}\left(\frac{c}{x^2}\right)}{8} + \frac{bc^3x^2}{8} + \frac{bcx^6}{24} + \frac{bx^8 \operatorname{atanh}\left(\frac{c}{x^2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(a+b*atanh(c/x**2)),x)
```

```
[Out] a*x**8/8 - b*c**4*atanh(c/x**2)/8 + b*c**3*x**2/8 + b*c*x**6/24 + b*x**8*at
anh(c/x**2)/8
```

Giac [A] time = 1.33246, size = 96, normalized size = 1.78

$$\frac{1}{16}bx^8\log\left(\frac{x^2 + c}{x^2 - c}\right) + \frac{1}{8}ax^8 + \frac{1}{24}bcx^6 + \frac{1}{8}bc^3x^2 - \frac{1}{16}bc^4\log\left(x^2 + c\right) + \frac{1}{16}bc^4\log\left(-x^2 + c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arctanh(c/x^2)),x, algorithm="giac")
```

```
[Out] 1/16*b*x^8*log((x^2 + c)/(x^2 - c)) + 1/8*a*x^8 + 1/24*b*c*x^6 + 1/8*b*c^3*
x^2 - 1/16*b*c^4*log(x^2 + c) + 1/16*b*c^4*log(-x^2 + c)
```

3.158 $\int x^5 \left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right) dx$

Optimal. Leaf size=45

$$\frac{1}{6}x^6\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{12}bc^3\log\left(c^2-x^4\right)+\frac{1}{12}bcx^4$$

```
[Out] (b*c*x^4)/12 + (x^6*(a + b*ArcTanh[c/x^2]))/6 + (b*c^3*Log[c^2 - x^4])/12
```

Rubi [A] time = 0.0329161, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6097, 263, 266, 43}

$$\frac{1}{6}x^6\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{12}bc^3\log\left(c^2-x^4\right)+\frac{1}{12}bcx^4$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*ArcTanh[c/x^2]),x]
```

```
[Out] (b*c*x^4)/12 + (x^6*(a + b*ArcTanh[c/x^2]))/6 + (b*c^3*Log[c^2 - x^4])/12
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^5\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)dx = \frac{1}{6}x^6\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{3}(bc)\int\frac{x^3}{1-\frac{c^2}{x^4}}dx$$
$$=\frac{1}{6}x^6\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{3}(bc)\int\frac{x^7}{-c^2+x^4}dx$$
$$=\frac{1}{6}x^6\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{12}(bc)\,\text{Subst}\left(\int\frac{x}{-c^2+x}dx,x,x^4\right)$$
$$=\frac{1}{6}x^6\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{12}(bc)\,\text{Subst}\left(\int\left(1-\frac{c^2}{c^2-x}\right)dx,x,x^4\right)$$
$$=\frac{1}{12}bcx^4+\frac{1}{6}x^6\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{12}bc^3\log\left(c^2-x^4\right)$$

Mathematica [A] time = 0.0119029, size = 50, normalized size = 1.11

$$\frac{ax^6}{6}+\frac{1}{12}bc^3\log\left(x^4-c^2\right)+\frac{1}{12}bcx^4+\frac{1}{6}bx^6\tanh^{-1}\left(\frac{c}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*ArcTanh[c/x^2]),x]
```

```
[Out] (b*c*x^4)/12 + (a*x^6)/6 + (b*x^6*ArcTanh[c/x^2])/6 + (b*c^3*Log[-c^2 + x^4
])/12
```

Maple [A] time = 0.019, size = 65, normalized size = 1.4

$$\frac{x^6a}{6}+\frac{bx^6}{6}\text{Artanh}\left(\frac{c}{x^2}\right)+\frac{bc^3}{12}\ln\left(1+\frac{c}{x^2}\right)+\frac{bcx^4}{12}-\frac{bc^3\ln\left(x^{-1}\right)}{3}+\frac{bc^3}{12}\ln\left(\frac{c}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arctanh(c/x^2)),x)
```

```
[Out] 1/6*x^6*a+1/6*b*x^6*arctanh(c/x^2)+1/12*b*c^3*ln(1+c/x^2)+1/12*b*c*x^4-1/3*
b*c^3*ln(1/x)+1/12*b*c^3*ln(c/x^2-1)
```

Maxima [A] time = 0.968106, size = 57, normalized size = 1.27

$$\frac{1}{6}ax^6+\frac{1}{12}\left(2x^6\,\text{artanh}\left(\frac{c}{x^2}\right)+\left(x^4+c^2\log\left(x^4-c^2\right)\right)c\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c/x^2)),x, algorithm="maxima")
```

```
[Out] 1/6*a*x^6 + 1/12*(2*x^6*arctanh(c/x^2) + (x^4 + c^2*log(x^4 - c^2))*c)*b
```

Fricas [A] time = 1.71104, size = 124, normalized size = 2.76

$$\frac{1}{12}bx^6\log\left(\frac{x^2+c}{x^2-c}\right)+\frac{1}{6}ax^6+\frac{1}{12}bcx^4+\frac{1}{12}bc^3\log\left(x^4-c^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c/x^2)),x, algorithm="fricas")
```

```
[Out] 1/12*b*x^6*log((x^2 + c)/(x^2 - c)) + 1/6*a*x^6 + 1/12*b*c*x^4 + 1/12*b*c^3
*log(x^4 - c^2)
```

Sympy [C] time = 13.1943, size = 75, normalized size = 1.67

$$\frac{ax^6}{6} + \frac{bc^3\log\left(-i\sqrt{c}+x\right)}{6} + \frac{bc^3\log\left(i\sqrt{c}+x\right)}{6} - \frac{bc^3\operatorname{atanh}\left(\frac{c}{x^2}\right)}{6} + \frac{bcx^4}{12} + \frac{bx^6\operatorname{atanh}\left(\frac{c}{x^2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*atanh(c/x**2)),x)
```

```
[Out] a*x**6/6 + b*c**3*log(-I*sqrt(c) + x)/6 + b*c**3*log(I*sqrt(c) + x)/6 - b*c
**3*atanh(c/x**2)/6 + b*c*x**4/12 + b*x**6*atanh(c/x**2)/6
```

Giac [A] time = 1.24473, size = 70, normalized size = 1.56

$$\frac{1}{12}\,bx^6\log\left(\frac{x^2+c}{x^2-c}\right) + \frac{1}{6}\,ax^6 + \frac{1}{12}\,bcx^4 + \frac{1}{12}\,bc^3\log\left(x^4-c^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c/x^2)),x, algorithm="giac")
```

```
[Out] 1/12*b*x^6*log((x^2 + c)/(x^2 - c)) + 1/6*a*x^6 + 1/12*b*c*x^4 + 1/12*b*c^3
*log(x^4 - c^2)
```

3.159 $\int x^3 \left(a+b \tanh^{-1}\left(\frac{c}{x^2}\right)\right) dx$

Optimal. Leaf size=43

$$\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)-\frac{1}{4}bc^2\tanh^{-1}\left(\frac{x^2}{c}\right)+\frac{1}{4}bcx^2$$

```
[Out] (b*c*x^2)/4 + (x^4*(a + b*ArcTanh[c/x^2]))/4 - (b*c^2*ArcTanh[x^2/c])/4
```

Rubi [A] time = 0.0297014, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6097, 263, 275, 321, 207}

$$\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)-\frac{1}{4}bc^2\tanh^{-1}\left(\frac{x^2}{c}\right)+\frac{1}{4}bcx^2$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcTanh[c/x^2]),x]
```

```
[Out] (b*c*x^2)/4 + (x^4*(a + b*ArcTanh[c/x^2]))/4 - (b*c^2*ArcTanh[x^2/c])/4
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int x^3 \left(a + b \tanh^{-1}\left(\frac{c}{x^2}\right)\right) dx &= \frac{1}{4}x^4 \left(a + b \tanh^{-1}\left(\frac{c}{x^2}\right)\right) + \frac{1}{2}(bc) \int \frac{x}{1 - \frac{c^2}{x^4}} dx \\ &= \frac{1}{4}x^4 \left(a + b \tanh^{-1}\left(\frac{c}{x^2}\right)\right) + \frac{1}{2}(bc) \int \frac{x^5}{-c^2 + x^4} dx \\ &= \frac{1}{4}x^4 \left(a + b \tanh^{-1}\left(\frac{c}{x^2}\right)\right) + \frac{1}{4}(bc) \operatorname{Subst}\left(\int \frac{x^2}{-c^2 + x^2} dx, x, x^2\right) \\ &= \frac{1}{4}bcx^2 + \frac{1}{4}x^4 \left(a + b \tanh^{-1}\left(\frac{c}{x^2}\right)\right) + \frac{1}{4}\left(bc^3\right) \operatorname{Subst}\left(\int \frac{1}{-c^2 + x^2} dx, x, x^2\right) \\ &= \frac{1}{4}bcx^2 + \frac{1}{4}x^4 \left(a + b \tanh^{-1}\left(\frac{c}{x^2}\right)\right) - \frac{1}{4}bc^2 \tanh^{-1}\left(\frac{x^2}{c}\right)\end{aligned}$$

Mathematica [A] time = 0.0111031, size = 62, normalized size = 1.44

$$\frac{ax^4}{4} + \frac{1}{8}bc^2 \log\left(x^2 - c\right) - \frac{1}{8}bc^2 \log\left(c + x^2\right) + \frac{1}{4}bcx^2 + \frac{1}{4}bx^4 \tanh^{-1}\left(\frac{c}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcTanh[c/x^2]),x]
```

```
[Out] (b*c*x^2)/4 + (a*x^4)/4 + (b*x^4*ArcTanh[c/x^2])/4 + (b*c^2*Log[-c + x^2])/
8 - (b*c^2*Log[c + x^2])/8
```

Maple [A] time = 0.006, size = 55, normalized size = 1.3

$$\frac{x^4 a}{4} + \frac{bx^4}{4} \operatorname{Artanh}\left(\frac{c}{x^2}\right) - \frac{bc^2}{8} \ln\left(1 + \frac{c}{x^2}\right) + \frac{bc^2}{8} \ln\left(\frac{c}{x^2} - 1\right) + \frac{bcx^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c/x^2)),x)
```

```
[Out] 1/4*x^4*a+1/4*arctanh(c/x^2)*b*x^4-1/8*b*c^2*ln(1+c/x^2)+1/8*b*c^2*ln(c/x^2
-1)+1/4*b*c*x^2
```

Maxima [A] time = 0.960415, size = 66, normalized size = 1.53

$$\frac{1}{4} ax^4 + \frac{1}{8}\left(2 x^4 \operatorname{artanh}\left(\frac{c}{x^2}\right) + \left(2 x^2 - c \log\left(x^2 + c\right) + c \log\left(x^2 - c\right)\right)c\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c/x^2)),x, algorithm="maxima")
```

```
[Out] 1/4*a*x^4 + 1/8*(2*x^4*arctanh(c/x^2) + (2*x^2 - c*log(x^2 + c) + c*log(x^2
 - c))*c)*b
```

Fricas [A] time = 1.585, size = 97, normalized size = 2.26

$$\frac{1}{4}\,ax^4 + \frac{1}{4}\,bcx^2 + \frac{1}{8}\left(bx^4 - bc^2\right)\log\left(\frac{x^2+c}{x^2-c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c/x^2)),x, algorithm="fricas")
```

```
[Out] 1/4*a*x^4 + 1/4*b*c*x^2 + 1/8*(b*x^4 - b*c^2)*log((x^2 + c)/(x^2 - c))
```

Sympy [A] time = 15.3665, size = 41, normalized size = 0.95

$$\frac{ax^4}{4} - \frac{bc^2\operatorname{atanh}\left(\frac{c}{x^2}\right)}{4} + \frac{bcx^2}{4} + \frac{bx^4\operatorname{atanh}\left(\frac{c}{x^2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c/x**2)),x)
```

```
[Out] a*x**4/4 - b*c**2*atanh(c/x**2)/4 + b*c*x**2/4 + b*x**4*atanh(c/x**2)/4
```

Giac [A] time = 1.25419, size = 84, normalized size = 1.95

$$\frac{1}{8}\,bx^4\log\left(\frac{x^2+c}{x^2-c}\right) + \frac{1}{4}\,ax^4 + \frac{1}{4}\,bcx^2 - \frac{1}{8}\,bc^2\log\left(x^2+c\right) + \frac{1}{8}\,bc^2\log\left(-x^2+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c/x^2)),x, algorithm="giac")
```

```
[Out] 1/8*b*x^4*log((x^2 + c)/(x^2 - c)) + 1/4*a*x^4 + 1/4*b*c*x^2 - 1/8*b*c^2*lo
g(x^2 + c) + 1/8*b*c^2*log(-x^2 + c)
```

3.160 $\int x \left(a+b \tanh^{-1}\left(\frac{c}{x^2}\right)\right) \, dx$

Optimal. Leaf size=34

$$\frac{1}{2} x^2 \left(a+b \tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{4} b c \log \left(c^2-x^4\right)$$

```
[Out] (x^2*(a + b*ArcTanh[c/x^2]))/2 + (b*c*Log[c^2 - x^4])/4
```

Rubi [A] time = 0.0180373, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6097, 263, 260}

$$\frac{1}{2} x^2 \left(a+b \tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{4} b c \log \left(c^2-x^4\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcTanh[c/x^2]),x]
```

```
[Out] (x^2*(a + b*ArcTanh[c/x^2]))/2 + (b*c*Log[c^2 - x^4])/4
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int x \left(a+b \tanh^{-1}\left(\frac{c}{x^2}\right)\right) \, dx &= \frac{1}{2} x^2 \left(a+b \tanh^{-1}\left(\frac{c}{x^2}\right)\right)+(b c) \int \frac{1}{\left(1-\frac{c^2}{x^4}\right) x} \, dx\\
&= \frac{1}{2} x^2 \left(a+b \tanh^{-1}\left(\frac{c}{x^2}\right)\right)+(b c) \int \frac{x^3}{-c^2+x^4} \, dx\\
&= \frac{1}{2} x^2 \left(a+b \tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{4} b c \log \left(c^2-x^4\right)
\end{aligned}$$

Mathematica [A] time = 0.0068589, size = 39, normalized size = 1.15

$$\frac{a x^2}{2}+\frac{1}{4} b c \log \left(x^4-c^2\right)+\frac{1}{2} b x^2 \tanh^{-1}\left(\frac{c}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcTanh[c/x^2]),x]
```

```
[Out] (a*x^2)/2 + (b*x^2*ArcTanh[c/x^2])/2 + (b*c*Log[-c^2 + x^4])/4
```

Maple [A] time = 0.016, size = 52, normalized size = 1.5

$$\frac{ax^2}{2} + \frac{bx^2}{2}\mathrm{Artanh}\left(\frac{c}{x^2}\right) + \frac{bc}{4}\ln\left(1 + \frac{c}{x^2}\right) - bc\ln\left(x^{-1}\right) + \frac{bc}{4}\ln\left(\frac{c}{x^2} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c/x^2)),x)
```

```
[Out] 1/2*a*x^2+1/2*b*x^2*arctanh(c/x^2)+1/4*b*c*ln(1+c/x^2)-b*c*ln(1/x)+1/4*b*c*
ln(c/x^2-1)
```

Maxima [A] time = 0.981602, size = 46, normalized size = 1.35

$$\frac{1}{2}\,ax^2 + \frac{1}{4}\left(2\,x^2\,\mathrm{artanh}\left(\frac{c}{x^2}\right) + c\log\left(x^4 - c^2\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c/x^2)),x, algorithm="maxima")
```

```
[Out] 1/2*a*x^2 + 1/4*(2*x^2*arctanh(c/x^2) + c*log(x^4 - c^2))*b
```

Fricas [A] time = 1.64731, size = 99, normalized size = 2.91

$$\frac{1}{4}\,bx^2\log\left(\frac{x^2 + c}{x^2 - c}\right) + \frac{1}{2}\,ax^2 + \frac{1}{4}\,bc\log\left(x^4 - c^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c/x^2)),x, algorithm="fricas")
```

```
[Out] 1/4*b*x^2*log((x^2 + c)/(x^2 - c)) + 1/2*a*x^2 + 1/4*b*c*log(x^4 - c^2)
```

Sympy [C] time = 10.1228, size = 61, normalized size = 1.79

$$\frac{ax^2}{2} + \frac{bc\log\left(-i\sqrt{c} + x\right)}{2} + \frac{bc\log\left(i\sqrt{c} + x\right)}{2} - \frac{bc\,\mathrm{atanh}\left(\frac{c}{x^2}\right)}{2} + \frac{bx^2\,\mathrm{atanh}\left(\frac{c}{x^2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c/x**2)),x)
```

```
[Out] a*x**2/2 + b*c*log(-I*sqrt(c) + x)/2 + b*c*log(I*sqrt(c) + x)/2 - b*c*atanh
(c/x**2)/2 + b*x**2*atanh(c/x**2)/2
```

Giac [A] time = 1.38631, size = 63, normalized size = 1.85

$$\frac{1}{2}ax^2 + \frac{1}{4}\left(x^2\log\left(-\frac{\frac{c}{x^2}+1}{\frac{c}{x^2}-1}\right) + c\log\left(\left|x^4-c^2\right|\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c/x^2)),x, algorithm="giac")
```

```
[Out] 1/2*a*x^2 + 1/4*(x^2*log(-(c/x^2 + 1)/(c/x^2 - 1)) + c*log(abs(x^4 - c^2)))
*b
```

3.161 $\int \frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{x}\,dx$

Optimal. Leaf size=30

$$\frac{1}{4}b\text{PolyLog}\left(2,-\frac{c}{x^2}\right)-\frac{1}{4}b\text{PolyLog}\left(2,\frac{c}{x^2}\right)+a\log(x)$$

```
[Out] a*Log[x] + (b*PolyLog[2, -(c/x^2)])/4 - (b*PolyLog[2, c/x^2])/4
```

Rubi [A] time = 0.0327894, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6095, 5912}

$$\frac{1}{4}b\text{PolyLog}\left(2,-\frac{c}{x^2}\right)-\frac{1}{4}b\text{PolyLog}\left(2,\frac{c}{x^2}\right)+a\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c/x^2])/x,x]
```

```
[Out] a*Log[x] + (b*PolyLog[2, -(c/x^2)])/4 - (b*PolyLog[2, c/x^2])/4
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[
1/n, Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
 n}, x] && IGtQ[p, 0]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{x}\,dx &= -\left(\frac{1}{2}\,\text{Subst}\left(\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx,x,\frac{1}{x^2}\right)\right)\\ &= a\log(x)+\frac{1}{4}b\text{Li}_2\left(-\frac{c}{x^2}\right)-\frac{1}{4}b\text{Li}_2\left(\frac{c}{x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.0139595, size = 28, normalized size = 0.93

$$\frac{1}{4}b\left(\text{PolyLog}\left(2,-\frac{c}{x^2}\right)-\text{PolyLog}\left(2,\frac{c}{x^2}\right)\right)+a\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c/x^2])/x,x]
```

```
[Out] a*Log[x] + (b*(PolyLog[2, -(c/x^2)] - PolyLog[2, c/x^2]))/4
```

Maple [B] time = 0.03, size = 154, normalized size = 5.1

$$-a\ln\left(x^{-1}\right)-b\ln\left(x^{-1}\right)\operatorname{Artanh}\left(\frac{c}{x^{2}}\right)+\frac{b\ln\left(x^{-1}\right)}{2}\ln\left(1+\frac{1}{x}\sqrt{-c}\right)+\frac{b\ln\left(x^{-1}\right)}{2}\ln\left(1-\frac{1}{x}\sqrt{-c}\right)+\frac{b}{2}\operatorname{dilog}\left(1+\frac{1}{x}\sqrt{-c}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x^2))/x,x)
```

```
[Out] -a*ln(1/x)-b*ln(1/x)*arctanh(c/x^2)+1/2*b*ln(1/x)*ln(1+(-c)^(1/2)/x)+1/2*b*
ln(1/x)*ln(1-(-c)^(1/2)/x)+1/2*b*dilog(1+(-c)^(1/2)/x)+1/2*b*dilog(1-(-c)^(
1/2)/x)-1/2*b*ln(1/x)*ln(1-1/x*c^(1/2))-1/2*b*ln(1/x)*ln(1+1/x*c^(1/2))-1/2
*b*dilog(1-1/x*c^(1/2))-1/2*b*dilog(1+1/x*c^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}b\int\frac{\log\left(\frac{c}{x^{2}}+1\right)-\log\left(-\frac{c}{x^{2}}+1\right)}{x}\,dx+a\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))/x,x, algorithm="maxima")
```

```
[Out] 1/2*b*integrate((log(c/x^2 + 1) - log(-c/x^2 + 1))/x, x) + a*log(x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b\,\mathrm{artanh}\left(\frac{c}{x^{2}}\right)+a}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))/x,x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c/x^2) + a)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{a+b\,\mathrm{atanh}\left(\frac{c}{x^{2}}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x**2))/x,x)
```

```
[Out] Integral((a + b*atanh(c/x**2))/x, x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\,\mathrm{artanh}\left(\frac{c}{x^2}\right) + a}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c/x^2) + a)/x, x)
```

3.162 $\int \frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{x^3}\,dx$

Optimal. Leaf size=37

$$-\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{2x^2}-\frac{b\log\left(1-\frac{c^2}{x^4}\right)}{4c}$$

```
[Out] -(a + b*ArcTanh[c/x^2])/(2*x^2) - (b*Log[1 - c^2/x^4])/(4*c)
```

Rubi [A] time = 0.0202372, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6097, 260}

$$-\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{2x^2}-\frac{b\log\left(1-\frac{c^2}{x^4}\right)}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c/x^2])/x^3,x]
```

```
[Out] -(a + b*ArcTanh[c/x^2])/(2*x^2) - (b*Log[1 - c^2/x^4])/(4*c)
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{x^3}\,dx &= -\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{2x^2}-(bc)\int \frac{1}{\left(1-\frac{c^2}{x^4}\right)x^5}\,dx\\ &= -\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{2x^2}-\frac{b\log\left(1-\frac{c^2}{x^4}\right)}{4c}\end{aligned}$$

Mathematica [A] time = 0.0089931, size = 42, normalized size = 1.14

$$-\frac{a}{2x^2}-\frac{b\log\left(1-\frac{c^2}{x^4}\right)}{4c}-\frac{b\tanh^{-1}\left(\frac{c}{x^2}\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c/x^2])/x^3,x]
```

```
[Out] -a/(2*x^2) - (b*ArcTanh[c/x^2])/(2*x^2) - (b*Log[1 - c^2/x^4])/(4*c)
```

Maple [A] time = 0.004, size = 37, normalized size = 1.

$$-\frac{a}{2x^2}-\frac{b}{2x^2}\mathrm{Artanh}\left(\frac{c}{x^2}\right)-\frac{b}{4c}\ln\left(1-\frac{c^2}{x^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x^2))/x^3,x)
```

```
[Out] -1/2*a/x^2-1/2*b/x^2*arctanh(c/x^2)-1/4*b*ln(1-c^2/x^4)/c
```

Maxima [A] time = 0.969016, size = 50, normalized size = 1.35

$$-\frac{b\left(\frac{2\,c\,\mathrm{artanh}\left(\frac{c}{x^2}\right)}{x^2}+\log\left(-\frac{c^2}{x^4}+1\right)\right)}{4\,c}-\frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))/x^3,x, algorithm="maxima")
```

```
[Out] -1/4*b*(2*c*arctanh(c/x^2)/x^2 + log(-c^2/x^4 + 1))/c - 1/2*a/x^2
```

Fricas [A] time = 1.77503, size = 126, normalized size = 3.41

$$-\frac{bx^2\log\left(x^4-c^2\right)-4\,bx^2\log\left(x\right)+bc\log\left(\frac{x^2+c}{x^2-c}\right)+2\,ac}{4\,cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(b*x^2*log(x^4 - c^2) - 4*b*x^2*log(x) + b*c*log((x^2 + c)/(x^2 - c))
+ 2*a*c)/(c*x^2)
```

Sympy [A] time = 18.5034, size = 76, normalized size = 2.05

$$\begin{cases} -\frac{a}{2x^2}-\frac{b\,\mathrm{atanh}\left(\frac{c}{x^2}\right)}{2x^2}+\frac{b\log\left(x\right)}{c}-\frac{b\log\left(-i\sqrt{c}+x\right)}{2c}-\frac{b\log\left(i\sqrt{c}+x\right)}{2c}+\frac{b\,\mathrm{atanh}\left(\frac{c}{x^2}\right)}{2c} & \text{for}\ c\neq 0\\ -\frac{a}{2x^2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x**2))/x**3,x)
```

```
[Out] Piecewise((-a/(2*x**2) - b*atanh(c/x**2)/(2*x**2) + b*log(x)/c - b*log(-I*s
qrt(c) + x)/(2*c) - b*log(I*sqrt(c) + x)/(2*c) + b*atanh(c/x**2)/(2*c), Ne(
```

```
c, 0)), (-a/(2*x**2), True))
```

Giac [A] time = 1.27518, size = 70, normalized size = 1.89

$$-\frac{b\log\left(x^4-c^2\right)}{4c}+\frac{b\log\left(x\right)}{c}-\frac{b\log\left(\frac{x^2+c}{x^2-c}\right)}{4x^2}-\frac{a}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))/x^3,x, algorithm="giac")
```

```
[Out] -1/4*b*log(x^4 - c^2)/c + b*log(x)/c - 1/4*b*log((x^2 + c)/(x^2 - c))/x^2 -
 1/2*a/x^2
```

3.163 $\int \frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{x^5} dx$

Optimal. Leaf size=45

$$-\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{4x^4}+\frac{b\tanh^{-1}\left(\frac{x^2}{c}\right)}{4c^2}-\frac{b}{4cx^2}$$

```
[Out] -b/(4*c*x^2) - (a + b*ArcTanh[c/x^2])/(4*x^4) + (b*ArcTanh[x^2/c])/(4*c^2)
```

Rubi [A] time = 0.0320788, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6097, 263, 275, 325, 207}

$$-\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{4x^4}+\frac{b\tanh^{-1}\left(\frac{x^2}{c}\right)}{4c^2}-\frac{b}{4cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c/x^2])/x^5,x]
```

```
[Out] -b/(4*c*x^2) - (a + b*ArcTanh[c/x^2])/(4*x^4) + (b*ArcTanh[x^2/c])/(4*c^2)
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{x^5}\,dx &= -\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{4x^4}-\frac{1}{2}(bc)\int \frac{1}{\left(1-\frac{c^2}{x^4}\right)x^7}\,dx \\
&= -\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{4x^4}-\frac{1}{2}(bc)\int \frac{1}{x^3\left(-c^2+x^4\right)}\,dx \\
&= -\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{4x^4}-\frac{1}{4}(bc)\,\text{Subst}\left(\int \frac{1}{x^2\left(-c^2+x^2\right)}\,dx,x,x^2\right) \\
&= -\frac{b}{4cx^2}-\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{4x^4}-\frac{b\,\text{Subst}\left(\int \frac{1}{-c^2+x^2}\,dx,x,x^2\right)}{4c} \\
&= -\frac{b}{4cx^2}-\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{4x^4}+\frac{b\tanh^{-1}\left(\frac{x^2}{c}\right)}{4c^2}
\end{aligned}
$$

Mathematica [A] time = 0.0113542, size = 64, normalized size = 1.42

$$
-\frac{a}{4x^4}-\frac{b\log\left(x^2-c\right)}{8c^2}+\frac{b\log\left(c+x^2\right)}{8c^2}-\frac{b}{4cx^2}-\frac{b\tanh^{-1}\left(\frac{c}{x^2}\right)}{4x^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c/x^2])/x^5,x]
```

```
[Out] -a/(4*x^4) - b/(4*c*x^2) - (b*ArcTanh[c/x^2])/(4*x^4) - (b*Log[-c + x^2])/(
8*c^2) + (b*Log[c + x^2])/(8*c^2)
```

Maple [A] time = 0.009, size = 57, normalized size = 1.3

$$
-\frac{a}{4\,x^4}-\frac{b}{4\,x^4}\text{Artanh}\left(\frac{c}{x^2}\right)-\frac{b}{4\,cx^2}-\frac{b}{8\,c^2}\ln\left(\frac{c}{x^2}-1\right)+\frac{b}{8\,c^2}\ln\left(1+\frac{c}{x^2}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x^2))/x^5,x)
```

```
[Out] -1/4*a/x^4-1/4*b/x^4*arctanh(c/x^2)-1/4*b/c/x^2-1/8*b/c^2*ln(c/x^2-1)+1/8*b
/c^2*ln(1+c/x^2)
```

Maxima [A] time = 0.967116, size = 76, normalized size = 1.69

$$
\frac{1}{8}\left(c\left(\frac{\log\left(x^2+c\right)}{c^3}-\frac{\log\left(x^2-c\right)}{c^3}-\frac{2}{c^2x^2}\right)-\frac{2\,\text{artanh}\left(\frac{c}{x^2}\right)}{x^4}\right)b-\frac{a}{4\,x^4}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))/x^5,x, algorithm="maxima")
```

```
[Out] 1/8*(c*(log(x^2 + c)/c^3 - log(x^2 - c)/c^3 - 2/(c^2*x^2)) - 2*arctanh(c/x^
2)/x^4)*b - 1/4*a/x^4
```

Fricas [A] time = 1.54632, size = 109, normalized size = 2.42

$$-\frac{2\,bcx^2 + 2\,ac^2 - \left(bx^4 - bc^2\right)\log\left(\frac{x^2+c}{x^2-c}\right)}{8\,c^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))/x^5,x, algorithm="fricas")
```

```
[Out] -1/8*(2*b*c*x^2 + 2*a*c^2 - (b*x^4 - b*c^2)*log((x^2 + c)/(x^2 - c)))/(c^2*
x^4)
```

Sympy [A] time = 45.3849, size = 49, normalized size = 1.09

$$\begin{cases} -\frac{a}{4x^4} - \frac{b\,\mathrm{atanh}\left(\frac{c}{x^2}\right)}{4x^4} - \frac{b}{4cx^2} + \frac{b\,\mathrm{atanh}\left(\frac{c}{x^2}\right)}{4c^2} & \text{for}\: c \neq 0 \\ -\frac{a}{4x^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x**2))/x**5,x)
```

```
[Out] Piecewise((-a/(4*x**4) - b*atanh(c/x**2)/(4*x**4) - b/(4*c*x**2) + b*atanh(
c/x**2)/(4*c**2), Ne(c, 0)), (-a/(4*x**4), True))
```

Giac [A] time = 1.2143, size = 89, normalized size = 1.98

$$\frac{b\log\left(x^2+c\right)}{8\,c^2} - \frac{b\log\left(-x^2+c\right)}{8\,c^2} - \frac{b\log\left(\frac{x^2+c}{x^2-c}\right)}{8\,x^4} - \frac{bx^2+ac}{4\,cx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))/x^5,x, algorithm="giac")
```

```
[Out] 1/8*b*log(x^2 + c)/c^2 - 1/8*b*log(-x^2 + c)/c^2 - 1/8*b*log((x^2 + c)/(x^2
 - c))/x^4 - 1/4*(b*x^2 + a*c)/(c*x^4)
```

3.164 $\int \frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{x^7}\,dx$

Optimal. Leaf size=57

$$-\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{6x^6}-\frac{b\log\left(c^2-x^4\right)}{12c^3}+\frac{b\log(x)}{3c^3}-\frac{b}{12cx^4}$$

```
[Out] -b/(12*c*x^4) - (a + b*ArcTanh[c/x^2])/(6*x^6) + (b*Log[x])/(3*c^3) - (b*Lo
g[c^2 - x^4])/(12*c^3)
```

Rubi [A] time = 0.0400527, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6097, 263, 266, 44}

$$-\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{6x^6}-\frac{b\log\left(c^2-x^4\right)}{12c^3}+\frac{b\log(x)}{3c^3}-\frac{b}{12cx^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c/x^2])/x^7,x]
```

```
[Out] -b/(12*c*x^4) - (a + b*ArcTanh[c/x^2])/(6*x^6) + (b*Log[x])/(3*c^3) - (b*Lo
g[c^2 - x^4])/(12*c^3)
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{x^7}\,dx = -\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{6x^6} - \frac{1}{3}(bc)\int \frac{1}{\left(1-\frac{c^2}{x^4}\right)x^9}\,dx$$
$$= -\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{6x^6} - \frac{1}{3}(bc)\int \frac{1}{x^5\left(-c^2+x^4\right)}\,dx$$
$$= -\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{6x^6} - \frac{1}{12}(bc)\,\text{Subst}\left(\int \frac{1}{x^2\left(-c^2+x\right)}\,dx, x, x^4\right)$$
$$= -\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{6x^6} - \frac{1}{12}(bc)\,\text{Subst}\left(\int \left(-\frac{1}{c^4\left(c^2-x\right)} - \frac{1}{c^2x^2} - \frac{1}{c^4x}\right)dx, x, x^4\right)$$
$$= -\frac{b}{12cx^4} - \frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{6x^6} + \frac{b\log(x)}{3c^3} - \frac{b\log\left(c^2-x^4\right)}{12c^3}$$

Mathematica [A] time = 0.012913, size = 62, normalized size = 1.09

$$-\frac{a}{6x^6} - \frac{b\log\left(x^4-c^2\right)}{12c^3} + \frac{b\log(x)}{3c^3} - \frac{b}{12cx^4} - \frac{b\tanh^{-1}\left(\frac{c}{x^2}\right)}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c/x^2])/x^7,x]
```

```
[Out] -a/(6*x^6) - b/(12*c*x^4) - (b*ArcTanh[c/x^2])/(6*x^6) + (b*Log[x])/(3*c^3)
 - (b*Log[-c^2 + x^4])/(12*c^3)
```

Maple [A] time = 0.006, size = 45, normalized size = 0.8

$$-\frac{a}{6\,x^6} - \frac{b}{6\,x^6}\text{Artanh}\left(\frac{c}{x^2}\right) - \frac{b}{12\,cx^4} - \frac{b}{12\,c^3}\ln\left(\frac{c^2}{x^4}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x^2))/x^7,x)
```

```
[Out] -1/6*a/x^6-1/6*b/x^6*arctanh(c/x^2)-1/12*b/c/x^4-1/12*b/c^3*ln(c^2/x^4-1)
```

Maxima [A] time = 0.979566, size = 74, normalized size = 1.3

$$-\frac{1}{12}\left(c\left(\frac{\log\left(x^4-c^2\right)}{c^4} - \frac{\log\left(x^4\right)}{c^4} + \frac{1}{c^2x^4}\right) + \frac{2\,\text{artanh}\left(\frac{c}{x^2}\right)}{x^6}\right)b - \frac{a}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))/x^7,x, algorithm="maxima")
```

```
[Out] -1/12*(c*(log(x^4 - c^2)/c^4 - log(x^4)/c^4 + 1/(c^2*x^4)) + 2*arctanh(c/x^
2)/x^6)*b - 1/6*a/x^6
```

Fricas [A] time = 1.67821, size = 151, normalized size = 2.65

$$-\frac{bx^6 \log\left(x^4-c^2\right)-4\,bx^6 \log\left(x\right)+bc^2x^2+bc^3 \log\left(\frac{x^2+c}{x^2-c}\right)+2\,ac^3}{12\,c^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))/x^7,x, algorithm="fricas")
```

```
[Out] -1/12*(b*x^6*log(x^4 - c^2) - 4*b*x^6*log(x) + b*c^2*x^2 + b*c^3*log((x^2 +
 c)/(x^2 - c)) + 2*a*c^3)/(c^3*x^6)
```

Sympy [A] time = 47.1008, size = 94, normalized size = 1.65

$$\begin{cases} -\frac{a}{6x^6} - \frac{b\operatorname{atanh}\left(\frac{c}{x^2}\right)}{6x^6} - \frac{b}{12cx^4} + \frac{b\log\left(x\right)}{3c^3} - \frac{b\log\left(-i\sqrt{c}+x\right)}{6c^3} - \frac{b\log\left(i\sqrt{c}+x\right)}{6c^3} + \frac{b\operatorname{atanh}\left(\frac{c}{x^2}\right)}{6c^3} & \text{for}\: c \neq 0 \\ -\frac{a}{6x^6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x**2))/x**7,x)
```

```
[Out] Piecewise((-a/(6*x**6) - b*atanh(c/x**2)/(6*x**6) - b/(12*c*x**4) + b*log(x
)/(3*c**3) - b*log(-I*sqrt(c) + x)/(6*c**3) - b*log(I*sqrt(c) + x)/(6*c**3)
 + b*atanh(c/x**2)/(6*c**3), Ne(c, 0)), (-a/(6*x**6), True))
```

Giac [A] time = 1.16046, size = 88, normalized size = 1.54

$$-\frac{b\log\left(x^4-c^2\right)}{12\,c^3}+\frac{b\log\left(x\right)}{3\,c^3}-\frac{b\log\left(\frac{x^2+c}{x^2-c}\right)}{12\,x^6}-\frac{bx^2+2\,ac}{12\,cx^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))/x^7,x, algorithm="giac")
```

```
[Out] -1/12*b*log(x^4 - c^2)/c^3 + 1/3*b*log(x)/c^3 - 1/12*b*log((x^2 + c)/(x^2 -
 c))/x^6 - 1/12*(b*x^2 + 2*a*c)/(c*x^6)
```

3.165 $\int x^4 \left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right) dx$

Optimal. Leaf size=63

$$\frac{1}{5}x^5\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{5}bc^{5/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)-\frac{1}{5}bc^{5/2}\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)+\frac{2}{15}bcx^3$$

```
[Out] (2*b*c*x^3)/15 + (b*c^(5/2)*ArcTan[x/Sqrt[c]])/5 + (x^5*(a + b*ArcTanh[c/x^
2]))/5 - (b*c^(5/2)*ArcTanh[x/Sqrt[c]])/5
```

Rubi [A] time = 0.0347141, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6097, 263, 321, 298, 203, 206}

$$\frac{1}{5}x^5\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{5}bc^{5/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)-\frac{1}{5}bc^{5/2}\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)+\frac{2}{15}bcx^3$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*ArcTanh[c/x^2]),x]
```

```
[Out] (2*b*c*x^3)/15 + (b*c^(5/2)*ArcTan[x/Sqrt[c]])/5 + (x^5*(a + b*ArcTanh[c/x^
2]))/5 - (b*c^(5/2)*ArcTanh[x/Sqrt[c]])/5
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int x^4\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)dx &= \frac{1}{5}x^5\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{5}(2bc)\int\frac{x^2}{1-\frac{c^2}{x^4}}\,dx\\ &= \frac{1}{5}x^5\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{5}(2bc)\int\frac{x^6}{-c^2+x^4}\,dx\\ &= \frac{2}{15}bcx^3+\frac{1}{5}x^5\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{5}\left(2bc^3\right)\int\frac{x^2}{-c^2+x^4}\,dx\\ &= \frac{2}{15}bcx^3+\frac{1}{5}x^5\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)-\frac{1}{5}\left(bc^3\right)\int\frac{1}{c-x^2}\,dx+\frac{1}{5}\left(bc^3\right)\int\frac{1}{c+x^2}\,dx\\ &= \frac{2}{15}bcx^3+\frac{1}{5}bc^{5/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)+\frac{1}{5}x^5\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)-\frac{1}{5}bc^{5/2}\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)\end{aligned}$$

Mathematica [A] time = 0.0205633, size = 88, normalized size = 1.4

$$\frac{ax^5}{5}+\frac{1}{10}bc^{5/2}\log\left(\sqrt{c}-x\right)-\frac{1}{10}bc^{5/2}\log\left(\sqrt{c}+x\right)+\frac{1}{5}bc^{5/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)+\frac{2}{15}bcx^3+\frac{1}{5}bx^5\tanh^{-1}\left(\frac{c}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*ArcTanh[c/x^2]),x]
```

```
[Out] (2*b*c*x^3)/15 + (a*x^5)/5 + (b*c^(5/2)*ArcTan[x/Sqrt[c]])/5 + (b*x^5*ArcTa
nh[c/x^2])/5 + (b*c^(5/2)*Log[Sqrt[c] - x])/10 - (b*c^(5/2)*Log[Sqrt[c] + x
])/10
```

Maple [A] time = 0.014, size = 53, normalized size = 0.8

$$\frac{ax^5}{5}+\frac{bx^5}{5}\mathrm{Artanh}\left(\frac{c}{x^2}\right)+\frac{b}{5}c^{\frac{5}{2}}\arctan\left(x\frac{1}{\sqrt{c}}\right)-\frac{b}{5}c^{\frac{5}{2}}\mathrm{Artanh}\left(\frac{1}{x}\sqrt{c}\right)+\frac{2\,bcx^3}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arctanh(c/x^2)),x)
```

```
[Out] 1/5*a*x^5+1/5*b*x^5*arctanh(c/x^2)+1/5*b*c^(5/2)*arctan(x/c^(1/2))-1/5*b*c^
(5/2)*arctanh(1/x*c^(1/2))+2/15*b*c*x^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c/x^2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.83608, size = 440, normalized size = 6.98

$$\left[\frac{1}{10}\,bx^5\log\left(\frac{x^2+c}{x^2-c}\right)+\frac{1}{5}\,ax^5+\frac{2}{15}\,bcx^3+\frac{1}{5}\,bc^{\frac{5}{2}}\arctan\left(\frac{x}{\sqrt{c}}\right)+\frac{1}{10}\,bc^{\frac{5}{2}}\log\left(\frac{x^2-2\sqrt{c}x+c}{x^2-c}\right),\frac{1}{10}\,bx^5\log\left(\frac{x^2+c}{x^2-c}\right)+\frac{1}{5}\,ax\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c/x^2)),x, algorithm="fricas")
```

```
[Out] [1/10*b*x^5*log((x^2 + c)/(x^2 - c)) + 1/5*a*x^5 + 2/15*b*c*x^3 + 1/5*b*c^(
5/2)*arctan(x/sqrt(c)) + 1/10*b*c^(5/2)*log((x^2 - 2*sqrt(c)*x + c)/(x^2 -
c)), 1/10*b*x^5*log((x^2 + c)/(x^2 - c)) + 1/5*a*x^5 + 2/15*b*c*x^3 + 1/5*b
*sqrt(-c)*c^2*arctan(sqrt(-c)*x/c) + 1/10*b*sqrt(-c)*c^2*log((x^2 + 2*sqrt(
-c)*x - c)/(x^2 + c))]
```

Sympy [A] time = 42.2058, size = 588, normalized size = 9.33

$$\begin{cases}\frac{ax^5}{5}\\ \frac{x^5(a-\infty b)}{5}\\ \frac{x^5(a+\infty b)}{5}\\ -\frac{6ac^{69}x^5}{-30c^{69}+30c^{67}x^4}+\frac{6ac^{67}x^9}{-30c^{69}+30c^{67}x^4}-\frac{6bc^{\frac{143}{2}}\log\left(-\sqrt{c}+x\right)}{-30c^{69}+30c^{67}x^4}+\frac{3bc^{\frac{143}{2}}\log\left(-i\sqrt{c}+x\right)}{-30c^{69}+30c^{67}x^4}+\frac{3ibc^{\frac{143}{2}}\log\left(-i\sqrt{c}+x\right)}{-30c^{69}+30c^{67}x^4}+\frac{3bc^{\frac{143}{2}}\log\left(i\sqrt{c}+x\right)}{-30c^{69}+30c^{67}x^4}-\frac{3ibc^{\frac{143}{2}}\log\left(i\sqrt{\right.}}{-30c^{69}+30c^{6}}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*atanh(c/x**2)),x)
```

```
[Out] Piecewise((a*x**5/5, Eq(c, 0)), (x**5*(a - oo*b)/5, Eq(c, -x**2)), (x**5*(a
 + oo*b)/5, Eq(c, x**2)), (-6*a*c**69*x**5/(-30*c**69 + 30*c**67*x**4) + 6*
a*c**67*x**9/(-30*c**69 + 30*c**67*x**4) - 6*b*c**(143/2)*log(-sqrt(c) + x)
/(-30*c**69 + 30*c**67*x**4) + 3*b*c**(143/2)*log(-I*sqrt(c) + x)/(-30*c**6
9 + 30*c**67*x**4) + 3*I*b*c**(143/2)*log(-I*sqrt(c) + x)/(-30*c**69 + 30*c
**67*x**4) + 3*b*c**(143/2)*log(I*sqrt(c) + x)/(-30*c**69 + 30*c**67*x**4)
- 3*I*b*c**(143/2)*log(I*sqrt(c) + x)/(-30*c**69 + 30*c**67*x**4) - 6*b*c**
(143/2)*atanh(c/x**2)/(-30*c**69 + 30*c**67*x**4) + 6*b*c**(139/2)*x**4*log
(-sqrt(c) + x)/(-30*c**69 + 30*c**67*x**4) - 3*b*c**(139/2)*x**4*log(-I*sqr
t(c) + x)/(-30*c**69 + 30*c**67*x**4) - 3*I*b*c**(139/2)*x**4*log(-I*sqrt(c
) + x)/(-30*c**69 + 30*c**67*x**4) - 3*b*c**(139/2)*x**4*log(I*sqrt(c) + x)
/(-30*c**69 + 30*c**67*x**4) + 3*I*b*c**(139/2)*x**4*log(I*sqrt(c) + x)/(-3
0*c**69 + 30*c**67*x**4) + 6*b*c**(139/2)*x**4*atanh(c/x**2)/(-30*c**69 + 3
0*c**67*x**4) - 4*b*c**70*x**3/(-30*c**69 + 30*c**67*x**4) - 6*b*c**69*x**5
*atanh(c/x**2)/(-30*c**69 + 30*c**67*x**4) + 4*b*c**68*x**7/(-30*c**69 + 30
*c**67*x**4) + 6*b*c**67*x**9*atanh(c/x**2)/(-30*c**69 + 30*c**67*x**4), Tr
ue))
```

Giac [A] time = 1.30346, size = 90, normalized size = 1.43

$$\frac{1}{10}\,bx^5\log\left(\frac{x^2+c}{x^2-c}\right)+\frac{1}{5}\,ax^5+\frac{2}{15}\,bcx^3+\frac{1}{5}\,bc^3\left(\frac{\arctan\left(\frac{x}{\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{\arctan\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c/x^2)),x, algorithm="giac")
```

```
[Out] 1/10*b*x^5*log((x^2 + c)/(x^2 - c)) + 1/5*a*x^5 + 2/15*b*c*x^3 + 1/5*b*c^3*
(arctan(x/sqrt(-c))/sqrt(-c) + arctan(x/sqrt(c))/sqrt(c))
```

3.166 $\int x^2 \left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right) dx$

Optimal. Leaf size=61

$$\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)-\frac{1}{3}bc^{3/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)-\frac{1}{3}bc^{3/2}\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)+\frac{2bcx}{3}$$

```
[Out] (2*b*c*x)/3 - (b*c^(3/2)*ArcTan[x/Sqrt[c]])/3 + (x^3*(a + b*ArcTanh[c/x^2])
)/3 - (b*c^(3/2)*ArcTanh[x/Sqrt[c]])/3
```

Rubi [A] time = 0.0331548, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6097, 193, 321, 212, 206, 203}

$$\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)-\frac{1}{3}bc^{3/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)-\frac{1}{3}bc^{3/2}\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)+\frac{2bcx}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcTanh[c/x^2]),x]
```

```
[Out] (2*b*c*x)/3 - (b*c^(3/2)*ArcTan[x/Sqrt[c]])/3 + (x^3*(a + b*ArcTanh[c/x^2])
)/3 - (b*c^(3/2)*ArcTanh[x/Sqrt[c]])/3
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)dx &= \frac{1}{3}x^3\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{3}(2bc)\int\frac{1}{1-\frac{c^2}{x^4}}\,dx\\ &= \frac{1}{3}x^3\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{3}(2bc)\int\frac{x^4}{-c^2+x^4}\,dx\\ &= \frac{2bcx}{3}+\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)+\frac{1}{3}\left(2bc^3\right)\int\frac{1}{-c^2+x^4}\,dx\\ &= \frac{2bcx}{3}+\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)-\frac{1}{3}\left(bc^2\right)\int\frac{1}{c-x^2}\,dx-\frac{1}{3}\left(bc^2\right)\int\frac{1}{c+x^2}\,dx\\ &= \frac{2bcx}{3}-\frac{1}{3}bc^{3/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)+\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)-\frac{1}{3}bc^{3/2}\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)\end{aligned}$$

Mathematica [A] time = 0.0183367, size = 86, normalized size = 1.41

$$\frac{ax^3}{3}+\frac{1}{6}bc^{3/2}\log\left(\sqrt{c}-x\right)-\frac{1}{6}bc^{3/2}\log\left(\sqrt{c}+x\right)-\frac{1}{3}bc^{3/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)+\frac{1}{3}bx^3\tanh^{-1}\left(\frac{c}{x^2}\right)+\frac{2bcx}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcTanh[c/x^2]),x]
```

```
[Out] (2*b*c*x)/3 + (a*x^3)/3 - (b*c^(3/2)*ArcTan[x/Sqrt[c]])/3 + (b*x^3*ArcTanh[
c/x^2])/3 + (b*c^(3/2)*Log[Sqrt[c] - x])/6 - (b*c^(3/2)*Log[Sqrt[c] + x])/6
```

Maple [A] time = 0.008, size = 51, normalized size = 0.8

$$\frac{x^3a}{3}+\frac{bx^3}{3}\mathrm{Artanh}\left(\frac{c}{x^2}\right)-\frac{b}{3}c^{\frac{3}{2}}\arctan\left(x\frac{1}{\sqrt{c}}\right)+\frac{2\,xbc}{3}-\frac{b}{3}c^{\frac{3}{2}}\mathrm{Artanh}\left(\frac{1}{x}\sqrt{c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c/x^2)),x)
```

```
[Out] 1/3*x^3*a+1/3*b*x^3*arctanh(c/x^2)-1/3*b*c^(3/2)*arctan(x/c^(1/2))+2/3*x*b*
c-1/3*b*c^(3/2)*arctanh(1/x*c^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c/x^2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.77425, size = 421, normalized size = 6.9

$$\left[\frac{1}{6}\,bx^3\log\left(\frac{x^2+c}{x^2-c}\right)+\frac{1}{3}\,ax^3-\frac{1}{3}\,bc^{\frac{3}{2}}\arctan\left(\frac{x}{\sqrt{c}}\right)+\frac{1}{6}\,bc^{\frac{3}{2}}\log\left(\frac{x^2-2\sqrt{c}x+c}{x^2-c}\right)+\frac{2}{3}\,bcx,\frac{1}{6}\,bx^3\log\left(\frac{x^2+c}{x^2-c}\right)+\frac{1}{3}\,ax^3+\frac{1}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c/x^2)),x, algorithm="fricas")
```

```
[Out] [1/6*b*x^3*log((x^2 + c)/(x^2 - c)) + 1/3*a*x^3 - 1/3*b*c^(3/2)*arctan(x/sq
rt(c)) + 1/6*b*c^(3/2)*log((x^2 - 2*sqrt(c)*x + c)/(x^2 - c)) + 2/3*b*c*x,
1/6*b*x^3*log((x^2 + c)/(x^2 - c)) + 1/3*a*x^3 + 1/3*b*sqrt(-c)*c*arctan(sq
rt(-c)*x/c) + 1/6*b*sqrt(-c)*c*log((x^2 - 2*sqrt(-c)*x - c)/(x^2 + c)) + 2/
3*b*c*x]
```

Sympy [A] time = 22.2281, size = 624, normalized size = 10.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c/x**2)),x)
```

```
[Out] Piecewise((a*x**3/3, Eq(c, 0)), (x**3*(a - oo*b)/3, Eq(c, -x**2)), (x**3*(a
 + oo*b)/3, Eq(c, x**2)), (-2*a*c**(29/2)*x**3/(-6*c**(29/2) + 6*c**(25/2)*
x**4) + 2*a*c**(25/2)*x**7/(-6*c**(29/2) + 6*c**(25/2)*x**4) - 4*b*c**(31/2
)*x/(-6*c**(29/2) + 6*c**(25/2)*x**4) - 2*b*c**(29/2)*x**3*atanh(c/x**2)/(-
6*c**(29/2) + 6*c**(25/2)*x**4) + 4*b*c**(27/2)*x**5/(-6*c**(29/2) + 6*c**(
25/2)*x**4) + 2*b*c**(25/2)*x**7*atanh(c/x**2)/(-6*c**(29/2) + 6*c**(25/2)*
x**4) - 2*b*c**16*log(-sqrt(c) + x)/(-6*c**(29/2) + 6*c**(25/2)*x**4) + b*c
**16*log(-I*sqrt(c) + x)/(-6*c**(29/2) + 6*c**(25/2)*x**4) - I*b*c**16*log(
-I*sqrt(c) + x)/(-6*c**(29/2) + 6*c**(25/2)*x**4) + b*c**16*log(I*sqrt(c) +
 x)/(-6*c**(29/2) + 6*c**(25/2)*x**4) + I*b*c**16*log(I*sqrt(c) + x)/(-6*c*
*(29/2) + 6*c**(25/2)*x**4) - 2*b*c**16*atanh(c/x**2)/(-6*c**(29/2) + 6*c**
(25/2)*x**4) + 2*b*c**14*x**4*log(-sqrt(c) + x)/(-6*c**(29/2) + 6*c**(25/2)
*x**4) - b*c**14*x**4*log(-I*sqrt(c) + x)/(-6*c**(29/2) + 6*c**(25/2)*x**4)
 + I*b*c**14*x**4*log(-I*sqrt(c) + x)/(-6*c**(29/2) + 6*c**(25/2)*x**4) - b
*c**14*x**4*log(I*sqrt(c) + x)/(-6*c**(29/2) + 6*c**(25/2)*x**4) - I*b*c**1
4*x**4*log(I*sqrt(c) + x)/(-6*c**(29/2) + 6*c**(25/2)*x**4) + 2*b*c**14*x**
4*atanh(c/x**2)/(-6*c**(29/2) + 6*c**(25/2)*x**4), True))
```

Giac [A] time = 1.25818, size = 93, normalized size = 1.52

$$\frac{1}{3}\,bc^3\left(\frac{\arctan\left(\frac{x}{\sqrt{-c}}\right)}{\sqrt{-c}c}-\frac{\arctan\left(\frac{x}{\sqrt{c}}\right)}{c^{\frac{3}{2}}}\right)+\frac{1}{6}\,bx^3\log\left(\frac{x^2+c}{x^2-c}\right)+\frac{1}{3}\,ax^3+\frac{2}{3}\,bcx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c/x^2)),x, algorithm="giac")
```

```
[Out] 1/3*b*c^3*(arctan(x/sqrt(-c))/(sqrt(-c)*c) - arctan(x/sqrt(c))/c^(3/2)) + 1
/6*b*x^3*log((x^2 + c)/(x^2 - c)) + 1/3*a*x^3 + 2/3*b*c*x
```

3.167 $\int \left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right) dx$

Optimal. Leaf size=44

$$ax+bx\tanh^{-1}\left(\frac{c}{x^2}\right)+b\sqrt{c}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)-b\sqrt{c}\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)$$

```
[Out] a*x + b*Sqrt[c]*ArcTan[x/Sqrt[c]] + b*x*ArcTanh[c/x^2] - b*Sqrt[c]*ArcTanh[
x/Sqrt[c]]
```

Rubi [A] time = 0.0244142, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6091, 263, 298, 203, 206}

$$ax+bx\tanh^{-1}\left(\frac{c}{x^2}\right)+b\sqrt{c}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)-b\sqrt{c}\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcTanh[c/x^2],x]
```

```
[Out] a*x + b*Sqrt[c]*ArcTan[x/Sqrt[c]] + b*x*ArcTanh[c/x^2] - b*Sqrt[c]*ArcTanh[
x/Sqrt[c]]
```

Rule 6091

```
Int[ArcTanh[(c_.)*(x_)^(n_)], x_Symbol] :> Simp[x*ArcTanh[c*x^n], x] - Dist
[c*n, Int[x^n/(1 - c^2*x^(2*n)), x], x] /; FreeQ[{c, n}, x]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)dx &= ax+b\int\tanh^{-1}\left(\frac{c}{x^2}\right)dx\\
&= ax+bx\tanh^{-1}\left(\frac{c}{x^2}\right)+(2bc)\int\frac{1}{\left(1-\frac{c^2}{x^4}\right)x^2}\,dx\\
&= ax+bx\tanh^{-1}\left(\frac{c}{x^2}\right)+(2bc)\int\frac{x^2}{-c^2+x^4}\,dx\\
&= ax+bx\tanh^{-1}\left(\frac{c}{x^2}\right)-(bc)\int\frac{1}{c-x^2}\,dx+(bc)\int\frac{1}{c+x^2}\,dx\\
&= ax+b\sqrt{c}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)+bx\tanh^{-1}\left(\frac{c}{x^2}\right)-b\sqrt{c}\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0158426, size = 54, normalized size = 1.23

$$ax+bx\tanh^{-1}\left(\frac{c}{x^2}\right)+\frac{1}{2}b\sqrt{c}\left(\log\left(\sqrt{c}-x\right)-\log\left(\sqrt{c}+x\right)+2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcTanh[c/x^2],x]
```

```
[Out] a*x + b*x*ArcTanh[c/x^2] + (b*Sqrt[c]*(2*ArcTan[x/Sqrt[c]] + Log[Sqrt[c] -
x] - Log[Sqrt[c] + x]))/2
```

Maple [A] time = 0.01, size = 39, normalized size = 0.9

$$ax+bx\mathrm{Artanh}\left(\frac{c}{x^2}\right)-\mathrm{Artanh}\left(\frac{1}{x}\sqrt{c}\right)\sqrt{c}b+b\arctan\left(x\frac{1}{\sqrt{c}}\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arctanh(c/x^2),x)
```

```
[Out] a*x+b*x*arctanh(c/x^2)-arctanh(1/x*c^(1/2))*c^(1/2)*b+b*arctan(x/c^(1/2))*c
^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c/x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.76386, size = 351, normalized size = 7.98

$$\left[\frac{1}{2}bx\log\left(\frac{x^2+c}{x^2-c}\right)+b\sqrt{c}\arctan\left(\frac{x}{\sqrt{c}}\right)+\frac{1}{2}b\sqrt{c}\log\left(\frac{x^2-2\sqrt{c}x+c}{x^2-c}\right)+ax,\frac{1}{2}bx\log\left(\frac{x^2+c}{x^2-c}\right)+b\sqrt{-c}\arctan\left(\frac{\sqrt{-c}x}{c}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c/x^2),x, algorithm="fricas")
```

```
[Out] [1/2*b*x*log((x^2 + c)/(x^2 - c)) + b*sqrt(c)*arctan(x/sqrt(c)) + 1/2*b*sqr
t(c)*log((x^2 - 2*sqrt(c)*x + c)/(x^2 - c)) + a*x, 1/2*b*x*log((x^2 + c)/(x
^2 - c)) + b*sqrt(-c)*arctan(sqrt(-c)*x/c) + 1/2*b*sqrt(-c)*log((x^2 + 2*sq
rt(-c)*x - c)/(x^2 + c)) + a*x]
```

Sympy [A] time = 16.7275, size = 473, normalized size = 10.75

$$ax + b\left(\begin{cases} 0 \\ -\infty x \\ \infty x \\ -\frac{2c^{\frac{21}{2}}x\,\operatorname{atanh}\left(\frac{c}{x^2}\right)}{-2c^{\frac{21}{2}}+2c^{\frac{17}{2}}x^4} + \frac{2c^{\frac{17}{2}}x^5\operatorname{atanh}\left(\frac{c}{x^2}\right)}{-2c^{\frac{21}{2}}+2c^{\frac{17}{2}}x^4} - \frac{2c^{11}\log\left(-\sqrt{c}+x\right)}{-2c^{\frac{21}{2}}+2c^{\frac{17}{2}}x^4} + \frac{c^{11}\log\left(-i\sqrt{c}+x\right)}{-2c^{\frac{21}{2}}+2c^{\frac{17}{2}}x^4} + \frac{ic^{11}\log\left(-i\sqrt{c}+x\right)}{-2c^{\frac{21}{2}}+2c^{\frac{17}{2}}x^4} + \frac{c^{11}\log\left(i\sqrt{c}+x\right)}{-2c^{\frac{21}{2}}+2c^{\frac{17}{2}}x^4} - \frac{ic^{11}\log\left(i\sqrt{c}+\right.}{-2c^{\frac{21}{2}}+2c^{\frac{17}{2}}} \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*atanh(c/x**2),x)
```

```
[Out] a*x + b*Piecewise((0, Eq(c, 0)), (-oo*x, Eq(c, -x**2)), (oo*x, Eq(c, x**2))
, (-2*c**(21/2)*x*atanh(c/x**2)/(-2*c**(21/2) + 2*c**(17/2)*x**4) + 2*c**(1
7/2)*x**5*atanh(c/x**2)/(-2*c**(21/2) + 2*c**(17/2)*x**4) - 2*c**11*log(-sq
rt(c) + x)/(-2*c**(21/2) + 2*c**(17/2)*x**4) + c**11*log(-I*sqrt(c) + x)/(-
2*c**(21/2) + 2*c**(17/2)*x**4) + I*c**11*log(-I*sqrt(c) + x)/(-2*c**(21/2)
 + 2*c**(17/2)*x**4) + c**11*log(I*sqrt(c) + x)/(-2*c**(21/2) + 2*c**(17/2)
*x**4) - I*c**11*log(I*sqrt(c) + x)/(-2*c**(21/2) + 2*c**(17/2)*x**4) - 2*c
**11*atanh(c/x**2)/(-2*c**(21/2) + 2*c**(17/2)*x**4) + 2*c**9*x**4*log(-sqr
t(c) + x)/(-2*c**(21/2) + 2*c**(17/2)*x**4) - c**9*x**4*log(-I*sqrt(c) + x)
/(-2*c**(21/2) + 2*c**(17/2)*x**4) - I*c**9*x**4*log(-I*sqrt(c) + x)/(-2*c*
*(21/2) + 2*c**(17/2)*x**4) - c**9*x**4*log(I*sqrt(c) + x)/(-2*c**(21/2) +
2*c**(17/2)*x**4) + I*c**9*x**4*log(I*sqrt(c) + x)/(-2*c**(21/2) + 2*c**(17
/2)*x**4) + 2*c**9*x**4*atanh(c/x**2)/(-2*c**(21/2) + 2*c**(17/2)*x**4), Tr
ue))
```

Giac [A] time = 1.21485, size = 77, normalized size = 1.75

$$\frac{1}{2}\left(2\,c\left(\frac{\arctan\left(\frac{x}{\sqrt{-c}}\right)}{\sqrt{-c}} + \frac{\arctan\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}}\right) + x\log\left(-\frac{\frac{c}{x^2}+1}{\frac{c}{x^2}-1}\right)\right)b + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c/x^2),x, algorithm="giac")
```

```
[Out] 1/2*(2*c*(arctan(x/sqrt(-c))/sqrt(-c) + arctan(x/sqrt(c))/sqrt(c)) + x*log(
-(c/x^2 + 1)/(c/x^2 - 1)))*b + a*x
```

3.168 $\int \frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{x^2}\,dx$

Optimal. Leaf size=46

$$-\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{x}+\frac{b\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}}+\frac{b\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}}$$

```
[Out] (b*ArcTan[x/Sqrt[c]])/Sqrt[c] - (a + b*ArcTanh[c/x^2])/x + (b*ArcTanh[x/Sqr
t[c]])/Sqrt[c]
```

Rubi [A] time = 0.0309239, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6097, 263, 212, 206, 203}

$$-\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{x}+\frac{b\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}}+\frac{b\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c/x^2])/x^2,x]
```

```
[Out] (b*ArcTan[x/Sqrt[c]])/Sqrt[c] - (a + b*ArcTanh[c/x^2])/x + (b*ArcTanh[x/Sqr
t[c]])/Sqrt[c]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{x^2}\,dx &= -\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{x}-(2bc)\int \frac{1}{\left(1-\frac{c^2}{x^4}\right)x^4}\,dx\\ &= -\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{x}-(2bc)\int \frac{1}{-c^2+x^4}\,dx\\ &= -\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{x}+b\int \frac{1}{c-x^2}\,dx+b\int \frac{1}{c+x^2}\,dx\\ &= \frac{b\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{x}+\frac{b\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.0180695, size = 72, normalized size = 1.57

$$-\frac{a}{x}-\frac{b\tanh^{-1}\left(\frac{c}{x^2}\right)}{x}-\frac{b\log\left(\sqrt{c}-x\right)}{2\sqrt{c}}+\frac{b\log\left(\sqrt{c}+x\right)}{2\sqrt{c}}+\frac{b\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c/x^2])/x^2,x]
```

```
[Out] -(a/x) + (b*ArcTan[x/Sqrt[c]])/Sqrt[c] - (b*ArcTanh[c/x^2])/x - (b*Log[Sqrt
[c] - x])/(2*Sqrt[c]) + (b*Log[Sqrt[c] + x])/(2*Sqrt[c])
```

Maple [A] time = 0.01, size = 44, normalized size = 1.

$$-\frac{a}{x}-\frac{b}{x}\mathrm{Artanh}\left(\frac{c}{x^2}\right)+b\arctan\left(x\frac{1}{\sqrt{c}}\right)\frac{1}{\sqrt{c}}+b\mathrm{Artanh}\left(\frac{1}{x}\sqrt{c}\right)\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x^2))/x^2,x)
```

```
[Out] -a/x-b/x*arctanh(c/x^2)+b*arctan(x/c^(1/2))/c^(1/2)+b/c^(1/2)*arctanh(1/x*c
^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.7739, size = 385, normalized size = 8.37

$$\left[\frac{2\,b\sqrt{c}x\arctan\left(\frac{x}{\sqrt{c}}\right)+b\sqrt{c}x\log\left(\frac{x^2+2\,\sqrt{c}x+c}{x^2-c}\right)-bc\log\left(\frac{x^2+c}{x^2-c}\right)-2\,ac}{2\,cx},-\frac{2\,b\sqrt{-c}x\arctan\left(\frac{\sqrt{-c}x}{c}\right)+b\sqrt{-c}x\log\left(\frac{x^2-2\,\sqrt{-c}x-}{x^2+c}\right.}{2\,cx}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*b*sqrt(c)*x*arctan(x/sqrt(c)) + b*sqrt(c)*x*log((x^2 + 2*sqrt(c)*x
+ c)/(x^2 - c)) - b*c*log((x^2 + c)/(x^2 - c)) - 2*a*c)/(c*x), -1/2*(2*b*sq
rt(-c)*x*arctan(sqrt(-c)*x/c) + b*sqrt(-c)*x*log((x^2 - 2*sqrt(-c)*x - c)/(
x^2 + c)) + b*c*log((x^2 + c)/(x^2 - c)) + 2*a*c)/(c*x)]
```

Sympy [A] time = 13.8796, size = 593, normalized size = 12.89

$$\begin{cases} -\frac{a}{x} \\ -\frac{a-\infty b}{x} \\ -\frac{a+\infty b}{x} \\ \frac{2ac^{\frac{39}{2}}}{-2c^{\frac{39}{2}}x+2c^{\frac{35}{2}}x^5}-\frac{2ac^{\frac{35}{2}}x^4}{-2c^{\frac{39}{2}}x+2c^{\frac{35}{2}}x^5}+\frac{2bc^{\frac{39}{2}}\operatorname{atanh}\left(\frac{c}{x^2}\right)}{-2c^{\frac{39}{2}}x+2c^{\frac{35}{2}}x^5}-\frac{2bc^{\frac{35}{2}}x^4\operatorname{atanh}\left(\frac{c}{x^2}\right)}{-2c^{\frac{39}{2}}x+2c^{\frac{35}{2}}x^5}+\frac{2bc^{19}x\log\left(-\sqrt{c}+x\right)}{-2c^{\frac{39}{2}}x+2c^{\frac{35}{2}}x^5}-\frac{bc^{19}x\log\left(-i\sqrt{c}+x\right)}{-2c^{\frac{39}{2}}x+2c^{\frac{35}{2}}x^5}+\frac{ibc^{19}x\log\left(-i\sqrt{c}\right.}{-2c^{\frac{39}{2}}x+2c^{\frac{35}{2}}x}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x**2))/x**2,x)
```

```
[Out] Piecewise((-a/x, Eq(c, 0)), (-(a - oo*b)/x, Eq(c, -x**2)), (-(a + oo*b)/x,
Eq(c, x**2)), (2*a*c**(39/2)/(-2*c**(39/2)*x + 2*c**(35/2)*x**5) - 2*a*c**(
35/2)*x**4/(-2*c**(39/2)*x + 2*c**(35/2)*x**5) + 2*b*c**(39/2)*atanh(c/x**2
)/(-2*c**(39/2)*x + 2*c**(35/2)*x**5) - 2*b*c**(35/2)*x**4*atanh(c/x**2)/(-
2*c**(39/2)*x + 2*c**(35/2)*x**5) + 2*b*c**19*x*log(-sqrt(c) + x)/(-2*c**(3
9/2)*x + 2*c**(35/2)*x**5) - b*c**19*x*log(-I*sqrt(c) + x)/(-2*c**(39/2)*x
+ 2*c**(35/2)*x**5) + I*b*c**19*x*log(-I*sqrt(c) + x)/(-2*c**(39/2)*x + 2*c
**(35/2)*x**5) - b*c**19*x*log(I*sqrt(c) + x)/(-2*c**(39/2)*x + 2*c**(35/2)
*x**5) - I*b*c**19*x*log(I*sqrt(c) + x)/(-2*c**(39/2)*x + 2*c**(35/2)*x**5)
 + 2*b*c**19*x*atanh(c/x**2)/(-2*c**(39/2)*x + 2*c**(35/2)*x**5) - 2*b*c**1
7*x**5*log(-sqrt(c) + x)/(-2*c**(39/2)*x + 2*c**(35/2)*x**5) + b*c**17*x**5
*log(-I*sqrt(c) + x)/(-2*c**(39/2)*x + 2*c**(35/2)*x**5) - I*b*c**17*x**5*l
og(-I*sqrt(c) + x)/(-2*c**(39/2)*x + 2*c**(35/2)*x**5) + b*c**17*x**5*log(I
*sqrt(c) + x)/(-2*c**(39/2)*x + 2*c**(35/2)*x**5) + I*b*c**17*x**5*log(I*sq
rt(c) + x)/(-2*c**(39/2)*x + 2*c**(35/2)*x**5) - 2*b*c**17*x**5*atanh(c/x**
2)/(-2*c**(39/2)*x + 2*c**(35/2)*x**5), True))
```

Giac [A] time = 1.23177, size = 84, normalized size = 1.83

$$-bc\left(\frac{\arctan\left(\frac{x}{\sqrt{-c}}\right)}{\sqrt{-c}c}-\frac{\arctan\left(\frac{x}{\sqrt{c}}\right)}{c^{\frac{3}{2}}}\right)-\frac{b\log\left(\frac{x^2+c}{x^2-c}\right)}{2\,x}-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))/x^2,x, algorithm="giac")
```

```
[Out] -b*c*(arctan(x/sqrt(-c))/(sqrt(-c)*c) - arctan(x/sqrt(c))/c^(3/2)) - 1/2*b*
log((x^2 + c)/(x^2 - c))/x - a/x
```

3.169 $\int \frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{x^4}\,dx$

Optimal. Leaf size=65

$$-\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{3x^3}-\frac{b\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}}+\frac{b\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}}-\frac{2b}{3cx}$$

```
[Out] (-2*b)/(3*c*x) - (b*ArcTan[x/Sqrt[c]])/(3*c^(3/2)) - (a + b*ArcTanh[c/x^2])
/(3*x^3) + (b*ArcTanh[x/Sqrt[c]])/(3*c^(3/2))
```

Rubi [A] time = 0.0371315, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6097, 263, 325, 298, 203, 206}

$$-\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{3x^3}-\frac{b\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}}+\frac{b\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}}-\frac{2b}{3cx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c/x^2])/x^4,x]
```

```
[Out] (-2*b)/(3*c*x) - (b*ArcTan[x/Sqrt[c]])/(3*c^(3/2)) - (a + b*ArcTanh[c/x^2])
/(3*x^3) + (b*ArcTanh[x/Sqrt[c]])/(3*c^(3/2))
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{x^4}\,dx &= -\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{3x^3}-\frac{1}{3}(2bc)\int \frac{1}{\left(1-\frac{c^2}{x^4}\right)x^6}\,dx\\ &= -\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{3x^3}-\frac{1}{3}(2bc)\int \frac{1}{x^2\left(-c^2+x^4\right)}\,dx\\ &= -\frac{2b}{3cx}-\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{3x^3}-\frac{(2b)\int \frac{x^2}{-c^2+x^4}\,dx}{3c}\\ &= -\frac{2b}{3cx}-\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{3x^3}+\frac{b\int \frac{1}{c-x^2}\,dx}{3c}-\frac{b\int \frac{1}{c+x^2}\,dx}{3c}\\ &= -\frac{2b}{3cx}-\frac{b\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}}-\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{3x^3}+\frac{b\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0305724, size = 90, normalized size = 1.38

$$-\frac{a}{3x^3}-\frac{b\log\left(\sqrt{c}-x\right)}{6c^{3/2}}+\frac{b\log\left(\sqrt{c}+x\right)}{6c^{3/2}}-\frac{b\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}}-\frac{b\tanh^{-1}\left(\frac{c}{x^2}\right)}{3x^3}-\frac{2b}{3cx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c/x^2])/x^4,x]
```

```
[Out] -a/(3*x^3) - (2*b)/(3*c*x) - (b*ArcTan[x/Sqrt[c]])/(3*c^(3/2)) - (b*ArcTanh
[c/x^2])/(3*x^3) - (b*Log[Sqrt[c] - x])/(6*c^(3/2)) + (b*Log[Sqrt[c] + x])/
(6*c^(3/2))
```

Maple [A] time = 0.012, size = 55, normalized size = 0.9

$$-\frac{a}{3x^3}-\frac{b}{3x^3}\mathrm{Artanh}\left(\frac{c}{x^2}\right)-\frac{2b}{3cx}-\frac{b}{3}\arctan\left(x\frac{1}{\sqrt{c}}\right)c^{-\frac{3}{2}}+\frac{b}{3}\mathrm{Artanh}\left(\frac{1}{x}\sqrt{c}\right)c^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x^2))/x^4,x)
```

```
[Out] -1/3*a/x^3-1/3*b/x^3*arctanh(c/x^2)-2/3*b/c/x-1/3*b*arctan(x/c^(1/2))/c^(3/
2)+1/3*b/c^(3/2)*arctanh(1/x*c^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.84796, size = 451, normalized size = 6.94

$$\left[-\frac{2\,b\sqrt{c}x^3\arctan\left(\frac{x}{\sqrt{c}}\right)-b\sqrt{c}x^3\log\left(\frac{x^2+2\,\sqrt{c}x+c}{x^2-c}\right)+4\,bcx^2+bc^2\log\left(\frac{x^2+c}{x^2-c}\right)+2\,ac^2}{6\,c^2x^3},-\frac{2\,b\sqrt{-c}x^3\arctan\left(\frac{\sqrt{-c}x}{c}\right)+b\sqrt{-c}\ldots}{\ldots}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))/x^4,x, algorithm="fricas")
```

```
[Out] [-1/6*(2*b*sqrt(c)*x^3*arctan(x/sqrt(c)) - b*sqrt(c)*x^3*log((x^2 + 2*sqrt(
c)*x + c)/(x^2 - c)) + 4*b*c*x^2 + b*c^2*log((x^2 + c)/(x^2 - c)) + 2*a*c^2
)/(c^2*x^3), -1/6*(2*b*sqrt(-c)*x^3*arctan(sqrt(-c)*x/c) + b*sqrt(-c)*x^3*l
og((x^2 + 2*sqrt(-c)*x - c)/(x^2 + c)) + 4*b*c*x^2 + b*c^2*log((x^2 + c)/(x
^2 - c)) + 2*a*c^2)/(c^2*x^3)]
```

Sympy [A] time = 25.6988, size = 706, normalized size = 10.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x**2))/x**4,x)
```

```
[Out] Piecewise((-a/(3*x**3), Eq(c, 0)), (-(a - oo*b)/(3*x**3), Eq(c, -x**2)), (-
(a + oo*b)/(3*x**3), Eq(c, x**2)), (2*a*c**(65/2)/(-6*c**(65/2)*x**3 + 6*c*
*(61/2)*x**7) - 2*a*c**(61/2)*x**4/(-6*c**(65/2)*x**3 + 6*c**(61/2)*x**7) +
 2*b*c**(65/2)*atanh(c/x**2)/(-6*c**(65/2)*x**3 + 6*c**(61/2)*x**7) + 4*b*c
**(63/2)*x**2/(-6*c**(65/2)*x**3 + 6*c**(61/2)*x**7) - 2*b*c**(61/2)*x**4*a
tanh(c/x**2)/(-6*c**(65/2)*x**3 + 6*c**(61/2)*x**7) - 4*b*c**(59/2)*x**6/(-
6*c**(65/2)*x**3 + 6*c**(61/2)*x**7) + 2*b*c**31*x**3*log(-sqrt(c) + x)/(-6
*c**(65/2)*x**3 + 6*c**(61/2)*x**7) - b*c**31*x**3*log(-I*sqrt(c) + x)/(-6*
c**(65/2)*x**3 + 6*c**(61/2)*x**7) - I*b*c**31*x**3*log(-I*sqrt(c) + x)/(-6
*c**(65/2)*x**3 + 6*c**(61/2)*x**7) - b*c**31*x**3*log(I*sqrt(c) + x)/(-6*c
**(65/2)*x**3 + 6*c**(61/2)*x**7) + I*b*c**31*x**3*log(I*sqrt(c) + x)/(-6*c
**(65/2)*x**3 + 6*c**(61/2)*x**7) + 2*b*c**31*x**3*atanh(c/x**2)/(-6*c**(65
/2)*x**3 + 6*c**(61/2)*x**7) - 2*b*c**29*x**7*log(-sqrt(c) + x)/(-6*c**(65/
2)*x**3 + 6*c**(61/2)*x**7) + b*c**29*x**7*log(-I*sqrt(c) + x)/(-6*c**(65/2
)*x**3 + 6*c**(61/2)*x**7) + I*b*c**29*x**7*log(-I*sqrt(c) + x)/(-6*c**(65/
2)*x**3 + 6*c**(61/2)*x**7) + b*c**29*x**7*log(I*sqrt(c) + x)/(-6*c**(65/2)
*x**3 + 6*c**(61/2)*x**7) - I*b*c**29*x**7*log(I*sqrt(c) + x)/(-6*c**(65/2)
*x**3 + 6*c**(61/2)*x**7) - 2*b*c**29*x**7*atanh(c/x**2)/(-6*c**(65/2)*x**3
 + 6*c**(61/2)*x**7), True))
```

Giac [A] time = 1.30144, size = 97, normalized size = 1.49

$$-\frac{1}{3}b\left(\frac{\arctan\left(\frac{x}{\sqrt{-c}}\right)}{\sqrt{-c}c}+\frac{\arctan\left(\frac{x}{\sqrt{c}}\right)}{c^{\frac{3}{2}}}\right)-\frac{b\log\left(\frac{x^2+c}{x^2-c}\right)}{6x^3}-\frac{2bx^2+ac}{3cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))/x^4,x, algorithm="giac")
```

```
[Out] -1/3*b*(arctan(x/sqrt(-c))/(sqrt(-c)*c) + arctan(x/sqrt(c))/c^(3/2)) - 1/6*
b*log((x^2 + c)/(x^2 - c))/x^3 - 1/3*(2*b*x^2 + a*c)/(c*x^3)
```

3.170 $\int \frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{x^6}\,dx$

Optimal. Leaf size=65

$$-\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{5x^5}+\frac{b\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{5c^{5/2}}+\frac{b\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)}{5c^{5/2}}-\frac{2b}{15cx^3}$$

```
[Out] (-2*b)/(15*c*x^3) + (b*ArcTan[x/Sqrt[c]])/(5*c^(5/2)) - (a + b*ArcTanh[c/x^
2])/(5*x^5) + (b*ArcTanh[x/Sqrt[c]])/(5*c^(5/2))
```

Rubi [A] time = 0.0375865, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6097, 263, 325, 212, 206, 203}

$$-\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{5x^5}+\frac{b\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{5c^{5/2}}+\frac{b\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)}{5c^{5/2}}-\frac{2b}{15cx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c/x^2])/x^6,x]
```

```
[Out] (-2*b)/(15*c*x^3) + (b*ArcTan[x/Sqrt[c]])/(5*c^(5/2)) - (a + b*ArcTanh[c/x^
2])/(5*x^5) + (b*ArcTanh[x/Sqrt[c]])/(5*c^(5/2))
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{x^6}\,dx &= -\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{5x^5}-\frac{1}{5}(2bc)\int \frac{1}{\left(1-\frac{c^2}{x^4}\right)x^8}\,dx\\
&= -\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{5x^5}-\frac{1}{5}(2bc)\int \frac{1}{x^4\left(-c^2+x^4\right)}\,dx\\
&= -\frac{2b}{15cx^3}-\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{5x^5}-\frac{(2b)\int \frac{1}{-c^2+x^4}\,dx}{5c}\\
&= -\frac{2b}{15cx^3}-\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{5x^5}+\frac{b\int \frac{1}{c-x^2}\,dx}{5c^2}+\frac{b\int \frac{1}{c+x^2}\,dx}{5c^2}\\
&= -\frac{2b}{15cx^3}+\frac{b\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{5c^{5/2}}-\frac{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}{5x^5}+\frac{b\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)}{5c^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.023624, size = 90, normalized size = 1.38

$$-\frac{a}{5x^5}-\frac{b\log\left(\sqrt{c}-x\right)}{10c^{5/2}}+\frac{b\log\left(\sqrt{c}+x\right)}{10c^{5/2}}+\frac{b\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{5c^{5/2}}-\frac{2b}{15cx^3}-\frac{b\tanh^{-1}\left(\frac{c}{x^2}\right)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c/x^2])/x^6,x]
```

```
[Out] -a/(5*x^5) - (2*b)/(15*c*x^3) + (b*ArcTan[x/Sqrt[c]])/(5*c^(5/2)) - (b*ArcT
anh[c/x^2])/(5*x^5) - (b*Log[Sqrt[c] - x])/(10*c^(5/2)) + (b*Log[Sqrt[c] +
x])/(10*c^(5/2))
```

Maple [A] time = 0.011, size = 55, normalized size = 0.9

$$-\frac{a}{5\,x^5}-\frac{b}{5\,x^5}\mathrm{Artanh}\left(\frac{c}{x^2}\right)-\frac{2\,b}{15\,cx^3}+\frac{b}{5}\arctan\left(x\frac{1}{\sqrt{c}}\right)c^{-\frac{5}{2}}+\frac{b}{5}\mathrm{Artanh}\left(\frac{1}{x}\sqrt{c}\right)c^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x^2))/x^6,x)
```

```
[Out] -1/5*a/x^5-1/5/x^5*b*arctanh(c/x^2)-2/15*b/c/x^3+1/5*b*arctan(x/c^(1/2))/c^
(5/2)+1/5*b/c^(5/2)*arctanh(1/x*c^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.7849, size = 468, normalized size = 7.2

$$\left[\frac{6\,b\sqrt{c}x^5\arctan\left(\frac{x}{\sqrt{c}}\right)+3\,b\sqrt{c}x^5\log\left(\frac{x^2+2\,\sqrt{c}x+c}{x^2-c}\right)-4\,bc^2x^2-3\,bc^3\log\left(\frac{x^2+c}{x^2-c}\right)-6\,ac^3}{30\,c^3x^5},-\frac{6\,b\sqrt{-c}x^5\arctan\left(\frac{\sqrt{-c}x}{c}\right)+3\,b\cdots}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))/x^6,x, algorithm="fricas")
```

```
[Out] [1/30*(6*b*sqrt(c)*x^5*arctan(x/sqrt(c)) + 3*b*sqrt(c)*x^5*log((x^2 + 2*sqr
t(c)*x + c)/(x^2 - c)) - 4*b*c^2*x^2 - 3*b*c^3*log((x^2 + c)/(x^2 - c)) - 6
*a*c^3)/(c^3*x^5), -1/30*(6*b*sqrt(-c)*x^5*arctan(sqrt(-c)*x/c) + 3*b*sqrt(
-c)*x^5*log((x^2 - 2*sqrt(-c)*x - c)/(x^2 + c)) + 4*b*c^2*x^2 + 3*b*c^3*log
((x^2 + c)/(x^2 - c)) + 6*a*c^3)/(c^3*x^5)]
```

Sympy [A] time = 52.4155, size = 668, normalized size = 10.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x**2))/x**6,x)
```

```
[Out] Piecewise((-a/(5*x**5), Eq(c, 0)), (-(a - oo*b)/(5*x**5), Eq(c, -x**2)), (-
(a + oo*b)/(5*x**5), Eq(c, x**2)), (6*a*c**71/(-30*c**71*x**5 + 30*c**69*x*
*9) - 6*a*c**69*x**4/(-30*c**71*x**5 + 30*c**69*x**9) + 6*b*c**(137/2)*x**5
*log(-sqrt(c) + x)/(-30*c**71*x**5 + 30*c**69*x**9) - 3*b*c**(137/2)*x**5*l
og(-I*sqrt(c) + x)/(-30*c**71*x**5 + 30*c**69*x**9) + 3*I*b*c**(137/2)*x**5
*log(-I*sqrt(c) + x)/(-30*c**71*x**5 + 30*c**69*x**9) - 3*b*c**(137/2)*x**5
*log(I*sqrt(c) + x)/(-30*c**71*x**5 + 30*c**69*x**9) - 3*I*b*c**(137/2)*x**
5*log(I*sqrt(c) + x)/(-30*c**71*x**5 + 30*c**69*x**9) + 6*b*c**(137/2)*x**5
*atanh(c/x**2)/(-30*c**71*x**5 + 30*c**69*x**9) - 6*b*c**(133/2)*x**9*log(-
sqrt(c) + x)/(-30*c**71*x**5 + 30*c**69*x**9) + 3*b*c**(133/2)*x**9*log(-I*
sqrt(c) + x)/(-30*c**71*x**5 + 30*c**69*x**9) - 3*I*b*c**(133/2)*x**9*log(-
I*sqrt(c) + x)/(-30*c**71*x**5 + 30*c**69*x**9) + 3*b*c**(133/2)*x**9*log(I
*sqrt(c) + x)/(-30*c**71*x**5 + 30*c**69*x**9) + 3*I*b*c**(133/2)*x**9*log(
I*sqrt(c) + x)/(-30*c**71*x**5 + 30*c**69*x**9) - 6*b*c**(133/2)*x**9*atanh
(c/x**2)/(-30*c**71*x**5 + 30*c**69*x**9) + 6*b*c**71*atanh(c/x**2)/(-30*c*
*71*x**5 + 30*c**69*x**9) + 4*b*c**70*x**2/(-30*c**71*x**5 + 30*c**69*x**9)
 - 6*b*c**69*x**4*atanh(c/x**2)/(-30*c**71*x**5 + 30*c**69*x**9) - 4*b*c**6
8*x**6/(-30*c**71*x**5 + 30*c**69*x**9), True))
```

Giac [A] time = 1.31428, size = 100, normalized size = 1.54

$$-\frac{1}{5}\,b\left(\frac{\arctan\left(\frac{x}{\sqrt{-c}}\right)}{\sqrt{-c}c^{2}}-\frac{\arctan\left(\frac{x}{\sqrt{c}}\right)}{c^{\frac{5}{2}}}\right)-\frac{b\log\left(\frac{x^{2}+c}{x^{2}-c}\right)}{10\,x^{5}}-\frac{2\,bx^{2}+3\,ac}{15\,cx^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))/x^6,x, algorithm="giac")
```

```
[Out] -1/5*b*(arctan(x/sqrt(-c))/(sqrt(-c)*c^2) - arctan(x/sqrt(c))/c^(5/2)) - 1/
10*b*log((x^2 + c)/(x^2 - c))/x^5 - 1/15*(2*b*x^2 + 3*a*c)/(c*x^5)
```

3.171 $\int x^3 \left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2 dx$

Optimal. Leaf size=94

$$-\frac{1}{4}c^2\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)^2+\frac{1}{4}x^4\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)^2+\frac{1}{2}bcx^2\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)+\frac{1}{4}b^2c^2\log\left(1-\frac{c^2}{x^4}\right)+b^2c^2\log$$

```
[Out] (b*c*x^2*(a + b*ArcCoth[x^2/c]))/2 - (c^2*(a + b*ArcCoth[x^2/c])^2)/4 + (x^
4*(a + b*ArcCoth[x^2/c])^2)/4 + (b^2*c^2*Log[1 - c^2/x^4])/4 + b^2*c^2*Log[
x]
```

Rubi [C] time = 1.29574, antiderivative size = 599, normalized size of antiderivative = 6.37, number of steps used = 59, number of rules used = 33, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 2.063, Rules used = {6099, 2454, 2398, 2411, 2347, 2344, 2301, 2316, 2315, 2314, 31, 2455, 263, 266, 43, 193, 6742, 30, 2557, 12, 2466, 2448, 2462, 260, 2416, 2394, 2393, 2391, 2410, 2395, 36, 29, 2390}

$$-\frac{1}{8}b^2c^2\text{PolyLog}\left(2,-\frac{c}{x^2}\right)-\frac{1}{8}b^2c^2\text{PolyLog}\left(2,\frac{c}{x^2}\right)-\frac{1}{8}b^2c^2\text{PolyLog}\left(2,\frac{c-x^2}{2c}\right)-\frac{1}{8}b^2c^2\text{PolyLog}\left(2,\frac{c+x^2}{2c}\right)+\frac{1}{8}b^2c$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^3*(a + b*ArcTanh[c/x^2])^2,x]
```

```
[Out] (a*b*c*x^2)/4 - (b^2*c*x^2*Log[1 - c/x^2])/8 + (b*c*(1 - c/x^2)*x^2*(2*a -
b*Log[1 - c/x^2]))/8 - (c^2*(2*a - b*Log[1 - c/x^2])^2)/16 + (x^4*(2*a - b*
Log[1 - c/x^2])^2)/16 + (b^2*c^2*Log[1 + c/x^2])/8 + (b^2*c*x^2*Log[1 + c/x
^2])/4 + (a*b*x^4*Log[1 + c/x^2])/4 - (b^2*x^4*Log[1 - c/x^2]*Log[1 + c/x^2
])/8 - (b^2*c^2*Log[1 + c/x^2]^2)/16 + (b^2*x^4*Log[1 + c/x^2]^2)/16 + (a*b
*c^2*Log[x])/2 + (b^2*c^2*Log[x])/2 + (b^2*c^2*Log[c - x^2])/8 + (b^2*c^2*L
og[1 + c/x^2]*Log[c - x^2])/8 + (b^2*c^2*Log[x^2/c]*Log[c - x^2])/8 - (a*b*
c^2*Log[c + x^2])/4 + (b^2*c^2*Log[c + x^2])/8 + (b^2*c^2*Log[1 - c/x^2]*Lo
g[c + x^2])/8 + (b^2*c^2*Log[-(x^2/c)]*Log[c + x^2])/8 - (b^2*c^2*Log[(c -
x^2)/(2*c)]*Log[c + x^2])/8 - (b^2*c^2*Log[c - x^2]*Log[(c + x^2)/(2*c)])/8
 - (b^2*c^2*PolyLog[2, -(c/x^2)])/8 - (b^2*c^2*PolyLog[2, c/x^2])/8 - (b^2*
c^2*PolyLog[2, (c - x^2)/(2*c)])/8 - (b^2*c^2*PolyLog[2, (c + x^2)/(2*c)])/
8 + (b^2*c^2*PolyLog[2, (c + x^2)/c])/8 + (b^2*c^2*PolyLog[2, 1 - x^2/c])/8
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
```

```
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2316

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[
((a + b*Log[-((c*d)/e)])*Log[d + e*x])/e, x] + Dist[b, Int[Log[-((e*x)/d)]/
(d + e*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[-((c*d)/e), 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
```

```
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2557

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] :> With[{z = IntHide[u, x]}, Dist[Log[v
]*Log[w], z, x] + (-Int[SimplifyIntegrand[(z*Log[w]*D[v, x])/v, x], x] - In
t[SimplifyIntegrand[(z*Log[v]*D[w, x])/w, x], x]) /; InverseFunctionFreeQ[z
, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
```

```
e, n, p}, x]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
```

```
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rubi steps

$$\int x^3 \Bigl(a + b\tanh^{-1}\Bigl(\frac{c}{x^2}\Bigr)\Bigr)^2 dx = \int \Bigl(\frac{1}{4}x^3 \Bigl(2a - b\log\Bigl(1 - \frac{c}{x^2}\Bigr)\Bigr)^2 - \frac{1}{2}bx^3 \Bigl(-2a + b\log\Bigl(1 - \frac{c}{x^2}\Bigr)\Bigr)\log\Bigl(1 + \frac{c}{x^2}\Bigr) + \frac{1}{4}b^2x^3 \ldots$$

$$= \frac{1}{4}\int x^3 \Bigl(2a - b\log\Bigl(1 - \frac{c}{x^2}\Bigr)\Bigr)^2 dx - \frac{1}{2}b\int x^3 \Bigl(-2a + b\log\Bigl(1 - \frac{c}{x^2}\Bigr)\Bigr)\log\Bigl(1 + \frac{c}{x^2}\Bigr) dx \ldots$$

$$= -\Bigl(\frac{1}{8}\,\mathrm{Subst}\Bigl(\int \frac{(2a - b\log(1 - cx))^2}{x^3} dx, x, \frac{1}{x^2}\Bigr)\Bigr) - \frac{1}{4}b\,\mathrm{Subst}\Bigl(\int x \Bigl(-2a + b\log\Bigl(1 - \ldots$$

$$= \frac{1}{16}x^4 \Bigl(2a - b\log\Bigl(1 - \frac{c}{x^2}\Bigr)\Bigr)^2 + \frac{1}{16}b^2x^4\log^2\Bigl(1 + \frac{c}{x^2}\Bigr) - \frac{1}{4}b\,\mathrm{Subst}\Bigl(\int \Bigl(-2ax\log\Bigl(1 + \ldots$$

$$= \frac{1}{16}x^4 \Bigl(2a - b\log\Bigl(1 - \frac{c}{x^2}\Bigr)\Bigr)^2 + \frac{1}{16}b^2x^4\log^2\Bigl(1 + \frac{c}{x^2}\Bigr) + \frac{1}{8}b\,\mathrm{Subst}\Biggl(\int \frac{2a - b\log(x)}{x\bigl(\frac{1}{c} - \frac{x}{c}\bigr)^2} dx \ldots$$

$$= \frac{1}{16}x^4 \Bigl(2a - b\log\Bigl(1 - \frac{c}{x^2}\Bigr)\Bigr)^2 + \frac{1}{4}abx^4\log\Bigl(1 + \frac{c}{x^2}\Bigr) - \frac{1}{8}b^2x^4\log\Bigl(1 - \frac{c}{x^2}\Bigr)\log\Bigl(1 + \frac{c}{x^2}\Bigr) \ldots$$

$$= \frac{1}{8}bc\Bigl(1 - \frac{c}{x^2}\Bigr)x^2\Bigl(2a - b\log\Bigl(1 - \frac{c}{x^2}\Bigr)\Bigr) + \frac{1}{16}x^4\Bigl(2a - b\log\Bigl(1 - \frac{c}{x^2}\Bigr)\Bigr)^2 + \frac{1}{8}b^2cx^2\log\Bigl(1 \ldots$$

$$= \frac{1}{8}bc\Bigl(1 - \frac{c}{x^2}\Bigr)x^2\Bigl(2a - b\log\Bigl(1 - \frac{c}{x^2}\Bigr)\Bigr) - \frac{1}{16}c^2\Bigl(2a - b\log\Bigl(1 - \frac{c}{x^2}\Bigr)\Bigr)^2 + \frac{1}{16}x^4\Bigl(2a - b\,\mathrm{l} \ldots$$

$$= \frac{1}{4}abcx^2 + \frac{1}{8}bc\Bigl(1 - \frac{c}{x^2}\Bigr)x^2\Bigl(2a - b\log\Bigl(1 - \frac{c}{x^2}\Bigr)\Bigr) - \frac{1}{16}c^2\Bigl(2a - b\log\Bigl(1 - \frac{c}{x^2}\Bigr)\Bigr)^2 + \frac{1}{16}x \ldots$$

$$= \frac{1}{4}abcx^2 - \frac{1}{8}b^2cx^2\log\Bigl(1 - \frac{c}{x^2}\Bigr) + \frac{1}{8}bc\Bigl(1 - \frac{c}{x^2}\Bigr)x^2\Bigl(2a - b\log\Bigl(1 - \frac{c}{x^2}\Bigr)\Bigr) - \frac{1}{16}c^2\Bigl(2a - \ldots$$

$$= \frac{1}{4}abcx^2 - \frac{1}{8}b^2cx^2\log\Bigl(1 - \frac{c}{x^2}\Bigr) + \frac{1}{8}bc\Bigl(1 - \frac{c}{x^2}\Bigr)x^2\Bigl(2a - b\log\Bigl(1 - \frac{c}{x^2}\Bigr)\Bigr) - \frac{1}{16}c^2\Bigl(2a - \ldots$$

$$= \frac{1}{4}abcx^2 - \frac{1}{8}b^2cx^2\log\Bigl(1 - \frac{c}{x^2}\Bigr) + \frac{1}{8}bc\Bigl(1 - \frac{c}{x^2}\Bigr)x^2\Bigl(2a - b\log\Bigl(1 - \frac{c}{x^2}\Bigr)\Bigr) - \frac{1}{16}c^2\Bigl(2a - \ldots$$

$$= \frac{1}{4}abcx^2 - \frac{1}{8}b^2cx^2\log\Bigl(1 - \frac{c}{x^2}\Bigr) + \frac{1}{8}bc\Bigl(1 - \frac{c}{x^2}\Bigr)x^2\Bigl(2a - b\log\Bigl(1 - \frac{c}{x^2}\Bigr)\Bigr) - \frac{1}{16}c^2\Bigl(2a - \ldots$$

$$= \frac{1}{4}abcx^2 - \frac{1}{8}b^2cx^2\log\Bigl(1 - \frac{c}{x^2}\Bigr) + \frac{1}{8}bc\Bigl(1 - \frac{c}{x^2}\Bigr)x^2\Bigl(2a - b\log\Bigl(1 - \frac{c}{x^2}\Bigr)\Bigr) - \frac{1}{16}c^2\Bigl(2a - \ldots$$

$$= \frac{1}{4}abcx^2 - \frac{1}{8}b^2cx^2\log\Bigl(1 - \frac{c}{x^2}\Bigr) + \frac{1}{8}bc\Bigl(1 - \frac{c}{x^2}\Bigr)x^2\Bigl(2a - b\log\Bigl(1 - \frac{c}{x^2}\Bigr)\Bigr) - \frac{1}{16}c^2\Bigl(2a - \ldots$$

Mathematica [A] time = 0.0604428, size = 104, normalized size = 1.11

$$\frac{1}{4}\left(a^2x^4 + bc^2(a+b)\log\left(x^2-c\right) - abc^2\log\left(c+x^2\right) + 2abcx^2 + 2bx^2\tanh^{-1}\left(\frac{c}{x^2}\right)\left(ax^2+bc\right) + b^2c^2\log\left(c+x^2\right) + b^2\left(x^4\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcTanh[c/x^2])^2,x]
```

```
[Out] (2*a*b*c*x^2 + a^2*x^4 + 2*b*x^2*(b*c + a*x^2)*ArcTanh[c/x^2] + b^2*(-c^2 +
 x^4)*ArcTanh[c/x^2]^2 + b*(a + b)*c^2*Log[-c + x^2] - a*b*c^2*Log[c + x^2]
 + b^2*c^2*Log[c + x^2])/4
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int x^3\left(a + b\text{Artanh}\left(\frac{c}{x^2}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c/x^2))^2,x)
```

```
[Out] int(x^3*(a+b*arctanh(c/x^2))^2,x)
```

Maxima [A] time = 0.978379, size = 212, normalized size = 2.26

$$\frac{1}{4}b^2x^4\,\text{artanh}\left(\frac{c}{x^2}\right)^2 + \frac{1}{4}a^2x^4 + \frac{1}{4}\left(2\,x^4\,\text{artanh}\left(\frac{c}{x^2}\right) + \left(2\,x^2 - c\log\left(x^2+c\right) + c\log\left(x^2-c\right)\right)c\right)ab + \frac{1}{16}\left(\left(\log\left(x^2+c\right)^2 -\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c/x^2))^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*x^4*arctanh(c/x^2)^2 + 1/4*a^2*x^4 + 1/4*(2*x^4*arctanh(c/x^2) + (2
*x^2 - c*log(x^2 + c) + c*log(x^2 - c))*c)*a*b + 1/16*((log(x^2 + c)^2 - 2*
(log(x^2 + c) - 2)*log(x^2 - c) + log(x^2 - c)^2 + 4*log(x^2 + c))*c^2 + 4*
(2*x^2 - c*log(x^2 + c) + c*log(x^2 - c))*c*arctanh(c/x^2))*b^2
```

Fricas [A] time = 1.81725, size = 278, normalized size = 2.96

$$\frac{1}{4}a^2x^4 + \frac{1}{2}abcx^2 - \frac{1}{4}\left(ab-b^2\right)c^2\log\left(x^2+c\right) + \frac{1}{4}\left(ab+b^2\right)c^2\log\left(x^2-c\right) + \frac{1}{16}\left(b^2x^4-b^2c^2\right)\log\left(\frac{x^2+c}{x^2-c}\right)^2 + \frac{1}{4}\left(abx^4+b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c/x^2))^2,x, algorithm="fricas")
```

```
[Out] 1/4*a^2*x^4 + 1/2*a*b*c*x^2 - 1/4*(a*b - b^2)*c^2*log(x^2 + c) + 1/4*(a*b +
 b^2)*c^2*log(x^2 - c) + 1/16*(b^2*x^4 - b^2*c^2)*log((x^2 + c)/(x^2 - c))^
2 + 1/4*(a*b*x^4 + b^2*c*x^2)*log((x^2 + c)/(x^2 - c))
```

Sympy [C] time = 14.7646, size = 151, normalized size = 1.61

$$\frac{a^2x^4}{4} - \frac{abc^2 \operatorname{atanh}\left(\frac{c}{x^2}\right)}{2} + \frac{abcx^2}{2} + \frac{abx^4 \operatorname{atanh}\left(\frac{c}{x^2}\right)}{2} + \frac{b^2c^2 \log\left(-i\sqrt{c}+x\right)}{2} + \frac{b^2c^2 \log\left(i\sqrt{c}+x\right)}{2} - \frac{b^2c^2 \operatorname{atanh}^2\left(\frac{c}{x^2}\right)}{4} \cdot$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c/x**2))**2,x)
```

```
[Out] a**2*x**4/4 - a*b*c**2*atanh(c/x**2)/2 + a*b*c*x**2/2 + a*b*x**4*atanh(c/x*
*2)/2 + b**2*c**2*log(-I*sqrt(c) + x)/2 + b**2*c**2*log(I*sqrt(c) + x)/2 -
b**2*c**2*atanh(c/x**2)**2/4 - b**2*c**2*atanh(c/x**2)/2 + b**2*c*x**2*atan
h(c/x**2)/2 + b**2*x**4*atanh(c/x**2)**2/4
```

Giac [A] time = 1.2751, size = 180, normalized size = 1.91

$$\frac{1}{4}a^2x^4 + \frac{1}{2}abcx^2 + \frac{1}{16}\left(b^2x^4 - b^2c^2\right)\log\left(\frac{x^2+c}{x^2-c}\right)^2 - \frac{1}{4}\left(abc^2 - b^2c^2\right)\log\left(x^2+c\right) + \frac{1}{4}\left(abc^2 + b^2c^2\right)\log\left(-x^2+c\right) + \frac{1}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c/x^2))^2,x, algorithm="giac")
```

```
[Out] 1/4*a^2*x^4 + 1/2*a*b*c*x^2 + 1/16*(b^2*x^4 - b^2*c^2)*log((x^2 + c)/(x^2 -
 c))^2 - 1/4*(a*b*c^2 - b^2*c^2)*log(x^2 + c) + 1/4*(a*b*c^2 + b^2*c^2)*log
(-x^2 + c) + 1/4*(a*b*x^4 + b^2*c*x^2)*log((x^2 + c)/(x^2 - c))
```

3.172 $\int x\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2\,dx$

Optimal. Leaf size=94

$$\frac{1}{2}b^2c\text{PolyLog}\left(2,\frac{2}{\frac{c}{x^2}+1}-1\right)+\frac{1}{2}x^2\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)^2-\frac{1}{2}c\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)^2-bc\log\left(2-\frac{2}{\frac{c}{x^2}+1}\right)\left(a+b\coth^{-1}\right.$$

```
[Out] -(c*(a + b*ArcCoth[x^2/c])^2)/2 + (x^2*(a + b*ArcCoth[x^2/c])^2)/2 - b*c*(a
 + b*ArcCoth[x^2/c])*Log[2 - 2/(1 + c/x^2)] + (b^2*c*PolyLog[2, -1 + 2/(1 +
 c/x^2)])/2
```

Rubi [B] time = 0.696246, antiderivative size = 404, normalized size of antiderivative = 4.3, number of steps used = 34, number of rules used = 19, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.357, Rules used = {6099, 2454, 2397, 2392, 2391, 2455, 263, 260, 6715, 2448, 31, 6742, 2556, 12, 2462, 2416, 2394, 2315, 2393}

$$\frac{1}{4}b^2c\text{PolyLog}\left(2,-\frac{c}{x^2}\right)-\frac{1}{4}b^2c\text{PolyLog}\left(2,\frac{c}{x^2}\right)-\frac{1}{4}b^2c\text{PolyLog}\left(2,\frac{c-x^2}{2c}\right)+\frac{1}{4}b^2c\text{PolyLog}\left(2,\frac{c+x^2}{2c}\right)-\frac{1}{4}b^2c\text{PolyLog}$$

Warning: Unable to verify antiderivative.

```
[In] Int[x*(a + b*ArcTanh[c/x^2])^2,x]
```

```
[Out] ((1 - c/x^2)*x^2*(2*a - b*Log[1 - c/x^2])^2)/8 + (a*b*x^2*Log[1 + c/x^2])/2
 - (b^2*x^2*Log[1 - c/x^2]*Log[1 + c/x^2])/4 + (b^2*(1 + c/x^2)*x^2*Log[1 +
 c/x^2]^2)/8 + a*b*c*Log[x] - (b^2*c*Log[1 - c/x^2]*Log[-c - x^2])/4 - (b^2
*c*Log[-(x^2/c)]*Log[-c - x^2])/4 + (b^2*c*Log[-c - x^2]*Log[(c - x^2)/(2*c
)])/4 + (b^2*c*Log[1 + c/x^2]*Log[-c + x^2])/4 + (b^2*c*Log[x^2/c]*Log[-c +
 x^2])/4 + (a*b*c*Log[c + x^2])/2 - (b^2*c*Log[-c + x^2]*Log[(c + x^2)/(2*c
)])/4 + (b^2*c*PolyLog[2, -(c/x^2)])/4 - (b^2*c*PolyLog[2, c/x^2])/4 - (b^2
*c*PolyLog[2, (c - x^2)/(2*c)])/4 + (b^2*c*PolyLog[2, (c + x^2)/(2*c)])/4 -
 (b^2*c*PolyLog[2, (c + x^2)/c])/4 + (b^2*c*PolyLog[2, 1 - x^2/c])/4
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2397

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_))^2, x_Symbol] :> Simp[((d + e*x)*(a + b*Log[c*(d + e*x)^n])^p)/((e*f
 - d*g)*(f + g*x)), x] - Dist[(b*e*n*p)/(e*f - d*g), Int[(a + b*Log[c*(d +
e*x)^n])^(p - 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] &&
 NeQ[e*f - d*g, 0] && GtQ[p, 0]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/(x_), x_Symbol] :> Simp[
(a + b*Log[c*d])*Log[x], x] + Dist[b, Int[Log[1 + (e*x)/d]/x, x], x] /; Fre
eQ[{a, b, c, d, e}, x] && GtQ[c*d, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2556

```
Int[Log[v_]*Log[w_], x_Symbol] :> Simp[x*Log[v]*Log[w], x] + (-Int[Simplify
Integrand[(x*Log[w]*D[v, x])/v, x], x] - Int[SimplifyIntegrand[(x*Log[v]*D[
w, x])/w, x], x]) /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w,
x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rubi steps

$$\int x\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2dx=\int\left(\frac{1}{4}x\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2-\frac{1}{2}bx\left(-2a+b\log\left(1-\frac{c}{x^2}\right)\right)\log\left(1+\frac{c}{x^2}\right)+\frac{1}{4}b^2x\log^2$$
$$=\frac{1}{4}\int x\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2dx-\frac{1}{2}b\int x\left(-2a+b\log\left(1-\frac{c}{x^2}\right)\right)\log\left(1+\frac{c}{x^2}\right)dx+$$
$$=-\left(\frac{1}{8}\text{Subst}\left(\int\frac{(2a-b\log(1-cx))^2}{x^2}dx,x,\frac{1}{x^2}\right)\right)-\frac{1}{4}b\,\text{Subst}\left(\int\left(-2a+b\log\left(1-\frac{c}{x}\right)\right.\right.$$
$$=\frac{1}{8}\left(1-\frac{c}{x^2}\right)x^2\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2+\frac{1}{8}b^2\left(1+\frac{c}{x^2}\right)x^2\log^2\left(1+\frac{c}{x^2}\right)-\frac{1}{4}b\,\text{Subst}\left(\int\right.$$
$$=\frac{1}{8}\left(1-\frac{c}{x^2}\right)x^2\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2+\frac{1}{8}b^2\left(1+\frac{c}{x^2}\right)x^2\log^2\left(1+\frac{c}{x^2}\right)+abc\log(x)+$$
$$=\frac{1}{8}\left(1-\frac{c}{x^2}\right)x^2\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2+\frac{1}{2}abx^2\log\left(1+\frac{c}{x^2}\right)-\frac{1}{4}b^2x^2\log\left(1-\frac{c}{x^2}\right)\log\left(\right.$$
$$=\frac{1}{8}\left(1-\frac{c}{x^2}\right)x^2\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2+\frac{1}{2}abx^2\log\left(1+\frac{c}{x^2}\right)-\frac{1}{4}b^2x^2\log\left(1-\frac{c}{x^2}\right)\log\left(\right.$$
$$=\frac{1}{8}\left(1-\frac{c}{x^2}\right)x^2\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2+\frac{1}{2}abx^2\log\left(1+\frac{c}{x^2}\right)-\frac{1}{4}b^2x^2\log\left(1-\frac{c}{x^2}\right)\log\left(\right.$$
$$=\frac{1}{8}\left(1-\frac{c}{x^2}\right)x^2\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2+\frac{1}{2}abx^2\log\left(1+\frac{c}{x^2}\right)-\frac{1}{4}b^2x^2\log\left(1-\frac{c}{x^2}\right)\log\left(\right.$$
$$=\frac{1}{8}\left(1-\frac{c}{x^2}\right)x^2\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2+\frac{1}{2}abx^2\log\left(1+\frac{c}{x^2}\right)-\frac{1}{4}b^2x^2\log\left(1-\frac{c}{x^2}\right)\log\left(\right.$$
$$=\frac{1}{8}\left(1-\frac{c}{x^2}\right)x^2\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2+\frac{1}{2}abx^2\log\left(1+\frac{c}{x^2}\right)-\frac{1}{4}b^2x^2\log\left(1-\frac{c}{x^2}\right)\log\left(\right.$$
$$=\frac{1}{8}\left(1-\frac{c}{x^2}\right)x^2\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2+\frac{1}{2}abx^2\log\left(1+\frac{c}{x^2}\right)-\frac{1}{4}b^2x^2\log\left(1-\frac{c}{x^2}\right)\log\left(\right.$$
$$=\frac{1}{8}\left(1-\frac{c}{x^2}\right)x^2\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2+\frac{1}{2}abx^2\log\left(1+\frac{c}{x^2}\right)-\frac{1}{4}b^2x^2\log\left(1-\frac{c}{x^2}\right)\log\left(\right.$$

Mathematica [A] time = 0.13381, size = 107, normalized size = 1.14

$$\frac{1}{2}\left(b^2c\text{PolyLog}\left(2,e^{-2\tanh^{-1}\left(\frac{c}{x^2}\right)}\right)+a\left(ax^2+bc\log\left(1-\frac{c^2}{x^4}\right)-2bc\log\left(\frac{c}{x^2}\right)\right)+2b\tanh^{-1}\left(\frac{c}{x^2}\right)\left(ax^2-bc\log\left(1-e^{-2\tanh}\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(a + b*ArcTanh[c/x^2])^2,x]
```

```
[Out] (b^2*(-c + x^2)*ArcTanh[c/x^2]^2 + 2*b*ArcTanh[c/x^2]*(a*x^2 - b*c*Log[1 -
E^(-2*ArcTanh[c/x^2])]) + a*(a*x^2 + b*c*Log[1 - c^2/x^4] - 2*b*c*Log[c/x^2
]) + b^2*c*PolyLog[2, E^(-2*ArcTanh[c/x^2])])/2
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int x\left(a+b\text{Artanh}\left(\frac{c}{x^2}\right)\right)^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c/x^2))^2,x)
```

```
[Out] int(x*(a+b*arctanh(c/x^2))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}a^2x^2 + \frac{1}{2}\left(2x^2\operatorname{artanh}\left(\frac{c}{x^2}\right) + c\log\left(x^4 - c^2\right)\right)ab + \frac{1}{8}\left(x^2\log\left(x^2 + c\right)^2 - 2\left(x^2 + c\right)\log\left(x^2 + c\right)\log\left(x^2 - c\right) + \left(x^2 - c\right)\log\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c/x^2))^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*x^2 + 1/2*(2*x^2*arctanh(c/x^2) + c*log(x^4 - c^2))*a*b + 1/8*(x^2*
log(x^2 + c)^2 - 2*(x^2 + c)*log(x^2 + c)*log(x^2 - c) + (x^2 - c)*log(x^2
- c)^2 + 2*integrate(2*(3*c*x^3 + c^2*x)*log(x^2 + c)/(x^4 - c^2), x))*b^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(b^2x\operatorname{artanh}\left(\frac{c}{x^2}\right)^2 + 2\,abx\operatorname{artanh}\left(\frac{c}{x^2}\right) + a^2x, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c/x^2))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x*arctanh(c/x^2)^2 + 2*a*b*x*arctanh(c/x^2) + a^2*x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(a + b\operatorname{atanh}\left(\frac{c}{x^2}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c/x**2))**2,x)
```

```
[Out] Integral(x*(a + b*atanh(c/x**2))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\operatorname{artanh}\left(\frac{c}{x^2}\right) + a\right)^2 x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c/x^2))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c/x^2) + a)^2*x, x)
```

3.173 $\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2}{x}\,dx$

Optimal. Leaf size=144

$$\frac{1}{2}b\text{PolyLog}\left(2,1-\frac{2}{1-\frac{c}{x^2}}\right)\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)-\frac{1}{2}b\text{PolyLog}\left(2,\frac{2}{1-\frac{c}{x^2}}-1\right)\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)-\frac{1}{4}b^2\text{PolyLog}\left(3,\right.$$

```
[Out] -((a + b*ArcCoth[x^2/c])^2*ArcTanh[1 - 2/(1 - c/x^2)]) + (b*(a + b*ArcCoth[
x^2/c])*PolyLog[2, 1 - 2/(1 - c/x^2)])/2 - (b*(a + b*ArcCoth[x^2/c])*PolyLo
g[2, -1 + 2/(1 - c/x^2)])/2 - (b^2*PolyLog[3, 1 - 2/(1 - c/x^2)])/4 + (b^2*
PolyLog[3, -1 + 2/(1 - c/x^2)])/4
```

Rubi [A] time = 0.32428, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6095, 5914, 6052, 5948, 6058, 6610}

$$\frac{1}{2}b\text{PolyLog}\left(2,1-\frac{2}{1-\frac{c}{x^2}}\right)\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)-\frac{1}{2}b\text{PolyLog}\left(2,\frac{2}{1-\frac{c}{x^2}}-1\right)\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)-\frac{1}{4}b^2\text{PolyLog}\left(3,\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c/x^2])^2/x,x]
```

```
[Out] -((a + b*ArcCoth[x^2/c])^2*ArcTanh[1 - 2/(1 - c/x^2)]) + (b*(a + b*ArcCoth[
x^2/c])*PolyLog[2, 1 - 2/(1 - c/x^2)])/2 - (b*(a + b*ArcCoth[x^2/c])*PolyLo
g[2, -1 + 2/(1 - c/x^2)])/2 - (b^2*PolyLog[3, 1 - 2/(1 - c/x^2)])/4 + (b^2*
PolyLog[3, -1 + 2/(1 - c/x^2)])/4
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[
1/n, Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
 n}, x] && IGtQ[p, 0]
```

Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
 e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
```

```
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2}{x}\,dx = -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x}\,dx, x, \frac{1}{x^2}\right)\right)$$

$$= -\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1-\frac{c}{x^2}}\right) + (2bc)\operatorname{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(cx)\right)\tanh^{-1}\left(\right.}{1-c^2x^2}\right.$$

$$= -\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1-\frac{c}{x^2}}\right) - (bc)\operatorname{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1-cx}\right)}{1-c^2x^2}\right.$$

$$= -\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1-\frac{c}{x^2}}\right) + \frac{1}{2}b\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)\operatorname{Li}_2\left(1-\frac{2}{1-\frac{c}{x^2}}\right) - \frac{1}{2}$$

$$= -\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1-\frac{c}{x^2}}\right) + \frac{1}{2}b\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)\operatorname{Li}_2\left(1-\frac{2}{1-\frac{c}{x^2}}\right) - \frac{1}{2}$$

Mathematica [A] time = 0.0664234, size = 177, normalized size = 1.23

$$\frac{1}{2}\left(4bc\left(\frac{1}{2}\left(\frac{b\operatorname{PolyLog}\left(3,\frac{-\frac{c}{x^2}-1}{\frac{c}{x^2}-1}\right)}{4c} - \frac{\operatorname{PolyLog}\left(2,\frac{-\frac{c}{x^2}-1}{\frac{c}{x^2}-1}\right)\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)}{2c}\right) + \frac{1}{2}\left(\frac{\operatorname{PolyLog}\left(2,\frac{\frac{c}{x^2}+1}{\frac{c}{x^2}-1}\right)\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right.\right.}{2c}\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c/x^2])^2/x,x]
```

```
[Out] (-2*ArcTanh[1 - 2/(1 - c/x^2)]*(a + b*ArcTanh[c/x^2])^2 + 4*b*c*((-((a + b*
ArcTanh[c/x^2])*PolyLog[2, (-1 - c/x^2)/(-1 + c/x^2)])/(2*c) + (b*PolyLog[3
, (-1 - c/x^2)/(-1 + c/x^2)])/(4*c))/2 + (((a + b*ArcTanh[c/x^2])*PolyLog[2
, (1 + c/x^2)/(-1 + c/x^2)])/(2*c) - (b*PolyLog[3, (1 + c/x^2)/(-1 + c/x^2)
])/(4*c))/2))/2
```

Maple [F] time = 0.483, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left(a+b\operatorname{Artanh}\left(\frac{c}{x^2}\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x^2))^2/x,x)
```

```
[Out] int((a+b*arctanh(c/x^2))^2/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$a^2\log(x) + \int \frac{b^2\left(\log\left(\frac{c}{x^2}+1\right)-\log\left(-\frac{c}{x^2}+1\right)\right)^2}{4x} + \frac{ab\left(\log\left(\frac{c}{x^2}+1\right)-\log\left(-\frac{c}{x^2}+1\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))^2/x,x, algorithm="maxima")
```

```
[Out] a^2*log(x) + integrate(1/4*b^2*(log(c/x^2 + 1) - log(-c/x^2 + 1))^2/x + a*b
*(log(c/x^2 + 1) - log(-c/x^2 + 1))/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2\,\text{artanh}\left(\frac{c}{x^2}\right)^2 + 2\,ab\,\text{artanh}\left(\frac{c}{x^2}\right) + a^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))^2/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c/x^2)^2 + 2*a*b*arctanh(c/x^2) + a^2)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + b\,\text{atanh}\left(\frac{c}{x^2}\right)\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x**2))**2/x,x)
```

```
[Out] Integral((a + b*atanh(c/x**2))**2/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\,\text{artanh}\left(\frac{c}{x^2}\right) + a\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))^2/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c/x^2) + a)^2/x, x)
```

3.174 $\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2}{x^3}\,dx$

Optimal. Leaf size=99

$$\frac{b^2\text{PolyLog}\left(2,1-\frac{2}{1-\frac{c}{x^2}}\right)}{2c}-\frac{\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)^2}{2c}-\frac{\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)^2}{2x^2}+\frac{b\log\left(\frac{2}{1-\frac{c}{x^2}}\right)\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)}{c}$$

```
[Out] -(a + b*ArcCoth[x^2/c])^2/(2*c) - (a + b*ArcCoth[x^2/c])^2/(2*x^2) + (b*(a
+ b*ArcCoth[x^2/c])*Log[2/(1 - c/x^2)])/c + (b^2*PolyLog[2, 1 - 2/(1 - c/x^
2)])/(2*c)
```

Rubi [B] time = 0.528957, antiderivative size = 207, normalized size of antiderivative = 2.09, number of steps used = 28, number of rules used = 12, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {6099, 2454, 2389, 2296, 2295, 6715, 2430, 43, 2416, 2394, 2393, 2391}

$$\frac{b^2\text{PolyLog}\left(2,\frac{1}{2}\left(1-\frac{c}{x^2}\right)\right)}{4c}-\frac{b^2\text{PolyLog}\left(2,\frac{1}{2}\left(\frac{c}{x^2}+1\right)\right)}{4c}-\frac{b\log\left(\frac{1}{2}\left(\frac{c}{x^2}+1\right)\right)\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)}{4c}-\frac{b\log\left(\frac{c}{x^2}+1\right)\left(2a\right.}{4x}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcTanh[c/x^2])^2/x^3,x]
```

```
[Out] ((1 - c/x^2)*(2*a - b*Log[1 - c/x^2])^2)/(8*c) - (b*(2*a - b*Log[1 - c/x^2]
)*Log[(1 + c/x^2)/2])/(4*c) - (b^2*Log[(1 - c/x^2)/2]*Log[1 + c/x^2])/(4*c)
 - (b*(2*a - b*Log[1 - c/x^2])*Log[1 + c/x^2])/(4*x^2) - (b^2*(1 + c/x^2)*L
og[1 + c/x^2]^2)/(8*c) + (b^2*PolyLog[2, (1 - c/x^2)/2])/(4*c) - (b^2*PolyL
og[2, (1 + c/x^2)/2])/(4*c)
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
```

```
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 2430

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.)), x_Symbol] :> Simp[x*(a + b*Log[c
*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]), x] + (-Dist[g*j*m, Int[(x*(a +
 b*Log[c*(d + e*x)^n])^p)/(i + j*x), x], x] - Dist[b*e*n*p, Int[(x*(a + b*L
og[c*(d + e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /
; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2}{x^3}\,dx = \int\left(\frac{\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{4x^3} - \frac{b\left(-2a+b\log\left(1-\frac{c}{x^2}\right)\right)\log\left(1+\frac{c}{x^2}\right)}{2x^3} + \frac{b^2\log^2\left(1+\frac{c}{x^2}\right)}{4x^3}\right) dx$$

$$= \frac{1}{4}\int \frac{\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{x^3}\,dx - \frac{1}{2}b\int \frac{\left(-2a+b\log\left(1-\frac{c}{x^2}\right)\right)\log\left(1+\frac{c}{x^2}\right)}{x^3}\,dx + \frac{1}{4}b^2\int$$

$$= -\left(\frac{1}{8}\operatorname{Subst}\left(\int (2a-b\log(1-cx))^2\,dx, x, \frac{1}{x^2}\right)\right) + \frac{1}{4}b\operatorname{Subst}\left(\int(-2a+b\log(1-cx))\log\right.$$

$$= -\frac{b\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)\log\left(1+\frac{c}{x^2}\right)}{4x^2} + \frac{\operatorname{Subst}\left(\int(2a-b\log(x))^2\,dx, x, 1-\frac{c}{x^2}\right)}{8c} - \frac{b^2\operatorname{Sub}}{}$$

$$= \frac{\left(1-\frac{c}{x^2}\right)\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{8c} - \frac{b\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)\log\left(1+\frac{c}{x^2}\right)}{4x^2} - \frac{b^2\left(1+\frac{c}{x^2}\right)\log^2\left(\right.}{8c}$$

$$= -\frac{ab}{2x^2} - \frac{b^2}{4x^2} + \frac{\left(1-\frac{c}{x^2}\right)\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{8c} + \frac{b^2\left(1+\frac{c}{x^2}\right)\log\left(1+\frac{c}{x^2}\right)}{4c} - \frac{b\left(2a-b\log\right.}{}$$

$$= -\frac{b^2}{2x^2} - \frac{b^2\left(1-\frac{c}{x^2}\right)\log\left(1-\frac{c}{x^2}\right)}{4c} + \frac{\left(1-\frac{c}{x^2}\right)\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{8c} - \frac{b\left(2a-b\log\left(1-\frac{c}{x^2}\right.\right.}{4c}$$

$$= -\frac{b^2}{4x^2} - \frac{b^2\left(1-\frac{c}{x^2}\right)\log\left(1-\frac{c}{x^2}\right)}{4c} + \frac{\left(1-\frac{c}{x^2}\right)\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{8c} - \frac{b\left(2a-b\log\left(1-\frac{c}{x^2}\right.\right.}{4c}$$

$$= \frac{\left(1-\frac{c}{x^2}\right)\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{8c} - \frac{b\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)\log\left(\frac{1}{2}\left(1+\frac{c}{x^2}\right)\right)}{4c} - \frac{b^2\log\left(\frac{1}{2}\left(1-\right.\right.}{}$$

Mathematica [A] time = 0.0899924, size = 114, normalized size = 1.15

$$-\frac{b^2\left(\operatorname{PolyLog}\left(2,-e^{-2\tanh^{-1}\left(\frac{c}{x^2}\right)}\right)+\tanh^{-1}\left(\frac{c}{x^2}\right)\left(\frac{c\tanh^{-1}\left(\frac{c}{x^2}\right)}{x^2}-\tanh^{-1}\left(\frac{c}{x^2}\right)-2\log\left(e^{-2\tanh^{-1}\left(\frac{c}{x^2}\right)}+1\right)\right)\right)}{2c} - \frac{a^2}{2x^2} - \frac{ab}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c/x^2])^2/x^3,x]
```

```
[Out] -a^2/(2*x^2) - (a*b*((c*ArcTanh[c/x^2])/x^2 - Log[1/Sqrt[1 - c^2/x^4]]))/c
- (b^2*(ArcTanh[c/x^2]*(-ArcTanh[c/x^2] + (c*ArcTanh[c/x^2])/x^2 - 2*Log[1
+ E^(-2*ArcTanh[c/x^2])]) + PolyLog[2, -E^(-2*ArcTanh[c/x^2])]))/(2*c)
```

Maple [A] time = 0.004, size = 144, normalized size = 1.5

$$-\frac{a^2}{2x^2} - \frac{b^2}{2x^2}\left(\operatorname{Artanh}\left(\frac{c}{x^2}\right)\right)^2 - \frac{b^2}{2c}\left(\operatorname{Artanh}\left(\frac{c}{x^2}\right)\right)^2 + \frac{b^2}{c}\operatorname{Artanh}\left(\frac{c}{x^2}\right)\ln\left(\left(1+\frac{c}{x^2}\right)^2\left(1-\frac{c^2}{x^4}\right)^{-1}+1\right) + \frac{b^2}{2c}\operatorname{polylog}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x^2))^2/x^3,x)
```

```
[Out] -1/2*a^2/x^2-1/2*arctanh(c/x^2)^2/x^2*b^2-1/2/c*b^2*arctanh(c/x^2)^2+1/c*ar
ctanh(c/x^2)*ln((1+c/x^2)^2/(1-c^2/x^4)+1)*b^2+1/2/c*polylog(2,-(1+c/x^2)^2
```

```
/(1-c^2/x^4))*b^2-a*b/x^2*arctanh(c/x^2)-1/2/c*a*b*ln(1-c^2/x^4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{8}\left(8\,c^3\int\frac{\log\left(x\right)^2}{cx^7-c^3x^3}\,dx+c^2\left(\frac{\log\left(x^2+c\right)}{c^3}+\frac{\log\left(x^2-c\right)}{c^3}-\frac{4\,\log\left(x\right)}{c^3}\right)-8\,c^2\int\frac{x^2\log\left(x^2+c\right)}{cx^7-c^3x^3}\,dx+8\,c^2\int\frac{x^2\log\left(x\right)}{cx^7-c^3x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/8*(8*c^3*integrate(log(x)^2/(c*x^7 - c^3*x^3), x) + c^2*(log(x^2 + c)/c^3
 + log(x^2 - c)/c^3 - 4*log(x)/c^3) - 8*c^2*integrate(x^2*log(x^2 + c)/(c*x
^7 - c^3*x^3), x) + 8*c^2*integrate(x^2*log(x)/(c*x^7 - c^3*x^3), x) + 2*c*
(log(x^2 - c)/c^2 - log(x^2)/c^2 + 1/(c*x^2))*log(-c/x^2 + 1) - c*(log(x^2
+ c)/c^2 - log(x^2 - c)/c^2) - 8*c*integrate(x^4*log(x)^2/(c*x^7 - c^3*x^3)
, x) - 4*c*integrate(x^4*log(x^2 + c)/(c*x^7 - c^3*x^3), x) + 16*c*integrat
e(x^4*log(x)/(c*x^7 - c^3*x^3), x) - log(-c/x^2 + 1)^2/x^2 - (x^2*log(x^2 -
 c)^2 + 4*x^2*log(x)^2 - 4*x^2*log(x) - 2*(2*x^2*log(x) - x^2)*log(x^2 - c)
 + 2*c)/(c*x^2) - (c*log(x^2 + c)^2 - 2*((x^2 + c)*log(x^2 + c) - 2*(x^2 +
c)*log(x) - c)*log(x^2 - c))/(c*x^2) - 4*integrate(x^6*log(x^2 + c)/(c*x^7
- c^3*x^3), x) + 8*integrate(x^6*log(x)/(c*x^7 - c^3*x^3), x))*b^2 - 1/2*a*
b*(2*c*arctanh(c/x^2)/x^2 + log(-c^2/x^4 + 1))/c - 1/2*a^2/x^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2\,\text{artanh}\left(\frac{c}{x^2}\right)^2+2\,ab\,\text{artanh}\left(\frac{c}{x^2}\right)+a^2}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))^2/x^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c/x^2)^2 + 2*a*b*arctanh(c/x^2) + a^2)/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+b\,\text{atanh}\left(\frac{c}{x^2}\right)\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x**2))**2/x**3,x)
```

```
[Out] Integral((a + b*atanh(c/x**2))**2/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \operatorname{artanh}\left(\frac{c}{x^2}\right) + a\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))^2/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c/x^2) + a)^2/x^3, x)
```

3.175 $\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2}{x^5}\,dx$

Optimal. Leaf size=97

$$\frac{\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)^2}{4c^2}-\frac{ab}{2cx^2}-\frac{\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)^2}{4x^4}-\frac{b^2\log\left(1-\frac{c^2}{x^4}\right)}{4c^2}-\frac{b^2\coth^{-1}\left(\frac{x^2}{c}\right)}{2cx^2}$$

```
[Out] -(a*b)/(2*c*x^2) - (b^2*ArcCoth[x^2/c])/(2*c*x^2) + (a + b*ArcCoth[x^2/c])^
2/(4*c^2) - (a + b*ArcCoth[x^2/c])^2/(4*x^4) - (b^2*Log[1 - c^2/x^4])/(4*c^
2)
```

Rubi [C] time = 1.5301, antiderivative size = 770, normalized size of antiderivative = 7.94, number of steps used = 67, number of rules used = 23, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.438, Rules used = {6099, 2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304, 2395, 43, 6742, 30, 2557, 12, 2466, 2462, 260, 2416, 2394, 2315, 2393, 2391}

$$\frac{b^2\text{PolyLog}\left(2,-\frac{c}{x^2}\right)}{8c^2}+\frac{b^2\text{PolyLog}\left(2,\frac{c}{x^2}\right)}{8c^2}+\frac{b^2\text{PolyLog}\left(2,\frac{c-x^2}{2c}\right)}{8c^2}+\frac{b^2\text{PolyLog}\left(2,\frac{c+x^2}{2c}\right)}{8c^2}-\frac{b^2\text{PolyLog}\left(2,\frac{c+x^2}{c}\right)}{8c^2}-\frac{b^2\text{Po}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcTanh[c/x^2])^2/x^5,x]
```

```
[Out] -(b^2*(1 - c/x^2)^2)/(32*c^2) - (b^2*(1 + c/x^2)^2)/(32*c^2) + (a*b)/(8*x^4
) + b^2/(16*x^4) - (3*a*b)/(4*c*x^2) + (b^2*Log[1 - c/x^2])/(16*c^2) - (3*b
^2*(1 - c/x^2)*Log[1 - c/x^2])/(8*c^2) - (b^2*Log[1 - c/x^2])/(16*x^4) - (b
*(1 - c/x^2)^2*(2*a - b*Log[1 - c/x^2]))/(16*c^2) + ((1 - c/x^2)*(2*a - b*L
og[1 - c/x^2])^2)/(8*c^2) - ((1 - c/x^2)^2*(2*a - b*Log[1 - c/x^2])^2)/(16*
c^2) - (b^2*(1 + c/x^2)*Log[1 + c/x^2])/(4*c^2) + (b^2*(1 + c/x^2)^2*Log[1
+ c/x^2])/(16*c^2) + (b^2*Log[1 - c/x^2]*Log[1 + c/x^2])/(8*x^4) + (b^2*(1
+ c/x^2)*Log[1 + c/x^2]^2)/(8*c^2) - (b^2*(1 + c/x^2)^2*Log[1 + c/x^2]^2)/(
16*c^2) - (b^2*Log[1 + c/x^2]*Log[c - x^2])/(8*c^2) - (b^2*Log[x^2/c]*Log[c
 - x^2])/(8*c^2) - (b^2*Log[1 - c/x^2]*Log[c + x^2])/(8*c^2) - (b^2*Log[-(x
^2/c)]*Log[c + x^2])/(8*c^2) + (b^2*Log[(c - x^2)/(2*c)]*Log[c + x^2])/(8*c
^2) + (b^2*Log[c - x^2]*Log[(c + x^2)/(2*c)])/(8*c^2) + (a*b*Log[(c + x^2)/
x^2])/(4*c^2) + (b^2*Log[(c + x^2)/x^2])/(16*c^2) - (b^2*(1 + c/x^2)*Log[(c
 + x^2)/x^2])/(8*c^2) - (a*b*Log[(c + x^2)/x^2])/(4*x^4) - (b^2*Log[(c + x^
2)/x^2])/(16*x^4) + (b^2*PolyLog[2, -(c/x^2)])/(8*c^2) + (b^2*PolyLog[2, c/
x^2])/(8*c^2) + (b^2*PolyLog[2, (c - x^2)/(2*c)])/(8*c^2) + (b^2*PolyLog[2,
 (c + x^2)/(2*c)])/(8*c^2) - (b^2*PolyLog[2, (c + x^2)/c])/(8*c^2) - (b^2*P
olyLog[2, 1 - x^2/c])/(8*c^2)
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
```

```
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2557

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] :> With[{z = IntHide[u, x]}, Dist[Log[v
]*Log[w], z, x] + (-Int[SimplifyIntegrand[(z*Log[w]*D[v, x])/v, x], x] - In
t[SimplifyIntegrand[(z*Log[v]*D[w, x])/w, x], x]) /; InverseFunctionFreeQ[z
, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2}{x^5}\,dx = \int \left(\frac{\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{4x^5} - \frac{b\left(-2a+b\log\left(1-\frac{c}{x^2}\right)\right)\log\left(1+\frac{c}{x^2}\right)}{2x^5} + \frac{b^2\log^2\left(1+\frac{c}{x^2}\right)}{4x^5}\right)dx$$

$$= \frac{1}{4}\int \frac{\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{x^5}\,dx - \frac{1}{2}b\int \frac{\left(-2a+b\log\left(1-\frac{c}{x^2}\right)\right)\log\left(1+\frac{c}{x^2}\right)}{x^5}\,dx + \frac{1}{4}b^2\int \frac{\log^2}{}$$

$$= -\left(\frac{1}{8}\text{Subst}\left(\int x(2a-b\log(1-cx))^2\,dx, x, \frac{1}{x^2}\right)\right) - \frac{1}{4}b\,\text{Subst}\left(\int \frac{\left(-2a+b\log\left(1-\frac{c}{x}\right)\right)\log}{x^3}\right.$$

$$= -\left(\frac{1}{8}\text{Subst}\left(\int \left(\frac{(2a-b\log(1-cx))^2}{c} - \frac{(1-cx)(2a-b\log(1-cx))^2}{c}\right)dx, x, \frac{1}{x^2}\right)\right) - \frac{1}{4}b\,\text{Su}$$

$$= \frac{1}{2}(ab)\,\text{Subst}\left(\int \frac{\log\left(1+\frac{c}{x}\right)}{x^3}\,dx, x, x^2\right) - \frac{1}{4}b^2\,\text{Subst}\left(\int \frac{\log\left(1-\frac{c}{x}\right)\log\left(1+\frac{c}{x}\right)}{x^3}\,dx, x, x^2\right) -$$

$$= \frac{b^2\log\left(1-\frac{c}{x^2}\right)\log\left(1+\frac{c}{x^2}\right)}{8x^4} - \frac{1}{2}(ab)\,\text{Subst}\left(\int x\log(1+cx)\,dx, x, \frac{1}{x^2}\right) + \frac{1}{4}b^2\,\text{Subst}\left(\int \frac{c\,\text{lo}}{2x}\right.$$

$$= \frac{\left(1-\frac{c}{x^2}\right)\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{8c^2} - \frac{\left(1-\frac{c}{x^2}\right)^2\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{16c^2} - \frac{ab\log\left(1+\frac{c}{x^2}\right)}{4x^4} + \frac{b^2\log}{}$$

$$= -\frac{b^2\left(1-\frac{c}{x^2}\right)^2}{32c^2} - \frac{b^2\left(1+\frac{c}{x^2}\right)^2}{32c^2} - \frac{ab}{2cx^2} + \frac{b^2}{4cx^2} - \frac{b\left(1-\frac{c}{x^2}\right)^2\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)}{16c^2} + \frac{\left(1-\frac{c}{x^2}\right)\left(\right.}{}$$

$$= -\frac{b^2\left(1-\frac{c}{x^2}\right)^2}{32c^2} - \frac{b^2\left(1+\frac{c}{x^2}\right)^2}{32c^2} + \frac{ab}{8x^4} - \frac{3ab}{4cx^2} - \frac{b^2\left(1-\frac{c}{x^2}\right)\log\left(1-\frac{c}{x^2}\right)}{4c^2} - \frac{b\left(1-\frac{c}{x^2}\right)^2\left(2a-b\right.}{16c^2}$$

$$= -\frac{b^2\left(1-\frac{c}{x^2}\right)^2}{32c^2} - \frac{b^2\left(1+\frac{c}{x^2}\right)^2}{32c^2} + \frac{ab}{8x^4} - \frac{3ab}{4cx^2} - \frac{b^2\left(1-\frac{c}{x^2}\right)\log\left(1-\frac{c}{x^2}\right)}{4c^2} - \frac{b\left(1-\frac{c}{x^2}\right)^2\left(2a-b\right.}{16c^2}$$

$$= -\frac{b^2\left(1-\frac{c}{x^2}\right)^2}{32c^2} - \frac{b^2\left(1+\frac{c}{x^2}\right)^2}{32c^2} + \frac{ab}{8x^4} - \frac{3ab}{4cx^2} - \frac{b^2\left(1-\frac{c}{x^2}\right)\log\left(1-\frac{c}{x^2}\right)}{4c^2} - \frac{b^2\log\left(1-\frac{c}{x^2}\right)}{16x^4} - \frac{b}{}$$

$$= -\frac{b^2\left(1-\frac{c}{x^2}\right)^2}{32c^2} - \frac{b^2\left(1+\frac{c}{x^2}\right)^2}{32c^2} + \frac{ab}{8x^4} - \frac{3ab}{4cx^2} - \frac{3b^2\left(1-\frac{c}{x^2}\right)\log\left(1-\frac{c}{x^2}\right)}{8c^2} - \frac{b^2\log\left(1-\frac{c}{x^2}\right)}{16x^4} - $$

$$= -\frac{b^2\left(1-\frac{c}{x^2}\right)^2}{32c^2} - \frac{b^2\left(1+\frac{c}{x^2}\right)^2}{32c^2} + \frac{ab}{8x^4} + \frac{b^2}{16x^4} - \frac{3ab}{4cx^2} + \frac{b^2\log\left(1-\frac{c}{x^2}\right)}{16c^2} - \frac{3b^2\left(1-\frac{c}{x^2}\right)\log\left(1-\right.}{8c^2}$$

$$= -\frac{b^2\left(1-\frac{c}{x^2}\right)^2}{32c^2} - \frac{b^2\left(1+\frac{c}{x^2}\right)^2}{32c^2} + \frac{ab}{8x^4} + \frac{b^2}{16x^4} - \frac{3ab}{4cx^2} + \frac{b^2\log\left(1-\frac{c}{x^2}\right)}{16c^2} - \frac{3b^2\left(1-\frac{c}{x^2}\right)\log\left(1-\right.}{8c^2}$$

$$= -\frac{b^2\left(1-\frac{c}{x^2}\right)^2}{32c^2} - \frac{b^2\left(1+\frac{c}{x^2}\right)^2}{32c^2} + \frac{ab}{8x^4} + \frac{b^2}{16x^4} - \frac{3ab}{4cx^2} + \frac{b^2\log\left(1-\frac{c}{x^2}\right)}{16c^2} - \frac{3b^2\left(1-\frac{c}{x^2}\right)\log\left(1-\right.}{8c^2}$$

Mathematica [A] time = 0.0809365, size = 131, normalized size = 1.35

$$-\frac{a^2c^2 + 2abcx^2 + abx^4\log\left(x^2-c\right) - abx^4\log\left(c+x^2\right) + 2bc\tanh^{-1}\left(\frac{c}{x^2}\right)\left(ac+bx^2\right) + b^2\left(c^2-x^4\right)\tanh^{-1}\left(\frac{c}{x^2}\right)^2 + b^2x^4\text{lo}}{4c^2x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c/x^2])^2/x^5,x]
```

```
[Out] -(a^2*c^2 + 2*a*b*c*x^2 + 2*b*c*(a*c + b*x^2)*ArcTanh[c/x^2] + b^2*(c^2 - x
^4)*ArcTanh[c/x^2]^2 - 4*b^2*x^4*Log[x] + a*b*x^4*Log[-c + x^2] + b^2*x^4*L
```

```
og[-c + x^2] - a*b*x^4*Log[c + x^2] + b^2*x^4*Log[c + x^2])/(4*c^2*x^4)
```

Maple [B] time = 88.027, size = 682002, normalized size = 7031.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x^2))^2/x^5,x)
```

```
[Out] result too large to display
```

Maxima [B] time = 1.05709, size = 247, normalized size = 2.55

$$\frac{1}{4}\left(c\left(\frac{\log\left(x^2+c\right)}{c^3}-\frac{\log\left(x^2-c\right)}{c^3}-\frac{2}{c^2x^2}\right)-\frac{2\,\operatorname{artanh}\left(\frac{c}{x^2}\right)}{x^4}\right)ab-\frac{1}{16}\left(c^2\left(\frac{\log\left(x^2+c\right)^2-2\left(\log\left(x^2+c\right)-2\right)\log\left(x^2-c\right)}{c^4}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))^2/x^5,x, algorithm="maxima")
```

```
[Out] 1/4*(c*(log(x^2 + c)/c^3 - log(x^2 - c)/c^3 - 2/(c^2*x^2)) - 2*arctanh(c/x^
2)/x^4)*a*b - 1/16*(c^2*((log(x^2 + c)^2 - 2*(log(x^2 + c) - 2)*log(x^2 - c
) + log(x^2 - c)^2 + 4*log(x^2 + c))/c^4 - 16*log(x)/c^4) - 4*c*(log(x^2 +
c)/c^3 - log(x^2 - c)/c^3 - 2/(c^2*x^2))*arctanh(c/x^2))*b^2 - 1/4*b^2*arct
anh(c/x^2)^2/x^4 - 1/4*a^2/x^4
```

Fricas [A] time = 1.7995, size = 308, normalized size = 3.18

$$\frac{16\,b^2x^4\log\left(x\right)+4\left(ab-b^2\right)x^4\log\left(x^2+c\right)-4\left(ab+b^2\right)x^4\log\left(x^2-c\right)-8\,abcx^2-4\,a^2c^2+\left(b^2x^4-b^2c^2\right)\log\left(\frac{x^2+c}{x^2-c}\right)^2-}{16\,c^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/16*(16*b^2*x^4*log(x) + 4*(a*b - b^2)*x^4*log(x^2 + c) - 4*(a*b + b^2)*x^
4*log(x^2 - c) - 8*a*b*c*x^2 - 4*a^2*c^2 + (b^2*x^4 - b^2*c^2)*log((x^2 + c
)/(x^2 - c))^2 - 4*(b^2*c*x^2 + a*b*c^2)*log((x^2 + c)/(x^2 - c)))/(c^2*x^4
)
```

Sympy [A] time = 41.7701, size = 172, normalized size = 1.77

$$\begin{cases}-\frac{a^2}{4x^4}-\frac{ab\operatorname{atanh}\left(\frac{c}{x^2}\right)}{2x^4}-\frac{ab}{2cx^2}+\frac{ab\operatorname{atanh}\left(\frac{c}{x^2}\right)}{2c^2}-\frac{b^2\operatorname{atanh}^2\left(\frac{c}{x^2}\right)}{4x^4}-\frac{b^2\operatorname{atanh}\left(\frac{c}{x^2}\right)}{2cx^2}+\frac{b^2\log\left(x\right)}{c^2}-\frac{b^2\log\left(-i\sqrt{c}+x\right)}{2c^2}-\frac{b^2\log\left(i\sqrt{c}+x\right)}{2c^2}+\frac{b^2\operatorname{ata}}{} \\ -\frac{a^2}{4x^4}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x**2))**2/x**5,x)
```

```
[Out] Piecewise((-a**2/(4*x**4) - a*b*atanh(c/x**2)/(2*x**4) - a*b/(2*c*x**2) + a
*b*atanh(c/x**2)/(2*c**2) - b**2*atanh(c/x**2)**2/(4*x**4) - b**2*atanh(c/x
**2)/(2*c*x**2) + b**2*log(x)/c**2 - b**2*log(-I*sqrt(c) + x)/(2*c**2) - b*
*2*log(I*sqrt(c) + x)/(2*c**2) + b**2*atanh(c/x**2)**2/(4*c**2) + b**2*atan
h(c/x**2)/(2*c**2), Ne(c, 0)), (-a**2/(4*x**4), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \operatorname{artanh}\left(\frac{c}{x^2}\right) + a\right)^2}{x^5} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))^2/x^5,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c/x^2) + a)^2/x^5, x)
```

3.176 $\int x^4 \left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2 dx$

Optimal. Leaf size=1214

result too large to display

```
[Out] (8*b^2*c^2*x)/15 + (2*a*b*c*x^3)/15 + (2*a*b*c^(5/2)*ArcTan[x/Sqrt[c]])/5 -
 (4*b^2*c^(5/2)*ArcTan[x/Sqrt[c]])/15 - (I/5)*b^2*c^(5/2)*ArcTan[x/Sqrt[c]]
^2 - (4*b^2*c^(5/2)*ArcTanh[x/Sqrt[c]])/15 + (b^2*c^(5/2)*ArcTanh[x/Sqrt[c]
]^2)/5 + (2*b^2*c^(5/2)*ArcTan[x/Sqrt[c]]*Log[2 - (2*Sqrt[c])/(Sqrt[c] - I*
x)])/5 - (b^2*c*x^3*Log[1 - c/x^2])/15 - (b^2*c^(5/2)*ArcTan[x/Sqrt[c]]*Log
[1 - c/x^2])/5 + (b*c*x^3*(2*a - b*Log[1 - c/x^2]))/15 - (b*c^(5/2)*ArcTanh
[x/Sqrt[c]]*(2*a - b*Log[1 - c/x^2]))/5 + (x^5*(2*a - b*Log[1 - c/x^2])^2)/
20 + (2*b^2*c*x^3*Log[1 + c/x^2])/15 + (a*b*x^5*Log[1 + c/x^2])/5 + (b^2*c^
(5/2)*ArcTan[x/Sqrt[c]]*Log[1 + c/x^2])/5 - (b^2*c^(5/2)*ArcTanh[x/Sqrt[c]]
*Log[1 + c/x^2])/5 - (b^2*x^5*Log[1 - c/x^2]*Log[1 + c/x^2])/10 + (b^2*x^5*
Log[1 + c/x^2]^2)/20 - (2*b^2*c^(5/2)*ArcTan[x/Sqrt[c]]*Log[(2*Sqrt[c])/(Sq
rt[c] - I*x)])/5 + (b^2*c^(5/2)*ArcTan[x/Sqrt[c]]*Log[((1 + I)*(Sqrt[c] - x
))/(Sqrt[c] - I*x)])/5 - (2*b^2*c^(5/2)*ArcTanh[x/Sqrt[c]]*Log[(2*Sqrt[c])/
(Sqrt[c] + x)])/5 + (b^2*c^(5/2)*ArcTanh[x/Sqrt[c]]*Log[(2*Sqrt[c]*(Sqrt[-c
] - x))/((Sqrt[-c] - Sqrt[c])*(Sqrt[c] + x))])/5 + (b^2*c^(5/2)*ArcTanh[x/S
qrt[c]]*Log[(2*Sqrt[c]*(Sqrt[-c] + x))/((Sqrt[-c] + Sqrt[c])*(Sqrt[c] + x))
])/5 + (b^2*c^(5/2)*ArcTan[x/Sqrt[c]]*Log[((1 - I)*(Sqrt[c] + x))/(Sqrt[c]
- I*x)])/5 + (2*b^2*c^(5/2)*ArcTanh[x/Sqrt[c]]*Log[2 - (2*Sqrt[c])/(Sqrt[c]
 + x)])/5 + (I/5)*b^2*c^(5/2)*PolyLog[2, 1 - (2*Sqrt[c])/(Sqrt[c] - I*x)] -
 (I/5)*b^2*c^(5/2)*PolyLog[2, -1 + (2*Sqrt[c])/(Sqrt[c] - I*x)] - (I/10)*b^
2*c^(5/2)*PolyLog[2, 1 - ((1 + I)*(Sqrt[c] - x))/(Sqrt[c] - I*x)] + (b^2*c^
(5/2)*PolyLog[2, -(x/Sqrt[c])])/5 - (I/5)*b^2*c^(5/2)*PolyLog[2, ((-I)*x)/S
qrt[c]] + (I/5)*b^2*c^(5/2)*PolyLog[2, (I*x)/Sqrt[c]] - (b^2*c^(5/2)*PolyLo
g[2, x/Sqrt[c]])/5 + (b^2*c^(5/2)*PolyLog[2, 1 - (2*Sqrt[c])/(Sqrt[c] + x)]
)/5 - (b^2*c^(5/2)*PolyLog[2, -1 + (2*Sqrt[c])/(Sqrt[c] + x)])/5 - (b^2*c^(
5/2)*PolyLog[2, 1 - (2*Sqrt[c]*(Sqrt[-c] - x))/((Sqrt[-c] - Sqrt[c])*(Sqrt[
c] + x))])/10 - (b^2*c^(5/2)*PolyLog[2, 1 - (2*Sqrt[c]*(Sqrt[-c] + x))/((Sq
rt[-c] + Sqrt[c])*(Sqrt[c] + x))])/10 - (I/10)*b^2*c^(5/2)*PolyLog[2, 1 - (
(1 - I)*(Sqrt[c] + x))/(Sqrt[c] - I*x)]
```

Rubi [A] time = 2.71234, antiderivative size = 1214, normalized size of antiderivative = 1., number of steps used = 97, number of rules used = 33, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 2.063, Rules used = {6099, 2457, 2476, 2448, 263, 207, 2455, 193, 321, 2470, 12, 260, 6688, 5988, 5932, 2447, 302, 6742, 203, 30, 2557, 5992, 5912, 5920, 2402, 2315, 204, 4928, 4848, 2391, 4856, 4924, 4868}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*ArcTanh[c/x^2])^2,x]
```


```
[Out] (8*b^2*c^2*x)/15 + (2*a*b*c*x^3)/15 + (2*a*b*c^(5/2)*ArcTan[x/Sqrt[c]])/5 -
 (4*b^2*c^(5/2)*ArcTan[x/Sqrt[c]])/15 - (I/5)*b^2*c^(5/2)*ArcTan[x/Sqrt[c]]
^2 - (4*b^2*c^(5/2)*ArcTanh[x/Sqrt[c]])/15 + (b^2*c^(5/2)*ArcTanh[x/Sqrt[c]
]^2)/5 + (2*b^2*c^(5/2)*ArcTan[x/Sqrt[c]]*Log[2 - (2*Sqrt[c])/(Sqrt[c] - I*
x)])/5 - (b^2*c*x^3*Log[1 - c/x^2])/15 - (b^2*c^(5/2)*ArcTan[x/Sqrt[c]]*Log
[1 - c/x^2])/5 + (b*c*x^3*(2*a - b*Log[1 - c/x^2]))/15 - (b*c^(5/2)*ArcTanh
[x/Sqrt[c]]*(2*a - b*Log[1 - c/x^2]))/5 + (x^5*(2*a - b*Log[1 - c/x^2])^2)/
20 + (2*b^2*c*x^3*Log[1 + c/x^2])/15 + (a*b*x^5*Log[1 + c/x^2])/5 + (b^2*c^
```

```
(5/2)*ArcTan[x/Sqrt[c]]*Log[1 + c/x^2])/5 - (b^2*c^(5/2)*ArcTanh[x/Sqrt[c]]
*Log[1 + c/x^2])/5 - (b^2*x^5*Log[1 - c/x^2]*Log[1 + c/x^2])/10 + (b^2*x^5*
Log[1 + c/x^2]^2)/20 - (2*b^2*c^(5/2)*ArcTan[x/Sqrt[c]]*Log[(2*Sqrt[c])/(Sq
rt[c] - I*x)])/5 + (b^2*c^(5/2)*ArcTan[x/Sqrt[c]]*Log[((1 + I)*(Sqrt[c] - x
))/(Sqrt[c] - I*x)])/5 - (2*b^2*c^(5/2)*ArcTanh[x/Sqrt[c]]*Log[(2*Sqrt[c])/
(Sqrt[c] + x)])/5 + (b^2*c^(5/2)*ArcTanh[x/Sqrt[c]]*Log[(2*Sqrt[c]*(Sqrt[-c
] - x))/((Sqrt[-c] - Sqrt[c])*(Sqrt[c] + x))])/5 + (b^2*c^(5/2)*ArcTanh[x/S
qrt[c]]*Log[(2*Sqrt[c]*(Sqrt[-c] + x))/((Sqrt[-c] + Sqrt[c])*(Sqrt[c] + x))
])/5 + (b^2*c^(5/2)*ArcTan[x/Sqrt[c]]*Log[((1 - I)*(Sqrt[c] + x))/(Sqrt[c]
- I*x)])/5 + (2*b^2*c^(5/2)*ArcTanh[x/Sqrt[c]]*Log[2 - (2*Sqrt[c])/(Sqrt[c]
 + x)])/5 + (I/5)*b^2*c^(5/2)*PolyLog[2, 1 - (2*Sqrt[c])/(Sqrt[c] - I*x)] -
 (I/5)*b^2*c^(5/2)*PolyLog[2, -1 + (2*Sqrt[c])/(Sqrt[c] - I*x)] - (I/10)*b^
2*c^(5/2)*PolyLog[2, 1 - ((1 + I)*(Sqrt[c] - x))/(Sqrt[c] - I*x)] + (b^2*c^
(5/2)*PolyLog[2, -(x/Sqrt[c])])/5 - (I/5)*b^2*c^(5/2)*PolyLog[2, ((-I)*x)/S
qrt[c]] + (I/5)*b^2*c^(5/2)*PolyLog[2, (I*x)/Sqrt[c]] - (b^2*c^(5/2)*PolyLo
g[2, x/Sqrt[c]])/5 + (b^2*c^(5/2)*PolyLog[2, 1 - (2*Sqrt[c])/(Sqrt[c] + x)]
)/5 - (b^2*c^(5/2)*PolyLog[2, -1 + (2*Sqrt[c])/(Sqrt[c] + x)])/5 - (b^2*c^(
5/2)*PolyLog[2, 1 - (2*Sqrt[c]*(Sqrt[-c] - x))/((Sqrt[-c] - Sqrt[c])*(Sqrt[
c] + x))])/10 - (b^2*c^(5/2)*PolyLog[2, 1 - (2*Sqrt[c]*(Sqrt[-c] + x))/((Sq
rt[-c] + Sqrt[c])*(Sqrt[c] + x))])/10 - (I/10)*b^2*c^(5/2)*PolyLog[2, 1 - (
(1 - I)*(Sqrt[c] + x))/(Sqrt[c] - I*x)]
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q
)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a +
 b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
```

```
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
```

```
2*d^2 - e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2557

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] :> With[{z = IntHide[u, x]}, Dist[Log[v
]*Log[w], z, x] + (-Int[SimplifyIntegrand[(z*Log[w]*D[v, x])/v, x], x] - In
t[SimplifyIntegrand[(z*Log[v]*D[w, x])/w, x], x]) /; InverseFunctionFreeQ[z
, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 5992

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
```

```
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
```

```
^2 + e^2, 0]
```

Rubi steps

$$\int x^4\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2 dx = \int\left(\frac{1}{4}x^4\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2-\frac{1}{2}bx^4\left(-2a+b\log\left(1-\frac{c}{x^2}\right)\right)\log\left(1+\frac{c}{x^2}\right)+\frac{1}{4}b^2x^4\right.$$

$$=\frac{1}{4}\int x^4\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2 dx-\frac{1}{2}b\int x^4\left(-2a+b\log\left(1-\frac{c}{x^2}\right)\right)\log\left(1+\frac{c}{x^2}\right)dx$$

$$=\frac{1}{20}x^5\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2+\frac{1}{20}b^2x^5\log^2\left(1+\frac{c}{x^2}\right)-\frac{1}{2}b\int\left(-2ax^4\log\left(1+\frac{c}{x^2}\right)+\right.$$

$$=\frac{1}{20}x^5\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2+\frac{1}{20}b^2x^5\log^2\left(1+\frac{c}{x^2}\right)+(ab)\int x^4\log\left(1+\frac{c}{x^2}\right)dx-$$

$$=\frac{1}{20}x^5\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2+\frac{1}{5}abx^5\log\left(1+\frac{c}{x^2}\right)-\frac{1}{10}b^2x^5\log\left(1-\frac{c}{x^2}\right)\log\left(1+\frac{c}{x^2}\right.$$

$$=\frac{2}{5}abc^2x+\frac{1}{15}bcx^3\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)-\frac{1}{5}bc^{5/2}\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)$$

$$=\frac{2}{5}abc^2x-\frac{1}{5}b^2c^2x\log\left(1-\frac{c}{x^2}\right)+\frac{1}{15}bcx^3\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)-\frac{1}{5}bc^{5/2}\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)$$

$$=\frac{4}{15}b^2c^2x+\frac{2}{15}abcx^3-\frac{2}{5}b^2c^{5/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)-\frac{1}{5}b^2c^2x\log\left(1-\frac{c}{x^2}\right)+\frac{1}{15}bcx^3\left(2a-b\log\right.$$

$$=\frac{4}{15}b^2c^2x+\frac{2}{15}abcx^3+\frac{2}{5}abc^{5/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)-\frac{8}{15}b^2c^{5/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)-\frac{1}{5}ib^2c^{5/2}\tan^{-1}\left(\right.$$

$$=\frac{4}{15}b^2c^2x+\frac{2}{15}abcx^3+\frac{2}{5}abc^{5/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)-\frac{8}{15}b^2c^{5/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)-\frac{1}{5}ib^2c^{5/2}\tan^{-1}\left(\right.$$

$$=\frac{8}{15}b^2c^2x+\frac{2}{15}abcx^3+\frac{2}{5}abc^{5/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)-\frac{2}{15}b^2c^{5/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)-\frac{1}{5}ib^2c^{5/2}\tan^{-1}\left(\right.$$

$$=\frac{8}{15}b^2c^2x+\frac{2}{15}abcx^3+\frac{2}{5}abc^{5/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)-\frac{4}{15}b^2c^{5/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)-\frac{1}{5}ib^2c^{5/2}\tan^{-1}\left(\right.$$

$$=\frac{8}{15}b^2c^2x+\frac{2}{15}abcx^3+\frac{2}{5}abc^{5/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)-\frac{4}{15}b^2c^{5/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)-\frac{1}{5}ib^2c^{5/2}\tan^{-1}\left(\right.$$

$$=\frac{8}{15}b^2c^2x+\frac{2}{15}abcx^3+\frac{2}{5}abc^{5/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)-\frac{4}{15}b^2c^{5/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)-\frac{1}{5}ib^2c^{5/2}\tan^{-1}\left(\right.$$

$$=\frac{8}{15}b^2c^2x+\frac{2}{15}abcx^3+\frac{2}{5}abc^{5/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)-\frac{4}{15}b^2c^{5/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)-\frac{1}{5}ib^2c^{5/2}\tan^{-1}\left(\right.$$

$$=\frac{8}{15}b^2c^2x+\frac{2}{15}abcx^3+\frac{2}{5}abc^{5/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)-\frac{4}{15}b^2c^{5/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)-\frac{1}{5}ib^2c^{5/2}\tan^{-1}\left(\right.$$

$$=\frac{8}{15}b^2c^2x+\frac{2}{15}abcx^3+\frac{2}{5}abc^{5/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)-\frac{4}{15}b^2c^{5/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)-\frac{1}{5}ib^2c^{5/2}\tan^{-1}\left(\right.$$

$$=\frac{8}{15}b^2c^2x+\frac{2}{15}abcx^3+\frac{2}{5}abc^{5/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)-\frac{4}{15}b^2c^{5/2}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)-\frac{1}{5}ib^2c^{5/2}\tan^{-1}\left(\right.$$

Mathematica [F] time = 5.64618, size = 0, normalized size = 0.

$$\int x^4 \left(a + b \tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2 dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^4*(a + b*ArcTanh[c/x^2])^2,x]
```

```
[Out] Integrate[x^4*(a + b*ArcTanh[c/x^2])^2, x]
```

Maple [F] time = 0.685, size = 0, normalized size = 0.

$$\int x^4 \left(a + b\mathrm{Artanh}\left(\frac{c}{x^2}\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arctanh(c/x^2))^2,x)
```

```
[Out] int(x^4*(a+b*arctanh(c/x^2))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c/x^2))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(b^2 x^4 \operatorname{artanh}\left(\frac{c}{x^2}\right)^2 + 2\, abx^4 \operatorname{artanh}\left(\frac{c}{x^2}\right) + a^2 x^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c/x^2))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^4*arctanh(c/x^2)^2 + 2*a*b*x^4*arctanh(c/x^2) + a^2*x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \left(a + b \operatorname{atanh}\left(\frac{c}{x^2}\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*atanh(c/x**2))**2,x)
```

```
[Out] Integral(x**4*(a + b*atanh(c/x**2))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \operatorname{artanh}\left(\frac{c}{x^2}\right) + a\right)^2 x^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c/x^2))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c/x^2) + a)^2*x^4, x)
```

3.177 $\int x^2 \left(a+b \tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2 \, dx$

Optimal. Leaf size=1172

result too large to display

```
[Out] (4*a*b*c*x)/3 - (2*a*b*c^(3/2)*ArcTan[x/Sqrt[c]])/3 + (4*b^2*c^(3/2)*ArcTan
[x/Sqrt[c]])/3 + (I/3)*b^2*c^(3/2)*ArcTan[x/Sqrt[c]]^2 - (4*b^2*c^(3/2)*Arc
Tanh[x/Sqrt[c]])/3 + (b^2*c^(3/2)*ArcTanh[x/Sqrt[c]]^2)/3 - (2*b^2*c^(3/2)*
ArcTan[x/Sqrt[c]]*Log[2 - (2*Sqrt[c])/(Sqrt[c] - I*x)])/3 - (2*b^2*c*x*Log[
1 - c/x^2])/3 + (b^2*c^(3/2)*ArcTan[x/Sqrt[c]]*Log[1 - c/x^2])/3 - (b*c^(3/
2)*ArcTanh[x/Sqrt[c]]*(2*a - b*Log[1 - c/x^2]))/3 + (x^3*(2*a - b*Log[1 - c
/x^2])^2)/12 + (2*b^2*c*x*Log[1 + c/x^2])/3 + (a*b*x^3*Log[1 + c/x^2])/3 -
(b^2*c^(3/2)*ArcTan[x/Sqrt[c]]*Log[1 + c/x^2])/3 - (b^2*c^(3/2)*ArcTanh[x/S
qrt[c]]*Log[1 + c/x^2])/3 - (b^2*x^3*Log[1 - c/x^2]*Log[1 + c/x^2])/6 + (b^
2*x^3*Log[1 + c/x^2]^2)/12 + (2*b^2*c^(3/2)*ArcTan[x/Sqrt[c]]*Log[(2*Sqrt[c
])/(Sqrt[c] - I*x)])/3 - (b^2*c^(3/2)*ArcTan[x/Sqrt[c]]*Log[((1 + I)*(Sqrt[
c] - x))/(Sqrt[c] - I*x)])/3 - (2*b^2*c^(3/2)*ArcTanh[x/Sqrt[c]]*Log[(2*Sqr
t[c])/(Sqrt[c] + x)])/3 + (b^2*c^(3/2)*ArcTanh[x/Sqrt[c]]*Log[(2*Sqrt[c]*(S
qrt[-c] - x))/((Sqrt[-c] - Sqrt[c])*(Sqrt[c] + x))])/3 + (b^2*c^(3/2)*ArcTa
nh[x/Sqrt[c]]*Log[(2*Sqrt[c]*(Sqrt[-c] + x))/((Sqrt[-c] + Sqrt[c])*(Sqrt[c]
 + x))])/3 - (b^2*c^(3/2)*ArcTan[x/Sqrt[c]]*Log[((1 - I)*(Sqrt[c] + x))/(Sq
rt[c] - I*x)])/3 + (2*b^2*c^(3/2)*ArcTanh[x/Sqrt[c]]*Log[2 - (2*Sqrt[c])/(S
qrt[c] + x)])/3 - (I/3)*b^2*c^(3/2)*PolyLog[2, 1 - (2*Sqrt[c])/(Sqrt[c] - I
*x)] + (I/3)*b^2*c^(3/2)*PolyLog[2, -1 + (2*Sqrt[c])/(Sqrt[c] - I*x)] + (I/
6)*b^2*c^(3/2)*PolyLog[2, 1 - ((1 + I)*(Sqrt[c] - x))/(Sqrt[c] - I*x)] + (b
^2*c^(3/2)*PolyLog[2, -(x/Sqrt[c])])/3 + (I/3)*b^2*c^(3/2)*PolyLog[2, ((-I)
*x)/Sqrt[c]] - (I/3)*b^2*c^(3/2)*PolyLog[2, (I*x)/Sqrt[c]] - (b^2*c^(3/2)*P
olyLog[2, x/Sqrt[c]])/3 + (b^2*c^(3/2)*PolyLog[2, 1 - (2*Sqrt[c])/(Sqrt[c]
+ x)])/3 - (b^2*c^(3/2)*PolyLog[2, -1 + (2*Sqrt[c])/(Sqrt[c] + x)])/3 - (b^
2*c^(3/2)*PolyLog[2, 1 - (2*Sqrt[c]*(Sqrt[-c] - x))/((Sqrt[-c] - Sqrt[c])*(
Sqrt[c] + x))])/6 - (b^2*c^(3/2)*PolyLog[2, 1 - (2*Sqrt[c]*(Sqrt[-c] + x))/
((Sqrt[-c] + Sqrt[c])*(Sqrt[c] + x))])/6 + (I/6)*b^2*c^(3/2)*PolyLog[2, 1 -
 ((1 - I)*(Sqrt[c] + x))/(Sqrt[c] - I*x)]
```

Rubi [A] time = 2.20796, antiderivative size = 1172, normalized size of antiderivative = 1., number of steps used = 79, number of rules used = 33, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 2.063, Rules used = {6099, 2457, 2471, 2448, 263, 207, 2470, 12, 260, 6688, 5988, 5932, 2447, 2455, 193, 321, 6742, 203, 30, 2557, 5992, 5912, 5920, 2402, 2315, 2476, 204, 4928, 4848, 2391, 4856, 4924, 4868}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcTanh[c/x^2])^2,x]
```

```
[Out] (4*a*b*c*x)/3 - (2*a*b*c^(3/2)*ArcTan[x/Sqrt[c]])/3 + (4*b^2*c^(3/2)*ArcTan
[x/Sqrt[c]])/3 + (I/3)*b^2*c^(3/2)*ArcTan[x/Sqrt[c]]^2 - (4*b^2*c^(3/2)*Arc
Tanh[x/Sqrt[c]])/3 + (b^2*c^(3/2)*ArcTanh[x/Sqrt[c]]^2)/3 - (2*b^2*c^(3/2)*
ArcTan[x/Sqrt[c]]*Log[2 - (2*Sqrt[c])/(Sqrt[c] - I*x)])/3 - (2*b^2*c*x*Log[
1 - c/x^2])/3 + (b^2*c^(3/2)*ArcTan[x/Sqrt[c]]*Log[1 - c/x^2])/3 - (b*c^(3/
2)*ArcTanh[x/Sqrt[c]]*(2*a - b*Log[1 - c/x^2]))/3 + (x^3*(2*a - b*Log[1 - c
/x^2])^2)/12 + (2*b^2*c*x*Log[1 + c/x^2])/3 + (a*b*x^3*Log[1 + c/x^2])/3 -
(b^2*c^(3/2)*ArcTan[x/Sqrt[c]]*Log[1 + c/x^2])/3 - (b^2*c^(3/2)*ArcTanh[x/S
qrt[c]]*Log[1 + c/x^2])/3 - (b^2*x^3*Log[1 - c/x^2]*Log[1 + c/x^2])/6 + (b^
```

```
2*x^3*Log[1 + c/x^2]^2)/12 + (2*b^2*c^(3/2)*ArcTan[x/Sqrt[c]]*Log[(2*Sqrt[c
])/(Sqrt[c] - I*x)])/3 - (b^2*c^(3/2)*ArcTan[x/Sqrt[c]]*Log[((1 + I)*(Sqrt[
c] - x))/(Sqrt[c] - I*x)])/3 - (2*b^2*c^(3/2)*ArcTanh[x/Sqrt[c]]*Log[(2*Sqr
t[c])/(Sqrt[c] + x)])/3 + (b^2*c^(3/2)*ArcTanh[x/Sqrt[c]]*Log[(2*Sqrt[c]*(S
qrt[-c] - x))/((Sqrt[-c] - Sqrt[c])*(Sqrt[c] + x))])/3 + (b^2*c^(3/2)*ArcTa
nh[x/Sqrt[c]]*Log[(2*Sqrt[c]*(Sqrt[-c] + x))/((Sqrt[-c] + Sqrt[c])*(Sqrt[c]
 + x))])/3 - (b^2*c^(3/2)*ArcTan[x/Sqrt[c]]*Log[((1 - I)*(Sqrt[c] + x))/(Sq
rt[c] - I*x)])/3 + (2*b^2*c^(3/2)*ArcTanh[x/Sqrt[c]]*Log[2 - (2*Sqrt[c])/(S
qrt[c] + x)])/3 - (I/3)*b^2*c^(3/2)*PolyLog[2, 1 - (2*Sqrt[c])/(Sqrt[c] - I
*x)] + (I/3)*b^2*c^(3/2)*PolyLog[2, -1 + (2*Sqrt[c])/(Sqrt[c] - I*x)] + (I/
6)*b^2*c^(3/2)*PolyLog[2, 1 - ((1 + I)*(Sqrt[c] - x))/(Sqrt[c] - I*x)] + (b
^2*c^(3/2)*PolyLog[2, -(x/Sqrt[c])])/3 + (I/3)*b^2*c^(3/2)*PolyLog[2, ((-I)
*x)/Sqrt[c]] - (I/3)*b^2*c^(3/2)*PolyLog[2, (I*x)/Sqrt[c]] - (b^2*c^(3/2)*P
olyLog[2, x/Sqrt[c]])/3 + (b^2*c^(3/2)*PolyLog[2, 1 - (2*Sqrt[c])/(Sqrt[c]
+ x)])/3 - (b^2*c^(3/2)*PolyLog[2, -1 + (2*Sqrt[c])/(Sqrt[c] + x)])/3 - (b^
2*c^(3/2)*PolyLog[2, 1 - (2*Sqrt[c]*(Sqrt[-c] - x))/((Sqrt[-c] - Sqrt[c])*(
Sqrt[c] + x))])/6 - (b^2*c^(3/2)*PolyLog[2, 1 - (2*Sqrt[c]*(Sqrt[-c] + x))/
((Sqrt[-c] + Sqrt[c])*(Sqrt[c] + x))])/6 + (I/6)*b^2*c^(3/2)*PolyLog[2, 1 -
 ((1 - I)*(Sqrt[c] + x))/(Sqrt[c] - I*x)]
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q
)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a +
 b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2471

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2557

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] :> With[{z = IntHide[u, x]}, Dist[Log[v
]*Log[w], z, x] + (-Int[SimplifyIntegrand[(z*Log[w]*D[v, x])/v, x], x] - In
t[SimplifyIntegrand[(z*Log[v]*D[w, x])/w, x], x]) /; InverseFunctionFreeQ[z
, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 5992

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rubi steps

$$\int x^2 \left(a + b \tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2 dx = \int \left(\frac{1}{4}x^2 \left(2a - b \log\left(1 - \frac{c}{x^2}\right)\right)^2 - \frac{1}{2}bx^2 \left(-2a + b \log\left(1 - \frac{c}{x^2}\right)\right) \log\left(1 + \frac{c}{x^2}\right) + \frac{1}{4}b^2x^2\right.$$

$$= \frac{1}{4}\int x^2 \left(2a - b \log\left(1 - \frac{c}{x^2}\right)\right)^2 dx - \frac{1}{2}b \int x^2 \left(-2a + b \log\left(1 - \frac{c}{x^2}\right)\right) \log\left(1 + \frac{c}{x^2}\right) dx$$

$$= \frac{1}{12}x^3 \left(2a - b \log\left(1 - \frac{c}{x^2}\right)\right)^2 + \frac{1}{12}b^2x^3 \log^2\left(1 + \frac{c}{x^2}\right) - \frac{1}{2}b \int \left(-2ax^2 \log\left(1 + \frac{c}{x^2}\right) + b\right.$$

$$= \frac{1}{12}x^3 \left(2a - b \log\left(1 - \frac{c}{x^2}\right)\right)^2 + \frac{1}{12}b^2x^3 \log^2\left(1 + \frac{c}{x^2}\right) + (ab) \int x^2 \log\left(1 + \frac{c}{x^2}\right) dx - \frac{1}{2}$$

$$= \frac{2}{3}abcx + \frac{1}{12}x^3 \left(2a - b \log\left(1 - \frac{c}{x^2}\right)\right)^2 + \frac{1}{3}abx^3 \log\left(1 + \frac{c}{x^2}\right) - \frac{1}{6}b^2x^3 \log\left(1 - \frac{c}{x^2}\right) \log$$

$$= \frac{2}{3}abcx - \frac{1}{3}b^2cx \log\left(1 - \frac{c}{x^2}\right) - \frac{1}{3}bc^{3/2} \tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)\left(2a - b \log\left(1 - \frac{c}{x^2}\right)\right) + \frac{1}{12}x^3 \left(2a\right.$$

$$= \frac{4}{3}abcx - \frac{1}{3}b^2cx \log\left(1 - \frac{c}{x^2}\right) - \frac{1}{3}bc^{3/2} \tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)\left(2a - b \log\left(1 - \frac{c}{x^2}\right)\right) + \frac{1}{12}x^3 \left(2a\right.$$

$$= \frac{4}{3}abcx - \frac{2}{3}abc^{3/2} \tan^{-1}\left(\frac{x}{\sqrt{c}}\right) + \frac{2}{3}b^2c^{3/2} \tan^{-1}\left(\frac{x}{\sqrt{c}}\right) - \frac{2}{3}b^2c^{3/2} \tanh^{-1}\left(\frac{x}{\sqrt{c}}\right) - \frac{1}{3}b^2cx$$

$$= \frac{4}{3}abcx - \frac{2}{3}abc^{3/2} \tan^{-1}\left(\frac{x}{\sqrt{c}}\right) + \frac{2}{3}b^2c^{3/2} \tan^{-1}\left(\frac{x}{\sqrt{c}}\right) + \frac{1}{3}ib^2c^{3/2} \tan^{-1}\left(\frac{x}{\sqrt{c}}\right)^2 - \frac{2}{3}b^2c^{3}$$

$$= \frac{4}{3}abcx - \frac{2}{3}abc^{3/2} \tan^{-1}\left(\frac{x}{\sqrt{c}}\right) + \frac{2}{3}b^2c^{3/2} \tan^{-1}\left(\frac{x}{\sqrt{c}}\right) + \frac{1}{3}ib^2c^{3/2} \tan^{-1}\left(\frac{x}{\sqrt{c}}\right)^2 - \frac{2}{3}b^2c^{3}$$

$$= \frac{4}{3}abcx - \frac{2}{3}abc^{3/2} \tan^{-1}\left(\frac{x}{\sqrt{c}}\right) + \frac{4}{3}b^2c^{3/2} \tan^{-1}\left(\frac{x}{\sqrt{c}}\right) + \frac{1}{3}ib^2c^{3/2} \tan^{-1}\left(\frac{x}{\sqrt{c}}\right)^2 - \frac{4}{3}b^2c^{3}$$

$$= \frac{4}{3}abcx - \frac{2}{3}abc^{3/2} \tan^{-1}\left(\frac{x}{\sqrt{c}}\right) + \frac{4}{3}b^2c^{3/2} \tan^{-1}\left(\frac{x}{\sqrt{c}}\right) + \frac{1}{3}ib^2c^{3/2} \tan^{-1}\left(\frac{x}{\sqrt{c}}\right)^2 - \frac{4}{3}b^2c^{3}$$

$$= \frac{4}{3}abcx - \frac{2}{3}abc^{3/2} \tan^{-1}\left(\frac{x}{\sqrt{c}}\right) + \frac{4}{3}b^2c^{3/2} \tan^{-1}\left(\frac{x}{\sqrt{c}}\right) + \frac{1}{3}ib^2c^{3/2} \tan^{-1}\left(\frac{x}{\sqrt{c}}\right)^2 - \frac{4}{3}b^2c^{3}$$

$$= \frac{4}{3}abcx - \frac{2}{3}abc^{3/2} \tan^{-1}\left(\frac{x}{\sqrt{c}}\right) + \frac{4}{3}b^2c^{3/2} \tan^{-1}\left(\frac{x}{\sqrt{c}}\right) + \frac{1}{3}ib^2c^{3/2} \tan^{-1}\left(\frac{x}{\sqrt{c}}\right)^2 - \frac{4}{3}b^2c^{3}$$

$$= \frac{4}{3}abcx - \frac{2}{3}abc^{3/2} \tan^{-1}\left(\frac{x}{\sqrt{c}}\right) + \frac{4}{3}b^2c^{3/2} \tan^{-1}\left(\frac{x}{\sqrt{c}}\right) + \frac{1}{3}ib^2c^{3/2} \tan^{-1}\left(\frac{x}{\sqrt{c}}\right)^2 - \frac{4}{3}b^2c^{3}$$

$$= \frac{4}{3}abcx - \frac{2}{3}abc^{3/2} \tan^{-1}\left(\frac{x}{\sqrt{c}}\right) + \frac{4}{3}b^2c^{3/2} \tan^{-1}\left(\frac{x}{\sqrt{c}}\right) + \frac{1}{3}ib^2c^{3/2} \tan^{-1}\left(\frac{x}{\sqrt{c}}\right)^2 - \frac{4}{3}b^2c^{3}$$

$$= \frac{4}{3}abcx - \frac{2}{3}abc^{3/2} \tan^{-1}\left(\frac{x}{\sqrt{c}}\right) + \frac{4}{3}b^2c^{3/2} \tan^{-1}\left(\frac{x}{\sqrt{c}}\right) + \frac{1}{3}ib^2c^{3/2} \tan^{-1}\left(\frac{x}{\sqrt{c}}\right)^2 - \frac{4}{3}b^2c^{3}$$

$$= \frac{4}{3}abcx - \frac{2}{3}abc^{3/2} \tan^{-1}\left(\frac{x}{\sqrt{c}}\right) + \frac{4}{3}b^2c^{3/2} \tan^{-1}\left(\frac{x}{\sqrt{c}}\right) + \frac{1}{3}ib^2c^{3/2} \tan^{-1}\left(\frac{x}{\sqrt{c}}\right)^2 - \frac{4}{3}b^2c^{3}$$

Mathematica [F] time = 2.97755, size = 0, normalized size = 0.

$$\int x^2 \left(a + b \tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2 dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2*(a + b*ArcTanh[c/x^2])^2,x]
```

```
[Out] Integrate[x^2*(a + b*ArcTanh[c/x^2])^2, x]
```

Maple [F] time = 0.641, size = 0, normalized size = 0.

$$\int x^2 \left(a + b\mathrm{Artanh}\left(\frac{c}{x^2}\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c/x^2))^2,x)
```

```
[Out] int(x^2*(a+b*arctanh(c/x^2))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c/x^2))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(b^2 x^2 \operatorname{artanh}\left(\frac{c}{x^2}\right)^2 + 2\,abx^2 \operatorname{artanh}\left(\frac{c}{x^2}\right) + a^2x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c/x^2))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^2*arctanh(c/x^2)^2 + 2*a*b*x^2*arctanh(c/x^2) + a^2*x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \left(a + b \operatorname{atanh}\left(\frac{c}{x^2}\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c/x**2))**2,x)
```

```
[Out] Integral(x**2*(a + b*atanh(c/x**2))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \operatorname{artanh}\left(\frac{c}{x^2}\right) + a\right)^2 x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c/x^2))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c/x^2) + a)^2*x^2, x)
```

3.178 $\int \left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2 dx$

Optimal. Leaf size=1549

result too large to display

```
[Out] a^2*x + 2*a*b*Sqrt[c]*ArcTan[x/Sqrt[c]] - 2*a*b*Sqrt[c]*ArcTanh[x/Sqrt[c]]
- a*b*x*Log[1 - c/x^2] - b^2*Sqrt[c]*ArcTan[x/Sqrt[c]]*Log[1 - c/x^2] + (b^
2*x*Log[1 - c/x^2]^2)/4 + a*b*x*Log[1 + c/x^2] - b^2*Sqrt[c]*ArcTanh[x/Sqrt
[c]]*Log[1 + c/x^2] - (b^2*x*Log[1 - c/x^2]*Log[1 + c/x^2])/2 + (b^2*x*Log[
1 + c/x^2]^2)/4 - (b^2*Sqrt[-c]*Log[1 + c/x^2]*Log[Sqrt[-c] - x])/2 + (b^2*
Sqrt[-c]*Log[Sqrt[-c] - x]^2)/4 - (b^2*Sqrt[c]*Log[1 - c/x^2]*Log[Sqrt[c] -
 x])/2 + (b^2*Sqrt[c]*Log[Sqrt[c] - x]^2)/4 - 2*b^2*Sqrt[c]*ArcTan[x/Sqrt[c
]]*Log[(2*Sqrt[c])/(Sqrt[c] - I*x)] + b^2*Sqrt[c]*ArcTan[x/Sqrt[c]]*Log[((1
 + I)*(Sqrt[c] - x))/(Sqrt[c] - I*x)] - b^2*Sqrt[-c]*Log[Sqrt[-c] - x]*Log[
x/Sqrt[-c]] - b^2*Sqrt[c]*Log[Sqrt[c] - x]*Log[x/Sqrt[c]] + (b^2*Sqrt[-c]*L
og[1 + c/x^2]*Log[Sqrt[-c] + x])/2 - (b^2*Sqrt[-c]*Log[(Sqrt[-c] - x)/(2*Sq
rt[-c])]*Log[Sqrt[-c] + x])/2 + b^2*Sqrt[-c]*Log[-(x/Sqrt[-c])]*Log[Sqrt[-c
] + x] - (b^2*Sqrt[-c]*Log[Sqrt[-c] + x]^2)/4 + (b^2*Sqrt[-c]*Log[Sqrt[-c]
- x]*Log[(Sqrt[-c] + x)/(2*Sqrt[-c])])/2 - 2*b^2*Sqrt[c]*ArcTanh[x/Sqrt[c]]
*Log[(2*Sqrt[c])/(Sqrt[c] + x)] + b^2*Sqrt[c]*ArcTanh[x/Sqrt[c]]*Log[(2*Sqr
t[c]*(Sqrt[-c] - x))/((Sqrt[-c] - Sqrt[c])*(Sqrt[c] + x))] + b^2*Sqrt[c]*Ar
cTanh[x/Sqrt[c]]*Log[(2*Sqrt[c]*(Sqrt[-c] + x))/((Sqrt[-c] + Sqrt[c])*(Sqrt
[c] + x))] + (b^2*Sqrt[c]*Log[1 - c/x^2]*Log[Sqrt[c] + x])/2 - (b^2*Sqrt[c]
*Log[(Sqrt[c] - x)/(2*Sqrt[c])]*Log[Sqrt[c] + x])/2 + b^2*Sqrt[c]*Log[-(x/S
qrt[c])]*Log[Sqrt[c] + x] - (b^2*Sqrt[c]*Log[Sqrt[c] + x]^2)/4 + (b^2*Sqrt[
c]*Log[Sqrt[c] - x]*Log[(Sqrt[c] + x)/(2*Sqrt[c])])/2 + b^2*Sqrt[c]*ArcTan[
x/Sqrt[c]]*Log[((1 - I)*(Sqrt[c] + x))/(Sqrt[c] - I*x)] + I*b^2*Sqrt[c]*Pol
yLog[2, 1 - (2*Sqrt[c])/(Sqrt[c] - I*x)] - (I/2)*b^2*Sqrt[c]*PolyLog[2, 1 -
 ((1 + I)*(Sqrt[c] - x))/(Sqrt[c] - I*x)] + b^2*Sqrt[c]*PolyLog[2, -(x/Sqrt
[c])] - I*b^2*Sqrt[c]*PolyLog[2, ((-I)*x)/Sqrt[c]] + I*b^2*Sqrt[c]*PolyLog[
2, (I*x)/Sqrt[c]] - b^2*Sqrt[c]*PolyLog[2, x/Sqrt[c]] - (b^2*Sqrt[c]*PolyLo
g[2, (Sqrt[c] + x)/(2*Sqrt[c])])/2 + (b^2*Sqrt[-c]*PolyLog[2, (1 - x/Sqrt[-
c])/2])/2 - b^2*Sqrt[-c]*PolyLog[2, 1 - x/Sqrt[-c]] + b^2*Sqrt[-c]*PolyLog[
2, 1 + x/Sqrt[-c]] - (b^2*Sqrt[-c]*PolyLog[2, (c - Sqrt[-c]*x)/(2*c)])/2 -
b^2*Sqrt[c]*PolyLog[2, 1 - x/Sqrt[c]] + (b^2*Sqrt[c]*PolyLog[2, 1/2 - x/(2*
Sqrt[c])])/2 + b^2*Sqrt[c]*PolyLog[2, 1 + x/Sqrt[c]] + b^2*Sqrt[c]*PolyLog[
2, 1 - (2*Sqrt[c])/(Sqrt[c] + x)] - (b^2*Sqrt[c]*PolyLog[2, 1 - (2*Sqrt[c]*
(Sqrt[-c] - x))/((Sqrt[-c] - Sqrt[c])*(Sqrt[c] + x))])/2 - (b^2*Sqrt[c]*Pol
yLog[2, 1 - (2*Sqrt[c]*(Sqrt[-c] + x))/((Sqrt[-c] + Sqrt[c])*(Sqrt[c] + x))
])/2 - (I/2)*b^2*Sqrt[c]*PolyLog[2, 1 - ((1 - I)*(Sqrt[c] + x))/(Sqrt[c] -
I*x)]
```

Rubi [A] time = 2.24891, antiderivative size = 1549, normalized size of antiderivative = 1., number of steps used = 99, number of rules used = 29, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 2.417, Rules used = {6093, 2448, 263, 207, 2450, 2476, 2462, 260, 2416, 2394, 2315, 2390, 2301, 2393, 2391, 203, 2556, 12, 2470, 6688, 5992, 5912, 5920, 2402, 2447, 204, 4928, 4848, 4856}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c/x^2])^2,x]
```

```
[Out] a^2*x + 2*a*b*Sqrt[c]*ArcTan[x/Sqrt[c]] - 2*a*b*Sqrt[c]*ArcTanh[x/Sqrt[c]]
- a*b*x*Log[1 - c/x^2] - b^2*Sqrt[c]*ArcTan[x/Sqrt[c]]*Log[1 - c/x^2] + (b^
```

```
2*x*Log[1 - c/x^2]^2)/4 + a*b*x*Log[1 + c/x^2] - b^2*Sqrt[c]*ArcTanh[x/Sqrt
[c]]*Log[1 + c/x^2] - (b^2*x*Log[1 - c/x^2]*Log[1 + c/x^2])/2 + (b^2*x*Log[
1 + c/x^2]^2)/4 - (b^2*Sqrt[-c]*Log[1 + c/x^2]*Log[Sqrt[-c] - x])/2 + (b^2*
Sqrt[-c]*Log[Sqrt[-c] - x]^2)/4 - (b^2*Sqrt[c]*Log[1 - c/x^2]*Log[Sqrt[c] -
 x])/2 + (b^2*Sqrt[c]*Log[Sqrt[c] - x]^2)/4 - 2*b^2*Sqrt[c]*ArcTan[x/Sqrt[c
]]*Log[(2*Sqrt[c])/(Sqrt[c] - I*x)] + b^2*Sqrt[c]*ArcTan[x/Sqrt[c]]*Log[((1
 + I)*(Sqrt[c] - x))/(Sqrt[c] - I*x)] - b^2*Sqrt[-c]*Log[Sqrt[-c] - x]*Log[
x/Sqrt[-c]] - b^2*Sqrt[c]*Log[Sqrt[c] - x]*Log[x/Sqrt[c]] + (b^2*Sqrt[-c]*L
og[1 + c/x^2]*Log[Sqrt[-c] + x])/2 - (b^2*Sqrt[-c]*Log[(Sqrt[-c] - x)/(2*Sq
rt[-c])]*Log[Sqrt[-c] + x])/2 + b^2*Sqrt[-c]*Log[-(x/Sqrt[-c])]*Log[Sqrt[-c
] + x] - (b^2*Sqrt[-c]*Log[Sqrt[-c] + x]^2)/4 + (b^2*Sqrt[-c]*Log[Sqrt[-c]
- x]*Log[(Sqrt[-c] + x)/(2*Sqrt[-c])])/2 - 2*b^2*Sqrt[c]*ArcTanh[x/Sqrt[c]]
*Log[(2*Sqrt[c])/(Sqrt[c] + x)] + b^2*Sqrt[c]*ArcTanh[x/Sqrt[c]]*Log[(2*Sqr
t[c]*(Sqrt[-c] - x))/((Sqrt[-c] - Sqrt[c])*(Sqrt[c] + x))] + b^2*Sqrt[c]*Ar
cTanh[x/Sqrt[c]]*Log[(2*Sqrt[c]*(Sqrt[-c] + x))/((Sqrt[-c] + Sqrt[c])*(Sqrt
[c] + x))] + (b^2*Sqrt[c]*Log[1 - c/x^2]*Log[Sqrt[c] + x])/2 - (b^2*Sqrt[c]
*Log[(Sqrt[c] - x)/(2*Sqrt[c])]*Log[Sqrt[c] + x])/2 + b^2*Sqrt[c]*Log[-(x/S
qrt[c])]*Log[Sqrt[c] + x] - (b^2*Sqrt[c]*Log[Sqrt[c] + x]^2)/4 + (b^2*Sqrt[
c]*Log[Sqrt[c] - x]*Log[(Sqrt[c] + x)/(2*Sqrt[c])])/2 + b^2*Sqrt[c]*ArcTan[
x/Sqrt[c]]*Log[((1 - I)*(Sqrt[c] + x))/(Sqrt[c] - I*x)] + I*b^2*Sqrt[c]*Pol
yLog[2, 1 - (2*Sqrt[c])/(Sqrt[c] - I*x)] - (I/2)*b^2*Sqrt[c]*PolyLog[2, 1 -
 ((1 + I)*(Sqrt[c] - x))/(Sqrt[c] - I*x)] + b^2*Sqrt[c]*PolyLog[2, -(x/Sqrt
[c])] - I*b^2*Sqrt[c]*PolyLog[2, ((-I)*x)/Sqrt[c]] + I*b^2*Sqrt[c]*PolyLog[
2, (I*x)/Sqrt[c]] - b^2*Sqrt[c]*PolyLog[2, x/Sqrt[c]] - (b^2*Sqrt[c]*PolyLo
g[2, (Sqrt[c] + x)/(2*Sqrt[c])])/2 + (b^2*Sqrt[-c]*PolyLog[2, (1 - x/Sqrt[-
c])/2])/2 - b^2*Sqrt[-c]*PolyLog[2, 1 - x/Sqrt[-c]] + b^2*Sqrt[-c]*PolyLog[
2, 1 + x/Sqrt[-c]] - (b^2*Sqrt[-c]*PolyLog[2, (c - Sqrt[-c]*x)/(2*c)])/2 -
b^2*Sqrt[c]*PolyLog[2, 1 - x/Sqrt[c]] + (b^2*Sqrt[c]*PolyLog[2, 1/2 - x/(2*
Sqrt[c])])/2 + b^2*Sqrt[c]*PolyLog[2, 1 + x/Sqrt[c]] + b^2*Sqrt[c]*PolyLog[
2, 1 - (2*Sqrt[c])/(Sqrt[c] + x)] - (b^2*Sqrt[c]*PolyLog[2, 1 - (2*Sqrt[c]*
(Sqrt[-c] - x))/((Sqrt[-c] - Sqrt[c])*(Sqrt[c] + x))])/2 - (b^2*Sqrt[c]*Pol
yLog[2, 1 - (2*Sqrt[c]*(Sqrt[-c] + x))/((Sqrt[-c] + Sqrt[c])*(Sqrt[c] + x))
])/2 - (I/2)*b^2*Sqrt[c]*PolyLog[2, 1 - ((1 - I)*(Sqrt[c] + x))/(Sqrt[c] -
I*x)]
```

Rule 6093

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.), x_Symbol] :> Int[Expand
Integrand[(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 - c*x^n])/2)^p, x], x] /; Fr
eeQ[{a, b, c, n}, x] && IGtQ[p, 0] && IntegerQ[n]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2450

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> Simp[x*(a + b*Log[c*(d + e*x^n)^p])^q, x] - Dist[b*e*n*p*q, Int[(x^n*
(a + b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c
, d, e, n, p}, x] && IGtQ[q, 0] && (EqQ[q, 1] || IntegerQ[n])
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2556

```
Int[Log[v_]*Log[w_], x_Symbol] :> Simp[x*Log[v]*Log[w], x] + (-Int[Simplify
Integrand[(x*Log[w]*D[v, x])/v, x], x] - Int[SimplifyIntegrand[(x*Log[v]*D[
w, x])/w, x], x]) /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w,
x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 5992

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
```

```
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rubi steps

$$\int \left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2 dx = \int \left(a^2-ab\log\left(1-\frac{c}{x^2}\right)+\frac{1}{4}b^2\log^2\left(1-\frac{c}{x^2}\right)+ab\log\left(1+\frac{c}{x^2}\right)-\frac{1}{2}b^2\log\left(1-\frac{c}{x^2}\right)\log\left(\right.\right.$$

$$= a^2x-(ab)\int \log\left(1-\frac{c}{x^2}\right)dx+(ab)\int \log\left(1+\frac{c}{x^2}\right)dx+\frac{1}{4}b^2\int \log^2\left(1-\frac{c}{x^2}\right)dx+\frac{1}{4}b$$

$$= a^2x-abx\log\left(1-\frac{c}{x^2}\right)+\frac{1}{4}b^2x\log^2\left(1-\frac{c}{x^2}\right)+abx\log\left(1+\frac{c}{x^2}\right)-\frac{1}{2}b^2x\log\left(1-\frac{c}{x^2}\right)\mathrm{lo}$$

$$= a^2x-abx\log\left(1-\frac{c}{x^2}\right)+\frac{1}{4}b^2x\log^2\left(1-\frac{c}{x^2}\right)+abx\log\left(1+\frac{c}{x^2}\right)-\frac{1}{2}b^2x\log\left(1-\frac{c}{x^2}\right)\mathrm{lo}$$

$$= a^2x+2ab\sqrt{c}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)-2ab\sqrt{c}\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)-abx\log\left(1-\frac{c}{x^2}\right)-b^2\sqrt{c}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)$$

$$= a^2x+2ab\sqrt{c}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)-2ab\sqrt{c}\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)-abx\log\left(1-\frac{c}{x^2}\right)-b^2\sqrt{c}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)$$

$$= a^2x+2ab\sqrt{c}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)-2ab\sqrt{c}\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)-abx\log\left(1-\frac{c}{x^2}\right)-b^2\sqrt{c}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)$$

$$= a^2x+2ab\sqrt{c}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)-2ab\sqrt{c}\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)-abx\log\left(1-\frac{c}{x^2}\right)-b^2\sqrt{c}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)$$

$$= a^2x+2ab\sqrt{c}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)-2ab\sqrt{c}\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)-abx\log\left(1-\frac{c}{x^2}\right)-b^2\sqrt{c}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)$$

$$= a^2x+2ab\sqrt{c}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)-2ab\sqrt{c}\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)-abx\log\left(1-\frac{c}{x^2}\right)-b^2\sqrt{c}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)$$

$$= a^2x+2ab\sqrt{c}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)-2ab\sqrt{c}\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)-abx\log\left(1-\frac{c}{x^2}\right)-b^2\sqrt{c}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)$$

$$= a^2x+2ab\sqrt{c}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)-2ab\sqrt{c}\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)-abx\log\left(1-\frac{c}{x^2}\right)-b^2\sqrt{c}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)$$

$$= a^2x+2ab\sqrt{c}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)-2ab\sqrt{c}\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)-abx\log\left(1-\frac{c}{x^2}\right)-b^2\sqrt{c}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)$$

$$= a^2x+2ab\sqrt{c}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)-2ab\sqrt{c}\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)-abx\log\left(1-\frac{c}{x^2}\right)-b^2\sqrt{c}\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)$$

Mathematica [A] time = 3.40588, size = 565, normalized size = 0.36

$$-\frac{1}{2}b^2x\sqrt{\frac{c}{x^2}}\left(-\mathrm{PolyLog}\left(2,\frac{1}{2}\left(1-\sqrt{\frac{c}{x^2}}\right)\right)+\mathrm{PolyLog}\left(2,\left(-\frac{1}{2}-\frac{i}{2}\right)\left(\sqrt{\frac{c}{x^2}}-1\right)\right)+\mathrm{PolyLog}\left(2,\left(-\frac{1}{2}+\frac{i}{2}\right)\left(\sqrt{\frac{c}{x^2}}-1\right)\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c/x^2])^2,x]
```

```
[Out] a^2*x - 2*a*b*Sqrt[c/x^2]*x*(ArcTan[Sqrt[c/x^2]] + ArcTanh[Sqrt[c/x^2]]) +
2*a*b*x*ArcTanh[c/x^2] - (b^2*Sqrt[c/x^2]*x*((-2*I)*ArcTan[Sqrt[c/x^2]]^2 +
 4*ArcTan[Sqrt[c/x^2]]*ArcTanh[c/x^2] - (2*ArcTanh[c/x^2]^2)/Sqrt[c/x^2] +
2*ArcTan[Sqrt[c/x^2]]*Log[1 + E^((4*I)*ArcTan[Sqrt[c/x^2]])] - 2*ArcTanh[c/
x^2]*Log[1 - Sqrt[c/x^2]] + Log[2]*Log[1 - Sqrt[c/x^2]] - Log[1 - Sqrt[c/x^
2]]^2/2 + Log[1 - Sqrt[c/x^2]]*Log[(1/2 + I/2)*(-I + Sqrt[c/x^2])] + 2*ArcT
anh[c/x^2]*Log[1 + Sqrt[c/x^2]] - Log[2]*Log[1 + Sqrt[c/x^2]] - Log[((1 + I
) - (1 - I)*Sqrt[c/x^2])/2]*Log[1 + Sqrt[c/x^2]] - Log[(-1/2 - I/2)*(I + Sq
rt[c/x^2])]*Log[1 + Sqrt[c/x^2]] + Log[1 + Sqrt[c/x^2]]^2/2 + Log[1 - Sqrt[
c/x^2]]*Log[((1 + I) + (1 - I)*Sqrt[c/x^2])/2] - (I/2)*PolyLog[2, -E^((4*I)
*ArcTan[Sqrt[c/x^2]])] - PolyLog[2, (1 - Sqrt[c/x^2])/2] + PolyLog[2, (-1/2
 - I/2)*(-1 + Sqrt[c/x^2])] + PolyLog[2, (-1/2 + I/2)*(-1 + Sqrt[c/x^2])] +
 PolyLog[2, (1 + Sqrt[c/x^2])/2] - PolyLog[2, (1/2 - I/2)*(1 + Sqrt[c/x^2])
] - PolyLog[2, (1/2 + I/2)*(1 + Sqrt[c/x^2])]))/2
```

Maple [F] time = 0.549, size = 0, normalized size = 0.

$$\int \left(a + b\text{Artanh}\left(\frac{c}{x^2}\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x^2))^2,x)
```

```
[Out] int((a+b*arctanh(c/x^2))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(b^2 \operatorname{artanh}\left(\frac{c}{x^2}\right)^2 + 2\,ab \operatorname{artanh}\left(\frac{c}{x^2}\right) + a^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*arctanh(c/x^2)^2 + 2*a*b*arctanh(c/x^2) + a^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b \operatorname{atanh}\left(\frac{c}{x^2}\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x**2))**2,x)
```

```
[Out] Integral((a + b*atanh(c/x**2))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\,\mathrm{artanh}\left(\frac{c}{x^2}\right)+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c/x^2) + a)^2, x)
```

3.179 $\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2}{x^2}\,dx$

Optimal. Leaf size=1117

result too large to display

```
[Out] (2*a*b)/x - (2*a*b*ArcCot[x/Sqrt[c]])/Sqrt[c] - (2*b^2*ArcCot[x/Sqrt[c]])/S
qrt[c] - (2*b^2*ArcCoth[x/Sqrt[c]])/Sqrt[c] - (2*b^2*ArcTan[x/Sqrt[c]])/Sqr
t[c] - (I*b^2*ArcTan[x/Sqrt[c]]^2)/Sqrt[c] + (2*b^2*ArcTanh[x/Sqrt[c]])/Sqr
t[c] - (b^2*ArcTanh[x/Sqrt[c]]^2)/Sqrt[c] + (2*b^2*ArcTan[x/Sqrt[c]]*Log[2
- (2*Sqrt[c])/(Sqrt[c] - I*x)])/Sqrt[c] - (b^2*Log[1 - c/x^2])/x + (b^2*Arc
Cot[x/Sqrt[c]]*Log[1 - c/x^2])/Sqrt[c] - (b*(2*a - b*Log[1 - c/x^2]))/x + (
b*ArcTanh[x/Sqrt[c]]*(2*a - b*Log[1 - c/x^2]))/Sqrt[c] - (2*a - b*Log[1 - c
/x^2])^2/(4*x) - (a*b*Log[1 + c/x^2])/x + (b^2*ArcCoth[x/Sqrt[c]]*Log[1 + c
/x^2])/Sqrt[c] + (b^2*ArcTan[x/Sqrt[c]]*Log[1 + c/x^2])/Sqrt[c] + (b^2*Log[
1 - c/x^2]*Log[1 + c/x^2])/(2*x) - (b^2*Log[1 + c/x^2]^2)/(4*x) + (2*b^2*Ar
cCot[x/Sqrt[c]]*Log[2/(1 - (I*Sqrt[c])/x)])/Sqrt[c] - (b^2*ArcCot[x/Sqrt[c]
]*Log[((1 + I)*(1 - Sqrt[c]/x))/(1 - (I*Sqrt[c])/x)])/Sqrt[c] + (2*b^2*ArcC
oth[x/Sqrt[c]]*Log[2/(1 + Sqrt[c]/x)])/Sqrt[c] - (b^2*ArcCoth[x/Sqrt[c]]*Lo
g[(-2*Sqrt[c]*(1 - Sqrt[-c]/x))/((Sqrt[-c] - Sqrt[c])*(1 + Sqrt[c]/x))])/Sq
rt[c] - (b^2*ArcCoth[x/Sqrt[c]]*Log[(2*Sqrt[c]*(1 + Sqrt[-c]/x))/((Sqrt[-c]
 + Sqrt[c])*(1 + Sqrt[c]/x))])/Sqrt[c] - (b^2*ArcCot[x/Sqrt[c]]*Log[((1 - I
)*(1 + Sqrt[c]/x))/(1 - (I*Sqrt[c])/x)])/Sqrt[c] - (2*b^2*ArcTanh[x/Sqrt[c]
]*Log[2 - (2*Sqrt[c])/(Sqrt[c] + x)])/Sqrt[c] - (I*b^2*PolyLog[2, 1 - 2/(1
- (I*Sqrt[c])/x)])/Sqrt[c] + ((I/2)*b^2*PolyLog[2, 1 - ((1 + I)*(1 - Sqrt[c
]/x))/(1 - (I*Sqrt[c])/x)])/Sqrt[c] - (b^2*PolyLog[2, 1 - 2/(1 + Sqrt[c]/x)
])/Sqrt[c] + (b^2*PolyLog[2, 1 + (2*Sqrt[c]*(1 - Sqrt[-c]/x))/((Sqrt[-c] -
Sqrt[c])*(1 + Sqrt[c]/x))])/(2*Sqrt[c]) + (b^2*PolyLog[2, 1 - (2*Sqrt[c]*(1
 + Sqrt[-c]/x))/((Sqrt[-c] + Sqrt[c])*(1 + Sqrt[c]/x))])/(2*Sqrt[c]) + ((I/
2)*b^2*PolyLog[2, 1 - ((1 - I)*(1 + Sqrt[c]/x))/(1 - (I*Sqrt[c])/x)])/Sqrt[
c] - (I*b^2*PolyLog[2, -1 + (2*Sqrt[c])/(Sqrt[c] - I*x)])/Sqrt[c] + (b^2*Po
lyLog[2, -1 + (2*Sqrt[c])/(Sqrt[c] + x)])/Sqrt[c]
```

Rubi [A] time = 2.15037, antiderivative size = 1117, normalized size of antiderivative = 1., number of steps used = 71, number of rules used = 29, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.812, Rules used = {6099, 2457, 2476, 2455, 263, 325, 207, 206, 2470, 12, 260, 6688, 5988, 5932, 2447, 6715, 2448, 321, 6742, 203, 2556, 5992, 5920, 2402, 2315, 4928, 4856, 4924, 4868}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c/x^2])^2/x^2,x]
```


```
[Out] (2*a*b)/x - (2*a*b*ArcCot[x/Sqrt[c]])/Sqrt[c] - (2*b^2*ArcCot[x/Sqrt[c]])/S
qrt[c] - (2*b^2*ArcCoth[x/Sqrt[c]])/Sqrt[c] - (2*b^2*ArcTan[x/Sqrt[c]])/Sqr
t[c] - (I*b^2*ArcTan[x/Sqrt[c]]^2)/Sqrt[c] + (2*b^2*ArcTanh[x/Sqrt[c]])/Sqr
t[c] - (b^2*ArcTanh[x/Sqrt[c]]^2)/Sqrt[c] + (2*b^2*ArcTan[x/Sqrt[c]]*Log[2
- (2*Sqrt[c])/(Sqrt[c] - I*x)])/Sqrt[c] - (b^2*Log[1 - c/x^2])/x + (b^2*Arc
Cot[x/Sqrt[c]]*Log[1 - c/x^2])/Sqrt[c] - (b*(2*a - b*Log[1 - c/x^2]))/x + (
b*ArcTanh[x/Sqrt[c]]*(2*a - b*Log[1 - c/x^2]))/Sqrt[c] - (2*a - b*Log[1 - c
/x^2])^2/(4*x) - (a*b*Log[1 + c/x^2])/x + (b^2*ArcCoth[x/Sqrt[c]]*Log[1 + c
/x^2])/Sqrt[c] + (b^2*ArcTan[x/Sqrt[c]]*Log[1 + c/x^2])/Sqrt[c] + (b^2*Log[
1 - c/x^2]*Log[1 + c/x^2])/(2*x) - (b^2*Log[1 + c/x^2]^2)/(4*x) + (2*b^2*Ar
```

```
cCot[x/Sqrt[c]]*Log[2/(1 - (I*Sqrt[c])/x)])/Sqrt[c] - (b^2*ArcCot[x/Sqrt[c]
]*Log[((1 + I)*(1 - Sqrt[c]/x))/(1 - (I*Sqrt[c])/x)])/Sqrt[c] + (2*b^2*ArcC
oth[x/Sqrt[c]]*Log[2/(1 + Sqrt[c]/x)])/Sqrt[c] - (b^2*ArcCoth[x/Sqrt[c]]*Lo
g[(-2*Sqrt[c]*(1 - Sqrt[-c]/x))/((Sqrt[-c] - Sqrt[c])*(1 + Sqrt[c]/x))])/Sq
rt[c] - (b^2*ArcCoth[x/Sqrt[c]]*Log[(2*Sqrt[c]*(1 + Sqrt[-c]/x))/((Sqrt[-c]
 + Sqrt[c])*(1 + Sqrt[c]/x))])/Sqrt[c] - (b^2*ArcCot[x/Sqrt[c]]*Log[((1 - I
)*(1 + Sqrt[c]/x))/(1 - (I*Sqrt[c])/x)])/Sqrt[c] - (2*b^2*ArcTanh[x/Sqrt[c]
]*Log[2 - (2*Sqrt[c])/(Sqrt[c] + x)])/Sqrt[c] - (I*b^2*PolyLog[2, 1 - 2/(1
- (I*Sqrt[c])/x)])/Sqrt[c] + ((I/2)*b^2*PolyLog[2, 1 - ((1 + I)*(1 - Sqrt[c
]/x))/(1 - (I*Sqrt[c])/x)])/Sqrt[c] - (b^2*PolyLog[2, 1 - 2/(1 + Sqrt[c]/x)
])/Sqrt[c] + (b^2*PolyLog[2, 1 + (2*Sqrt[c]*(1 - Sqrt[-c]/x))/((Sqrt[-c] -
Sqrt[c])*(1 + Sqrt[c]/x))])/(2*Sqrt[c]) + (b^2*PolyLog[2, 1 - (2*Sqrt[c]*(1
 + Sqrt[-c]/x))/((Sqrt[-c] + Sqrt[c])*(1 + Sqrt[c]/x))])/(2*Sqrt[c]) + ((I/
2)*b^2*PolyLog[2, 1 - ((1 - I)*(1 + Sqrt[c]/x))/(1 - (I*Sqrt[c])/x)])/Sqrt[
c] - (I*b^2*PolyLog[2, -1 + (2*Sqrt[c])/(Sqrt[c] - I*x)])/Sqrt[c] + (b^2*Po
lyLog[2, -1 + (2*Sqrt[c])/(Sqrt[c] + x)])/Sqrt[c]
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q
)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a +
 b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2556

```
Int[Log[v_]*Log[w_], x_Symbol] :> Simp[x*Log[v]*Log[w], x] + (-Int[Simplify
Integrand[(x*Log[w]*D[v, x])/v, x], x] - Int[SimplifyIntegrand[(x*Log[v]*D[
w, x])/w, x], x]) /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w,
x]
```

Rule 5992

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2}{x^2}\,dx = \int \left(\frac{\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{4x^2} - \frac{b\left(-2a+b\log\left(1-\frac{c}{x^2}\right)\right)\log\left(1+\frac{c}{x^2}\right)}{2x^2} + \frac{b^2\log^2\left(1+\frac{c}{x^2}\right)}{4x^2}\right) dx$$

$$= \frac{1}{4}\int \frac{\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{x^2}\,dx - \frac{1}{2}b\int \frac{\left(-2a+b\log\left(1-\frac{c}{x^2}\right)\right)\log\left(1+\frac{c}{x^2}\right)}{x^2}\,dx + \frac{1}{4}b^2\int \frac{l}{}$$

$$= -\frac{\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{4x} - \frac{b^2\log^2\left(1+\frac{c}{x^2}\right)}{4x} + \frac{1}{2}b\,\mathrm{Subst}\left(\int \left(-2a+b\log\left(1-cx^2\right)\right)\log\left(1\right.\right.$$

$$= -\frac{\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{4x} - \frac{b^2\log^2\left(1+\frac{c}{x^2}\right)}{4x} + \frac{1}{2}b\,\mathrm{Subst}\left(\int \left(-2a\log\left(1+cx^2\right)+b\log\left(1\right.\right.\right.$$

$$= -\frac{\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{4x} - \frac{b^2\log^2\left(1+\frac{c}{x^2}\right)}{4x} + b\int \frac{2a-b\log\left(1-\frac{c}{x^2}\right)}{x^2}\,dx + b\int \frac{2a-b\log}{c-}$$

$$= -\frac{b\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)}{x} + \frac{b\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)}{\sqrt{c}} - \frac{\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{4x}$$

$$= \frac{2ab}{x} - \frac{b\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)}{x} + \frac{b\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)}{\sqrt{c}} - \frac{\left(2a-b\log\left(1-\frac{c}{x}\right.\right.}{4x}$$

$$= \frac{2ab}{x} - \frac{4b^2}{x} - \frac{2ab\cot^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{b\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)}{x} + \frac{b\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)\left(2a-b\log\left(1-\right.\right.}{\sqrt{c}}$$

$$= \frac{2ab}{x} - \frac{4b^2}{x} - \frac{2ab\cot^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{2b^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{ib^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)^2}{\sqrt{c}} + \frac{2b^2\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} -$$

$$= \frac{2ab}{x} - \frac{4b^2}{x} - \frac{2ab\cot^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{2b^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{ib^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)^2}{\sqrt{c}} + \frac{2b^2\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} -$$

$$= \frac{2ab}{x} - \frac{2ab\cot^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{2b^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{ib^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)^2}{\sqrt{c}} + \frac{2b^2\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{b^2\tan}{}$$

$$= \frac{2ab}{x} - \frac{2ab\cot^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{2b^2\cot^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{2b^2\coth^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{2b^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{ib^2\tan}{}$$

$$= \frac{2ab}{x} - \frac{2ab\cot^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{2b^2\cot^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{2b^2\coth^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{2b^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{ib^2\tan}{}$$

$$= \frac{2ab}{x} - \frac{2ab\cot^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{2b^2\cot^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{2b^2\coth^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{2b^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{ib^2\tan}{}$$

$$= \frac{2ab}{x} - \frac{2ab\cot^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{2b^2\cot^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{2b^2\coth^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{2b^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{ib^2\tan}{}$$

$$= \frac{2ab}{x} - \frac{2ab\cot^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{2b^2\cot^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{2b^2\coth^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{2b^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{ib^2\tan}{}$$

Mathematica [A] time = 3.03704, size = 568, normalized size = 0.51

$$b^2\Big(-\text{PolyLog}\Big(2,\tfrac{1}{2}\Big(1-\sqrt{\tfrac{c}{x^2}}\Big)\Big)+\text{PolyLog}\Big(2,\Big(-\tfrac{1}{2}-\tfrac{i}{2}\Big)\Big(\sqrt{\tfrac{c}{x^2}}-1\Big)\Big)+\text{PolyLog}\Big(2,\Big(-\tfrac{1}{2}+\tfrac{i}{2}\Big)\Big(\sqrt{\tfrac{c}{x^2}}-1\Big)\Big)+\text{PolyLog}\Big(2,\tfrac{1}{2}\Big(\sqrt{\tfrac{c}{x^2}}+1\Big)\Big)-\text{PolyLog}\Big(2,\Big(\tfrac{1}{2}-\tfrac{i}{2}\Big)\Big(\sqrt{\tfrac{c}{x^2}}+1\Big)\Big)-\text{PolyLo}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c/x^2])^2/x^2,x]
```

```
[Out] (-2*a^2 - (4*a*b*(ArcTan[Sqrt[c/x^2]] - ArcTanh[Sqrt[c/x^2]]))/Sqrt[c/x^2]
- 4*a*b*ArcTanh[c/x^2] + (b^2*((2*I)*ArcTan[Sqrt[c/x^2]]^2 - 4*ArcTan[Sqrt[
c/x^2]]*ArcTanh[c/x^2] - 2*Sqrt[c/x^2]*ArcTanh[c/x^2]^2 - 2*ArcTan[Sqrt[c/x
^2]]*Log[1 + E^((4*I)*ArcTan[Sqrt[c/x^2]])] - 2*ArcTanh[c/x^2]*Log[1 - Sqrt
[c/x^2]] + Log[2]*Log[1 - Sqrt[c/x^2]] - Log[1 - Sqrt[c/x^2]]^2/2 + Log[1 -
 Sqrt[c/x^2]]*Log[(1/2 + I/2)*(-I + Sqrt[c/x^2])] + 2*ArcTanh[c/x^2]*Log[1
+ Sqrt[c/x^2]] - Log[2]*Log[1 + Sqrt[c/x^2]] - Log[((1 + I) - (1 - I)*Sqrt[
c/x^2])/2]*Log[1 + Sqrt[c/x^2]] - Log[(-1/2 - I/2)*(I + Sqrt[c/x^2])]*Log[1
 + Sqrt[c/x^2]] + Log[1 + Sqrt[c/x^2]]^2/2 + Log[1 - Sqrt[c/x^2]]*Log[((1 +
 I) + (1 - I)*Sqrt[c/x^2])/2] + (I/2)*PolyLog[2, -E^((4*I)*ArcTan[Sqrt[c/x^
2]])] - PolyLog[2, (1 - Sqrt[c/x^2])/2] + PolyLog[2, (-1/2 - I/2)*(-1 + Sqr
t[c/x^2])] + PolyLog[2, (-1/2 + I/2)*(-1 + Sqrt[c/x^2])] + PolyLog[2, (1 +
Sqrt[c/x^2])/2] - PolyLog[2, (1/2 - I/2)*(1 + Sqrt[c/x^2])] - PolyLog[2, (1
/2 + I/2)*(1 + Sqrt[c/x^2])]))/Sqrt[c/x^2])/(2*x)
```

Maple [F] time = 0.656, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left(a+b\text{Artanh}\left(\frac{c}{x^2}\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x^2))^2/x^2,x)
```

```
[Out] int((a+b*arctanh(c/x^2))^2/x^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))^2/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2\operatorname{artanh}\left(\frac{c}{x^2}\right)^2+2\,ab\operatorname{artanh}\left(\frac{c}{x^2}\right)+a^2}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))^2/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c/x^2)^2 + 2*a*b*arctanh(c/x^2) + a^2)/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+b\operatorname{atanh}\left(\frac{c}{x^2}\right)\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x**2))**2/x**2,x)
```

```
[Out] Integral((a + b*atanh(c/x**2))**2/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\operatorname{artanh}\left(\frac{c}{x^2}\right)+a\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c/x^2) + a)^2/x^2, x)
```

3.180 $\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2}{x^4}\,dx$

Optimal. Leaf size=1263

result too large to display

```
[Out] (2*a*b)/(9*x^3) - (2*a*b)/(3*c*x) - (2*a*b*ArcTan[x/Sqrt[c]])/(3*c^(3/2)) +
 (4*b^2*ArcTan[x/Sqrt[c]])/(3*c^(3/2)) + ((I/3)*b^2*ArcTan[x/Sqrt[c]]^2)/c^
(3/2) + (4*b^2*ArcTanh[x/Sqrt[c]])/(3*c^(3/2)) - (b^2*ArcTanh[x/Sqrt[c]]^2)
/(3*c^(3/2)) - (2*b^2*ArcTan[x/Sqrt[c]]*Log[2 - (2*Sqrt[c])/(Sqrt[c] - I*x)
])/(3*c^(3/2)) - (b^2*Log[1 - c/x^2])/(9*x^3) + (b^2*Log[1 - c/x^2])/(3*c*x
) + (b^2*ArcTan[x/Sqrt[c]]*Log[1 - c/x^2])/(3*c^(3/2)) - (b*(2*a - b*Log[1
- c/x^2]))/(9*x^3) - (b*(2*a - b*Log[1 - c/x^2]))/(3*c*x) + (b*ArcTanh[x/Sq
rt[c]]*(2*a - b*Log[1 - c/x^2]))/(3*c^(3/2)) - (2*a - b*Log[1 - c/x^2])^2/(
12*x^3) - (a*b*Log[1 + c/x^2])/(3*x^3) - (2*b^2*Log[1 + c/x^2])/(3*c*x) - (
b^2*ArcTan[x/Sqrt[c]]*Log[1 + c/x^2])/(3*c^(3/2)) + (b^2*ArcTanh[x/Sqrt[c]]
*Log[1 + c/x^2])/(3*c^(3/2)) + (b^2*Log[1 - c/x^2]*Log[1 + c/x^2])/(6*x^3)
- (b^2*Log[1 + c/x^2]^2)/(12*x^3) + (2*b^2*ArcTan[x/Sqrt[c]]*Log[(2*Sqrt[c]
)/(Sqrt[c] - I*x)])/(3*c^(3/2)) - (b^2*ArcTan[x/Sqrt[c]]*Log[((1 + I)*(Sqrt
[c] - x))/(Sqrt[c] - I*x)])/(3*c^(3/2)) + (2*b^2*ArcTanh[x/Sqrt[c]]*Log[(2*
Sqrt[c])/(Sqrt[c] + x)])/(3*c^(3/2)) - (b^2*ArcTanh[x/Sqrt[c]]*Log[(2*Sqrt[
c]*(Sqrt[-c] - x))/((Sqrt[-c] - Sqrt[c])*(Sqrt[c] + x))])/(3*c^(3/2)) - (b^
2*ArcTanh[x/Sqrt[c]]*Log[(2*Sqrt[c]*(Sqrt[-c] + x))/((Sqrt[-c] + Sqrt[c])*(
Sqrt[c] + x))])/(3*c^(3/2)) - (b^2*ArcTan[x/Sqrt[c]]*Log[((1 - I)*(Sqrt[c]
+ x))/(Sqrt[c] - I*x)])/(3*c^(3/2)) - (2*b^2*ArcTanh[x/Sqrt[c]]*Log[2 - (2*
Sqrt[c])/(Sqrt[c] + x)])/(3*c^(3/2)) - ((I/3)*b^2*PolyLog[2, 1 - (2*Sqrt[c]
)/(Sqrt[c] - I*x)])/c^(3/2) + ((I/3)*b^2*PolyLog[2, -1 + (2*Sqrt[c])/(Sqrt[
c] - I*x)])/c^(3/2) + ((I/6)*b^2*PolyLog[2, 1 - ((1 + I)*(Sqrt[c] - x))/(Sq
rt[c] - I*x)])/c^(3/2) - (b^2*PolyLog[2, -(x/Sqrt[c])])/(3*c^(3/2)) + ((I/3
)*b^2*PolyLog[2, ((-I)*x)/Sqrt[c]])/c^(3/2) - ((I/3)*b^2*PolyLog[2, (I*x)/S
qrt[c]])/c^(3/2) + (b^2*PolyLog[2, x/Sqrt[c]])/(3*c^(3/2)) - (b^2*PolyLog[2
, 1 - (2*Sqrt[c])/(Sqrt[c] + x)])/(3*c^(3/2)) + (b^2*PolyLog[2, -1 + (2*Sqr
t[c])/(Sqrt[c] + x)])/(3*c^(3/2)) + (b^2*PolyLog[2, 1 - (2*Sqrt[c]*(Sqrt[-c
] - x))/((Sqrt[-c] - Sqrt[c])*(Sqrt[c] + x))])/(6*c^(3/2)) + (b^2*PolyLog[2
, 1 - (2*Sqrt[c]*(Sqrt[-c] + x))/((Sqrt[-c] + Sqrt[c])*(Sqrt[c] + x))])/(6*
c^(3/2)) + ((I/6)*b^2*PolyLog[2, 1 - ((1 - I)*(Sqrt[c] + x))/(Sqrt[c] - I*x
)])/c^(3/2)
```

Rubi [A] time = 2.58476, antiderivative size = 1263, normalized size of antiderivative = 1., number of steps used = 104, number of rules used = 30, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.875, Rules used = {6099, 2457, 2476, 2455, 263, 325, 207, 206, 2470, 12, 260, 6688, 5988, 5932, 2447, 6742, 203, 30, 2557, 5992, 5912, 5920, 2402, 2315, 4928, 4848, 2391, 4856, 4924, 4868}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c/x^2])^2/x^4,x]
```

```
[Out] (2*a*b)/(9*x^3) - (2*a*b)/(3*c*x) - (2*a*b*ArcTan[x/Sqrt[c]])/(3*c^(3/2)) +
 (4*b^2*ArcTan[x/Sqrt[c]])/(3*c^(3/2)) + ((I/3)*b^2*ArcTan[x/Sqrt[c]]^2)/c^
(3/2) + (4*b^2*ArcTanh[x/Sqrt[c]])/(3*c^(3/2)) - (b^2*ArcTanh[x/Sqrt[c]]^2)
/(3*c^(3/2)) - (2*b^2*ArcTan[x/Sqrt[c]]*Log[2 - (2*Sqrt[c])/(Sqrt[c] - I*x)
])/(3*c^(3/2)) - (b^2*Log[1 - c/x^2])/(9*x^3) + (b^2*Log[1 - c/x^2])/(3*c*x
```

```
) + (b^2*ArcTan[x/Sqrt[c]]*Log[1 - c/x^2])/(3*c^(3/2)) - (b*(2*a - b*Log[1
- c/x^2]))/(9*x^3) - (b*(2*a - b*Log[1 - c/x^2]))/(3*c*x) + (b*ArcTanh[x/Sq
rt[c]]*(2*a - b*Log[1 - c/x^2]))/(3*c^(3/2)) - (2*a - b*Log[1 - c/x^2])^2/(
12*x^3) - (a*b*Log[1 + c/x^2])/(3*x^3) - (2*b^2*Log[1 + c/x^2])/(3*c*x) - (
b^2*ArcTan[x/Sqrt[c]]*Log[1 + c/x^2])/(3*c^(3/2)) + (b^2*ArcTanh[x/Sqrt[c]]
*Log[1 + c/x^2])/(3*c^(3/2)) + (b^2*Log[1 - c/x^2]*Log[1 + c/x^2])/(6*x^3)
- (b^2*Log[1 + c/x^2]^2)/(12*x^3) + (2*b^2*ArcTan[x/Sqrt[c]]*Log[(2*Sqrt[c]
)/(Sqrt[c] - I*x)])/(3*c^(3/2)) - (b^2*ArcTan[x/Sqrt[c]]*Log[((1 + I)*(Sqrt
[c] - x))/(Sqrt[c] - I*x)])/(3*c^(3/2)) + (2*b^2*ArcTanh[x/Sqrt[c]]*Log[(2*
Sqrt[c])/(Sqrt[c] + x)])/(3*c^(3/2)) - (b^2*ArcTanh[x/Sqrt[c]]*Log[(2*Sqrt[
c]*(Sqrt[-c] - x))/((Sqrt[-c] - Sqrt[c])*(Sqrt[c] + x))])/(3*c^(3/2)) - (b^
2*ArcTanh[x/Sqrt[c]]*Log[(2*Sqrt[c]*(Sqrt[-c] + x))/((Sqrt[-c] + Sqrt[c])*(
Sqrt[c] + x))])/(3*c^(3/2)) - (b^2*ArcTan[x/Sqrt[c]]*Log[((1 - I)*(Sqrt[c]
+ x))/(Sqrt[c] - I*x)])/(3*c^(3/2)) - (2*b^2*ArcTanh[x/Sqrt[c]]*Log[2 - (2*
Sqrt[c])/(Sqrt[c] + x)])/(3*c^(3/2)) - ((I/3)*b^2*PolyLog[2, 1 - (2*Sqrt[c]
)/(Sqrt[c] - I*x)])/c^(3/2) + ((I/3)*b^2*PolyLog[2, -1 + (2*Sqrt[c])/(Sqrt[
c] - I*x)])/c^(3/2) + ((I/6)*b^2*PolyLog[2, 1 - ((1 + I)*(Sqrt[c] - x))/(Sq
rt[c] - I*x)])/c^(3/2) - (b^2*PolyLog[2, -(x/Sqrt[c])])/(3*c^(3/2)) + ((I/3
)*b^2*PolyLog[2, ((-I)*x)/Sqrt[c]])/c^(3/2) - ((I/3)*b^2*PolyLog[2, (I*x)/S
qrt[c]])/c^(3/2) + (b^2*PolyLog[2, x/Sqrt[c]])/(3*c^(3/2)) - (b^2*PolyLog[2
, 1 - (2*Sqrt[c])/(Sqrt[c] + x)])/(3*c^(3/2)) + (b^2*PolyLog[2, -1 + (2*Sqr
t[c])/(Sqrt[c] + x)])/(3*c^(3/2)) + (b^2*PolyLog[2, 1 - (2*Sqrt[c]*(Sqrt[-c
] - x))/((Sqrt[-c] - Sqrt[c])*(Sqrt[c] + x))])/(6*c^(3/2)) + (b^2*PolyLog[2
, 1 - (2*Sqrt[c]*(Sqrt[-c] + x))/((Sqrt[-c] + Sqrt[c])*(Sqrt[c] + x))])/(6*
c^(3/2)) + ((I/6)*b^2*PolyLog[2, 1 - ((1 - I)*(Sqrt[c] + x))/(Sqrt[c] - I*x
)])/c^(3/2)
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q
)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a +
 b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
```

```
2*d^2 - e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2557

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] :> With[{z = IntHide[u, x]}, Dist[Log[v
]*Log[w], z, x] + (-Int[SimplifyIntegrand[(z*Log[w]*D[v, x])/v, x], x] - In
t[SimplifyIntegrand[(z*Log[v]*D[w, x])/w, x], x]) /; InverseFunctionFreeQ[z
, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 5992

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
```

```
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2}{x^4}\,dx = \int\left(\frac{\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{4x^4} - \frac{b\left(-2a+b\log\left(1-\frac{c}{x^2}\right)\right)\log\left(1+\frac{c}{x^2}\right)}{2x^4} + \frac{b^2\log^2\left(1+\frac{c}{x^2}\right)}{4x^4}\right) dx$$

$$= \frac{1}{4}\int \frac{\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{x^4}\,dx - \frac{1}{2}b\int \frac{\left(-2a+b\log\left(1-\frac{c}{x^2}\right)\right)\log\left(1+\frac{c}{x^2}\right)}{x^4}\,dx + \frac{1}{4}b^2\int$$

$$= -\frac{\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{12x^3} - \frac{b^2\log^2\left(1+\frac{c}{x^2}\right)}{12x^3} - \frac{1}{2}b\int\left(-\frac{2a\log\left(1+\frac{c}{x^2}\right)}{x^4} + \frac{b\log\left(1-\frac{c}{x^2}\right)\log}{x^4}\right.$$

$$= -\frac{\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{12x^3} - \frac{b^2\log^2\left(1+\frac{c}{x^2}\right)}{12x^3} + (ab)\int \frac{\log\left(1+\frac{c}{x^2}\right)}{x^4}\,dx - \frac{1}{2}b^2\int \frac{\log\left(1-\frac{c}{x^2}\right)}{}$$

$$= -\frac{\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{12x^3} - \frac{ab\log\left(1+\frac{c}{x^2}\right)}{3x^3} + \frac{b^2\log\left(1-\frac{c}{x^2}\right)\log\left(1+\frac{c}{x^2}\right)}{6x^3} - \frac{b^2\log^2\left(1+\frac{c}{x^2}\right)}{12x^3}$$

$$= -\frac{b\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)}{9x^3} - \frac{b\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)}{3cx} + \frac{b\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)}{3c^{3/2}}$$

$$= \frac{2ab}{9x^3} - \frac{b\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)}{9x^3} - \frac{b\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)}{3cx} + \frac{b\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)\left(2a-b\log\left(1-\right.\right.}{3c^{3/2}}$$

$$= \frac{2ab}{9x^3} - \frac{4b^2}{27x^3} - \frac{2ab}{3cx} - \frac{b\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)}{9x^3} - \frac{b\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)}{3cx} + \frac{b\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)}{}$$

$$= \frac{2ab}{9x^3} - \frac{4b^2}{27x^3} - \frac{2ab}{3cx} - \frac{2ab\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}} + \frac{2b^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}} + \frac{ib^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)^2}{3c^{3/2}} + \frac{2b^2\tanh^{-1}}{3c^{3/2}}$$

$$= \frac{2ab}{9x^3} - \frac{4b^2}{27x^3} - \frac{2ab}{3cx} - \frac{2ab\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}} + \frac{8b^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{9c^{3/2}} + \frac{ib^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)^2}{3c^{3/2}} + \frac{8b^2\tanh^{-1}}{9c^{3/2}}$$

$$= \frac{2ab}{9x^3} - \frac{2ab}{3cx} - \frac{2ab\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}} + \frac{8b^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{9c^{3/2}} + \frac{ib^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)^2}{3c^{3/2}} + \frac{8b^2\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)}{9c^{3/2}} -$$

$$= \frac{2ab}{9x^3} - \frac{2ab}{3cx} - \frac{2ab\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}} + \frac{14b^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{9c^{3/2}} + \frac{ib^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)^2}{3c^{3/2}} + \frac{14b^2\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)}{9c^{3/2}}$$

$$= \frac{2ab}{9x^3} - \frac{2ab}{3cx} - \frac{2ab\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}} + \frac{4b^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}} + \frac{ib^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)^2}{3c^{3/2}} + \frac{4b^2\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}} -$$

$$= \frac{2ab}{9x^3} - \frac{2ab}{3cx} - \frac{2ab\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}} + \frac{4b^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}} + \frac{ib^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)^2}{3c^{3/2}} + \frac{4b^2\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}} -$$

$$= \frac{2ab}{9x^3} - \frac{2ab}{3cx} - \frac{2ab\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}} + \frac{4b^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}} + \frac{ib^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)^2}{3c^{3/2}} + \frac{4b^2\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}} -$$

$$= \frac{2ab}{9x^3} - \frac{2ab}{3cx} - \frac{2ab\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}} + \frac{4b^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}} + \frac{ib^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)^2}{3c^{3/2}} + \frac{4b^2\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}} -$$

$$= \frac{2ab}{9x^3} - \frac{2ab}{3cx} - \frac{2ab\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}} + \frac{4b^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}} + \frac{ib^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)^2}{3c^{3/2}} + \frac{4b^2\tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}} -$$

Mathematica [F] time = 2.6617, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2}{x^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcTanh[c/x^2])^2/x^4,x]
```

```
[Out] Integrate[(a + b*ArcTanh[c/x^2])^2/x^4, x]
```

Maple [F] time = 0.603, size = 0, normalized size = 0.

$$\int \frac{1}{x^4}\left(a+b\mathrm{Artanh}\left(\frac{c}{x^2}\right)\right)^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x^2))^2/x^4,x)
```

```
[Out] int((a+b*arctanh(c/x^2))^2/x^4,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))^2/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^2\,\mathrm{artanh}\left(\frac{c}{x^2}\right)^2+2\,ab\,\mathrm{artanh}\left(\frac{c}{x^2}\right)+a^2}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))^2/x^4,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c/x^2)^2 + 2*a*b*arctanh(c/x^2) + a^2)/x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+b\,\mathrm{atanh}\left(\frac{c}{x^2}\right)\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x**2))**2/x**4,x)
```

```
[Out] Integral((a + b*atanh(c/x**2))**2/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \operatorname{artanh}\left(\frac{c}{x^2}\right) + a\right)^2}{x^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))^2/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c/x^2) + a)^2/x^4, x)
```

3.181 $\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2}{x^6}\,dx$

Optimal. Leaf size=1337

result too large to display

```
[Out] (2*a*b)/(25*x^5) - (2*a*b)/(15*c*x^3) + (2*a*b)/(5*c^2*x) - (8*b^2)/(15*c^2
*x) + (2*a*b*ArcTan[x/Sqrt[c]])/(5*c^(5/2)) - (4*b^2*ArcTan[x/Sqrt[c]])/(15
*c^(5/2)) - ((I/5)*b^2*ArcTan[x/Sqrt[c]]^2)/c^(5/2) + (4*b^2*ArcTanh[x/Sqrt
[c]])/(15*c^(5/2)) - (b^2*ArcTanh[x/Sqrt[c]]^2)/(5*c^(5/2)) + (2*b^2*ArcTan
[x/Sqrt[c]]*Log[2 - (2*Sqrt[c])/(Sqrt[c] - I*x)])/(5*c^(5/2)) - (b^2*Log[1
- c/x^2])/(25*x^5) + (b^2*Log[1 - c/x^2])/(15*c*x^3) - (b^2*Log[1 - c/x^2])
/(5*c^2*x) - (b^2*ArcTan[x/Sqrt[c]]*Log[1 - c/x^2])/(5*c^(5/2)) - (b*(2*a -
 b*Log[1 - c/x^2]))/(25*x^5) - (b*(2*a - b*Log[1 - c/x^2]))/(15*c*x^3) - (b
*(2*a - b*Log[1 - c/x^2]))/(5*c^2*x) + (b*ArcTanh[x/Sqrt[c]]*(2*a - b*Log[1
 - c/x^2]))/(5*c^(5/2)) - (2*a - b*Log[1 - c/x^2])^2/(20*x^5) - (a*b*Log[1
+ c/x^2])/(5*x^5) - (2*b^2*Log[1 + c/x^2])/(15*c*x^3) + (b^2*ArcTan[x/Sqrt[
c]]*Log[1 + c/x^2])/(5*c^(5/2)) + (b^2*ArcTanh[x/Sqrt[c]]*Log[1 + c/x^2])/(
5*c^(5/2)) + (b^2*Log[1 - c/x^2]*Log[1 + c/x^2])/(10*x^5) - (b^2*Log[1 + c/
x^2]^2)/(20*x^5) - (2*b^2*ArcTan[x/Sqrt[c]]*Log[(2*Sqrt[c])/(Sqrt[c] - I*x)
])/(5*c^(5/2)) + (b^2*ArcTan[x/Sqrt[c]]*Log[((1 + I)*(Sqrt[c] - x))/(Sqrt[c
] - I*x)])/(5*c^(5/2)) + (2*b^2*ArcTanh[x/Sqrt[c]]*Log[(2*Sqrt[c])/(Sqrt[c]
 + x)])/(5*c^(5/2)) - (b^2*ArcTanh[x/Sqrt[c]]*Log[(2*Sqrt[c]*(Sqrt[-c] - x)
)/((Sqrt[-c] - Sqrt[c])*(Sqrt[c] + x))])/(5*c^(5/2)) - (b^2*ArcTanh[x/Sqrt[
c]]*Log[(2*Sqrt[c]*(Sqrt[-c] + x))/((Sqrt[-c] + Sqrt[c])*(Sqrt[c] + x))])/(
5*c^(5/2)) + (b^2*ArcTan[x/Sqrt[c]]*Log[((1 - I)*(Sqrt[c] + x))/(Sqrt[c] -
I*x)])/(5*c^(5/2)) - (2*b^2*ArcTanh[x/Sqrt[c]]*Log[2 - (2*Sqrt[c])/(Sqrt[c]
 + x)])/(5*c^(5/2)) + ((I/5)*b^2*PolyLog[2, 1 - (2*Sqrt[c])/(Sqrt[c] - I*x)
])/c^(5/2) - ((I/5)*b^2*PolyLog[2, -1 + (2*Sqrt[c])/(Sqrt[c] - I*x)])/c^(5/
2) - ((I/10)*b^2*PolyLog[2, 1 - ((1 + I)*(Sqrt[c] - x))/(Sqrt[c] - I*x)])/c
^(5/2) - (b^2*PolyLog[2, -(x/Sqrt[c])])/(5*c^(5/2)) - ((I/5)*b^2*PolyLog[2,
 ((-I)*x)/Sqrt[c]])/c^(5/2) + ((I/5)*b^2*PolyLog[2, (I*x)/Sqrt[c]])/c^(5/2)
 + (b^2*PolyLog[2, x/Sqrt[c]])/(5*c^(5/2)) - (b^2*PolyLog[2, 1 - (2*Sqrt[c]
)/(Sqrt[c] + x)])/(5*c^(5/2)) + (b^2*PolyLog[2, -1 + (2*Sqrt[c])/(Sqrt[c] +
 x)])/(5*c^(5/2)) + (b^2*PolyLog[2, 1 - (2*Sqrt[c]*(Sqrt[-c] - x))/((Sqrt[-
c] - Sqrt[c])*(Sqrt[c] + x))])/(10*c^(5/2)) + (b^2*PolyLog[2, 1 - (2*Sqrt[c
]*(Sqrt[-c] + x))/((Sqrt[-c] + Sqrt[c])*(Sqrt[c] + x))])/(10*c^(5/2)) - ((I
/10)*b^2*PolyLog[2, 1 - ((1 - I)*(Sqrt[c] + x))/(Sqrt[c] - I*x)])/c^(5/2)
```

Rubi [A] time = 2.78992, antiderivative size = 1337, normalized size of antiderivative = 1., number of steps used = 129, number of rules used = 30, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.875, Rules used = {6099, 2457, 2476, 2455, 263, 325, 207, 206, 2470, 12, 260, 6688, 5988, 5932, 2447, 6742, 203, 30, 2557, 5992, 5912, 5920, 2402, 2315, 4928, 4848, 2391, 4856, 4924, 4868}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c/x^2])^2/x^6,x]
```

```
[Out] (2*a*b)/(25*x^5) - (2*a*b)/(15*c*x^3) + (2*a*b)/(5*c^2*x) - (8*b^2)/(15*c^2
*x) + (2*a*b*ArcTan[x/Sqrt[c]])/(5*c^(5/2)) - (4*b^2*ArcTan[x/Sqrt[c]])/(15
*c^(5/2)) - ((I/5)*b^2*ArcTan[x/Sqrt[c]]^2)/c^(5/2) + (4*b^2*ArcTanh[x/Sqrt
[c]])/(15*c^(5/2)) - (b^2*ArcTanh[x/Sqrt[c]]^2)/(5*c^(5/2)) + (2*b^2*ArcTan
```

```
[x/Sqrt[c]]*Log[2 - (2*Sqrt[c])/(Sqrt[c] - I*x)])/(5*c^(5/2)) - (b^2*Log[1
- c/x^2])/(25*x^5) + (b^2*Log[1 - c/x^2])/(15*c*x^3) - (b^2*Log[1 - c/x^2])
/(5*c^2*x) - (b^2*ArcTan[x/Sqrt[c]]*Log[1 - c/x^2])/(5*c^(5/2)) - (b*(2*a -
 b*Log[1 - c/x^2]))/(25*x^5) - (b*(2*a - b*Log[1 - c/x^2]))/(15*c*x^3) - (b
*(2*a - b*Log[1 - c/x^2]))/(5*c^2*x) + (b*ArcTanh[x/Sqrt[c]]*(2*a - b*Log[1
 - c/x^2]))/(5*c^(5/2)) - (2*a - b*Log[1 - c/x^2])^2/(20*x^5) - (a*b*Log[1
+ c/x^2])/(5*x^5) - (2*b^2*Log[1 + c/x^2])/(15*c*x^3) + (b^2*ArcTan[x/Sqrt[
c]]*Log[1 + c/x^2])/(5*c^(5/2)) + (b^2*ArcTanh[x/Sqrt[c]]*Log[1 + c/x^2])/(
5*c^(5/2)) + (b^2*Log[1 - c/x^2]*Log[1 + c/x^2])/(10*x^5) - (b^2*Log[1 + c/
x^2]^2)/(20*x^5) - (2*b^2*ArcTan[x/Sqrt[c]]*Log[(2*Sqrt[c])/(Sqrt[c] - I*x)
])/(5*c^(5/2)) + (b^2*ArcTan[x/Sqrt[c]]*Log[((1 + I)*(Sqrt[c] - x))/(Sqrt[c
] - I*x)])/(5*c^(5/2)) + (2*b^2*ArcTanh[x/Sqrt[c]]*Log[(2*Sqrt[c])/(Sqrt[c]
 + x)])/(5*c^(5/2)) - (b^2*ArcTanh[x/Sqrt[c]]*Log[(2*Sqrt[c]*(Sqrt[-c] - x)
)/((Sqrt[-c] - Sqrt[c])*(Sqrt[c] + x))])/(5*c^(5/2)) - (b^2*ArcTanh[x/Sqrt[
c]]*Log[(2*Sqrt[c]*(Sqrt[-c] + x))/((Sqrt[-c] + Sqrt[c])*(Sqrt[c] + x))])/(
5*c^(5/2)) + (b^2*ArcTan[x/Sqrt[c]]*Log[((1 - I)*(Sqrt[c] + x))/(Sqrt[c] -
I*x)])/(5*c^(5/2)) - (2*b^2*ArcTanh[x/Sqrt[c]]*Log[2 - (2*Sqrt[c])/(Sqrt[c]
 + x)])/(5*c^(5/2)) + ((I/5)*b^2*PolyLog[2, 1 - (2*Sqrt[c])/(Sqrt[c] - I*x)
])/c^(5/2) - ((I/5)*b^2*PolyLog[2, -1 + (2*Sqrt[c])/(Sqrt[c] - I*x)])/c^(5/
2) - ((I/10)*b^2*PolyLog[2, 1 - ((1 + I)*(Sqrt[c] - x))/(Sqrt[c] - I*x)])/c
^(5/2) - (b^2*PolyLog[2, -(x/Sqrt[c])])/(5*c^(5/2)) - ((I/5)*b^2*PolyLog[2,
 ((-I)*x)/Sqrt[c]])/c^(5/2) + ((I/5)*b^2*PolyLog[2, (I*x)/Sqrt[c]])/c^(5/2)
 + (b^2*PolyLog[2, x/Sqrt[c]])/(5*c^(5/2)) - (b^2*PolyLog[2, 1 - (2*Sqrt[c]
)/(Sqrt[c] + x)])/(5*c^(5/2)) + (b^2*PolyLog[2, -1 + (2*Sqrt[c])/(Sqrt[c] +
 x)])/(5*c^(5/2)) + (b^2*PolyLog[2, 1 - (2*Sqrt[c]*(Sqrt[-c] - x))/((Sqrt[-
c] - Sqrt[c])*(Sqrt[c] + x))])/(10*c^(5/2)) + (b^2*PolyLog[2, 1 - (2*Sqrt[c
]*(Sqrt[-c] + x))/((Sqrt[-c] + Sqrt[c])*(Sqrt[c] + x))])/(10*c^(5/2)) - ((I
/10)*b^2*PolyLog[2, 1 - ((1 - I)*(Sqrt[c] + x))/(Sqrt[c] - I*x)])/c^(5/2)
```

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
 c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q
)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a +
 b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
```

```
2*d^2 - e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2557

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] :> With[{z = IntHide[u, x]}, Dist[Log[v
]*Log[w], z, x] + (-Int[SimplifyIntegrand[(z*Log[w]*D[v, x])/v, x], x] - In
t[SimplifyIntegrand[(z*Log[v]*D[w, x])/w, x], x]) /; InverseFunctionFreeQ[z
, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 5992

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
```

c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2}{x^6}\,dx = \int \left(\frac{\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{4x^6} - \frac{b\left(-2a+b\log\left(1-\frac{c}{x^2}\right)\right)\log\left(1+\frac{c}{x^2}\right)}{2x^6} + \frac{b^2\log^2\left(1+\frac{c}{x^2}\right)}{4x^6}\right) dx$$

$$= \frac{1}{4}\int \frac{\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{x^6}\,dx - \frac{1}{2}b\int \frac{\left(-2a+b\log\left(1-\frac{c}{x^2}\right)\right)\log\left(1+\frac{c}{x^2}\right)}{x^6}\,dx + \frac{1}{4}b^2\int$$

$$= -\frac{\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{20x^5} - \frac{b^2\log^2\left(1+\frac{c}{x^2}\right)}{20x^5} - \frac{1}{2}b\int\left(-\frac{2a\log\left(1+\frac{c}{x^2}\right)}{x^6} + \frac{b\log\left(1-\frac{c}{x^2}\right)\log}{x^6}\right.$$

$$= -\frac{\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{20x^5} - \frac{b^2\log^2\left(1+\frac{c}{x^2}\right)}{20x^5} + (ab)\int \frac{\log\left(1+\frac{c}{x^2}\right)}{x^6}\,dx - \frac{1}{2}b^2\int \frac{\log\left(1-\right.}{}$$

$$= -\frac{\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{20x^5} - \frac{ab\log\left(1+\frac{c}{x^2}\right)}{5x^5} + \frac{b^2\log\left(1-\frac{c}{x^2}\right)\log\left(1+\frac{c}{x^2}\right)}{10x^5} - \frac{b^2\log^2\left(1+\right.}{20x^5}$$

$$= -\frac{b\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)}{25x^5} - \frac{b\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)}{15cx^3} - \frac{b\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)}{5c^2x} + \frac{b\tanh^{-1}\left(\right.}{}$$

$$= \frac{2ab}{25x^5} - \frac{b\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)}{25x^5} - \frac{b\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)}{15cx^3} - \frac{b\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)}{5c^2x} + \frac{b\,\mathrm{ta}}{}$$

$$= \frac{2ab}{25x^5} - \frac{4b^2}{125x^5} - \frac{2ab}{15cx^3} - \frac{4b^2}{5c^2x} - \frac{b\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)}{25x^5} - \frac{b\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)}{15cx^3} - \frac{b\left(2\right.}{}$$

$$= \frac{2ab}{25x^5} - \frac{4b^2}{125x^5} - \frac{2ab}{15cx^3} + \frac{2ab}{5c^2x} - \frac{16b^2}{15c^2x} - \frac{2b^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{5c^{5/2}} - \frac{ib^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)^2}{5c^{5/2}} + \frac{2b^2\,\mathrm{tanh}}{5c}$$

$$= \frac{2ab}{25x^5} - \frac{4b^2}{125x^5} - \frac{2ab}{15cx^3} + \frac{2ab}{5c^2x} - \frac{92b^2}{75c^2x} + \frac{2ab\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{5c^{5/2}} - \frac{8b^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{15c^{5/2}} - \frac{ib^2\tan^{-}}{5c^{5}}$$

$$= \frac{2ab}{25x^5} - \frac{2ab}{15cx^3} + \frac{2ab}{5c^2x} - \frac{32b^2}{75c^2x} + \frac{2ab\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{5c^{5/2}} - \frac{46b^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{75c^{5/2}} - \frac{ib^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)^2}{5c^{5/2}}$$

$$= \frac{2ab}{25x^5} - \frac{2ab}{15cx^3} + \frac{2ab}{5c^2x} - \frac{52b^2}{75c^2x} + \frac{2ab\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{5c^{5/2}} - \frac{16b^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{75c^{5/2}} - \frac{ib^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)^2}{5c^{5/2}}$$

$$= \frac{2ab}{25x^5} - \frac{2ab}{15cx^3} + \frac{2ab}{5c^2x} - \frac{8b^2}{15c^2x} + \frac{2ab\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{5c^{5/2}} - \frac{26b^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{75c^{5/2}} - \frac{ib^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)^2}{5c^{5/2}}$$

$$= \frac{2ab}{25x^5} - \frac{2ab}{15cx^3} + \frac{2ab}{5c^2x} - \frac{8b^2}{15c^2x} + \frac{2ab\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{5c^{5/2}} - \frac{4b^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{15c^{5/2}} - \frac{ib^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)^2}{5c^{5/2}} +$$

$$= \frac{2ab}{25x^5} - \frac{2ab}{15cx^3} + \frac{2ab}{5c^2x} - \frac{8b^2}{15c^2x} + \frac{2ab\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{5c^{5/2}} - \frac{4b^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{15c^{5/2}} - \frac{ib^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)^2}{5c^{5/2}} +$$

$$= \frac{2ab}{25x^5} - \frac{2ab}{15cx^3} + \frac{2ab}{5c^2x} - \frac{8b^2}{15c^2x} + \frac{2ab\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{5c^{5/2}} - \frac{4b^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{15c^{5/2}} - \frac{ib^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)^2}{5c^{5/2}} +$$

$$= \frac{2ab}{25x^5} - \frac{2ab}{15cx^3} + \frac{2ab}{5c^2x} - \frac{8b^2}{15c^2x} + \frac{2ab\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{5c^{5/2}} - \frac{4b^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{15c^{5/2}} - \frac{ib^2\tan^{-1}\left(\frac{x}{\sqrt{c}}\right)^2}{5c^{5/2}} +$$

Mathematica [F] time = 2.60961, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2}{x^6}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcTanh[c/x^2])^2/x^6,x]
```

```
[Out] Integrate[(a + b*ArcTanh[c/x^2])^2/x^6, x]
```

Maple [F] time = 0.665, size = 0, normalized size = 0.

$$\int \frac{1}{x^6}\left(a+b\,\mathrm{Artanh}\left(\frac{c}{x^2}\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x^2))^2/x^6,x)
```

```
[Out] int((a+b*arctanh(c/x^2))^2/x^6,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))^2/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^2\,\mathrm{artanh}\left(\frac{c}{x^2}\right)^2+2\,ab\,\mathrm{artanh}\left(\frac{c}{x^2}\right)+a^2}{x^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))^2/x^6,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c/x^2)^2 + 2*a*b*arctanh(c/x^2) + a^2)/x^6, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+b\,\mathrm{atanh}\left(\frac{c}{x^2}\right)\right)^2}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x**2))**2/x**6,x)
```

```
[Out] Integral((a + b*atanh(c/x**2))**2/x**6, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\operatorname{artanh}\left(\frac{c}{x^2}\right)+a\right)^2}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))^2/x^6,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c/x^2) + a)^2/x^6, x)
```

3.182 $\int (dx)^m \left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^3 \, dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left((dx)^m \left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^3,x\right)$$

```
[Out] Unintegrable[(d*x)^m*(a + b*ArcTanh[c/x^2])^3, x]
```

Rubi [A] time = 0.0248424, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (dx)^m \left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m*(a + b*ArcTanh[c/x^2])^3,x]
```

```
[Out] Defer[Int][(d*x)^m*(a + b*ArcTanh[c/x^2])^3, x]
```

Rubi steps

$$\int (dx)^m \left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^3 \, dx = \int (dx)^m \left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^3 \, dx$$

Mathematica [A] time = 2.43724, size = 0, normalized size = 0.

$$\int (dx)^m \left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m*(a + b*ArcTanh[c/x^2])^3,x]
```

```
[Out] Integrate[(d*x)^m*(a + b*ArcTanh[c/x^2])^3, x]
```

Maple [A] time = 0.664, size = 0, normalized size = 0.

$$\int (dx)^m \left(a+b\text{Artanh}\left(\frac{c}{x^2}\right)\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctanh(c/x^2))^3,x)
```

```
[Out] int((d*x)^m*(a+b*arctanh(c/x^2))^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c/x^2))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^3 \operatorname{artanh}\left(\frac{c}{x^2}\right)^3 + 3\,ab^2 \operatorname{artanh}\left(\frac{c}{x^2}\right)^2 + 3\,a^2 b \operatorname{artanh}\left(\frac{c}{x^2}\right) + a^3\right)(dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c/x^2))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctanh(c/x^2)^3 + 3*a*b^2*arctanh(c/x^2)^2 + 3*a^2*b*arctanh
(c/x^2) + a^3)*(d*x)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atanh(c/x**2))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b \operatorname{artanh}\left(\frac{c}{x^2}\right) + a\right)^3 (dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c/x^2))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c/x^2) + a)^3*(d*x)^m, x)
```

3.183 $\int (dx)^m \left(a + b \tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2 dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left((dx)^m \left(a + b \tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2, x\right)$$

```
[Out] Unintegrable[(d*x)^m*(a + b*ArcTanh[c/x^2])^2, x]
```

Rubi [A] time = 0.0247062, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (dx)^m \left(a + b \tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2 dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m*(a + b*ArcTanh[c/x^2])^2,x]

[Out] Defer[Int][(d*x)^m*(a + b*ArcTanh[c/x^2])^2, x]
```

Rubi steps

$$\int (dx)^m \left(a + b \tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2 dx = \int (dx)^m \left(a + b \tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2 dx$$

Mathematica [A] time = 1.62497, size = 0, normalized size = 0.

$$\int (dx)^m \left(a + b \tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2 dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m*(a + b*ArcTanh[c/x^2])^2,x]

[Out] Integrate[(d*x)^m*(a + b*ArcTanh[c/x^2])^2, x]
```

Maple [A] time = 0.38, size = 0, normalized size = 0.

$$\int (dx)^m \left(a + b\mathrm{Artanh}\left(\frac{c}{x^2}\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctanh(c/x^2))^2,x)

[Out] int((d*x)^m*(a+b*arctanh(c/x^2))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c/x^2))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2 \operatorname{artanh}\left(\frac{c}{x^2}\right)^2 + 2\,ab \operatorname{artanh}\left(\frac{c}{x^2}\right) + a^2\right)(dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c/x^2))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c/x^2)^2 + 2*a*b*arctanh(c/x^2) + a^2)*(d*x)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atanh(c/x**2))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b \operatorname{artanh}\left(\frac{c}{x^2}\right) + a\right)^2 (dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c/x^2))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c/x^2) + a)^2*(d*x)^m, x)
```

3.184 $\int (dx)^m \left(a + b\tanh^{-1}\left(\frac{c}{x^2}\right)\right) dx$

Optimal. Leaf size=75

$$\frac{(dx)^{m+1}\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)}{d(m+1)}-\frac{2bcd(dx)^{m-1}\text{Hypergeometric2F1}\left(1,\frac{1-m}{4},\frac{5-m}{4},\frac{c^2}{x^4}\right)}{1-m^2}$$

```
[Out] ((d*x)^(1 + m)*(a + b*ArcTanh[c/x^2]))/(d*(1 + m)) - (2*b*c*d*(d*x)^(-1 + m
)*Hypergeometric2F1[1, (1 - m)/4, (5 - m)/4, c^2/x^4])/(1 - m^2)
```

Rubi [A] time = 0.0557798, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6097, 16, 339, 364}

$$\frac{(dx)^{m+1}\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)}{d(m+1)}-\frac{2bcd(dx)^{m-1}\,{}_2F_1\left(1,\frac{1-m}{4};\frac{5-m}{4};\frac{c^2}{x^4}\right)}{1-m^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*ArcTanh[c/x^2]),x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*ArcTanh[c/x^2]))/(d*(1 + m)) - (2*b*c*d*(d*x)^(-1 + m
)*Hypergeometric2F1[1, (1 - m)/4, (5 - m)/4, c^2/x^4])/(1 - m^2)
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 339

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Dist[((c
*x)^(m + 1)*(1/x)^(m + 1))/c, Subst[Int[(a + b/x^n)^p/x^(m + 2), x], x, 1/x
], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] &&  !RationalQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int (dx)^m \left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right) dx = \frac{(dx)^{1+m}\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)}{d(1+m)} + \frac{(2bc)\int \frac{(dx)^{1+m}}{\left(1-\frac{c^2}{x^4}\right)x^3} dx}{d(1+m)}$$
$$= \frac{(dx)^{1+m}\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)}{d(1+m)} + \frac{(2bcd^2)\int \frac{(dx)^{-2+m}}{1-\frac{c^2}{x^4}} dx}{1+m}$$
$$= \frac{(dx)^{1+m}\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)}{d(1+m)} - \frac{\left(2bcd\left(\frac{1}{x}\right)^{-1+m}(dx)^{-1+m}\right)\text{Subst}\left(\int \frac{x^{-m}}{1-c^2x^4} dx, x, \frac{1}{x}\right)}{1+m}$$
$$= \frac{(dx)^{1+m}\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)}{d(1+m)} - \frac{2bcd(dx)^{-1+m}\,{}_2F_1\left(1,\frac{1-m}{4};\frac{5-m}{4};\frac{c^2}{x^4}\right)}{1-m^2}$$

Mathematica [A] time = 0.0673567, size = 68, normalized size = 0.91

$$\frac{(dx)^m\left(2bc\text{Hypergeometric2F1}\left(1,\frac{1}{4}-\frac{m}{4},\frac{5}{4}-\frac{m}{4},\frac{c^2}{x^4}\right)+(m-1)x^2\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)\right)}{(m-1)(m+1)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a + b*ArcTanh[c/x^2]),x]
```

```
[Out] ((d*x)^m*((-1 + m)*x^2*(a + b*ArcTanh[c/x^2]) + 2*b*c*Hypergeometric2F1[1,
1/4 - m/4, 5/4 - m/4, c^2/x^4]))/((-1 + m)*(1 + m)*x)
```

Maple [F] time = 0.294, size = 0, normalized size = 0.

$$\int (dx)^m \left(a+b\text{Artanh}\left(\frac{c}{x^2}\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctanh(c/x^2)),x)
```

```
[Out] int((d*x)^m*(a+b*arctanh(c/x^2)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c/x^2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\,\text{artanh}\left(\frac{c}{x^2}\right)+a\right)(dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c/x^2)),x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c/x^2) + a)*(d*x)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atanh(c/x**2)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \operatorname{artanh}\left(\frac{c}{x^2}\right) + a\right) (dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c/x^2)),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c/x^2) + a)*(d*x)^m, x)
```

3.185 $\int \frac{(dx)^m}{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{(dx)^m}{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)},x\right)$$

```
[Out] Unintegrable[(d*x)^m/(a + b*ArcTanh[c/x^2]), x]
```

Rubi [A] time = 0.027983, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(dx)^m}{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m/(a + b*ArcTanh[c/x^2]),x]
```

```
[Out] Defer[Int][(d*x)^m/(a + b*ArcTanh[c/x^2]), x]
```

Rubi steps

$$\int \frac{(dx)^m}{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}\,dx=\int \frac{(dx)^m}{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}\,dx$$

Mathematica [A] time = 0.402949, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{a+b\tanh^{-1}\left(\frac{c}{x^2}\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m/(a + b*ArcTanh[c/x^2]),x]
```

```
[Out] Integrate[(d*x)^m/(a + b*ArcTanh[c/x^2]), x]
```

Maple [A] time = 0.185, size = 0, normalized size = 0.

$$\int (dx)^m\left(a+b\text{Artanh}\left(\frac{c}{x^2}\right)\right)^{-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a+b*arctanh(c/x^2)),x)
```

```
[Out] int((d*x)^m/(a+b*arctanh(c/x^2)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{b\,\mathrm{artanh}\left(\frac{c}{x^2}\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c/x^2)),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/(b*arctanh(c/x^2) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(dx)^m}{b\,\mathrm{artanh}\left(\frac{c}{x^2}\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c/x^2)),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(b*arctanh(c/x^2) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*atanh(c/x**2)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{b\,\mathrm{artanh}\left(\frac{c}{x^2}\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c/x^2)),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b*arctanh(c/x^2) + a), x)
```

3.186 $\int \frac{(dx)^m}{\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{(dx)^m}{\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2},x\right)$$

```
[Out] Unintegrable[(d*x)^m/(a + b*ArcTanh[c/x^2])^2, x]
```

Rubi [A] time = 0.0263933, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(dx)^m}{\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m/(a + b*ArcTanh[c/x^2])^2,x]
```

```
[Out] Defer[Int][(d*x)^m/(a + b*ArcTanh[c/x^2])^2, x]
```

Rubi steps

$$\int \frac{(dx)^m}{\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2}\,dx=\int \frac{(dx)^m}{\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2}\,dx$$

Mathematica [A] time = 0.389505, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left(a+b\tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m/(a + b*ArcTanh[c/x^2])^2,x]
```

```
[Out] Integrate[(d*x)^m/(a + b*ArcTanh[c/x^2])^2, x]
```

Maple [A] time = 0.146, size = 0, normalized size = 0.

$$\int (dx)^m \left(a+b\text{Artanh}\left(\frac{c}{x^2}\right)\right)^{-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a+b*arctanh(c/x^2))^2,x)
```

```
[Out] int((d*x)^m/(a+b*arctanh(c/x^2))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{\left(d^m x^4 - c^2 d^m\right)x^m}{b^2 cx \log\left(x^2 + c\right) - b^2 cx \log\left(x^2 - c\right) + 2\,abcx} + \int -\frac{\left(d^m (m + 3)x^4 - c^2 d^m (m - 1)\right)x^m}{b^2 cx^2 \log\left(x^2 + c\right) - b^2 cx^2 \log\left(x^2 - c\right) + 2\,abcx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c/x^2))^2,x, algorithm="maxima")
```

```
[Out] (d^m*x^4 - c^2*d^m)*x^m/(b^2*c*x*log(x^2 + c) - b^2*c*x*log(x^2 - c) + 2*a*
b*c*x) + integrate(-(d^m*(m + 3)*x^4 - c^2*d^m*(m - 1))*x^m/(b^2*c*x^2*log(
x^2 + c) - b^2*c*x^2*log(x^2 - c) + 2*a*b*c*x^2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx)^m}{b^2 \operatorname{artanh}\left(\frac{c}{x^2}\right)^2 + 2\,ab \operatorname{artanh}\left(\frac{c}{x^2}\right) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c/x^2))^2,x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(b^2*arctanh(c/x^2)^2 + 2*a*b*arctanh(c/x^2) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*atanh(c/x**2))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left(b \operatorname{artanh}\left(\frac{c}{x^2}\right) + a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c/x^2))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b*arctanh(c/x^2) + a)^2, x)
```

3.187 $\int x^3 \left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right) dx$

Optimal. Leaf size=88

$$\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)+\frac{bx^{5/2}}{20c^3}+\frac{bx^{3/2}}{12c^5}+\frac{b\sqrt{x}}{4c^7}-\frac{b\tanh^{-1}\left(c\sqrt{x}\right)}{4c^8}+\frac{bx^{7/2}}{28c}$$

```
[Out] (b*Sqrt[x])/(4*c^7) + (b*x^(3/2))/(12*c^5) + (b*x^(5/2))/(20*c^3) + (b*x^(7
/2))/(28*c) - (b*ArcTanh[c*Sqrt[x]])/(4*c^8) + (x^4*(a + b*ArcTanh[c*Sqrt[x
]]))/4
```

Rubi [A] time = 0.0415064, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6097, 50, 63, 206}

$$\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)+\frac{bx^{5/2}}{20c^3}+\frac{bx^{3/2}}{12c^5}+\frac{b\sqrt{x}}{4c^7}-\frac{b\tanh^{-1}\left(c\sqrt{x}\right)}{4c^8}+\frac{bx^{7/2}}{28c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcTanh[c*Sqrt[x]]),x]
```

```
[Out] (b*Sqrt[x])/(4*c^7) + (b*x^(3/2))/(12*c^5) + (b*x^(5/2))/(20*c^3) + (b*x^(7
/2))/(28*c) - (b*ArcTanh[c*Sqrt[x]])/(4*c^8) + (x^4*(a + b*ArcTanh[c*Sqrt[x
]]))/4
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int x^3\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)dx &= \frac{1}{4}x^4\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)-\frac{1}{8}(bc)\int\frac{x^{7/2}}{1-c^2x}\,dx\\ &= \frac{bx^{7/2}}{28c}+\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)-\frac{b\int\frac{x^{5/2}}{1-c^2x}\,dx}{8c}\\ &= \frac{bx^{5/2}}{20c^3}+\frac{bx^{7/2}}{28c}+\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)-\frac{b\int\frac{x^{3/2}}{1-c^2x}\,dx}{8c^3}\\ &= \frac{bx^{3/2}}{12c^5}+\frac{bx^{5/2}}{20c^3}+\frac{bx^{7/2}}{28c}+\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)-\frac{b\int\frac{\sqrt{x}}{1-c^2x}\,dx}{8c^5}\\ &= \frac{b\sqrt{x}}{4c^7}+\frac{bx^{3/2}}{12c^5}+\frac{bx^{5/2}}{20c^3}+\frac{bx^{7/2}}{28c}+\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)-\frac{b\int\frac{1}{\sqrt{x}(1-c^2x)}\,dx}{8c^7}\\ &= \frac{b\sqrt{x}}{4c^7}+\frac{bx^{3/2}}{12c^5}+\frac{bx^{5/2}}{20c^3}+\frac{bx^{7/2}}{28c}+\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)-\frac{b\,\mathrm{Subst}\left(\int\frac{1}{1-c^2x^2}\,dx,x,\sqrt{x}\right)}{4c^7}\\ &= \frac{b\sqrt{x}}{4c^7}+\frac{bx^{3/2}}{12c^5}+\frac{bx^{5/2}}{20c^3}+\frac{bx^{7/2}}{28c}-\frac{b\tanh^{-1}\left(c\sqrt{x}\right)}{4c^8}+\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)\end{aligned}$$

Mathematica [A] time = 0.031173, size = 114, normalized size = 1.3

$$\frac{ax^4}{4}+\frac{bx^{5/2}}{20c^3}+\frac{bx^{3/2}}{12c^5}+\frac{b\sqrt{x}}{4c^7}+\frac{b\log\left(1-c\sqrt{x}\right)}{8c^8}-\frac{b\log\left(c\sqrt{x}+1\right)}{8c^8}+\frac{bx^{7/2}}{28c}+\frac{1}{4}bx^4\tanh^{-1}\left(c\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcTanh[c*Sqrt[x]]),x]
```

```
[Out] (b*Sqrt[x])/(4*c^7) + (b*x^(3/2))/(12*c^5) + (b*x^(5/2))/(20*c^3) + (b*x^(7
/2))/(28*c) + (a*x^4)/4 + (b*x^4*ArcTanh[c*Sqrt[x]])/4 + (b*Log[1 - c*Sqrt[
x]])/(8*c^8) - (b*Log[1 + c*Sqrt[x]])/(8*c^8)
```

Maple [A] time = 0.026, size = 84, normalized size = 1.

$$\frac{x^4a}{4}+\frac{bx^4}{4}\mathrm{Artanh}\left(c\sqrt{x}\right)+\frac{b}{28\,c}x^{\frac{7}{2}}+\frac{b}{20\,c^3}x^{\frac{5}{2}}+\frac{b}{12\,c^5}x^{\frac{3}{2}}+\frac{b}{4\,c^7}\sqrt{x}+\frac{b}{8\,c^8}\ln\left(c\sqrt{x}-1\right)-\frac{b}{8\,c^8}\ln\left(1+c\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c*x^(1/2))),x)
```

```
[Out] 1/4*x^4*a+1/4*b*x^4*arctanh(c*x^(1/2))+1/28*b*x^(7/2)/c+1/20*b*x^(5/2)/c^3+
1/12*b*x^(3/2)/c^5+1/4*b*x^(1/2)/c^7+1/8/c^8*b*ln(c*x^(1/2)-1)-1/8/c^8*b*ln
(1+c*x^(1/2))
```

Maxima [A] time = 1.023, size = 116, normalized size = 1.32

$$\frac{1}{4}\,ax^4+\frac{1}{840}\left(210\,x^4\,\mathrm{artanh}\left(c\sqrt{x}\right)+c\left(\frac{2\left(15\,c^6x^{\frac{7}{2}}+21\,c^4x^{\frac{5}{2}}+35\,c^2x^{\frac{3}{2}}+105\,\sqrt{x}\right)}{c^8}-\frac{105\,\log\left(c\sqrt{x}+1\right)}{c^9}+\frac{105\,\log\left(c\sqrt{x}-\right.}{c^9}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^(1/2))),x, algorithm="maxima")
```

```
[Out] 1/4*a*x^4 + 1/840*(210*x^4*arctanh(c*sqrt(x)) + c*(2*(15*c^6*x^(7/2) + 21*c
^4*x^(5/2) + 35*c^2*x^(3/2) + 105*sqrt(x))/c^8 - 105*log(c*sqrt(x) + 1)/c^9
 + 105*log(c*sqrt(x) - 1)/c^9))*b
```

Fricas [A] time = 1.78655, size = 213, normalized size = 2.42

$$\frac{210\,ac^{8}x^{4}+105\left(bc^{8}x^{4}-b\right)\log\left(-\frac{c^{2}x+2\,c\sqrt{x}+1}{c^{2}x-1}\right)+2\left(15\,bc^{7}x^{3}+21\,bc^{5}x^{2}+35\,bc^{3}x+105\,bc\right)\sqrt{x}}{840\,c^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^(1/2))),x, algorithm="fricas")
```

```
[Out] 1/840*(210*a*c^8*x^4 + 105*(b*c^8*x^4 - b)*log(-(c^2*x + 2*c*sqrt(x) + 1)/(
c^2*x - 1)) + 2*(15*b*c^7*x^3 + 21*b*c^5*x^2 + 35*b*c^3*x + 105*b*c)*sqrt(x
))/c^8
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{3}\left(a+b\operatorname{atanh}\left(c\sqrt{x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c*x**(1/2))),x)
```

```
[Out] Integral(x**3*(a + b*atanh(c*sqrt(x))), x)
```

Giac [A] time = 1.20434, size = 142, normalized size = 1.61

$$\frac{1}{4}\,ax^{4}+\frac{1}{840}\left(105\,x^{4}\log\left(-\frac{c\sqrt{x}+1}{c\sqrt{x}-1}\right)-c\left(\frac{105\,\log\left(\left|c\sqrt{x}+1\right|\right)}{c^{9}}-\frac{105\,\log\left(\left|c\sqrt{x}-1\right|\right)}{c^{9}}-\frac{2\left(15\,c^{12}x^{\frac{7}{2}}+21\,c^{10}x^{\frac{5}{2}}+35\,c^{8}x\right.}{c^{14}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^(1/2))),x, algorithm="giac")
```

```
[Out] 1/4*a*x^4 + 1/840*(105*x^4*log(-(c*sqrt(x) + 1)/(c*sqrt(x) - 1)) - c*(105*l
og(abs(c*sqrt(x) + 1))/c^9 - 105*log(abs(c*sqrt(x) - 1))/c^9 - 2*(15*c^12*x
^(7/2) + 21*c^10*x^(5/2) + 35*c^8*x^(3/2) + 105*c^6*sqrt(x))/c^14))*b
```

3.188 $\int x^2 \left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right) dx$

Optimal. Leaf size=75

$$\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)+\frac{bx^{3/2}}{9c^3}+\frac{b\sqrt{x}}{3c^5}-\frac{b\tanh^{-1}\left(c\sqrt{x}\right)}{3c^6}+\frac{bx^{5/2}}{15c}$$

```
[Out] (b*Sqrt[x])/(3*c^5) + (b*x^(3/2))/(9*c^3) + (b*x^(5/2))/(15*c) - (b*ArcTanh
[c*Sqrt[x]])/(3*c^6) + (x^3*(a + b*ArcTanh[c*Sqrt[x]]))/3
```

Rubi [A] time = 0.0345661, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6097, 50, 63, 206}

$$\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)+\frac{bx^{3/2}}{9c^3}+\frac{b\sqrt{x}}{3c^5}-\frac{b\tanh^{-1}\left(c\sqrt{x}\right)}{3c^6}+\frac{bx^{5/2}}{15c}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcTanh[c*Sqrt[x]]),x]
```


```
[Out] (b*Sqrt[x])/(3*c^5) + (b*x^(3/2))/(9*c^3) + (b*x^(5/2))/(15*c) - (b*ArcTanh
[c*Sqrt[x]])/(3*c^6) + (x^3*(a + b*ArcTanh[c*Sqrt[x]]))/3
```


Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int x^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)dx &= \frac{1}{3}x^3\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)-\frac{1}{6}(bc)\int\frac{x^{5/2}}{1-c^2x}dx\\
&= \frac{bx^{5/2}}{15c}+\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)-\frac{b\int\frac{x^{3/2}}{1-c^2x}dx}{6c}\\
&= \frac{bx^{3/2}}{9c^3}+\frac{bx^{5/2}}{15c}+\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)-\frac{b\int\frac{\sqrt{x}}{1-c^2x}dx}{6c^3}\\
&= \frac{b\sqrt{x}}{3c^5}+\frac{bx^{3/2}}{9c^3}+\frac{bx^{5/2}}{15c}+\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)-\frac{b\int\frac{1}{\sqrt{x}(1-c^2x)}dx}{6c^5}\\
&= \frac{b\sqrt{x}}{3c^5}+\frac{bx^{3/2}}{9c^3}+\frac{bx^{5/2}}{15c}+\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)-\frac{b\,\text{Subst}\left(\int\frac{1}{1-c^2x^2}dx,x,\sqrt{x}\right)}{3c^5}\\
&= \frac{b\sqrt{x}}{3c^5}+\frac{bx^{3/2}}{9c^3}+\frac{bx^{5/2}}{15c}-\frac{b\tanh^{-1}\left(c\sqrt{x}\right)}{3c^6}+\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)
\end{aligned}$$

Mathematica [A] time = 0.0236325, size = 101, normalized size = 1.35

$$\frac{ax^3}{3}+\frac{bx^{3/2}}{9c^3}+\frac{b\sqrt{x}}{3c^5}+\frac{b\log\left(1-c\sqrt{x}\right)}{6c^6}-\frac{b\log\left(c\sqrt{x}+1\right)}{6c^6}+\frac{bx^{5/2}}{15c}+\frac{1}{3}bx^3\tanh^{-1}\left(c\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcTanh[c*Sqrt[x]]),x]
```

```
[Out] (b*Sqrt[x])/(3*c^5) + (b*x^(3/2))/(9*c^3) + (b*x^(5/2))/(15*c) + (a*x^3)/3
+ (b*x^3*ArcTanh[c*Sqrt[x]])/3 + (b*Log[1 - c*Sqrt[x]])/(6*c^6) - (b*Log[1
+ c*Sqrt[x]])/(6*c^6)
```

Maple [A] time = 0.026, size = 75, normalized size = 1.

$$\frac{x^3a}{3}+\frac{bx^3}{3}\text{Artanh}\left(c\sqrt{x}\right)+\frac{b}{15\,c}x^{\frac{5}{2}}+\frac{b}{9\,c^3}x^{\frac{3}{2}}+\frac{b}{3\,c^5}\sqrt{x}+\frac{b}{6\,c^6}\ln\left(c\sqrt{x}-1\right)-\frac{b}{6\,c^6}\ln\left(1+c\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c*x^(1/2))),x)
```

```
[Out] 1/3*x^3*a+1/3*b*x^3*arctanh(c*x^(1/2))+1/15*b*x^(5/2)/c+1/9*b*x^(3/2)/c^3+1
/3*b*x^(1/2)/c^5+1/6/c^6*b*ln(c*x^(1/2)-1)-1/6/c^6*b*ln(1+c*x^(1/2))
```

Maxima [A] time = 0.958278, size = 105, normalized size = 1.4

$$\frac{1}{3}ax^3+\frac{1}{90}\left(30\,x^3\,\text{artanh}\left(c\sqrt{x}\right)+c\left(\frac{2\left(3\,c^4x^{\frac{5}{2}}+5\,c^2x^{\frac{3}{2}}+15\,\sqrt{x}\right)}{c^6}-\frac{15\,\log\left(c\sqrt{x}+1\right)}{c^7}+\frac{15\,\log\left(c\sqrt{x}-1\right)}{c^7}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^(1/2))),x, algorithm="maxima")
```

```
[Out] 1/3*a*x^3 + 1/90*(30*x^3*arctanh(c*sqrt(x)) + c*(2*(3*c^4*x^(5/2) + 5*c^2*x
^(3/2) + 15*sqrt(x))/c^6 - 15*log(c*sqrt(x) + 1)/c^7 + 15*log(c*sqrt(x) - 1
)/c^7))*b
```

Fricas [A] time = 1.77937, size = 185, normalized size = 2.47

$$\frac{30\,ac^6x^3 + 15\left(bc^6x^3 - b\right)\log\left(-\frac{c^2x+2\,c\sqrt{x}+1}{c^2x-1}\right) + 2\left(3\,bc^5x^2 + 5\,bc^3x + 15\,bc\right)\sqrt{x}}{90\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^(1/2))),x, algorithm="fricas")
```

```
[Out] 1/90*(30*a*c^6*x^3 + 15*(b*c^6*x^3 - b)*log(-(c^2*x + 2*c*sqrt(x) + 1)/(c^2
*x - 1)) + 2*(3*b*c^5*x^2 + 5*b*c^3*x + 15*b*c)*sqrt(x))/c^6
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\left(a + b\operatorname{atanh}\left(c\sqrt{x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c*x**(1/2))),x)
```

```
[Out] Integral(x**2*(a + b*atanh(c*sqrt(x))), x)
```

Giac [A] time = 1.17049, size = 131, normalized size = 1.75

$$\frac{1}{3}\,ax^3 + \frac{1}{90}\left(15\,x^3\log\left(-\frac{c\sqrt{x}+1}{c\sqrt{x}-1}\right) - c\left(\frac{15\,\log\left(\left|c\sqrt{x}+1\right|\right)}{c^7} - \frac{15\,\log\left(\left|c\sqrt{x}-1\right|\right)}{c^7} - \frac{2\left(3\,c^8x^{\frac{5}{2}} + 5\,c^6x^{\frac{3}{2}} + 15\,c^4\sqrt{x}\right)}{c^{10}}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^(1/2))),x, algorithm="giac")
```

```
[Out] 1/3*a*x^3 + 1/90*(15*x^3*log(-(c*sqrt(x) + 1)/(c*sqrt(x) - 1)) - c*(15*log(
abs(c*sqrt(x) + 1))/c^7 - 15*log(abs(c*sqrt(x) - 1))/c^7 - 2*(3*c^8*x^(5/2)
 + 5*c^6*x^(3/2) + 15*c^4*sqrt(x))/c^10))*b
```

3.189 $\int x\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)dx$

Optimal. Leaf size=62

$$\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)+\frac{b\sqrt{x}}{2c^3}-\frac{b\tanh^{-1}\left(c\sqrt{x}\right)}{2c^4}+\frac{bx^{3/2}}{6c}$$

```
[Out] (b*Sqrt[x])/(2*c^3) + (b*x^(3/2))/(6*c) - (b*ArcTanh[c*Sqrt[x]])/(2*c^4) +
(x^2*(a + b*ArcTanh[c*Sqrt[x]]))/2
```

Rubi [A] time = 0.0238849, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6097, 50, 63, 206}

$$\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)+\frac{b\sqrt{x}}{2c^3}-\frac{b\tanh^{-1}\left(c\sqrt{x}\right)}{2c^4}+\frac{bx^{3/2}}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcTanh[c*Sqrt[x]]),x]
```

```
[Out] (b*Sqrt[x])/(2*c^3) + (b*x^(3/2))/(6*c) - (b*ArcTanh[c*Sqrt[x]])/(2*c^4) +
(x^2*(a + b*ArcTanh[c*Sqrt[x]]))/2
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)dx=\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)-\frac{1}{4}(bc)\int\frac{x^{3/2}}{1-c^2x}\,dx$$
$$=\frac{bx^{3/2}}{6c}+\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)-\frac{b\int\frac{\sqrt{x}}{1-c^2x}\,dx}{4c}$$
$$=\frac{b\sqrt{x}}{2c^3}+\frac{bx^{3/2}}{6c}+\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)-\frac{b\int\frac{1}{\sqrt{x}(1-c^2x)}\,dx}{4c^3}$$
$$=\frac{b\sqrt{x}}{2c^3}+\frac{bx^{3/2}}{6c}+\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)-\frac{b\,\mathrm{Subst}\left(\int\frac{1}{1-c^2x^2}\,dx,x,\sqrt{x}\right)}{2c^3}$$
$$=\frac{b\sqrt{x}}{2c^3}+\frac{bx^{3/2}}{6c}-\frac{b\tanh^{-1}\left(c\sqrt{x}\right)}{2c^4}+\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)$$

Mathematica [A] time = 0.0208938, size = 88, normalized size = 1.42

$$\frac{ax^2}{2}+\frac{b\sqrt{x}}{2c^3}+\frac{b\log\left(1-c\sqrt{x}\right)}{4c^4}-\frac{b\log\left(c\sqrt{x}+1\right)}{4c^4}+\frac{bx^{3/2}}{6c}+\frac{1}{2}bx^2\tanh^{-1}\left(c\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcTanh[c*Sqrt[x]]),x]
```

```
[Out] (b*Sqrt[x])/(2*c^3) + (b*x^(3/2))/(6*c) + (a*x^2)/2 + (b*x^2*ArcTanh[c*Sqrt
[x]])/2 + (b*Log[1 - c*Sqrt[x]])/(4*c^4) - (b*Log[1 + c*Sqrt[x]])/(4*c^4)
```

Maple [A] time = 0.027, size = 66, normalized size = 1.1

$$\frac{ax^2}{2}+\frac{bx^2}{2}\mathrm{Artanh}\left(c\sqrt{x}\right)+\frac{b}{6c}x^{\frac{3}{2}}+\frac{b}{2c^3}\sqrt{x}+\frac{b}{4c^4}\ln\left(c\sqrt{x}-1\right)-\frac{b}{4c^4}\ln\left(1+c\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c*x^(1/2))),x)
```

```
[Out] 1/2*a*x^2+1/2*b*x^2*arctanh(c*x^(1/2))+1/6*b*x^(3/2)/c+1/2*b*x^(1/2)/c^3+1/
4/c^4*b*ln(c*x^(1/2)-1)-1/4/c^4*b*ln(1+c*x^(1/2))
```

Maxima [A] time = 1.00239, size = 93, normalized size = 1.5

$$\frac{1}{2}ax^2+\frac{1}{12}\left(6x^2\,\mathrm{artanh}\left(c\sqrt{x}\right)+c\left(\frac{2\left(c^2x^{\frac{3}{2}}+3\sqrt{x}\right)}{c^4}-\frac{3\log\left(c\sqrt{x}+1\right)}{c^5}+\frac{3\log\left(c\sqrt{x}-1\right)}{c^5}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^(1/2))),x, algorithm="maxima")
```

```
[Out] 1/2*a*x^2 + 1/12*(6*x^2*arctanh(c*sqrt(x)) + c*(2*(c^2*x^(3/2) + 3*sqrt(x))
/c^4 - 3*log(c*sqrt(x) + 1)/c^5 + 3*log(c*sqrt(x) - 1)/c^5))*b
```

Fricas [A] time = 1.76095, size = 159, normalized size = 2.56

$$\frac{6\,ac^4x^2 + 3\left(bc^4x^2 - b\right)\log\left(-\frac{c^2x+2\,c\sqrt{x}+1}{c^2x-1}\right) + 2\left(bc^3x + 3\,bc\right)\sqrt{x}}{12\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^(1/2))),x, algorithm="fricas")
```

```
[Out] 1/12*(6*a*c^4*x^2 + 3*(b*c^4*x^2 - b)*log(-(c^2*x + 2*c*sqrt(x) + 1)/(c^2*x
 - 1)) + 2*(b*c^3*x + 3*b*c)*sqrt(x))/c^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(a + b\,\mathrm{atanh}\left(c\sqrt{x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c*x**(1/2))),x)
```

```
[Out] Integral(x*(a + b*atanh(c*sqrt(x))), x)
```

Giac [A] time = 1.19396, size = 119, normalized size = 1.92

$$\frac{1}{2}\,ax^2 + \frac{1}{12}\left(3\,x^2\log\left(-\frac{c\sqrt{x}+1}{c\sqrt{x}-1}\right) - c\left(\frac{3\,\log\left(\left|c\sqrt{x}+1\right|\right)}{c^5} - \frac{3\,\log\left(\left|c\sqrt{x}-1\right|\right)}{c^5} - \frac{2\left(c^4x^{\frac{3}{2}} + 3\,c^2\sqrt{x}\right)}{c^6}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^(1/2))),x, algorithm="giac")
```

```
[Out] 1/2*a*x^2 + 1/12*(3*x^2*log(-(c*sqrt(x) + 1)/(c*sqrt(x) - 1)) - c*(3*log(ab
s(c*sqrt(x) + 1))/c^5 - 3*log(abs(c*sqrt(x) - 1))/c^5 - 2*(c^4*x^(3/2) + 3*
c^2*sqrt(x))/c^6))*b
```

3.190 $\int \left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right) dx$

Optimal. Leaf size=39

$$ax-\frac{b\tanh^{-1}\left(c\sqrt{x}\right)}{c^2}+\frac{b\sqrt{x}}{c}+bx\tanh^{-1}\left(c\sqrt{x}\right)$$

```
[Out] (b*Sqrt[x])/c + a*x - (b*ArcTanh[c*Sqrt[x]])/c^2 + b*x*ArcTanh[c*Sqrt[x]]
```

Rubi [A] time = 0.0204042, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6091, 50, 63, 206}

$$ax-\frac{b\tanh^{-1}\left(c\sqrt{x}\right)}{c^2}+\frac{b\sqrt{x}}{c}+bx\tanh^{-1}\left(c\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcTanh[c*Sqrt[x]],x]
```

```
[Out] (b*Sqrt[x])/c + a*x - (b*ArcTanh[c*Sqrt[x]])/c^2 + b*x*ArcTanh[c*Sqrt[x]]
```

Rule 6091

```
Int[ArcTanh[(c_.)*(x_)^(n_)], x_Symbol] :> Simp[x*ArcTanh[c*x^n], x] - Dist
[c*n, Int[x^n/(1 - c^2*x^(2*n)), x], x] /; FreeQ[{c, n}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)dx &= ax+b\int \tanh^{-1}\left(c\sqrt{x}\right)dx\\ &= ax+bx\tanh^{-1}\left(c\sqrt{x}\right)-\frac{1}{2}(bc)\int \frac{\sqrt{x}}{1-c^2x}\,dx\\ &= \frac{b\sqrt{x}}{c}+ax+bx\tanh^{-1}\left(c\sqrt{x}\right)-\frac{b\int \frac{1}{\sqrt{x}(1-c^2x)}\,dx}{2c}\\ &= \frac{b\sqrt{x}}{c}+ax+bx\tanh^{-1}\left(c\sqrt{x}\right)-\frac{b\,\text{Subst}\left(\int \frac{1}{1-c^2x^2}\,dx,x,\sqrt{x}\right)}{c}\\ &= \frac{b\sqrt{x}}{c}+ax-\frac{b\tanh^{-1}\left(c\sqrt{x}\right)}{c^2}+bx\tanh^{-1}\left(c\sqrt{x}\right)\end{aligned}$$

Mathematica [A] time = 0.0244588, size = 42, normalized size = 1.08

$$ax-bc\left(\frac{\tanh^{-1}\left(c\sqrt{x}\right)}{c^3}-\frac{\sqrt{x}}{c^2}\right)+bx\tanh^{-1}\left(c\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcTanh[c*Sqrt[x]],x]
```

```
[Out] a*x + b*x*ArcTanh[c*Sqrt[x]] - b*c*(-(Sqrt[x]/c^2) + ArcTanh[c*Sqrt[x]]/c^3
)
```

Maple [A] time = 0.024, size = 50, normalized size = 1.3

$$ax+bx\text{Artanh}\left(c\sqrt{x}\right)+\frac{b}{c}\sqrt{x}+\frac{b}{2c^2}\ln\left(c\sqrt{x}-1\right)-\frac{b}{2c^2}\ln\left(1+c\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arctanh(c*x^(1/2)),x)
```

```
[Out] a*x+b*x*arctanh(c*x^(1/2))+b*x^(1/2)/c+1/2*b/c^2*ln(c*x^(1/2)-1)-1/2*b/c^2*
ln(1+c*x^(1/2))
```

Maxima [A] time = 0.971143, size = 72, normalized size = 1.85

$$\frac{1}{2}\left(c\left(\frac{2\sqrt{x}}{c^2}-\frac{\log\left(c\sqrt{x}+1\right)}{c^3}+\frac{\log\left(c\sqrt{x}-1\right)}{c^3}\right)+2\,x\,\text{artanh}\left(c\sqrt{x}\right)\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c*x^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/2*(c*(2*sqrt(x)/c^2 - log(c*sqrt(x) + 1)/c^3 + log(c*sqrt(x) - 1)/c^3) +
2*x*arctanh(c*sqrt(x)))*b + a*x
```

Fricas [A] time = 1.71946, size = 131, normalized size = 3.36

$$\frac{2\,ac^2x + 2\,bc\sqrt{x} + \left(bc^2x - b\right)\log\left(-\frac{c^2x+2\,c\sqrt{x}+1}{c^2x-1}\right)}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c*x^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*c^2*x + 2*b*c*sqrt(x) + (b*c^2*x - b)*log(-(c^2*x + 2*c*sqrt(x) +
1)/(c^2*x - 1)))/c^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\operatorname{atanh}\left(c\sqrt{x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*atanh(c*x**(1/2)),x)
```

```
[Out] Integral(a + b*atanh(c*sqrt(x)), x)
```

Giac [A] time = 1.22042, size = 90, normalized size = 2.31

$$\frac{1}{2}\left(c\left(\frac{2\,\sqrt{x}}{c^2} - \frac{\log\left(\left|c\sqrt{x}+1\right|\right)}{c^3} + \frac{\log\left(\left|c\sqrt{x}-1\right|\right)}{c^3}\right) + x\log\left(-\frac{c\sqrt{x}+1}{c\sqrt{x}-1}\right)\right)b + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c*x^(1/2)),x, algorithm="giac")
```

```
[Out] 1/2*(c*(2*sqrt(x)/c^2 - log(abs(c*sqrt(x) + 1))/c^3 + log(abs(c*sqrt(x) - 1
))/c^3) + x*log(-(c*sqrt(x) + 1)/(c*sqrt(x) - 1)))*b + a*x
```

3.191 $\int \frac{a+b\tanh^{-1}(c\sqrt{x})}{x}\,dx$

Optimal. Leaf size=29

$$-b\text{PolyLog}\left(2,-c\sqrt{x}\right)+b\text{PolyLog}\left(2,c\sqrt{x}\right)+a\log(x)$$

```
[Out] a*Log[x] - b*PolyLog[2, -(c*Sqrt[x])] + b*PolyLog[2, c*Sqrt[x]]
```

Rubi [A] time = 0.0315208, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6095, 5912}

$$-b\text{PolyLog}\left(2,-c\sqrt{x}\right)+b\text{PolyLog}\left(2,c\sqrt{x}\right)+a\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*Sqrt[x]])/x,x]
```

```
[Out] a*Log[x] - b*PolyLog[2, -(c*Sqrt[x])] + b*PolyLog[2, c*Sqrt[x]]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[
1/n, Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
 n}, x] && IGtQ[p, 0]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{x}\,dx &= 2\,\text{Subst}\left(\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx,x,\sqrt{x}\right)\\ &= a\log(x)-b\text{Li}_2\left(-c\sqrt{x}\right)+b\text{Li}_2\left(c\sqrt{x}\right)\end{aligned}$$

Mathematica [A] time = 0.0114324, size = 29, normalized size = 1.

$$-b\text{PolyLog}\left(2,-c\sqrt{x}\right)+b\text{PolyLog}\left(2,c\sqrt{x}\right)+a\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*Sqrt[x]])/x,x]
```

```
[Out] a*Log[x] - b*PolyLog[2, -(c*Sqrt[x])] + b*PolyLog[2, c*Sqrt[x]]
```

Maple [B] time = 0.036, size = 63, normalized size = 2.2

$$2a\ln\left(c\sqrt{x}\right)+2b\ln\left(c\sqrt{x}\right)\operatorname{Artanh}\left(c\sqrt{x}\right)-b\operatorname{dilog}\left(c\sqrt{x}\right)-b\operatorname{dilog}\left(1+c\sqrt{x}\right)-b\ln\left(c\sqrt{x}\right)\ln\left(1+c\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^(1/2)))/x,x)
```

```
[Out] 2*a*ln(c*x^(1/2))+2*b*ln(c*x^(1/2))*arctanh(c*x^(1/2))-b*dilog(c*x^(1/2))-b
*dilog(1+c*x^(1/2))-b*ln(c*x^(1/2))*ln(1+c*x^(1/2))
```

Maxima [B] time = 1.49312, size = 82, normalized size = 2.83

$$-\left(\log\left(c\sqrt{x}\right)\log\left(-c\sqrt{x}+1\right)+\operatorname{Li}_2\left(-c\sqrt{x}+1\right)\right)b+\left(\log\left(c\sqrt{x}+1\right)\log\left(-c\sqrt{x}\right)+\operatorname{Li}_2\left(c\sqrt{x}+1\right)\right)b+a\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))/x,x, algorithm="maxima")
```

```
[Out] -(log(c*sqrt(x))*log(-c*sqrt(x) + 1) + dilog(-c*sqrt(x) + 1))*b + (log(c*sq
rt(x) + 1)*log(-c*sqrt(x)) + dilog(c*sqrt(x) + 1))*b + a*log(x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{b\operatorname{artanh}\left(c\sqrt{x}\right)+a}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))/x,x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*sqrt(x)) + a)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{a+b\operatorname{atanh}\left(c\sqrt{x}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**(1/2)))/x,x)
```

```
[Out] Integral((a + b*atanh(c*sqrt(x)))/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{b\operatorname{artanh}\left(c\sqrt{x}\right)+a}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*sqrt(x)) + a)/x, x)
```

3.192 $\int \frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{x^2}\,dx$

Optimal. Leaf size=40

$$-\frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{x}+bc^2\tanh^{-1}\left(c\sqrt{x}\right)-\frac{bc}{\sqrt{x}}$$

```
[Out] -((b*c)/Sqrt[x]) + b*c^2*ArcTanh[c*Sqrt[x]] - (a + b*ArcTanh[c*Sqrt[x]])/x
```

Rubi [A] time = 0.0229893, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6097, 51, 63, 206}

$$-\frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{x}+bc^2\tanh^{-1}\left(c\sqrt{x}\right)-\frac{bc}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*Sqrt[x]])/x^2,x]
```

```
[Out] -((b*c)/Sqrt[x]) + b*c^2*ArcTanh[c*Sqrt[x]] - (a + b*ArcTanh[c*Sqrt[x]])/x
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{x^2}\,dx &= -\frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{x}+\frac{1}{2}(bc)\int \frac{1}{x^{3/2}\left(1-c^2x\right)}\,dx \\
&= -\frac{bc}{\sqrt{x}}-\frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{x}+\frac{1}{2}\left(bc^3\right)\int \frac{1}{\sqrt{x}\left(1-c^2x\right)}\,dx \\
&= -\frac{bc}{\sqrt{x}}-\frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{x}+\left(bc^3\right)\operatorname{Subst}\left(\int \frac{1}{1-c^2x^2}\,dx,x,\sqrt{x}\right) \\
&= -\frac{bc}{\sqrt{x}}+bc^2\tanh^{-1}\left(c\sqrt{x}\right)-\frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{x}
\end{aligned}
$$

Mathematica [A] time = 0.0241474, size = 67, normalized size = 1.68

$$
-\frac{a}{x}-\frac{1}{2}bc^2\log\left(1-c\sqrt{x}\right)+\frac{1}{2}bc^2\log\left(c\sqrt{x}+1\right)-\frac{bc}{\sqrt{x}}-\frac{b\tanh^{-1}\left(c\sqrt{x}\right)}{x}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*Sqrt[x]])/x^2,x]
```

```
[Out] -(a/x) - (b*c)/Sqrt[x] - (b*ArcTanh[c*Sqrt[x]])/x - (b*c^2*Log[1 - c*Sqrt[x
]])/2 + (b*c^2*Log[1 + c*Sqrt[x]])/2
```

Maple [A] time = 0.033, size = 55, normalized size = 1.4

$$
-\frac{a}{x}-\frac{b}{x}\operatorname{Artanh}\left(c\sqrt{x}\right)-bc\frac{1}{\sqrt{x}}-\frac{c^2b}{2}\ln\left(c\sqrt{x}-1\right)+\frac{c^2b}{2}\ln\left(1+c\sqrt{x}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^(1/2)))/x^2,x)
```

```
[Out] -a/x-b/x*arctanh(c*x^(1/2))-b*c/x^(1/2)-1/2*c^2*b*ln(c*x^(1/2)-1)+1/2*c^2*b
*ln(1+c*x^(1/2))
```

Maxima [A] time = 0.961342, size = 69, normalized size = 1.72

$$
\frac{1}{2}\left(\left(c\log\left(c\sqrt{x}+1\right)-c\log\left(c\sqrt{x}-1\right)-\frac{2}{\sqrt{x}}\right)c-\frac{2\,\operatorname{artanh}\left(c\sqrt{x}\right)}{x}\right)b-\frac{a}{x}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*((c*log(c*sqrt(x) + 1) - c*log(c*sqrt(x) - 1) - 2/sqrt(x))*c - 2*arctan
h(c*sqrt(x))/x)*b - a/x
```

Fricas [A] time = 1.74626, size = 122, normalized size = 3.05

$$-\frac{2\,bc\sqrt{x} - \left(bc^2x - b\right)\log\left(-\frac{c^2x+2\,c\sqrt{x}+1}{c^2x-1}\right) + 2\,a}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*c*sqrt(x) - (b*c^2*x - b)*log(-(c^2*x + 2*c*sqrt(x) + 1)/(c^2*x -
 1)) + 2*a)/x
```

Sympy [A] time = 40.8538, size = 231, normalized size = 5.78

$$\begin{cases} -\frac{a}{x} + \frac{b\,\mathrm{atanh}\left(\sqrt{x}\sqrt{\frac{1}{x}}\right)}{x} & \text{for}\: c = -\sqrt{\frac{1}{x}} \\ -\frac{a}{x} - \frac{b\,\mathrm{atanh}\left(\sqrt{x}\sqrt{\frac{1}{x}}\right)}{x} & \text{for}\: c = \sqrt{\frac{1}{x}} \\ -\frac{ac^4x^{\frac{5}{2}}}{c^2x^{\frac{5}{2}} - x^{\frac{3}{2}}} + \frac{a\sqrt{x}}{c^2x^{\frac{5}{2}} - x^{\frac{3}{2}}} + \frac{bc^4x^{\frac{5}{2}}\,\mathrm{atanh}\left(c\sqrt{x}\right)}{c^2x^{\frac{5}{2}} - x^{\frac{3}{2}}} - \frac{bc^3x^2}{c^2x^{\frac{5}{2}} - x^{\frac{3}{2}}} - \frac{2bc^2x^{\frac{3}{2}}\,\mathrm{atanh}\left(c\sqrt{x}\right)}{c^2x^{\frac{5}{2}} - x^{\frac{3}{2}}} + \frac{bcx}{c^2x^{\frac{5}{2}} - x^{\frac{3}{2}}} + \frac{b\sqrt{x}\,\mathrm{atanh}\left(c\sqrt{x}\right)}{c^2x^{\frac{5}{2}} - x^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**(1/2)))/x**2,x)
```

```
[Out] Piecewise((-a/x + b*atanh(sqrt(x)*sqrt(1/x))/x, Eq(c, -sqrt(1/x))), (-a/x -
 b*atanh(sqrt(x)*sqrt(1/x))/x, Eq(c, sqrt(1/x))), (-a*c**4*x**(5/2)/(c**2*x
**(5/2) - x**(3/2)) + a*sqrt(x)/(c**2*x**(5/2) - x**(3/2)) + b*c**4*x**(5/2
)*atanh(c*sqrt(x))/(c**2*x**(5/2) - x**(3/2)) - b*c**3*x**2/(c**2*x**(5/2)
- x**(3/2)) - 2*b*c**2*x**(3/2)*atanh(c*sqrt(x))/(c**2*x**(5/2) - x**(3/2))
 + b*c*x/(c**2*x**(5/2) - x**(3/2)) + b*sqrt(x)*atanh(c*sqrt(x))/(c**2*x**(
5/2) - x**(3/2)), True))
```

Giac [A] time = 1.33223, size = 90, normalized size = 2.25

$$\frac{1}{2}\,bc^2\log\left(c\sqrt{x}+1\right) - \frac{1}{2}\,bc^2\log\left(c\sqrt{x}-1\right) - \frac{b\log\left(-\frac{c\sqrt{x}+1}{c\sqrt{x}-1}\right)}{2\,x} - \frac{bc\sqrt{x}+a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))/x^2,x, algorithm="giac")
```

```
[Out] 1/2*b*c^2*log(c*sqrt(x) + 1) - 1/2*b*c^2*log(c*sqrt(x) - 1) - 1/2*b*log(-(c
*sqrt(x) + 1)/(c*sqrt(x) - 1))/x - (b*c*sqrt(x) + a)/x
```

3.193 $\int \frac{a+b\tanh^{-1}(c\sqrt{x})}{x^3}\,dx$

Optimal. Leaf size=60

$$-\frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{2x^2}-\frac{bc^3}{2\sqrt{x}}+\frac{1}{2}bc^4\tanh^{-1}\left(c\sqrt{x}\right)-\frac{bc}{6x^{3/2}}$$

```
[Out] -(b*c)/(6*x^(3/2)) - (b*c^3)/(2*Sqrt[x]) + (b*c^4*ArcTanh[c*Sqrt[x]])/2 - (
a + b*ArcTanh[c*Sqrt[x]])/(2*x^2)
```

Rubi [A] time = 0.0263707, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6097, 51, 63, 206}

$$-\frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{2x^2}-\frac{bc^3}{2\sqrt{x}}+\frac{1}{2}bc^4\tanh^{-1}\left(c\sqrt{x}\right)-\frac{bc}{6x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*Sqrt[x]])/x^3,x]
```

```
[Out] -(b*c)/(6*x^(3/2)) - (b*c^3)/(2*Sqrt[x]) + (b*c^4*ArcTanh[c*Sqrt[x]])/2 - (
a + b*ArcTanh[c*Sqrt[x]])/(2*x^2)
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{x^3}\,dx = -\frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{2x^2} + \frac{1}{4}(bc)\int \frac{1}{x^{5/2}\left(1-c^2x\right)}\,dx$$
$$= -\frac{bc}{6x^{3/2}} - \frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{2x^2} + \frac{1}{4}\left(bc^3\right)\int \frac{1}{x^{3/2}\left(1-c^2x\right)}\,dx$$
$$= -\frac{bc}{6x^{3/2}} - \frac{bc^3}{2\sqrt{x}} - \frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{2x^2} + \frac{1}{4}\left(bc^5\right)\int \frac{1}{\sqrt{x}\left(1-c^2x\right)}\,dx$$
$$= -\frac{bc}{6x^{3/2}} - \frac{bc^3}{2\sqrt{x}} - \frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{2x^2} + \frac{1}{2}\left(bc^5\right)\text{Subst}\left(\int \frac{1}{1-c^2x^2}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{bc}{6x^{3/2}} - \frac{bc^3}{2\sqrt{x}} + \frac{1}{2}bc^4\tanh^{-1}\left(c\sqrt{x}\right) - \frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{2x^2}$$

Mathematica [A] time = 0.0258193, size = 86, normalized size = 1.43

$$-\frac{a}{2x^2} - \frac{bc^3}{2\sqrt{x}} - \frac{1}{4}bc^4\log\left(1-c\sqrt{x}\right) + \frac{1}{4}bc^4\log\left(c\sqrt{x}+1\right) - \frac{bc}{6x^{3/2}} - \frac{b\tanh^{-1}\left(c\sqrt{x}\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*Sqrt[x]])/x^3,x]
```

```
[Out] -a/(2*x^2) - (b*c)/(6*x^(3/2)) - (b*c^3)/(2*Sqrt[x]) - (b*ArcTanh[c*Sqrt[x]
])/(2*x^2) - (b*c^4*Log[1 - c*Sqrt[x]])/4 + (b*c^4*Log[1 + c*Sqrt[x]])/4
```

Maple [A] time = 0.033, size = 64, normalized size = 1.1

$$-\frac{a}{2x^2} - \frac{b}{2x^2}\text{Artanh}\left(c\sqrt{x}\right) - \frac{c^4b}{4}\ln\left(c\sqrt{x}-1\right) - \frac{bc}{6}x^{-\frac{3}{2}} - \frac{bc^3}{2}\frac{1}{\sqrt{x}} + \frac{c^4b}{4}\ln\left(1+c\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^(1/2)))/x^3,x)
```

```
[Out] -1/2*a/x^2-1/2*b/x^2*arctanh(c*x^(1/2))-1/4*c^4*b*ln(c*x^(1/2)-1)-1/6*b*c/x
^(3/2)-1/2*b*c^3/x^(1/2)+1/4*c^4*b*ln(1+c*x^(1/2))
```

Maxima [A] time = 0.956247, size = 86, normalized size = 1.43

$$\frac{1}{12}\left(\left(3c^3\log\left(c\sqrt{x}+1\right) - 3c^3\log\left(c\sqrt{x}-1\right) - \frac{2\left(3c^2x+1\right)}{x^{\frac{3}{2}}}\right)c - \frac{6\,\text{artanh}\left(c\sqrt{x}\right)}{x^2}\right)b - \frac{a}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))/x^3,x, algorithm="maxima")
```

```
[Out] 1/12*((3*c^3*log(c*sqrt(x) + 1) - 3*c^3*log(c*sqrt(x) - 1) - 2*(3*c^2*x + 1
)/x^(3/2))*c - 6*arctanh(c*sqrt(x))/x^2)*b - 1/2*a/x^2
```

Fricas [A] time = 1.65234, size = 149, normalized size = 2.48

$$\frac{3\left(bc^4x^2-b\right)\log\left(-\frac{c^2x+2\,c\sqrt{x}+1}{c^2x-1}\right)-2\left(3\,bc^3x+bc\right)\sqrt{x}-6\,a}{12\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))/x^3,x, algorithm="fricas")
```

```
[Out] 1/12*(3*(b*c^4*x^2 - b)*log(-(c^2*x + 2*c*sqrt(x) + 1)/(c^2*x - 1)) - 2*(3*
b*c^3*x + b*c)*sqrt(x) - 6*a)/x^2
```

Sympy [A] time = 179.21, size = 342, normalized size = 5.7

$$\begin{cases} -\frac{a}{2x^2}+\frac{b\operatorname{atanh}\left(\sqrt{x}\sqrt{\frac{1}{x}}\right)}{2x^2} \\ -\frac{a}{2x^2}-\frac{b\operatorname{atanh}\left(\sqrt{x}\sqrt{\frac{1}{x}}\right)}{2x^2} \\ -\frac{3ac^2x^{\frac{3}{2}}}{6c^2x^{\frac{7}{2}}-6x^{\frac{5}{2}}}+\frac{3a\sqrt{x}}{6c^2x^{\frac{7}{2}}-6x^{\frac{5}{2}}}+\frac{3bc^6x^{\frac{7}{2}}\operatorname{atanh}\left(c\sqrt{x}\right)}{6c^2x^{\frac{7}{2}}-6x^{\frac{5}{2}}}-\frac{3bc^5x^3}{6c^2x^{\frac{7}{2}}-6x^{\frac{5}{2}}}-\frac{3bc^4x^{\frac{5}{2}}\operatorname{atanh}\left(c\sqrt{x}\right)}{6c^2x^{\frac{7}{2}}-6x^{\frac{5}{2}}}+\frac{2bc^3x^2}{6c^2x^{\frac{7}{2}}-6x^{\frac{5}{2}}}-\frac{3bc^2x^{\frac{3}{2}}\operatorname{atanh}\left(c\sqrt{x}\right)}{6c^2x^{\frac{7}{2}}-6x^{\frac{5}{2}}}+\frac{bcx}{6c^2x^{\frac{7}{2}}-6x^{\frac{5}{2}}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**(1/2)))/x**3,x)
```

```
[Out] Piecewise((-a/(2*x**2) + b*atanh(sqrt(x)*sqrt(1/x))/(2*x**2), Eq(c, -sqrt(1
/x))), (-a/(2*x**2) - b*atanh(sqrt(x)*sqrt(1/x))/(2*x**2), Eq(c, sqrt(1/x))
), (-3*a*c**2*x**(3/2)/(6*c**2*x**(7/2) - 6*x**(5/2)) + 3*a*sqrt(x)/(6*c**2
*x**(7/2) - 6*x**(5/2)) + 3*b*c**6*x**(7/2)*atanh(c*sqrt(x))/(6*c**2*x**(7/
2) - 6*x**(5/2)) - 3*b*c**5*x**3/(6*c**2*x**(7/2) - 6*x**(5/2)) - 3*b*c**4*
x**(5/2)*atanh(c*sqrt(x))/(6*c**2*x**(7/2) - 6*x**(5/2)) + 2*b*c**3*x**2/(6
*c**2*x**(7/2) - 6*x**(5/2)) - 3*b*c**2*x**(3/2)*atanh(c*sqrt(x))/(6*c**2*x
**(7/2) - 6*x**(5/2)) + b*c*x/(6*c**2*x**(7/2) - 6*x**(5/2)) + 3*b*sqrt(x)*
atanh(c*sqrt(x))/(6*c**2*x**(7/2) - 6*x**(5/2)), True))
```

Giac [A] time = 1.32737, size = 105, normalized size = 1.75

$$\frac{1}{4}\,bc^4\log\left(c\sqrt{x}+1\right)-\frac{1}{4}\,bc^4\log\left(c\sqrt{x}-1\right)-\frac{b\log\left(-\frac{c\sqrt{x}+1}{c\sqrt{x}-1}\right)}{4\,x^2}-\frac{3\,bc^3x^{\frac{3}{2}}+bc\sqrt{x}+3\,a}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))/x^3,x, algorithm="giac")
```

```
[Out] 1/4*b*c^4*log(c*sqrt(x) + 1) - 1/4*b*c^4*log(c*sqrt(x) - 1) - 1/4*b*log(-(c
*sqrt(x) + 1)/(c*sqrt(x) - 1))/x^2 - 1/6*(3*b*c^3*x^(3/2) + b*c*sqrt(x) + 3
*a)/x^2
```

3.194 $\int \frac{a+b\tanh^{-1}(c\sqrt{x})}{x^4}\,dx$

Optimal. Leaf size=73

$$-\frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{3x^3}-\frac{bc^3}{9x^{3/2}}-\frac{bc^5}{3\sqrt{x}}+\frac{1}{3}bc^6\tanh^{-1}\left(c\sqrt{x}\right)-\frac{bc}{15x^{5/2}}$$

```
[Out] -(b*c)/(15*x^(5/2)) - (b*c^3)/(9*x^(3/2)) - (b*c^5)/(3*Sqrt[x]) + (b*c^6*Ar
cTanh[c*Sqrt[x]])/3 - (a + b*ArcTanh[c*Sqrt[x]])/(3*x^3)
```

Rubi [A] time = 0.0335061, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6097, 51, 63, 206}

$$-\frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{3x^3}-\frac{bc^3}{9x^{3/2}}-\frac{bc^5}{3\sqrt{x}}+\frac{1}{3}bc^6\tanh^{-1}\left(c\sqrt{x}\right)-\frac{bc}{15x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*Sqrt[x]])/x^4,x]
```

```
[Out] -(b*c)/(15*x^(5/2)) - (b*c^3)/(9*x^(3/2)) - (b*c^5)/(3*Sqrt[x]) + (b*c^6*Ar
cTanh[c*Sqrt[x]])/3 - (a + b*ArcTanh[c*Sqrt[x]])/(3*x^3)
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{x^4}\,dx = -\frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{3x^3} + \frac{1}{6}(bc)\int \frac{1}{x^{7/2}\left(1-c^2x\right)}\,dx$$
$$= -\frac{bc}{15x^{5/2}} - \frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{3x^3} + \frac{1}{6}\left(bc^3\right)\int \frac{1}{x^{5/2}\left(1-c^2x\right)}\,dx$$
$$= -\frac{bc}{15x^{5/2}} - \frac{bc^3}{9x^{3/2}} - \frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{3x^3} + \frac{1}{6}\left(bc^5\right)\int \frac{1}{x^{3/2}\left(1-c^2x\right)}\,dx$$
$$= -\frac{bc}{15x^{5/2}} - \frac{bc^3}{9x^{3/2}} - \frac{bc^5}{3\sqrt{x}} - \frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{3x^3} + \frac{1}{6}\left(bc^7\right)\int \frac{1}{\sqrt{x}\left(1-c^2x\right)}\,dx$$
$$= -\frac{bc}{15x^{5/2}} - \frac{bc^3}{9x^{3/2}} - \frac{bc^5}{3\sqrt{x}} - \frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{3x^3} + \frac{1}{3}\left(bc^7\right)\mathrm{Subst}\left(\int \frac{1}{1-c^2x^2}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{bc}{15x^{5/2}} - \frac{bc^3}{9x^{3/2}} - \frac{bc^5}{3\sqrt{x}} + \frac{1}{3}bc^6\tanh^{-1}\left(c\sqrt{x}\right) - \frac{a+b\tanh^{-1}\left(c\sqrt{x}\right)}{3x^3}$$

Mathematica [A] time = 0.0302242, size = 99, normalized size = 1.36

$$-\frac{a}{3x^3} - \frac{bc^3}{9x^{3/2}} - \frac{bc^5}{3\sqrt{x}} - \frac{1}{6}bc^6\log\left(1-c\sqrt{x}\right) + \frac{1}{6}bc^6\log\left(c\sqrt{x}+1\right) - \frac{bc}{15x^{5/2}} - \frac{b\tanh^{-1}\left(c\sqrt{x}\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*Sqrt[x]])/x^4,x]
```

```
[Out] -a/(3*x^3) - (b*c)/(15*x^(5/2)) - (b*c^3)/(9*x^(3/2)) - (b*c^5)/(3*Sqrt[x])
 - (b*ArcTanh[c*Sqrt[x]])/(3*x^3) - (b*c^6*Log[1 - c*Sqrt[x]])/6 + (b*c^6*L
og[1 + c*Sqrt[x]])/6
```

Maple [A] time = 0.037, size = 73, normalized size = 1.

$$-\frac{a}{3\,x^3} - \frac{b}{3\,x^3}\mathrm{Artanh}\left(c\sqrt{x}\right) - \frac{c^6b}{6}\ln\left(c\sqrt{x}-1\right) - \frac{bc}{15}x^{-\frac{5}{2}} - \frac{bc^3}{9}x^{-\frac{3}{2}} - \frac{bc^5}{3}\frac{1}{\sqrt{x}} + \frac{c^6b}{6}\ln\left(1+c\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^(1/2)))/x^4,x)
```

```
[Out] -1/3*a/x^3-1/3*b/x^3*arctanh(c*x^(1/2))-1/6*c^6*b*ln(c*x^(1/2)-1)-1/15*b*c/
x^(5/2)-1/9*b*c^3/x^(3/2)-1/3*b*c^5/x^(1/2)+1/6*c^6*b*ln(1+c*x^(1/2))
```

Maxima [A] time = 0.957319, size = 97, normalized size = 1.33

$$\frac{1}{90}\left(\left(15\,c^5\log\left(c\sqrt{x}+1\right) - 15\,c^5\log\left(c\sqrt{x}-1\right) - \frac{2\left(15\,c^4x^2+5\,c^2x+3\right)}{x^{\frac{5}{2}}}\right)c - \frac{30\,\mathrm{artanh}\left(c\sqrt{x}\right)}{x^3}\right)b - \frac{a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))/x^4,x, algorithm="maxima")
```

```
[Out] 1/90*((15*c^5*log(c*sqrt(x) + 1) - 15*c^5*log(c*sqrt(x) - 1) - 2*(15*c^4*x^
2 + 5*c^2*x + 3)/x^(5/2))*c - 30*arctanh(c*sqrt(x))/x^3)*b - 1/3*a/x^3
```

Fricas [A] time = 1.62725, size = 174, normalized size = 2.38

$$\frac{15\left(bc^6x^3-b\right)\log\left(-\frac{c^2x+2c\sqrt{x}+1}{c^2x-1}\right)-2\left(15bc^5x^2+5bc^3x+3bc\right)\sqrt{x}-30a}{90x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))/x^4,x, algorithm="fricas")
```

```
[Out] 1/90*(15*(b*c^6*x^3 - b)*log(-(c^2*x + 2*c*sqrt(x) + 1)/(c^2*x - 1)) - 2*(1
5*b*c^5*x^2 + 5*b*c^3*x + 3*b*c)*sqrt(x) - 30*a)/x^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**(1/2)))/x**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.39889, size = 119, normalized size = 1.63

$$\frac{1}{6}bc^6\log\left(c\sqrt{x}+1\right)-\frac{1}{6}bc^6\log\left(c\sqrt{x}-1\right)-\frac{b\log\left(-\frac{c\sqrt{x}+1}{c\sqrt{x}-1}\right)}{6x^3}-\frac{15bc^5x^{\frac{5}{2}}+5bc^3x^{\frac{3}{2}}+3bc\sqrt{x}+15a}{45x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))/x^4,x, algorithm="giac")
```

```
[Out] 1/6*b*c^6*log(c*sqrt(x) + 1) - 1/6*b*c^6*log(c*sqrt(x) - 1) - 1/6*b*log(-(c
*sqrt(x) + 1)/(c*sqrt(x) - 1))/x^3 - 1/45*(15*b*c^5*x^(5/2) + 5*b*c^3*x^(3/
2) + 3*b*c*sqrt(x) + 15*a)/x^3
```

3.195 $\int x^3 \left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2 dx$

Optimal. Leaf size=211

$$\frac{bx^{5/2}\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{10c^3}+\frac{bx^{3/2}\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{6c^5}+\frac{ab\sqrt{x}}{2c^7}-\frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{4c^8}+\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(c\sqrt{x}\right.\right.$$

```
[Out] (a*b*Sqrt[x])/(2*c^7) + (71*b^2*x)/(420*c^6) + (3*b^2*x^2)/(70*c^4) + (b^2*
x^3)/(84*c^2) + (b^2*Sqrt[x]*ArcTanh[c*Sqrt[x]])/(2*c^7) + (b*x^(3/2)*(a +
b*ArcTanh[c*Sqrt[x]]))/(6*c^5) + (b*x^(5/2)*(a + b*ArcTanh[c*Sqrt[x]]))/(10
*c^3) + (b*x^(7/2)*(a + b*ArcTanh[c*Sqrt[x]]))/(14*c) - (a + b*ArcTanh[c*Sq
rt[x]])^2/(4*c^8) + (x^4*(a + b*ArcTanh[c*Sqrt[x]])^2)/4 + (44*b^2*Log[1 -
c^2*x])/(105*c^8)
```

Rubi [F] time = 0.0251889, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^3 \left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2 dx$$

Verification is Not applicable to the result.

```
[In] Int[x^3*(a + b*ArcTanh[c*Sqrt[x]])^2,x]
```

```
[Out] Defer[Int][x^3*(a + b*ArcTanh[c*Sqrt[x]])^2, x]
```

Rubi steps

$$\int x^3 \left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2 dx=\int x^3 \left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2 dx$$

Mathematica [A] time = 0.112294, size = 224, normalized size = 1.06

$$\frac{105a^2c^8x^4+30abc^7x^{7/2}+42abc^5x^{5/2}+70abc^3x^{3/2}+2bc\sqrt{x}\tanh^{-1}\left(c\sqrt{x}\right)\left(105ac^7x^{7/2}+b\left(15c^6x^3+21c^4x^2+35c^2x+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcTanh[c*Sqrt[x]])^2,x]
```

```
[Out] (210*a*b*c*Sqrt[x] + 71*b^2*c^2*x + 70*a*b*c^3*x^(3/2) + 18*b^2*c^4*x^2 + 4
2*a*b*c^5*x^(5/2) + 5*b^2*c^6*x^3 + 30*a*b*c^7*x^(7/2) + 105*a^2*c^8*x^4 +
2*b*c*Sqrt[x]*(105*a*c^7*x^(7/2) + b*(105 + 35*c^2*x + 21*c^4*x^2 + 15*c^6*
x^3))*ArcTanh[c*Sqrt[x]] + 105*b^2*(-1 + c^8*x^4)*ArcTanh[c*Sqrt[x]]^2 + b*
(105*a + 176*b)*Log[1 - c*Sqrt[x]] - 105*a*b*Log[1 + c*Sqrt[x]] + 176*b^2*L
og[1 + c*Sqrt[x]])/(420*c^8)
```

Maple [B] time = 0.056, size = 396, normalized size = 1.9

$$\frac{b^2}{2c^7}\text{Artanh}\left(c\sqrt{x}\right)\sqrt{x}+\frac{ab}{10c^3}x^{\frac{5}{2}}+\frac{ab}{14c}x^{\frac{7}{2}}+\frac{b^2}{14c}\text{Artanh}\left(c\sqrt{x}\right)x^{\frac{7}{2}}+\frac{abx^4}{2}\text{Artanh}\left(c\sqrt{x}\right)-\frac{b^2}{4c^8}\text{Artanh}\left(c\sqrt{x}\right)\ln\left(1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c*x^(1/2)))^2,x)
```

```
[Out] 1/14/c*b^2*arctanh(c*x^(1/2))*x^(7/2)+1/2*a*b*x^4*arctanh(c*x^(1/2))-1/4/c^
8*b^2*arctanh(c*x^(1/2))*ln(1+c*x^(1/2))+1/10/c^3*a*b*x^(5/2)+1/14/c*x^(7/2
)*a*b+1/10/c^3*b^2*arctanh(c*x^(1/2))*x^(5/2)+1/6/c^5*b^2*arctanh(c*x^(1/2)
)*x^(3/2)+1/6/c^5*a*b*x^(3/2)-1/8/c^8*b^2*ln(c*x^(1/2)-1)*ln(1/2+1/2*c*x^(1
/2))+1/8/c^8*b^2*ln(-1/2*c*x^(1/2)+1/2)*ln(1/2+1/2*c*x^(1/2))+1/4/c^8*b^2*a
rctanh(c*x^(1/2))*ln(c*x^(1/2)-1)-1/8/c^8*b^2*ln(-1/2*c*x^(1/2)+1/2)*ln(1+c
*x^(1/2))+1/4/c^8*a*b*ln(c*x^(1/2)-1)-1/4/c^8*a*b*ln(1+c*x^(1/2))+71/420*b^
2*x/c^6+1/84*b^2*x^3/c^2+1/4*b^2*x^4*arctanh(c*x^(1/2))^2+1/16/c^8*b^2*ln(1
+c*x^(1/2))^2+44/105/c^8*b^2*ln(c*x^(1/2)-1)+44/105/c^8*b^2*ln(1+c*x^(1/2))
+1/16/c^8*b^2*ln(c*x^(1/2)-1)^2+3/70*b^2*x^2/c^4+1/4*a^2*x^4+1/2*a*b*x^(1/2
)/c^7+1/2*b^2*arctanh(c*x^(1/2))*x^(1/2)/c^7
```

Maxima [A] time = 0.987937, size = 358, normalized size = 1.7

$$\frac{1}{4}\,b^2x^4\,\mathrm{artanh}\left(c\sqrt{x}\right)^2+\frac{1}{4}\,a^2x^4+\frac{1}{420}\left(210\,x^4\,\mathrm{artanh}\left(c\sqrt{x}\right)+c\left(\frac{2\left(15\,c^6x^{\frac{7}{2}}+21\,c^4x^{\frac{5}{2}}+35\,c^2x^{\frac{3}{2}}+105\,\sqrt{x}\right)}{c^8}-\frac{105\,\log\left(c\right.}{c^9}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^(1/2)))^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*x^4*arctanh(c*sqrt(x))^2 + 1/4*a^2*x^4 + 1/420*(210*x^4*arctanh(c*s
qrt(x)) + c*(2*(15*c^6*x^(7/2) + 21*c^4*x^(5/2) + 35*c^2*x^(3/2) + 105*sqrt
(x))/c^8 - 105*log(c*sqrt(x) + 1)/c^9 + 105*log(c*sqrt(x) - 1)/c^9))*a*b +
1/1680*(4*c*(2*(15*c^6*x^(7/2) + 21*c^4*x^(5/2) + 35*c^2*x^(3/2) + 105*sqrt
(x))/c^8 - 105*log(c*sqrt(x) + 1)/c^9 + 105*log(c*sqrt(x) - 1)/c^9)*arctanh
(c*sqrt(x)) + (20*c^6*x^3 + 72*c^4*x^2 + 284*c^2*x - 2*(105*log(c*sqrt(x) -
 1) - 352)*log(c*sqrt(x) + 1) + 105*log(c*sqrt(x) + 1)^2 + 105*log(c*sqrt(x
) - 1)^2 + 704*log(c*sqrt(x) - 1))/c^8)*b^2
```

Fricas [A] time = 2.12589, size = 662, normalized size = 3.14

$$\frac{420\,a^2c^8x^4+20\,b^2c^6x^3+72\,b^2c^4x^2+284\,b^2c^2x+105\left(b^2c^8x^4-b^2\right)\log\left(-\frac{c^2x+2\,c\sqrt{x}+1}{c^2x-1}\right)^2+4\left(105\,abc^8-105\,ab+176\,b^2\right)\mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^(1/2)))^2,x, algorithm="fricas")
```

```
[Out] 1/1680*(420*a^2*c^8*x^4 + 20*b^2*c^6*x^3 + 72*b^2*c^4*x^2 + 284*b^2*c^2*x +
 105*(b^2*c^8*x^4 - b^2)*log(-(c^2*x + 2*c*sqrt(x) + 1)/(c^2*x - 1))^2 + 4*
(105*a*b*c^8 - 105*a*b + 176*b^2)*log(c*sqrt(x) + 1) - 4*(105*a*b*c^8 - 105
*a*b - 176*b^2)*log(c*sqrt(x) - 1) + 4*(105*a*b*c^8*x^4 - 105*a*b*c^8 + (15
*b^2*c^7*x^3 + 21*b^2*c^5*x^2 + 35*b^2*c^3*x + 105*b^2*c)*sqrt(x))*log(-(c^
2*x + 2*c*sqrt(x) + 1)/(c^2*x - 1)) + 8*(15*a*b*c^7*x^3 + 21*a*b*c^5*x^2 +
35*a*b*c^3*x + 105*a*b*c)*sqrt(x))/c^8
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\left(a+b\operatorname{atanh}\left(c\sqrt{x}\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c*x**(1/2)))**2,x)
```

```
[Out] Integral(x**3*(a + b*atanh(c*sqrt(x)))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\operatorname{artanh}\left(c\sqrt{x}\right)+a\right)^2 x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^(1/2)))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*sqrt(x)) + a)^2*x^3, x)
```

3.196 $\int x^2 \left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2 dx$

Optimal. Leaf size=173

$$\frac{2bx^{3/2}\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{9c^3}+\frac{2ab\sqrt{x}}{3c^5}-\frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{3c^6}+\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2+\frac{2bx^{5/2}\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{15c}$$

```
[Out] (2*a*b*Sqrt[x])/(3*c^5) + (8*b^2*x)/(45*c^4) + (b^2*x^2)/(30*c^2) + (2*b^2*
Sqrt[x]*ArcTanh[c*Sqrt[x]])/(3*c^5) + (2*b*x^(3/2)*(a + b*ArcTanh[c*Sqrt[x]
]))/(9*c^3) + (2*b*x^(5/2)*(a + b*ArcTanh[c*Sqrt[x]]))/(15*c) - (a + b*ArcT
anh[c*Sqrt[x]])^2/(3*c^6) + (x^3*(a + b*ArcTanh[c*Sqrt[x]])^2)/3 + (23*b^2*
Log[1 - c^2*x])/(45*c^6)
```

Rubi [F] time = 0.023916, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^2 \left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2 dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2*(a + b*ArcTanh[c*Sqrt[x]])^2,x]
```

```
[Out] Defer[Int][x^2*(a + b*ArcTanh[c*Sqrt[x]])^2, x]
```

Rubi steps

$$\int x^2 \left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2 dx=\int x^2 \left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2 dx$$

Mathematica [A] time = 0.103839, size = 194, normalized size = 1.12

$$\frac{30a^2c^6x^3+12abc^5x^{5/2}+20abc^3x^{3/2}+4bc\sqrt{x}\tanh^{-1}\left(c\sqrt{x}\right)\left(15ac^5x^{5/2}+b\left(3c^4x^2+5c^2x+15\right)\right)+60abc\sqrt{x}+2b(15a+2}{90c^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcTanh[c*Sqrt[x]])^2,x]
```

```
[Out] (60*a*b*c*Sqrt[x] + 16*b^2*c^2*x + 20*a*b*c^3*x^(3/2) + 3*b^2*c^4*x^2 + 12*
a*b*c^5*x^(5/2) + 30*a^2*c^6*x^3 + 4*b*c*Sqrt[x]*(15*a*c^5*x^(5/2) + b*(15
+ 5*c^2*x + 3*c^4*x^2))*ArcTanh[c*Sqrt[x]] + 30*b^2*(-1 + c^6*x^3)*ArcTanh[
c*Sqrt[x]]^2 + 2*b*(15*a + 23*b)*Log[1 - c*Sqrt[x]] - 30*a*b*Log[1 + c*Sqrt
[x]] + 46*b^2*Log[1 + c*Sqrt[x]])/(90*c^6)
```

Maple [B] time = 0.049, size = 358, normalized size = 2.1

$$\frac{x^3a^2}{3}+\frac{x^3b^2}{3}\left(\text{Artanh}\left(c\sqrt{x}\right)\right)^2+\frac{2\,b^2}{15\,c}\text{Artanh}\left(c\sqrt{x}\right)x^{\frac{5}{2}}+\frac{2\,b^2}{9\,c^3}\text{Artanh}\left(c\sqrt{x}\right)x^{\frac{3}{2}}+\frac{2\,b^2}{3\,c^5}\text{Artanh}\left(c\sqrt{x}\right)\sqrt{x}+\frac{b^2}{3\,c^6}\text{Artanl}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c*x^(1/2)))^2,x)
```

```
[Out] 1/3*x^3*a^2+1/3*b^2*x^3*arctanh(c*x^(1/2))^2+2/15/c*b^2*arctanh(c*x^(1/2))*
x^(5/2)+2/9/c^3*b^2*arctanh(c*x^(1/2))*x^(3/2)+2/3*b^2*arctanh(c*x^(1/2))*x
^(1/2)/c^5+1/3/c^6*b^2*arctanh(c*x^(1/2))*ln(c*x^(1/2)-1)-1/3/c^6*b^2*arcta
nh(c*x^(1/2))*ln(1+c*x^(1/2))-1/6/c^6*b^2*ln(c*x^(1/2)-1)*ln(1/2+1/2*c*x^(1
/2))+1/12/c^6*b^2*ln(c*x^(1/2)-1)^2+1/6/c^6*b^2*ln(-1/2*c*x^(1/2)+1/2)*ln(1
/2+1/2*c*x^(1/2))-1/6/c^6*b^2*ln(-1/2*c*x^(1/2)+1/2)*ln(1+c*x^(1/2))+1/12/c
^6*b^2*ln(1+c*x^(1/2))^2+1/30*b^2*x^2/c^2+8/45*b^2*x/c^4+23/45/c^6*b^2*ln(c
*x^(1/2)-1)+23/45/c^6*b^2*ln(1+c*x^(1/2))+2/3*a*b*x^3*arctanh(c*x^(1/2))+2/
15/c*a*b*x^(5/2)+2/9/c^3*a*b*x^(3/2)+2/3*a*b*x^(1/2)/c^5+1/3/c^6*a*b*ln(c*x
^(1/2)-1)-1/3/c^6*a*b*ln(1+c*x^(1/2))
```

Maxima [A] time = 1.00531, size = 325, normalized size = 1.88

$$\frac{1}{3}b^2x^3\operatorname{artanh}\left(c\sqrt{x}\right)^2+\frac{1}{3}a^2x^3+\frac{1}{45}\left(30\,x^3\operatorname{artanh}\left(c\sqrt{x}\right)+c\left(\frac{2\left(3\,c^4x^{\frac{5}{2}}+5\,c^2x^{\frac{3}{2}}+15\,\sqrt{x}\right)}{c^6}-\frac{15\,\log\left(c\sqrt{x}+1\right)}{c^7}+\frac{15}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^(1/2)))^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*x^3*arctanh(c*sqrt(x))^2 + 1/3*a^2*x^3 + 1/45*(30*x^3*arctanh(c*sqr
t(x)) + c*(2*(3*c^4*x^(5/2) + 5*c^2*x^(3/2) + 15*sqrt(x))/c^6 - 15*log(c*sq
rt(x) + 1)/c^7 + 15*log(c*sqrt(x) - 1)/c^7))*a*b + 1/180*(4*c*(2*(3*c^4*x^(
5/2) + 5*c^2*x^(3/2) + 15*sqrt(x))/c^6 - 15*log(c*sqrt(x) + 1)/c^7 + 15*log
(c*sqrt(x) - 1)/c^7)*arctanh(c*sqrt(x)) + (6*c^4*x^2 + 32*c^2*x - 2*(15*log
(c*sqrt(x) - 1) - 46)*log(c*sqrt(x) + 1) + 15*log(c*sqrt(x) + 1)^2 + 15*log
(c*sqrt(x) - 1)^2 + 92*log(c*sqrt(x) - 1))/c^6)*b^2
```

Fricas [A] time = 2.18744, size = 567, normalized size = 3.28

$$\frac{60\,a^2c^6x^3+6\,b^2c^4x^2+32\,b^2c^2x+15\left(b^2c^6x^3-b^2\right)\log\left(-\frac{c^2x+2\,c\sqrt{x}+1}{c^2x-1}\right)^2+4\left(15\,abc^6-15\,ab+23\,b^2\right)\log\left(c\sqrt{x}+1\right)-4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^(1/2)))^2,x, algorithm="fricas")
```

```
[Out] 1/180*(60*a^2*c^6*x^3 + 6*b^2*c^4*x^2 + 32*b^2*c^2*x + 15*(b^2*c^6*x^3 - b^
2)*log(-(c^2*x + 2*c*sqrt(x) + 1)/(c^2*x - 1))^2 + 4*(15*a*b*c^6 - 15*a*b +
 23*b^2)*log(c*sqrt(x) + 1) - 4*(15*a*b*c^6 - 15*a*b - 23*b^2)*log(c*sqrt(x
) - 1) + 4*(15*a*b*c^6*x^3 - 15*a*b*c^6 + (3*b^2*c^5*x^2 + 5*b^2*c^3*x + 15
*b^2*c)*sqrt(x))*log(-(c^2*x + 2*c*sqrt(x) + 1)/(c^2*x - 1)) + 8*(3*a*b*c^5
*x^2 + 5*a*b*c^3*x + 15*a*b*c)*sqrt(x))/c^6
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\left(a+b\operatorname{atanh}\left(c\sqrt{x}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c*x**(1/2)))**2,x)

[Out] Integral(x**2*(a + b*atanh(c*sqrt(x)))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \operatorname{artanh}\left(c\sqrt{x}\right) + a\right)^2 x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^(1/2)))^2,x, algorithm="giac")

[Out] integrate((b*arctanh(c*sqrt(x)) + a)^2*x^2, x)
```

3.197 $\int x\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2\,dx$

Optimal. Leaf size=129

$$\frac{ab\sqrt{x}}{c^3}-\frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{2c^4}+\frac{bx^{3/2}\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{3c}+\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2+\frac{b^2x}{6c^2}+\frac{2b^2\log\left(1-c^2x\right)}{3c^4}$$

```
[Out] (a*b*Sqrt[x])/c^3 + (b^2*x)/(6*c^2) + (b^2*Sqrt[x]*ArcTanh[c*Sqrt[x]])/c^3
+ (b*x^(3/2)*(a + b*ArcTanh[c*Sqrt[x]]))/(3*c) - (a + b*ArcTanh[c*Sqrt[x]])
^2/(2*c^4) + (x^2*(a + b*ArcTanh[c*Sqrt[x]])^2)/2 + (2*b^2*Log[1 - c^2*x])/
(3*c^4)
```

Rubi [F] time = 0.013963, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x*(a + b*ArcTanh[c*Sqrt[x]])^2,x]
```

```
[Out] Defer[Int][x*(a + b*ArcTanh[c*Sqrt[x]])^2, x]
```

Rubi steps

$$\int x\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2\,dx=\int x\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2\,dx$$

Mathematica [A] time = 0.0859147, size = 160, normalized size = 1.24

$$\frac{3a^2c^4x^2+2abc^3x^{3/2}+2bc\sqrt{x}\tanh^{-1}\left(c\sqrt{x}\right)\left(3ac^3x^{3/2}+b\left(c^2x+3\right)\right)+6abc\sqrt{x}+b(3a+4b)\log\left(1-c\sqrt{x}\right)-3ab\log\left(c\right.}{6c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcTanh[c*Sqrt[x]])^2,x]
```

```
[Out] (6*a*b*c*Sqrt[x] + b^2*c^2*x + 2*a*b*c^3*x^(3/2) + 3*a^2*c^4*x^2 + 2*b*c*Sq
rt[x]*(3*a*c^3*x^(3/2) + b*(3 + c^2*x))*ArcTanh[c*Sqrt[x]] + 3*b^2*(-1 + c^
4*x^2)*ArcTanh[c*Sqrt[x]]^2 + b*(3*a + 4*b)*Log[1 - c*Sqrt[x]] - 3*a*b*Log[
1 + c*Sqrt[x]] + 4*b^2*Log[1 + c*Sqrt[x]])/(6*c^4)
```

Maple [B] time = 0.051, size = 317, normalized size = 2.5

$$\frac{a^2x^2}{2}+\frac{b^2x^2}{2}\left(\mathrm{Artanh}\left(c\sqrt{x}\right)\right)^2+\frac{b^2}{3c}\mathrm{Artanh}\left(c\sqrt{x}\right)x^{\frac{3}{2}}+\frac{b^2}{c^3}\mathrm{Artanh}\left(c\sqrt{x}\right)\sqrt{x}+\frac{b^2}{2c^4}\mathrm{Artanh}\left(c\sqrt{x}\right)\ln\left(c\sqrt{x}-1\right)-\frac{b}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c*x^(1/2)))^2,x)
```

```
[Out] 1/2*a^2*x^2+1/2*b^2*x^2*arctanh(c*x^(1/2))^2+1/3/c*b^2*arctanh(c*x^(1/2))*x
^(3/2)+b^2*arctanh(c*x^(1/2))*x^(1/2)/c^3+1/2/c^4*b^2*arctanh(c*x^(1/2))*ln
(c*x^(1/2)-1)-1/2/c^4*b^2*arctanh(c*x^(1/2))*ln(1+c*x^(1/2))-1/4/c^4*b^2*ln
(c*x^(1/2)-1)*ln(1/2+1/2*c*x^(1/2))+1/8/c^4*b^2*ln(c*x^(1/2)-1)^2+1/4/c^4*b
^2*ln(-1/2*c*x^(1/2)+1/2)*ln(1/2+1/2*c*x^(1/2))-1/4/c^4*b^2*ln(-1/2*c*x^(1/
2)+1/2)*ln(1+c*x^(1/2))+1/8/c^4*b^2*ln(1+c*x^(1/2))^2+1/6*b^2*x/c^2+2/3/c^4
*b^2*ln(c*x^(1/2)-1)+2/3/c^4*b^2*ln(1+c*x^(1/2))+a*b*x^2*arctanh(c*x^(1/2))
+1/3*a*b*x^(3/2)/c+a*b*x^(1/2)/c^3+1/2/c^4*a*b*ln(c*x^(1/2)-1)-1/2/c^4*a*b*
ln(1+c*x^(1/2))
```

Maxima [B] time = 1.02672, size = 290, normalized size = 2.25

$$\frac{1}{2}\,b^2x^2\,\operatorname{artanh}\left(c\sqrt{x}\right)^2+\frac{1}{2}\,a^2x^2+\frac{1}{6}\left(6\,x^2\,\operatorname{artanh}\left(c\sqrt{x}\right)+c\left(\frac{2\left(c^2x^{\frac{3}{2}}+3\,\sqrt{x}\right)}{c^4}-\frac{3\,\log\left(c\sqrt{x}+1\right)}{c^5}+\frac{3\,\log\left(c\sqrt{x}-1\right)}{c^5}\right)\right)ab$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^(1/2)))^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^2*arctanh(c*sqrt(x))^2 + 1/2*a^2*x^2 + 1/6*(6*x^2*arctanh(c*sqrt(
x)) + c*(2*(c^2*x^(3/2) + 3*sqrt(x))/c^4 - 3*log(c*sqrt(x) + 1)/c^5 + 3*log
(c*sqrt(x) - 1)/c^5))*a*b + 1/24*(4*c*(2*(c^2*x^(3/2) + 3*sqrt(x))/c^4 - 3*
log(c*sqrt(x) + 1)/c^5 + 3*log(c*sqrt(x) - 1)/c^5)*arctanh(c*sqrt(x)) + (4*
c^2*x - 2*(3*log(c*sqrt(x) - 1) - 8)*log(c*sqrt(x) + 1) + 3*log(c*sqrt(x) +
 1)^2 + 3*log(c*sqrt(x) - 1)^2 + 16*log(c*sqrt(x) - 1))/c^4)*b^2
```

Fricas [A] time = 2.20403, size = 479, normalized size = 3.71

$$\frac{12\,a^2c^4x^2+4\,b^2c^2x+3\left(b^2c^4x^2-b^2\right)\log\left(-\frac{c^2x+2\,c\sqrt{x}+1}{c^2x-1}\right)^2+4\left(3\,abc^4-3\,ab+4\,b^2\right)\log\left(c\sqrt{x}+1\right)-4\left(3\,abc^4-3\,ab-4\,b^2\right.}{24\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^(1/2)))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(12*a^2*c^4*x^2 + 4*b^2*c^2*x + 3*(b^2*c^4*x^2 - b^2)*log(-(c^2*x + 2*
c*sqrt(x) + 1)/(c^2*x - 1))^2 + 4*(3*a*b*c^4 - 3*a*b + 4*b^2)*log(c*sqrt(x)
 + 1) - 4*(3*a*b*c^4 - 3*a*b - 4*b^2)*log(c*sqrt(x) - 1) + 4*(3*a*b*c^4*x^2
 - 3*a*b*c^4 + (b^2*c^3*x + 3*b^2*c)*sqrt(x))*log(-(c^2*x + 2*c*sqrt(x) + 1
)/(c^2*x - 1)) + 8*(a*b*c^3*x + 3*a*b*c)*sqrt(x))/c^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(a+b\operatorname{atanh}\left(c\sqrt{x}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c*x**(1/2)))**2,x)
```

```
[Out] Integral(x*(a + b*atanh(c*sqrt(x)))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \operatorname{artanh}\left(c\sqrt{x}\right) + a\right)^2 x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^(1/2)))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*sqrt(x)) + a)^2*x, x)
```

3.198 $\int \left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2\,dx$

Optimal. Leaf size=85

$$-\frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{c^2}+\frac{2ab\sqrt{x}}{c}+x\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2+\frac{b^2\log\left(1-c^2x\right)}{c^2}+\frac{2b^2\sqrt{x}\tanh^{-1}\left(c\sqrt{x}\right)}{c}$$

```
[Out] (2*a*b*Sqrt[x])/c + (2*b^2*Sqrt[x]*ArcTanh[c*Sqrt[x]])/c - (a + b*ArcTanh[c
*Sqrt[x]])^2/c^2 + x*(a + b*ArcTanh[c*Sqrt[x]])^2 + (b^2*Log[1 - c^2*x])/c^
2
```

Rubi [F] time = 0.0063505, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcTanh[c*Sqrt[x]])^2,x]
```

```
[Out] Defer[Int][(a + b*ArcTanh[c*Sqrt[x]])^2, x]
```

Rubi steps

$$\int \left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2\,dx=\int \left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2\,dx$$

Mathematica [A] time = 0.0614956, size = 115, normalized size = 1.35

$$\frac{a^2c^2x+2abc\sqrt{x}+b(a+b)\log\left(1-c\sqrt{x}\right)-ab\log\left(c\sqrt{x}+1\right)+2bc\sqrt{x}\tanh^{-1}\left(c\sqrt{x}\right)\left(ac\sqrt{x}+b\right)+b^2\left(c^2x-1\right)\tanh^{-1}\left(c\right.}{c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*Sqrt[x]])^2,x]
```

```
[Out] (2*a*b*c*Sqrt[x] + a^2*c^2*x + 2*b*c*(b + a*c*Sqrt[x])*Sqrt[x]*ArcTanh[c*Sq
rt[x]] + b^2*(-1 + c^2*x)*ArcTanh[c*Sqrt[x]]^2 + b*(a + b)*Log[1 - c*Sqrt[x
]] - a*b*Log[1 + c*Sqrt[x]] + b^2*Log[1 + c*Sqrt[x]])/c^2
```

Maple [B] time = 0.05, size = 272, normalized size = 3.2

$$a^2x+b^2x\left(\operatorname{Artanh}\left(c\sqrt{x}\right)\right)^2+2\,\frac{b^2\operatorname{Artanh}\left(c\sqrt{x}\right)\sqrt{x}}{c}+\frac{b^2}{c^2}\operatorname{Artanh}\left(c\sqrt{x}\right)\ln\left(c\sqrt{x}-1\right)-\frac{b^2}{c^2}\operatorname{Artanh}\left(c\sqrt{x}\right)\ln\left(1+c\sqrt{x}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^(1/2)))^2,x)
```

```
[Out] a^2*x+b^2*x*arctanh(c*x^(1/2))^2+2*b^2*arctanh(c*x^(1/2))*x^(1/2)/c+1/c^2*b
^2*arctanh(c*x^(1/2))*ln(c*x^(1/2)-1)-1/c^2*b^2*arctanh(c*x^(1/2))*ln(1+c*x
^(1/2))-1/2/c^2*b^2*ln(c*x^(1/2)-1)*ln(1/2+1/2*c*x^(1/2))+1/4/c^2*b^2*ln(c*
x^(1/2)-1)^2+1/c^2*b^2*ln(c*x^(1/2)-1)+1/c^2*b^2*ln(1+c*x^(1/2))-1/2/c^2*b^
2*ln(-1/2*c*x^(1/2)+1/2)*ln(1+c*x^(1/2))+1/2/c^2*b^2*ln(-1/2*c*x^(1/2)+1/2)
*ln(1/2+1/2*c*x^(1/2))+1/4/c^2*b^2*ln(1+c*x^(1/2))^2+2*a*b*x*arctanh(c*x^(1
/2))+2*a*b*x^(1/2)/c+1/c^2*a*b*ln(c*x^(1/2)-1)-1/c^2*a*b*ln(1+c*x^(1/2))
```

Maxima [B] time = 1.01408, size = 236, normalized size = 2.78

$$\left(c\left(\frac{2\sqrt{x}}{c^2} - \frac{\log\left(c\sqrt{x}+1\right)}{c^3} + \frac{\log\left(c\sqrt{x}-1\right)}{c^3}\right) + 2x\,\mathrm{artanh}\left(c\sqrt{x}\right)\right)ab + \frac{1}{4}\left(4c\left(\frac{2\sqrt{x}}{c^2} - \frac{\log\left(c\sqrt{x}+1\right)}{c^3} + \frac{\log\left(c\sqrt{x}-1\right)}{c^3}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^2,x, algorithm="maxima")
```

```
[Out] (c*(2*sqrt(x)/c^2 - log(c*sqrt(x) + 1)/c^3 + log(c*sqrt(x) - 1)/c^3) + 2*x*
arctanh(c*sqrt(x)))*a*b + 1/4*(4*c*(2*sqrt(x)/c^2 - log(c*sqrt(x) + 1)/c^3
+ log(c*sqrt(x) - 1)/c^3)*arctanh(c*sqrt(x)) + 4*x*arctanh(c*sqrt(x))^2 - (
2*(log(c*sqrt(x) - 1) - 2)*log(c*sqrt(x) + 1) - log(c*sqrt(x) + 1)^2 - log(
c*sqrt(x) - 1)^2 - 4*log(c*sqrt(x) - 1))/c^2)*b^2 + a^2*x
```

Fricas [B] time = 1.99055, size = 382, normalized size = 4.49

$$\frac{4\,a^2c^2x + 8\,abc\sqrt{x} + \left(b^2c^2x - b^2\right)\log\left(-\frac{c^2x+2\,c\sqrt{x}+1}{c^2x-1}\right)^2 + 4\left(abc^2 - ab + b^2\right)\log\left(c\sqrt{x}+1\right) - 4\left(abc^2 - ab - b^2\right)\log\left(c\sqrt{x}\right.}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*a^2*c^2*x + 8*a*b*c*sqrt(x) + (b^2*c^2*x - b^2)*log(-(c^2*x + 2*c*sq
rt(x) + 1)/(c^2*x - 1))^2 + 4*(a*b*c^2 - a*b + b^2)*log(c*sqrt(x) + 1) - 4*
(a*b*c^2 - a*b - b^2)*log(c*sqrt(x) - 1) + 4*(a*b*c^2*x - a*b*c^2 + b^2*c*s
qrt(x))*log(-(c^2*x + 2*c*sqrt(x) + 1)/(c^2*x - 1)))/c^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\,\mathrm{atanh}\left(c\sqrt{x}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**(1/2)))**2,x)
```

```
[Out] Integral((a + b*atanh(c*sqrt(x)))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\,\mathrm{artanh}\left(c\sqrt{x}\right)+a\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*sqrt(x)) + a)^2, x)
```

3.199 $\int \frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{x}\,dx$

Optimal. Leaf size=145

$$-2b\text{PolyLog}\left(2,1-\frac{2}{1-c\sqrt{x}}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)+2b\text{PolyLog}\left(2,\frac{2}{1-c\sqrt{x}}-1\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)+b^2\text{PolyLo}$$

```
[Out] 4*ArcTanh[1 - 2/(1 - c*Sqrt[x])]*(a + b*ArcTanh[c*Sqrt[x]])^2 - 2*b*(a + b*
ArcTanh[c*Sqrt[x]])*PolyLog[2, 1 - 2/(1 - c*Sqrt[x])] + 2*b*(a + b*ArcTanh[
c*Sqrt[x]])*PolyLog[2, -1 + 2/(1 - c*Sqrt[x])] + b^2*PolyLog[3, 1 - 2/(1 -
c*Sqrt[x])] - b^2*PolyLog[3, -1 + 2/(1 - c*Sqrt[x])]
```

Rubi [A] time = 0.317312, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6095, 5914, 6052, 5948, 6058, 6610}

$$-2b\text{PolyLog}\left(2,1-\frac{2}{1-c\sqrt{x}}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)+2b\text{PolyLog}\left(2,\frac{2}{1-c\sqrt{x}}-1\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)+b^2\text{PolyLo}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*Sqrt[x]])^2/x,x]
```

```
[Out] 4*ArcTanh[1 - 2/(1 - c*Sqrt[x])]*(a + b*ArcTanh[c*Sqrt[x]])^2 - 2*b*(a + b*
ArcTanh[c*Sqrt[x]])*PolyLog[2, 1 - 2/(1 - c*Sqrt[x])] + 2*b*(a + b*ArcTanh[
c*Sqrt[x]])*PolyLog[2, -1 + 2/(1 - c*Sqrt[x])] + b^2*PolyLog[3, 1 - 2/(1 -
c*Sqrt[x])] - b^2*PolyLog[3, -1 + 2/(1 - c*Sqrt[x])]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[
1/n, Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
 n}, x] && IGtQ[p, 0]
```

Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
 e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{x}\,dx = 2\,\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x}\,dx, x, \sqrt{x}\right)$$

$$= 4\tanh^{-1}\left(1-\frac{2}{1-c\sqrt{x}}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2 - (8bc)\,\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(cx)\right)\tan}{1-c^2x^2}\right.$$

$$= 4\tanh^{-1}\left(1-\frac{2}{1-c\sqrt{x}}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2 + (4bc)\,\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(cx)\right)\log}{1-c^2x^2}\right.$$

$$= 4\tanh^{-1}\left(1-\frac{2}{1-c\sqrt{x}}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2 - 2b\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)\text{Li}_2\left(1-\frac{2}{1-c\sqrt{\ }}\right.$$

$$= 4\tanh^{-1}\left(1-\frac{2}{1-c\sqrt{x}}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2 - 2b\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)\text{Li}_2\left(1-\frac{2}{1-c\sqrt{\ }}\right.$$

Mathematica [A] time = 0.0829929, size = 164, normalized size = 1.13

$$4\tanh^{-1}\left(\frac{2}{c\sqrt{x}-1}+1\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2 - b\left(-2\text{PolyLog}\left(2,\frac{c\sqrt{x}+1}{1-c\sqrt{x}}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right) + 2\text{PolyLog}\left(2,\frac{c\sqrt{x}}{c\sqrt{x}}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*Sqrt[x]])^2/x,x]
```

```
[Out] 4*ArcTanh[1 + 2/(-1 + c*Sqrt[x])]*(a + b*ArcTanh[c*Sqrt[x]])^2 - b*(-2*(a +
 b*ArcTanh[c*Sqrt[x]])*PolyLog[2, (1 + c*Sqrt[x])/(1 - c*Sqrt[x])] + 2*(a +
 b*ArcTanh[c*Sqrt[x]])*PolyLog[2, (1 + c*Sqrt[x])/(-1 + c*Sqrt[x])] + b*(Po
lyLog[3, (1 + c*Sqrt[x])/(1 - c*Sqrt[x])] - PolyLog[3, (1 + c*Sqrt[x])/(-1
+ c*Sqrt[x])]))
```

Maple [C] time = 0.316, size = 742, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^(1/2)))^2/x,x)
```

```
[Out] 2*a^2*ln(c*x^(1/2))+2*b^2*ln(c*x^(1/2))*arctanh(c*x^(1/2))^2-2*b^2*arctanh(
c*x^(1/2))*polylog(2,-(1+c*x^(1/2))^2/(-c^2*x+1))+b^2*polylog(3,-(1+c*x^(1/
2))^2/(-c^2*x+1))-2*b^2*arctanh(c*x^(1/2))^2*ln((1+c*x^(1/2))^2/(-c^2*x+1)-
1)+2*b^2*arctanh(c*x^(1/2))^2*ln(1-(1+c*x^(1/2))/(-c^2*x+1)^(1/2))+4*b^2*ar
ctanh(c*x^(1/2))*polylog(2,(1+c*x^(1/2))/(-c^2*x+1)^(1/2))-4*b^2*polylog(3,
(1+c*x^(1/2))/(-c^2*x+1)^(1/2))+2*b^2*arctanh(c*x^(1/2))^2*ln(1+(1+c*x^(1/2
))/(-c^2*x+1)^(1/2))+4*b^2*arctanh(c*x^(1/2))*polylog(2,-(1+c*x^(1/2))/(-c^
2*x+1)^(1/2))-4*b^2*polylog(3,-(1+c*x^(1/2))/(-c^2*x+1)^(1/2))-I*b^2*Pi*csg
n(I*((1+c*x^(1/2))^2/(-c^2*x+1)-1))*csgn(I*((1+c*x^(1/2))^2/(-c^2*x+1)-1)/(
(1+c*x^(1/2))^2/(-c^2*x+1)+1))^2*arctanh(c*x^(1/2))^2-I*b^2*Pi*csgn(I/((1+c
*x^(1/2))^2/(-c^2*x+1)+1))*csgn(I*((1+c*x^(1/2))^2/(-c^2*x+1)-1)/((1+c*x^(1
/2))^2/(-c^2*x+1)+1))^2*arctanh(c*x^(1/2))^2+I*b^2*Pi*csgn(I*((1+c*x^(1/2))
^2/(-c^2*x+1)-1))*csgn(I/((1+c*x^(1/2))^2/(-c^2*x+1)+1))*csgn(I*((1+c*x^(1/
2))^2/(-c^2*x+1)-1)/((1+c*x^(1/2))^2/(-c^2*x+1)+1))*arctanh(c*x^(1/2))^2+I*
b^2*Pi*csgn(I*((1+c*x^(1/2))^2/(-c^2*x+1)-1)/((1+c*x^(1/2))^2/(-c^2*x+1)+1)
)^3*arctanh(c*x^(1/2))^2-2*a*b*ln(c*x^(1/2))*ln(1+c*x^(1/2))+4*a*b*ln(c*x^(
1/2))*arctanh(c*x^(1/2))-2*a*b*dilog(1+c*x^(1/2))-2*a*b*dilog(c*x^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{4}b^2\int\frac{\log\left(c\sqrt{x}+1\right)^2}{x}\,dx-\frac{1}{2}b^2\int\frac{\log\left(c\sqrt{x}+1\right)\log\left(-c\sqrt{x}+1\right)}{x}\,dx+\frac{1}{4}b^2\int\frac{\log\left(-c\sqrt{x}+1\right)^2}{x}\,dx+ab\int\frac{\log\left(c\sqrt{x}\right.}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^2/x,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*integrate(log(c*sqrt(x) + 1)^2/x, x) - 1/2*b^2*integrate(log(c*sqrt
(x) + 1)*log(-c*sqrt(x) + 1)/x, x) + 1/4*b^2*integrate(log(-c*sqrt(x) + 1)^
2/x, x) + a*b*integrate(log(c*sqrt(x) + 1)/x, x) - a*b*integrate(log(-c*sqr
t(x) + 1)/x, x) + a^2*log(x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2\operatorname{artanh}\left(c\sqrt{x}\right)^2+2\,ab\operatorname{artanh}\left(c\sqrt{x}\right)+a^2}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^2/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*sqrt(x))^2 + 2*a*b*arctanh(c*sqrt(x)) + a^2)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+b\operatorname{atanh}\left(c\sqrt{x}\right)\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**(1/2)))**2/x,x)
```

```
[Out] Integral((a + b*atanh(c*sqrt(x)))**2/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\,\text{artanh}\left(c\sqrt{x}\right) + a\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^2/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*sqrt(x)) + a)^2/x, x)
```

3.200 $\int \frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{x^2}\,dx$

Optimal. Leaf size=85

$$c^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2-\frac{2bc\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{\sqrt{x}}-\frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{x}+b^2c^2\log(x)-b^2c^2\log\left(1-c^2x\right)$$

```
[Out] (-2*b*c*(a + b*ArcTanh[c*Sqrt[x]]))/Sqrt[x] + c^2*(a + b*ArcTanh[c*Sqrt[x]]
)^2 - (a + b*ArcTanh[c*Sqrt[x]])^2/x + b^2*c^2*Log[x] - b^2*c^2*Log[1 - c^2
*x]
```

Rubi [F] time = 0.0238263, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcTanh[c*Sqrt[x]])^2/x^2,x]
```

```
[Out] Defer[Int][(a + b*ArcTanh[c*Sqrt[x]])^2/x^2, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{x^2}\,dx=\int \frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{x^2}\,dx$$

Mathematica [A] time = 0.110467, size = 129, normalized size = 1.52

$$-\frac{a^2-abc^2x\log\left(c\sqrt{x}+1\right)+bc^2x(a+b)\log\left(1-c\sqrt{x}\right)+2abc\sqrt{x}+2b\tanh^{-1}\left(c\sqrt{x}\right)\left(a+bc\sqrt{x}\right)+b^2c^2x\log\left(c\sqrt{x}+1\right.}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*Sqrt[x]])^2/x^2,x]
```

```
[Out] -((a^2 + 2*a*b*c*Sqrt[x] + 2*b*(a + b*c*Sqrt[x])*ArcTanh[c*Sqrt[x]] - b^2*(
-1 + c^2*x)*ArcTanh[c*Sqrt[x]]^2 + b*(a + b)*c^2*x*Log[1 - c*Sqrt[x]] - a*b
*c^2*x*Log[1 + c*Sqrt[x]] + b^2*c^2*x*Log[1 + c*Sqrt[x]] - b^2*c^2*x*Log[x]
)/x)
```

Maple [B] time = 0.054, size = 292, normalized size = 3.4

$$-\frac{a^2}{x}-\frac{b^2}{x}\left(\mathrm{Artanh}\left(c\sqrt{x}\right)\right)^2-2\frac{b^2c\,\mathrm{Artanh}\left(c\sqrt{x}\right)}{\sqrt{x}}-c^2b^2\mathrm{Artanh}\left(c\sqrt{x}\right)\ln\left(c\sqrt{x}-1\right)+c^2b^2\mathrm{Artanh}\left(c\sqrt{x}\right)\ln\left(1+c\sqrt{\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^(1/2)))^2/x^2,x)
```

```
[Out] -a^2/x-b^2/x*arctanh(c*x^(1/2))^2-2*c*b^2/x^(1/2)*arctanh(c*x^(1/2))-c^2*b^
2*arctanh(c*x^(1/2))*ln(c*x^(1/2)-1)+c^2*b^2*arctanh(c*x^(1/2))*ln(1+c*x^(1
/2))+1/2*c^2*b^2*ln(c*x^(1/2)-1)*ln(1/2+1/2*c*x^(1/2))-1/4*c^2*b^2*ln(c*x^(
1/2)-1)^2-c^2*b^2*ln(c*x^(1/2)-1)+2*c^2*b^2*ln(c*x^(1/2))-c^2*b^2*ln(1+c*x^
(1/2))+1/2*c^2*b^2*ln(-1/2*c*x^(1/2)+1/2)*ln(1+c*x^(1/2))-1/2*c^2*b^2*ln(-1
/2*c*x^(1/2)+1/2)*ln(1/2+1/2*c*x^(1/2))-1/4*c^2*b^2*ln(1+c*x^(1/2))^2-2*a*b
/x*arctanh(c*x^(1/2))-2*c*a*b/x^(1/2)-c^2*a*b*ln(c*x^(1/2)-1)+c^2*a*b*ln(1+
c*x^(1/2))
```

Maxima [B] time = 1.01967, size = 235, normalized size = 2.76

$$\left(\left(c\log\left(c\sqrt{x}+1\right)-c\log\left(c\sqrt{x}-1\right)-\frac{2}{\sqrt{x}}\right)c-\frac{2\,\text{artanh}\left(c\sqrt{x}\right)}{x}\right)ab+\frac{1}{4}\left(\left(2\left(\log\left(c\sqrt{x}-1\right)-2\right)\log\left(c\sqrt{x}+1\right)-\log\left(c\sqrt{x}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^2/x^2,x, algorithm="maxima")
```

```
[Out] ((c*log(c*sqrt(x) + 1) - c*log(c*sqrt(x) - 1) - 2/sqrt(x))*c - 2*arctanh(c*
sqrt(x))/x)*a*b + 1/4*((2*(log(c*sqrt(x) - 1) - 2)*log(c*sqrt(x) + 1) - log
(c*sqrt(x) + 1)^2 - log(c*sqrt(x) - 1)^2 - 4*log(c*sqrt(x) - 1) + 4*log(x))
*c^2 + 4*(c*log(c*sqrt(x) + 1) - c*log(c*sqrt(x) - 1) - 2/sqrt(x))*c*arctan
h(c*sqrt(x)))*b^2 - b^2*arctanh(c*sqrt(x))^2/x - a^2/x
```

Fricas [B] time = 1.83377, size = 375, normalized size = 4.41

$$\frac{8\,b^{2}c^{2}x\log\left(\sqrt{x}\right)+4\left(ab-b^{2}\right)c^{2}x\log\left(c\sqrt{x}+1\right)-4\left(ab+b^{2}\right)c^{2}x\log\left(c\sqrt{x}-1\right)-8\,abc\sqrt{x}+\left(b^{2}c^{2}x-b^{2}\right)\log\left(-\frac{c^{2}x+2\,c\sqrt{x}+}{c^{2}x-1}\right.}{4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^2/x^2,x, algorithm="fricas")
```

```
[Out] 1/4*(8*b^2*c^2*x*log(sqrt(x)) + 4*(a*b - b^2)*c^2*x*log(c*sqrt(x) + 1) - 4*
(a*b + b^2)*c^2*x*log(c*sqrt(x) - 1) - 8*a*b*c*sqrt(x) + (b^2*c^2*x - b^2)*
log(-(c^2*x + 2*c*sqrt(x) + 1)/(c^2*x - 1))^2 - 4*a^2 - 4*(b^2*c*sqrt(x) +
a*b)*log(-(c^2*x + 2*c*sqrt(x) + 1)/(c^2*x - 1)))/x
```

Sympy [A] time = 42.9305, size = 680, normalized size = 8.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**(1/2)))**2/x**2,x)
```

```
[Out] Piecewise((-a**2/x + 2*a*b*atanh(sqrt(x)*sqrt(1/x))/x - b**2*atanh(sqrt(x)*
sqrt(1/x))**2/x, Eq(c, -sqrt(1/x))), (-a**2/x - 2*a*b*atanh(sqrt(x)*sqrt(1/
x))/x - b**2*atanh(sqrt(x)*sqrt(1/x))**2/x, Eq(c, sqrt(1/x))), (-a**2/x, Eq
(c, 0)), (-a**2*c**4*x**(5/2)/(c**2*x**(5/2) - x**(3/2)) + a**2*sqrt(x)/(c*
*2*x**(5/2) - x**(3/2)) + 2*a*b*c**4*x**(5/2)*atanh(c*sqrt(x))/(c**2*x**(5/
2) - x**(3/2)) - 2*a*b*c**3*x**2/(c**2*x**(5/2) - x**(3/2)) - 4*a*b*c**2*x*
*(3/2)*atanh(c*sqrt(x))/(c**2*x**(5/2) - x**(3/2)) + 2*a*b*c*x/(c**2*x**(5/
2) - x**(3/2)) + 2*a*b*sqrt(x)*atanh(c*sqrt(x))/(c**2*x**(5/2) - x**(3/2))
+ b**2*c**4*x**(5/2)*log(x)/(c**2*x**(5/2) - x**(3/2)) - 2*b**2*c**4*x**(5/
2)*log(sqrt(x) - 1/c)/(c**2*x**(5/2) - x**(3/2)) + b**2*c**4*x**(5/2)*atanh
(c*sqrt(x))**2/(c**2*x**(5/2) - x**(3/2)) - 2*b**2*c**4*x**(5/2)*atanh(c*sq
rt(x))/(c**2*x**(5/2) - x**(3/2)) - 2*b**2*c**3*x**2*atanh(c*sqrt(x))/(c**2
*x**(5/2) - x**(3/2)) - b**2*c**2*x**(3/2)*log(x)/(c**2*x**(5/2) - x**(3/2)
) + 2*b**2*c**2*x**(3/2)*log(sqrt(x) - 1/c)/(c**2*x**(5/2) - x**(3/2)) - 2*
b**2*c**2*x**(3/2)*atanh(c*sqrt(x))**2/(c**2*x**(5/2) - x**(3/2)) + 2*b**2*
c**2*x**(3/2)*atanh(c*sqrt(x))/(c**2*x**(5/2) - x**(3/2)) + 2*b**2*c*x*atan
h(c*sqrt(x))/(c**2*x**(5/2) - x**(3/2)) + b**2*sqrt(x)*atanh(c*sqrt(x))**2/
(c**2*x**(5/2) - x**(3/2)), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \operatorname{artanh}\left(c\sqrt{x}\right) + a\right)^2}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*sqrt(x)) + a)^2/x^2, x)
```

3.201 $\int \frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{x^3}\,dx$

Optimal. Leaf size=133

$$\frac{1}{2}c^4\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2-\frac{bc^3\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{\sqrt{x}}-\frac{bc\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{3x^{3/2}}-\frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{2x^2}-\frac{b^2c^2}{6x}+\frac{2}{3}b$$

```
[Out] -(b^2*c^2)/(6*x) - (b*c*(a + b*ArcTanh[c*Sqrt[x]]))/(3*x^(3/2)) - (b*c^3*(a
 + b*ArcTanh[c*Sqrt[x]]))/Sqrt[x] + (c^4*(a + b*ArcTanh[c*Sqrt[x]])^2)/2 -
(a + b*ArcTanh[c*Sqrt[x]])^2/(2*x^2) + (2*b^2*c^4*Log[x])/3 - (2*b^2*c^4*Lo
g[1 - c^2*x])/3
```

Rubi [F] time = 0.0240127, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcTanh[c*Sqrt[x]])^2/x^3,x]
```

```
[Out] Defer[Int][(a + b*ArcTanh[c*Sqrt[x]])^2/x^3, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{x^3}\,dx=\int \frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{x^3}\,dx$$

Mathematica [A] time = 0.127832, size = 178, normalized size = 1.34

$$-\frac{3a^2+6abc^3x^{3/2}+bc^4x^2(3a+4b)\log\left(1-c\sqrt{x}\right)-3abc^4x^2\log\left(c\sqrt{x}+1\right)+2b\tanh^{-1}\left(c\sqrt{x}\right)\left(3a+bc\sqrt{x}\left(3c^2x+1\right)\right)+2}{6x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*Sqrt[x]])^2/x^3,x]
```

```
[Out] -(3*a^2 + 2*a*b*c*Sqrt[x] + b^2*c^2*x + 6*a*b*c^3*x^(3/2) + 2*b*(3*a + b*c*
Sqrt[x]*(1 + 3*c^2*x))*ArcTanh[c*Sqrt[x]] - 3*b^2*(-1 + c^4*x^2)*ArcTanh[c*
Sqrt[x]]^2 + b*(3*a + 4*b)*c^4*x^2*Log[1 - c*Sqrt[x]] - 3*a*b*c^4*x^2*Log[1
 + c*Sqrt[x]] + 4*b^2*c^4*x^2*Log[1 + c*Sqrt[x]] - 4*b^2*c^4*x^2*Log[x])/(6
*x^2)
```

Maple [B] time = 0.062, size = 332, normalized size = 2.5

$$-\frac{a^2}{2x^2}-\frac{b^2}{2x^2}\left(\mathrm{Artanh}\left(c\sqrt{x}\right)\right)^2-\frac{c^4b^2}{2}\mathrm{Artanh}\left(c\sqrt{x}\right)\ln\left(c\sqrt{x}-1\right)-\frac{b^2c}{3}\mathrm{Artanh}\left(c\sqrt{x}\right)x^{-\frac{3}{2}}-c^3b^2\mathrm{Artanh}\left(c\sqrt{x}\right)\frac{1}{\sqrt{x}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^(1/2)))^2/x^3,x)
```

```
[Out] -1/2*a^2/x^2-1/2*b^2/x^2*arctanh(c*x^(1/2))^2-1/2*c^4*b^2*arctanh(c*x^(1/2)
)*ln(c*x^(1/2)-1)-1/3*c*b^2/x^(3/2)*arctanh(c*x^(1/2))-c^3*b^2/x^(1/2)*arct
anh(c*x^(1/2))+1/2*c^4*b^2*arctanh(c*x^(1/2))*ln(1+c*x^(1/2))-1/8*c^4*b^2*l
n(c*x^(1/2)-1)^2+1/4*c^4*b^2*ln(c*x^(1/2)-1)*ln(1/2+1/2*c*x^(1/2))-1/4*c^4*
b^2*ln(-1/2*c*x^(1/2)+1/2)*ln(1/2+1/2*c*x^(1/2))+1/4*c^4*b^2*ln(-1/2*c*x^(1
/2)+1/2)*ln(1+c*x^(1/2))-1/8*c^4*b^2*ln(1+c*x^(1/2))^2-2/3*c^4*b^2*ln(c*x^(
1/2)-1)-1/6*b^2*c^2/x+4/3*c^4*b^2*ln(c*x^(1/2))-2/3*c^4*b^2*ln(1+c*x^(1/2))
-a*b/x^2*arctanh(c*x^(1/2))-1/2*c^4*a*b*ln(c*x^(1/2)-1)-1/3*c*a*b/x^(3/2)-c
^3*a*b/x^(1/2)+1/2*c^4*a*b*ln(1+c*x^(1/2))
```

Maxima [B] time = 0.984568, size = 316, normalized size = 2.38

$$\frac{1}{6}\left(\left(3\,c^3\log\left(c\sqrt{x}+1\right)-3\,c^3\log\left(c\sqrt{x}-1\right)-\frac{2\left(3\,c^2x+1\right)}{x^{\frac{3}{2}}}\right)c-\frac{6\,\mathrm{artanh}\left(c\sqrt{x}\right)}{x^2}\right)ab+\frac{1}{24}\left(\left(16\,c^2\log\left(x\right)-\frac{3\,c^2x\log\left(c\right.}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/6*((3*c^3*log(c*sqrt(x) + 1) - 3*c^3*log(c*sqrt(x) - 1) - 2*(3*c^2*x + 1)
/x^(3/2))*c - 6*arctanh(c*sqrt(x))/x^2)*a*b + 1/24*((16*c^2*log(x) - (3*c^2
*x*log(c*sqrt(x) + 1)^2 + 3*c^2*x*log(c*sqrt(x) - 1)^2 + 16*c^2*x*log(c*sqr
t(x) - 1) - 2*(3*c^2*x*log(c*sqrt(x) - 1) - 8*c^2*x)*log(c*sqrt(x) + 1) + 4
)/x)*c^2 + 4*(3*c^3*log(c*sqrt(x) + 1) - 3*c^3*log(c*sqrt(x) - 1) - 2*(3*c^
2*x + 1)/x^(3/2))*c*arctanh(c*sqrt(x)))*b^2 - 1/2*b^2*arctanh(c*sqrt(x))^2/
x^2 - 1/2*a^2/x^2
```

Fricas [A] time = 1.92422, size = 471, normalized size = 3.54

$$\frac{32\,b^2c^4x^2\log\left(\sqrt{x}\right)+4\left(3\,ab-4\,b^2\right)c^4x^2\log\left(c\sqrt{x}+1\right)-4\left(3\,ab+4\,b^2\right)c^4x^2\log\left(c\sqrt{x}-1\right)-4\,b^2c^2x+3\left(b^2c^4x^2-b^2\right)}{24\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/24*(32*b^2*c^4*x^2*log(sqrt(x)) + 4*(3*a*b - 4*b^2)*c^4*x^2*log(c*sqrt(x)
 + 1) - 4*(3*a*b + 4*b^2)*c^4*x^2*log(c*sqrt(x) - 1) - 4*b^2*c^2*x + 3*(b^2
*c^4*x^2 - b^2)*log(-(c^2*x + 2*c*sqrt(x) + 1)/(c^2*x - 1))^2 - 12*a^2 - 4*
(3*a*b + (3*b^2*c^3*x + b^2*c)*sqrt(x))*log(-(c^2*x + 2*c*sqrt(x) + 1)/(c^2
*x - 1)) - 8*(3*a*b*c^3*x + a*b*c)*sqrt(x))/x^2
```

Sympy [A] time = 179.052, size = 972, normalized size = 7.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**(1/2)))**2/x**3,x)
```

```
[Out] Piecewise((-a**2/(2*x**2) + a*b*atanh(sqrt(x)*sqrt(1/x))/x**2 - b**2*atanh(
sqrt(x)*sqrt(1/x))**2/(2*x**2), Eq(c, -sqrt(1/x))), (-a**2/(2*x**2) - a*b*a
tanh(sqrt(x)*sqrt(1/x))/x**2 - b**2*atanh(sqrt(x)*sqrt(1/x))**2/(2*x**2), E
q(c, sqrt(1/x))), (-a**2/(2*x**2), Eq(c, 0)), (-3*a**2*c**2*x**(3/2)/(6*c**
2*x**(7/2) - 6*x**(5/2)) + 3*a**2*sqrt(x)/(6*c**2*x**(7/2) - 6*x**(5/2)) +
6*a*b*c**6*x**(7/2)*atanh(c*sqrt(x))/(6*c**2*x**(7/2) - 6*x**(5/2)) - 6*a*b
*c**5*x**3/(6*c**2*x**(7/2) - 6*x**(5/2)) - 6*a*b*c**4*x**(5/2)*atanh(c*sqr
t(x))/(6*c**2*x**(7/2) - 6*x**(5/2)) + 4*a*b*c**3*x**2/(6*c**2*x**(7/2) - 6
*x**(5/2)) - 6*a*b*c**2*x**(3/2)*atanh(c*sqrt(x))/(6*c**2*x**(7/2) - 6*x**(
5/2)) + 2*a*b*c*x/(6*c**2*x**(7/2) - 6*x**(5/2)) + 6*a*b*sqrt(x)*atanh(c*sq
rt(x))/(6*c**2*x**(7/2) - 6*x**(5/2)) + 4*b**2*c**6*x**(7/2)*log(x)/(6*c**2
*x**(7/2) - 6*x**(5/2)) - 8*b**2*c**6*x**(7/2)*log(sqrt(x) - 1/c)/(6*c**2*x
**(7/2) - 6*x**(5/2)) + 3*b**2*c**6*x**(7/2)*atanh(c*sqrt(x))**2/(6*c**2*x*
*(7/2) - 6*x**(5/2)) - 8*b**2*c**6*x**(7/2)*atanh(c*sqrt(x))/(6*c**2*x**(7/
2) - 6*x**(5/2)) - b**2*c**6*x**(7/2)/(6*c**2*x**(7/2) - 6*x**(5/2)) - 6*b*
*2*c**5*x**3*atanh(c*sqrt(x))/(6*c**2*x**(7/2) - 6*x**(5/2)) - 4*b**2*c**4*
x**(5/2)*log(x)/(6*c**2*x**(7/2) - 6*x**(5/2)) + 8*b**2*c**4*x**(5/2)*log(s
qrt(x) - 1/c)/(6*c**2*x**(7/2) - 6*x**(5/2)) - 3*b**2*c**4*x**(5/2)*atanh(c
*sqrt(x))**2/(6*c**2*x**(7/2) - 6*x**(5/2)) + 8*b**2*c**4*x**(5/2)*atanh(c*
sqrt(x))/(6*c**2*x**(7/2) - 6*x**(5/2)) + 4*b**2*c**3*x**2*atanh(c*sqrt(x))
/(6*c**2*x**(7/2) - 6*x**(5/2)) - 3*b**2*c**2*x**(3/2)*atanh(c*sqrt(x))**2/
(6*c**2*x**(7/2) - 6*x**(5/2)) + b**2*c**2*x**(3/2)/(6*c**2*x**(7/2) - 6*x*
*(5/2)) + 2*b**2*c*x*atanh(c*sqrt(x))/(6*c**2*x**(7/2) - 6*x**(5/2)) + 3*b*
*2*sqrt(x)*atanh(c*sqrt(x))**2/(6*c**2*x**(7/2) - 6*x**(5/2)), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\,\mathrm{artanh}\left(c\sqrt{x}\right)+a\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^2/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*sqrt(x)) + a)^2/x^3, x)
```

3.202 $\int x^3 \left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3\,dx$

Optimal. Leaf size=374

$$-\frac{44b^3\text{PolyLog}\left(2,1-\frac{2}{1-c\sqrt{x}}\right)}{35c^8}+\frac{b^2x^3\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{28c^2}+\frac{9b^2x^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{70c^4}+\frac{71b^2x\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{140c^6}$$

```
[Out] (47*b^3*Sqrt[x])/(70*c^7) + (23*b^3*x^(3/2))/(420*c^5) + (b^3*x^(5/2))/(140
*c^3) - (47*b^3*ArcTanh[c*Sqrt[x]])/(70*c^8) + (71*b^2*x*(a + b*ArcTanh[c*S
qrt[x]]))/(140*c^6) + (9*b^2*x^2*(a + b*ArcTanh[c*Sqrt[x]]))/(70*c^4) + (b^
2*x^3*(a + b*ArcTanh[c*Sqrt[x]]))/(28*c^2) + (44*b*(a + b*ArcTanh[c*Sqrt[x]
])^2)/(35*c^8) + (3*b*Sqrt[x]*(a + b*ArcTanh[c*Sqrt[x]])^2)/(4*c^7) + (b*x^
(3/2)*(a + b*ArcTanh[c*Sqrt[x]])^2)/(4*c^5) + (3*b*x^(5/2)*(a + b*ArcTanh[c
*Sqrt[x]])^2)/(20*c^3) + (3*b*x^(7/2)*(a + b*ArcTanh[c*Sqrt[x]])^2)/(28*c)
- (a + b*ArcTanh[c*Sqrt[x]])^3/(4*c^8) + (x^4*(a + b*ArcTanh[c*Sqrt[x]])^3)
/4 - (88*b^2*(a + b*ArcTanh[c*Sqrt[x]])*Log[2/(1 - c*Sqrt[x])])/(35*c^8) -
(44*b^3*PolyLog[2, 1 - 2/(1 - c*Sqrt[x])])/(35*c^8)
```

Rubi [F] time = 0.0233636, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^3\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^3*(a + b*ArcTanh[c*Sqrt[x]])^3,x]
```

```
[Out] Defer[Int][x^3*(a + b*ArcTanh[c*Sqrt[x]])^3, x]
```

Rubi steps

$$\int x^3\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3\,dx=\int x^3\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3\,dx$$

Mathematica [A] time = 1.1936, size = 418, normalized size = 1.12

$$1056b^3\text{PolyLog}\left(2,-e^{-2\tanh^{-1}\left(c\sqrt{x}\right)}\right)+6b\tanh^{-1}\left(c\sqrt{x}\right)\left(105a^2c^8x^4+2abc\sqrt{x}\left(15c^6x^3+21c^4x^2+35c^2x+105\right)+b^2\left(\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*(a + b*ArcTanh[c*Sqrt[x]])^3,x]
```

```
[Out] (-564*a*b^2 + 630*a^2*b*c*Sqrt[x] + 564*b^3*c*Sqrt[x] + 426*a*b^2*c^2*x + 2
10*a^2*b*c^3*x^(3/2) + 46*b^3*c^3*x^(3/2) + 108*a*b^2*c^4*x^2 + 126*a^2*b*c
^5*x^(5/2) + 6*b^3*c^5*x^(5/2) + 30*a*b^2*c^6*x^3 + 90*a^2*b*c^7*x^(7/2) +
210*a^3*c^8*x^4 + 6*b^2*(b*(-176 + 105*c*Sqrt[x] + 35*c^3*x^(3/2) + 21*c^5*
x^(5/2) + 15*c^7*x^(7/2)) + 105*a*(-1 + c^8*x^4))*ArcTanh[c*Sqrt[x]]^2 + 21
0*b^3*(-1 + c^8*x^4)*ArcTanh[c*Sqrt[x]]^3 + 6*b*ArcTanh[c*Sqrt[x]]*(105*a^2
*c^8*x^4 + b^2*(-94 + 71*c^2*x + 18*c^4*x^2 + 5*c^6*x^3) + 2*a*b*c*Sqrt[x]*
(105 + 35*c^2*x + 21*c^4*x^2 + 15*c^6*x^3) - 352*b^2*Log[1 + E^(-2*ArcTanh[
```

```
c*Sqrt[x]])]) + 315*a^2*b*Log[1 - c*Sqrt[x]] - 315*a^2*b*Log[1 + c*Sqrt[x]]
 + 1056*a*b^2*Log[1 - c^2*x] + 1056*b^3*PolyLog[2, -E^(-2*ArcTanh[c*Sqrt[x]
])])/(840*c^8)
```

Maple [C] time = 1.179, size = 1518, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c*x^(1/2)))^3,x)
```

```
[Out] -1/4/c^8*b^3*arctanh(c*x^(1/2))^3+1/4*b^3*x^4*arctanh(c*x^(1/2))^3-11/15/c^
8*b^3-47/70*b^3*arctanh(c*x^(1/2))/c^8+47/70*b^3*x^(1/2)/c^7+23/420*b^3*x^(
3/2)/c^5+1/140*b^3*x^(5/2)/c^3+1/28*a*b^2*x^3/c^2+9/70/c^4*x^2*a*b^2+71/140
/c^6*b^2*x*a+3/4/c^7*a^2*b*x^(1/2)+3/20/c^3*a^2*b*x^(5/2)+1/4/c^5*a^2*b*x^(
3/2)+3/16/c^8*a*b^2*ln(c*x^(1/2)-1)^2+3/16/c^8*a*b^2*ln(1+c*x^(1/2))^2+44/3
5/c^8*a*b^2*ln(c*x^(1/2)-1)+44/35/c^8*a*b^2*ln(1+c*x^(1/2))+3/8/c^8*a^2*b*l
n(c*x^(1/2)-1)-3/8/c^8*a^2*b*ln(1+c*x^(1/2))-88/35/c^8*b^3*arctanh(c*x^(1/2
))*ln(1+I*(1+c*x^(1/2))/(-c^2*x+1)^(1/2))-88/35/c^8*b^3*arctanh(c*x^(1/2))*
ln(1-I*(1+c*x^(1/2))/(-c^2*x+1)^(1/2))+3/8/c^8*b^3*arctanh(c*x^(1/2))^2*ln(
c*x^(1/2)-1)-3/8/c^8*b^3*arctanh(c*x^(1/2))^2*ln(1+c*x^(1/2))+3/4/c^8*b^3*a
rctanh(c*x^(1/2))^2*ln((1+c*x^(1/2))/(-c^2*x+1)^(1/2))+3/28/c*x^(7/2)*a^2*b
+9/70/c^4*b^3*arctanh(c*x^(1/2))*x^2+71/140/c^6*b^3*arctanh(c*x^(1/2))*x+1/
28/c^2*b^3*arctanh(c*x^(1/2))*x^3+3/4/c^7*b^3*arctanh(c*x^(1/2))^2*x^(1/2)+
3/28/c*b^3*arctanh(c*x^(1/2))^2*x^(7/2)+3/20/c^3*b^3*arctanh(c*x^(1/2))^2*x
^(5/2)+1/4/c^5*b^3*arctanh(c*x^(1/2))^2*x^(3/2)+3/4*a*b^2*x^4*arctanh(c*x^(
1/2))^2+3/4*a^2*b*x^4*arctanh(c*x^(1/2))+1/4*x^4*a^3+3/8/c^8*a*b^2*ln(-1/2*
c*x^(1/2)+1/2)*ln(1/2+1/2*c*x^(1/2))+3/4/c^8*a*b^2*arctanh(c*x^(1/2))*ln(c*
x^(1/2)-1)-3/4/c^8*a*b^2*arctanh(c*x^(1/2))*ln(1+c*x^(1/2))-3/8/c^8*a*b^2*l
n(c*x^(1/2)-1)*ln(1/2+1/2*c*x^(1/2))+3/14/c*a*b^2*arctanh(c*x^(1/2))*x^(7/2
)+3/10/c^3*a*b^2*arctanh(c*x^(1/2))*x^(5/2)+1/2/c^5*a*b^2*arctanh(c*x^(1/2)
)*x^(3/2)+3/2/c^7*a*b^2*x^(1/2)*arctanh(c*x^(1/2))-3/8/c^8*a*b^2*ln(-1/2*c*
x^(1/2)+1/2)*ln(1+c*x^(1/2))-3/8*I/c^8*b^3*Pi*arctanh(c*x^(1/2))^2-3/16*I/c
^8*b^3*Pi*csgn(I/((1+c*x^(1/2))^2/(-c^2*x+1)+1))*csgn(I*(1+c*x^(1/2))^2/(c^
2*x-1))*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1)/((1+c*x^(1/2))^2/(-c^2*x+1)+1))*ar
ctanh(c*x^(1/2))^2+44/35/c^8*b^3*arctanh(c*x^(1/2))^2-88/35/c^8*b^3*dilog(1
-I*(1+c*x^(1/2))/(-c^2*x+1)^(1/2))-88/35/c^8*b^3*dilog(1+I*(1+c*x^(1/2))/(-
c^2*x+1)^(1/2))+3/16*I/c^8*b^3*Pi*csgn(I/((1+c*x^(1/2))^2/(-c^2*x+1)+1))*cs
gn(I*(1+c*x^(1/2))^2/(c^2*x-1)/((1+c*x^(1/2))^2/(-c^2*x+1)+1))^2*arctanh(c*
x^(1/2))^2-3/16*I/c^8*b^3*Pi*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1))*csgn(I*(1+c*
x^(1/2))^2/(c^2*x-1)/((1+c*x^(1/2))^2/(-c^2*x+1)+1))^2*arctanh(c*x^(1/2))^2
+3/8*I/c^8*b^3*Pi*csgn(I*(1+c*x^(1/2))/(-c^2*x+1)^(1/2))*csgn(I*(1+c*x^(1/2
))^2/(c^2*x-1))^2*arctanh(c*x^(1/2))^2+3/16*I/c^8*b^3*Pi*csgn(I*(1+c*x^(1/2
))/(-c^2*x+1)^(1/2))^2*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1))*arctanh(c*x^(1/2))
^2+3/16*I/c^8*b^3*Pi*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1))^3*arctanh(c*x^(1/2))
^2+3/8*I/c^8*b^3*Pi*csgn(I/((1+c*x^(1/2))^2/(-c^2*x+1)+1))^2*arctanh(c*x^(1
/2))^2-3/8*I/c^8*b^3*Pi*csgn(I/((1+c*x^(1/2))^2/(-c^2*x+1)+1))^3*arctanh(c*
x^(1/2))^2+3/16*I/c^8*b^3*Pi*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1)/((1+c*x^(1/2)
)^2/(-c^2*x+1)+1))^3*arctanh(c*x^(1/2))^2
```

Maxima [B] time = 4.15179, size = 2662, normalized size = 7.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^(1/2)))^3,x, algorithm="maxima")
```

```
[Out] 1/4*a^3*x^4 - 1/26880*a*b^2*c*((315*c^7*x^4 + 500*c^5*x^3 + 1002*c^3*x^2 +
3684*c*x - 12*(105*c^7*x^4 + 120*c^6*x^(7/2) + 140*c^5*x^3 + 168*c^4*x^(5/2
) + 210*c^3*x^2 + 280*c^2*x^(3/2) + 420*c*x + 840*sqrt(x))*log(c*sqrt(x) +
1))/c^8 - 6396*log(c*sqrt(x) + 1)/c^9 - 6396*log(c*sqrt(x) - 1)/c^9) - 1/22
40*(840*x^4*log(c*sqrt(x) + 1) - c*((105*c^7*x^4 - 120*c^6*x^(7/2) + 140*c^
5*x^3 - 168*c^4*x^(5/2) + 210*c^3*x^2 - 280*c^2*x^(3/2) + 420*c*x - 840*sqr
t(x))/c^8 + 840*log(c*sqrt(x) + 1)/c^9))*a*b^2*log(-c*sqrt(x) + 1) + 1/2240
*(840*x^4*log(c*sqrt(x) + 1) - c*((105*c^7*x^4 - 120*c^6*x^(7/2) + 140*c^5*
x^3 - 168*c^4*x^(5/2) + 210*c^3*x^2 - 280*c^2*x^(3/2) + 420*c*x - 840*sqrt(
x))/c^8 + 840*log(c*sqrt(x) + 1)/c^9))*a^2*b - 1/2240*(840*x^4*log(-c*sqrt(
x) + 1) - c*((105*c^7*x^4 + 120*c^6*x^(7/2) + 140*c^5*x^3 + 168*c^4*x^(5/2)
 + 210*c^3*x^2 + 280*c^2*x^(3/2) + 420*c*x + 840*sqrt(x))/c^8 + 840*log(c*s
qrt(x) - 1)/c^9))*a^2*b + 1/1881600*(11025*(32*log(-c*sqrt(x) + 1)^2 - 8*lo
g(-c*sqrt(x) + 1) + 1)*(c*sqrt(x) - 1)^8 + 57600*(49*log(-c*sqrt(x) + 1)^2
- 14*log(-c*sqrt(x) + 1) + 2)*(c*sqrt(x) - 1)^7 + 548800*(18*log(-c*sqrt(x)
 + 1)^2 - 6*log(-c*sqrt(x) + 1) + 1)*(c*sqrt(x) - 1)^6 + 790272*(25*log(-c*
sqrt(x) + 1)^2 - 10*log(-c*sqrt(x) + 1) + 2)*(c*sqrt(x) - 1)^5 + 3087000*(8
*log(-c*sqrt(x) + 1)^2 - 4*log(-c*sqrt(x) + 1) + 1)*(c*sqrt(x) - 1)^4 + 219
5200*(9*log(-c*sqrt(x) + 1)^2 - 6*log(-c*sqrt(x) + 1) + 2)*(c*sqrt(x) - 1)^
3 + 4939200*(2*log(-c*sqrt(x) + 1)^2 - 2*log(-c*sqrt(x) + 1) + 1)*(c*sqrt(x
) - 1)^2 + 2822400*(log(-c*sqrt(x) + 1)^2 - 2*log(-c*sqrt(x) + 1) + 2)*(c*s
qrt(x) - 1))*a*b^2/c^8 - 1/3161088000*(385875*(256*log(-c*sqrt(x) + 1)^3 -
96*log(-c*sqrt(x) + 1)^2 + 24*log(-c*sqrt(x) + 1) - 3)*(c*sqrt(x) - 1)^8 +
2304000*(343*log(-c*sqrt(x) + 1)^3 - 147*log(-c*sqrt(x) + 1)^2 + 42*log(-c*
sqrt(x) + 1) - 6)*(c*sqrt(x) - 1)^7 + 76832000*(36*log(-c*sqrt(x) + 1)^3 -
18*log(-c*sqrt(x) + 1)^2 + 6*log(-c*sqrt(x) + 1) - 1)*(c*sqrt(x) - 1)^6 + 4
4255232*(125*log(-c*sqrt(x) + 1)^3 - 75*log(-c*sqrt(x) + 1)^2 + 30*log(-c*s
qrt(x) + 1) - 6)*(c*sqrt(x) - 1)^5 + 216090000*(32*log(-c*sqrt(x) + 1)^3 -
24*log(-c*sqrt(x) + 1)^2 + 12*log(-c*sqrt(x) + 1) - 3)*(c*sqrt(x) - 1)^4 +
614656000*(9*log(-c*sqrt(x) + 1)^3 - 9*log(-c*sqrt(x) + 1)^2 + 6*log(-c*sqr
t(x) + 1) - 2)*(c*sqrt(x) - 1)^3 + 691488000*(4*log(-c*sqrt(x) + 1)^3 - 6*l
og(-c*sqrt(x) + 1)^2 + 6*log(-c*sqrt(x) + 1) - 3)*(c*sqrt(x) - 1)^2 + 79027
2000*(log(-c*sqrt(x) + 1)^3 - 3*log(-c*sqrt(x) + 1)^2 + 6*log(-c*sqrt(x) +
1) - 6)*(c*sqrt(x) - 1))*b^3/c^8 + 44/35*(log(c*sqrt(x) + 1)*log(-1/2*c*sqr
t(x) + 1/2) + dilog(1/2*c*sqrt(x) + 1/2))*b^3/c^8 - 1881559/3763200*b^3*log
(c*sqrt(x) - 1)/c^8 + 1/2240*(2283*a*b^2 - 752*b^3)*log(c*sqrt(x) + 1)/c^8
+ 1/3161088000*(1157625*(16*a*b^2*c^8 - b^3*c^8)*x^4 - 27000*(1680*a*b^2*c^
7 + 169*b^3*c^7)*x^(7/2) + 3500*(24528*a*b^2*c^6 - 3565*b^3*c^6)*x^3 + 9878
4000*(b^3*c^8*x^4 - b^3)*log(c*sqrt(x) + 1)^3 - 168*(895440*a*b^2*c^5 + 442
69*b^3*c^5)*x^(5/2) + 210*(1248240*a*b^2*c^4 - 334699*b^3*c^4)*x^2 + 564480
0*(105*a*b^2*c^8*x^4 + 15*b^3*c^7*x^(7/2) + 21*b^3*c^5*x^(5/2) + 35*b^3*c^3
*x^(3/2) + 105*b^3*c*sqrt(x) - 105*a*b^2 + 176*b^3)*log(c*sqrt(x) + 1)^2 -
352800*(105*b^3*c^8*x^4 - 120*b^3*c^7*x^(7/2) + 140*b^3*c^6*x^3 - 168*b^3*c
^5*x^(5/2) + 210*b^3*c^4*x^2 - 280*b^3*c^3*x^(3/2) + 420*b^3*c^2*x - 840*b^
3*c*sqrt(x) + 533*b^3 - 840*(b^3*c^8*x^4 - b^3)*log(c*sqrt(x) + 1))*log(-c*
sqrt(x) + 1)^2 - 280*(1718640*a*b^2*c^3 + 2899*b^3*c^3)*x^(3/2) + 420*(2424
240*a*b^2*c^2 - 1227199*b^3*c^2)*x - 1411200*(105*a*b^2*c^8*x^4 - 120*a*b^2
*c^7*x^(7/2) - 168*a*b^2*c^5*x^(5/2) - 280*a*b^2*c^3*x^(3/2) - 840*a*b^2*c*
sqrt(x) + 20*(7*a*b^2*c^6 - 2*b^3*c^6)*x^3 + 6*(35*a*b^2*c^4 - 24*b^3*c^4)*
x^2 + 4*(105*a*b^2*c^2 - 142*b^3*c^2)*x)*log(c*sqrt(x) + 1) + 840*(11025*b^
3*c^8*x^4 + 27000*b^3*c^7*x^(7/2) - 16100*b^3*c^6*x^3 + 89544*b^3*c^5*x^(5/
2) - 85890*b^3*c^4*x^2 + 286440*b^3*c^3*x^(3/2) - 348180*b^3*c^2*x + 191772
0*b^3*c*sqrt(x) - 352800*(b^3*c^8*x^4 - b^3)*log(c*sqrt(x) + 1)^2 - 13440*(
15*b^3*c^7*x^(7/2) + 21*b^3*c^5*x^(5/2) + 35*b^3*c^3*x^(3/2) + 105*b^3*c*sq
rt(x) + 176*b^3)*log(c*sqrt(x) + 1))*log(-c*sqrt(x) + 1) - 840*(3835440*a*b
^2*c + 618199*b^3*c)*sqrt(x))/c^8
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(b^3x^3\,\text{artanh}\left(c\sqrt{x}\right)^3+3\,ab^2x^3\,\text{artanh}\left(c\sqrt{x}\right)^2+3\,a^2bx^3\,\text{artanh}\left(c\sqrt{x}\right)+a^3x^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^(1/2)))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^3*arctanh(c*sqrt(x))^3 + 3*a*b^2*x^3*arctanh(c*sqrt(x))^2 +
3*a^2*b*x^3*arctanh(c*sqrt(x)) + a^3*x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c*x**(1/2)))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\,\text{artanh}\left(c\sqrt{x}\right)+a\right)^3x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^(1/2)))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*sqrt(x)) + a)^3*x^3, x)
```

3.203 $\int x^2 \left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3 dx$

Optimal. Leaf size=304

$$-\frac{23b^3\text{PolyLog}\left(2,1-\frac{2}{1-c\sqrt{x}}\right)}{15c^6}+\frac{b^2x^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{10c^2}+\frac{8b^2x\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{15c^4}-\frac{46b^2\log\left(\frac{2}{1-c\sqrt{x}}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{15c^6}$$

```
[Out] (19*b^3*Sqrt[x])/(30*c^5) + (b^3*x^(3/2))/(30*c^3) - (19*b^3*ArcTanh[c*Sqrt
[x]])/(30*c^6) + (8*b^2*x*(a + b*ArcTanh[c*Sqrt[x]]))/(15*c^4) + (b^2*x^2*(
a + b*ArcTanh[c*Sqrt[x]]))/(10*c^2) + (23*b*(a + b*ArcTanh[c*Sqrt[x]])^2)/(
15*c^6) + (b*Sqrt[x]*(a + b*ArcTanh[c*Sqrt[x]])^2)/c^5 + (b*x^(3/2)*(a + b*
ArcTanh[c*Sqrt[x]])^2)/(3*c^3) + (b*x^(5/2)*(a + b*ArcTanh[c*Sqrt[x]])^2)/(
5*c) - (a + b*ArcTanh[c*Sqrt[x]])^3/(3*c^6) + (x^3*(a + b*ArcTanh[c*Sqrt[x]
])^3)/3 - (46*b^2*(a + b*ArcTanh[c*Sqrt[x]])*Log[2/(1 - c*Sqrt[x])])/(15*c^
6) - (23*b^3*PolyLog[2, 1 - 2/(1 - c*Sqrt[x])])/(15*c^6)
```

Rubi [F] time = 0.0237346, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3 dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2*(a + b*ArcTanh[c*Sqrt[x]])^3,x]
```

```
[Out] Defer[Int][x^2*(a + b*ArcTanh[c*Sqrt[x]])^3, x]
```

Rubi steps

$$\int x^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3 dx=\int x^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3 dx$$

Mathematica [A] time = 0.789276, size = 351, normalized size = 1.15

$$46b^3\text{PolyLog}\left(2,-e^{-2\tanh^{-1}\left(c\sqrt{x}\right)}\right)+b\tanh^{-1}\left(c\sqrt{x}\right)\left(30a^2c^6x^3+4abc\sqrt{x}\left(3c^4x^2+5c^2x+15\right)+b^2\left(3c^4x^2+16c^2x-1\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(a + b*ArcTanh[c*Sqrt[x]])^3,x]
```

```
[Out] (-19*a*b^2 + 30*a^2*b*c*Sqrt[x] + 19*b^3*c*Sqrt[x] + 16*a*b^2*c^2*x + 10*a^
2*b*c^3*x^(3/2) + b^3*c^3*x^(3/2) + 3*a*b^2*c^4*x^2 + 6*a^2*b*c^5*x^(5/2) +
 10*a^3*c^6*x^3 + 2*b^2*(b*(-23 + 15*c*Sqrt[x] + 5*c^3*x^(3/2) + 3*c^5*x^(5
/2)) + 15*a*(-1 + c^6*x^3))*ArcTanh[c*Sqrt[x]]^2 + 10*b^3*(-1 + c^6*x^3)*Ar
cTanh[c*Sqrt[x]]^3 + b*ArcTanh[c*Sqrt[x]]*(30*a^2*c^6*x^3 + 4*a*b*c*Sqrt[x]
*(15 + 5*c^2*x + 3*c^4*x^2) + b^2*(-19 + 16*c^2*x + 3*c^4*x^2) - 92*b^2*Log
[1 + E^(-2*ArcTanh[c*Sqrt[x]])]) + 15*a^2*b*Log[1 - c*Sqrt[x]] - 15*a^2*b*L
og[1 + c*Sqrt[x]] + 46*a*b^2*Log[1 - c^2*x] + 46*b^3*PolyLog[2, -E^(-2*ArcT
anh[c*Sqrt[x]])])/(30*c^6)
```

Maple [C] time = 0.326, size = 1423, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c*x^(1/2)))^3,x)
```

```
[Out] -1/4*I/c^6*b^3*Pi*csgn(I/((1+c*x^(1/2))^2/(-c^2*x+1)+1))*csgn(I*(1+c*x^(1/2
))^2/(c^2*x-1))*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1)/((1+c*x^(1/2))^2/(-c^2*x+1
)+1))*arctanh(c*x^(1/2))^2-19/30*b^3*arctanh(c*x^(1/2))/c^6+19/30*b^3*x^(1/
2)/c^5+1/3*b^3*x^3*arctanh(c*x^(1/2))^3+23/15/c^6*b^3*arctanh(c*x^(1/2))^2-
46/15/c^6*b^3*dilog(1-I*(1+c*x^(1/2))/(-c^2*x+1)^(1/2))-46/15/c^6*b^3*dilog
(1+I*(1+c*x^(1/2))/(-c^2*x+1)^(1/2))-1/3/c^6*b^3*arctanh(c*x^(1/2))^3+1/3*x
^3*a^3+1/30*b^3*x^(3/2)/c^3+1/2*I/c^6*b^3*Pi*csgn(I/((1+c*x^(1/2))^2/(-c^2*
x+1)+1))^2*arctanh(c*x^(1/2))^2+1/4*I/c^6*b^3*Pi*csgn(I*(1+c*x^(1/2))^2/(c^
2*x-1)/((1+c*x^(1/2))^2/(-c^2*x+1)+1))^3*arctanh(c*x^(1/2))^2+1/4*I/c^6*b^3
*Pi*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1))^3*arctanh(c*x^(1/2))^2-1/2*I/c^6*b^3*
Pi*csgn(I/((1+c*x^(1/2))^2/(-c^2*x+1)+1))^3*arctanh(c*x^(1/2))^2+a*b^2*x^3*
arctanh(c*x^(1/2))^2+a^2*b*x^3*arctanh(c*x^(1/2))+1/10/c^2*b^3*arctanh(c*x^
(1/2))*x^2+8/15/c^4*b^3*arctanh(c*x^(1/2))*x+1/c^5*b^3*arctanh(c*x^(1/2))^2
*x^(1/2)+1/5/c*b^3*arctanh(c*x^(1/2))^2*x^(5/2)+1/3/c^3*b^3*arctanh(c*x^(1/
2))^2*x^(3/2)+1/c^5*a^2*b*x^(1/2)+1/5/c*a^2*b*x^(5/2)+1/3/c^3*a^2*b*x^(3/2)
-46/15/c^6*b^3*arctanh(c*x^(1/2))*ln(1+I*(1+c*x^(1/2))/(-c^2*x+1)^(1/2))-46
/15/c^6*b^3*arctanh(c*x^(1/2))*ln(1-I*(1+c*x^(1/2))/(-c^2*x+1)^(1/2))+1/2/c
^6*b^3*arctanh(c*x^(1/2))^2*ln(c*x^(1/2)-1)-1/2/c^6*b^3*arctanh(c*x^(1/2))^
2*ln(1+c*x^(1/2))+1/c^6*b^3*arctanh(c*x^(1/2))^2*ln((1+c*x^(1/2))/(-c^2*x+1
)^(1/2))+1/4/c^6*a*b^2*ln(c*x^(1/2)-1)^2+1/4/c^6*a*b^2*ln(1+c*x^(1/2))^2+23
/15/c^6*a*b^2*ln(c*x^(1/2)-1)+23/15/c^6*a*b^2*ln(1+c*x^(1/2))+1/2/c^6*a^2*b
*ln(c*x^(1/2)-1)-1/2/c^6*a^2*b*ln(1+c*x^(1/2))+8/15*a*b^2*x/c^4-1/4*I/c^6*b
^3*Pi*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1))*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1)/((
1+c*x^(1/2))^2/(-c^2*x+1)+1))^2*arctanh(c*x^(1/2))^2-2/3/c^6*b^3+1/4*I/c^6*
b^3*Pi*csgn(I*(1+c*x^(1/2))/(-c^2*x+1)^(1/2))^2*csgn(I*(1+c*x^(1/2))^2/(c^2
*x-1))*arctanh(c*x^(1/2))^2+1/4*I/c^6*b^3*Pi*csgn(I/((1+c*x^(1/2))^2/(-c^2*
x+1)+1))*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1)/((1+c*x^(1/2))^2/(-c^2*x+1)+1))^2
*arctanh(c*x^(1/2))^2+1/2*I/c^6*b^3*Pi*csgn(I*(1+c*x^(1/2))/(-c^2*x+1)^(1/2
))*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1))^2*arctanh(c*x^(1/2))^2-1/2/c^6*a*b^2*l
n(-1/2*c*x^(1/2)+1/2)*ln(1+c*x^(1/2))+1/2/c^6*a*b^2*ln(-1/2*c*x^(1/2)+1/2)*
ln(1/2+1/2*c*x^(1/2))+1/c^6*a*b^2*arctanh(c*x^(1/2))*ln(c*x^(1/2)-1)-1/c^6*
a*b^2*arctanh(c*x^(1/2))*ln(1+c*x^(1/2))-1/2/c^6*a*b^2*ln(c*x^(1/2)-1)*ln(1
/2+1/2*c*x^(1/2))+2/5/c*a*b^2*arctanh(c*x^(1/2))*x^(5/2)+2/3/c^3*a*b^2*arct
anh(c*x^(1/2))*x^(3/2)+2/c^5*a*b^2*x^(1/2)*arctanh(c*x^(1/2))-1/2*I/c^6*b^3
*Pi*arctanh(c*x^(1/2))^2+1/10/c^2*x^2*a*b^2
```

Maxima [B] time = 3.73239, size = 2132, normalized size = 7.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^(1/2)))^3,x, algorithm="maxima")
```

```
[Out] 1/3*a^3*x^3 - 1/720*a*b^2*c*((20*c^5*x^3 + 39*c^3*x^2 + 138*c*x - 6*(10*c^5
*x^3 + 12*c^4*x^(5/2) + 15*c^3*x^2 + 20*c^2*x^(3/2) + 30*c*x + 60*sqrt(x))*
log(c*sqrt(x) + 1))/c^6 - 222*log(c*sqrt(x) + 1)/c^7 - 222*log(c*sqrt(x) -
```

```
1)/c^7) - 1/120*(60*x^3*log(c*sqrt(x) + 1) - c*((10*c^5*x^3 - 12*c^4*x^(5/2
) + 15*c^3*x^2 - 20*c^2*x^(3/2) + 30*c*x - 60*sqrt(x))/c^6 + 60*log(c*sqrt(
x) + 1)/c^7))*a*b^2*log(-c*sqrt(x) + 1) + 1/120*(60*x^3*log(c*sqrt(x) + 1)
- c*((10*c^5*x^3 - 12*c^4*x^(5/2) + 15*c^3*x^2 - 20*c^2*x^(3/2) + 30*c*x -
60*sqrt(x))/c^6 + 60*log(c*sqrt(x) + 1)/c^7))*a^2*b - 1/120*(60*x^3*log(-c*
sqrt(x) + 1) - c*((10*c^5*x^3 + 12*c^4*x^(5/2) + 15*c^3*x^2 + 20*c^2*x^(3/2
) + 30*c*x + 60*sqrt(x))/c^6 + 60*log(c*sqrt(x) - 1)/c^7))*a^2*b + 1/7200*(
100*(18*log(-c*sqrt(x) + 1)^2 - 6*log(-c*sqrt(x) + 1) + 1)*(c*sqrt(x) - 1)^
6 + 432*(25*log(-c*sqrt(x) + 1)^2 - 10*log(-c*sqrt(x) + 1) + 2)*(c*sqrt(x)
- 1)^5 + 3375*(8*log(-c*sqrt(x) + 1)^2 - 4*log(-c*sqrt(x) + 1) + 1)*(c*sqrt
(x) - 1)^4 + 4000*(9*log(-c*sqrt(x) + 1)^2 - 6*log(-c*sqrt(x) + 1) + 2)*(c*
sqrt(x) - 1)^3 + 13500*(2*log(-c*sqrt(x) + 1)^2 - 2*log(-c*sqrt(x) + 1) + 1
)*(c*sqrt(x) - 1)^2 + 10800*(log(-c*sqrt(x) + 1)^2 - 2*log(-c*sqrt(x) + 1)
+ 2)*(c*sqrt(x) - 1))*a*b^2/c^6 - 1/864000*(1000*(36*log(-c*sqrt(x) + 1)^3
- 18*log(-c*sqrt(x) + 1)^2 + 6*log(-c*sqrt(x) + 1) - 1)*(c*sqrt(x) - 1)^6 +
 1728*(125*log(-c*sqrt(x) + 1)^3 - 75*log(-c*sqrt(x) + 1)^2 + 30*log(-c*sqr
t(x) + 1) - 6)*(c*sqrt(x) - 1)^5 + 16875*(32*log(-c*sqrt(x) + 1)^3 - 24*log
(-c*sqrt(x) + 1)^2 + 12*log(-c*sqrt(x) + 1) - 3)*(c*sqrt(x) - 1)^4 + 80000*
(9*log(-c*sqrt(x) + 1)^3 - 9*log(-c*sqrt(x) + 1)^2 + 6*log(-c*sqrt(x) + 1)
- 2)*(c*sqrt(x) - 1)^3 + 135000*(4*log(-c*sqrt(x) + 1)^3 - 6*log(-c*sqrt(x)
 + 1)^2 + 6*log(-c*sqrt(x) + 1) - 3)*(c*sqrt(x) - 1)^2 + 216000*(log(-c*sqr
t(x) + 1)^3 - 3*log(-c*sqrt(x) + 1)^2 + 6*log(-c*sqrt(x) + 1) - 6)*(c*sqrt(
x) - 1))*b^3/c^6 + 23/15*(log(c*sqrt(x) + 1)*log(-1/2*c*sqrt(x) + 1/2) + di
log(1/2*c*sqrt(x) + 1/2))*b^3/c^6 - 8929/14400*b^3*log(c*sqrt(x) - 1)/c^6 +
 1/120*(147*a*b^2 - 38*b^3)*log(c*sqrt(x) + 1)/c^6 + 1/864000*(1000*(12*a*b
^2*c^6 - b^3*c^6)*x^3 + 36000*(b^3*c^6*x^3 - b^3)*log(c*sqrt(x) + 1)^3 - 48
*(660*a*b^2*c^5 + 91*b^3*c^5)*x^(5/2) + 15*(4440*a*b^2*c^4 - 919*b^3*c^4)*x
^2 + 14400*(15*a*b^2*c^6*x^3 + 3*b^3*c^5*x^(5/2) + 5*b^3*c^3*x^(3/2) + 15*b
^3*c*sqrt(x) - 15*a*b^2 + 23*b^3)*log(c*sqrt(x) + 1)^2 - 1800*(10*b^3*c^6*x
^3 - 12*b^3*c^5*x^(5/2) + 15*b^3*c^4*x^2 - 20*b^3*c^3*x^(3/2) + 30*b^3*c^2*
x - 60*b^3*c*sqrt(x) + 37*b^3 - 60*(b^3*c^6*x^3 - b^3)*log(c*sqrt(x) + 1))*
log(-c*sqrt(x) + 1)^2 - 20*(6840*a*b^2*c^3 + 619*b^3*c^3)*x^(3/2) + 870*(36
0*a*b^2*c^2 - 161*b^3*c^2)*x - 7200*(10*a*b^2*c^6*x^3 - 12*a*b^2*c^5*x^(5/2
) - 20*a*b^2*c^3*x^(3/2) - 60*a*b^2*c*sqrt(x) + 3*(5*a*b^2*c^4 - 2*b^3*c^4)
*x^2 + 2*(15*a*b^2*c^2 - 16*b^3*c^2)*x)*log(c*sqrt(x) + 1) + 60*(100*b^3*c^
6*x^3 + 264*b^3*c^5*x^(5/2) - 165*b^3*c^4*x^2 + 1140*b^3*c^3*x^(3/2) - 1230
*b^3*c^2*x + 8820*b^3*c*sqrt(x) - 1800*(b^3*c^6*x^3 - b^3)*log(c*sqrt(x) +
1)^2 - 480*(3*b^3*c^5*x^(5/2) + 5*b^3*c^3*x^(3/2) + 15*b^3*c*sqrt(x) + 23*b
^3)*log(c*sqrt(x) + 1))*log(-c*sqrt(x) + 1) - 60*(17640*a*b^2*c + 4369*b^3*
c)*sqrt(x))/c^6
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(b^3x^2\,\mathrm{artanh}\left(c\sqrt{x}\right)^3+3\,ab^2x^2\,\mathrm{artanh}\left(c\sqrt{x}\right)^2+3\,a^2bx^2\,\mathrm{artanh}\left(c\sqrt{x}\right)+a^3x^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^(1/2)))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^2*arctanh(c*sqrt(x))^3 + 3*a*b^2*x^2*arctanh(c*sqrt(x))^2 +
3*a^2*b*x^2*arctanh(c*sqrt(x)) + a^3*x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\left(a+b\,\mathrm{atanh}\left(c\sqrt{x}\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c*x**(1/2)))**3,x)
```

```
[Out] Integral(x**2*(a + b*atanh(c*sqrt(x)))**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \operatorname{artanh}\left(c\sqrt{x}\right) + a\right)^3 x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^(1/2)))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*sqrt(x)) + a)^3*x^2, x)
```

3.204 $\int x\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3\,dx$

Optimal. Leaf size=234

$$-\frac{2b^3\text{PolyLog}\left(2,1-\frac{2}{1-c\sqrt{x}}\right)}{c^4}+\frac{b^2x\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{2c^2}-\frac{4b^2\log\left(\frac{2}{1-c\sqrt{x}}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{c^4}+\frac{3b\sqrt{x}\left(a+b\tanh\right.}{2c^3}$$

```
[Out] (b^3*Sqrt[x])/(2*c^3) - (b^3*ArcTanh[c*Sqrt[x]])/(2*c^4) + (b^2*x*(a + b*Ar
cTanh[c*Sqrt[x]]))/(2*c^2) + (2*b*(a + b*ArcTanh[c*Sqrt[x]])^2)/c^4 + (3*b*
Sqrt[x]*(a + b*ArcTanh[c*Sqrt[x]])^2)/(2*c^3) + (b*x^(3/2)*(a + b*ArcTanh[c
*Sqrt[x]])^2)/(2*c) - (a + b*ArcTanh[c*Sqrt[x]])^3/(2*c^4) + (x^2*(a + b*Ar
cTanh[c*Sqrt[x]])^3)/2 - (4*b^2*(a + b*ArcTanh[c*Sqrt[x]])*Log[2/(1 - c*Sqr
t[x])])/c^4 - (2*b^3*PolyLog[2, 1 - 2/(1 - c*Sqrt[x])])/c^4
```

Rubi [F] time = 0.0142464, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x*(a + b*ArcTanh[c*Sqrt[x]])^3,x]
```

```
[Out] Defer[Int][x*(a + b*ArcTanh[c*Sqrt[x]])^3, x]
```

Rubi steps

$$\int x\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3\,dx=\int x\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3\,dx$$

Mathematica [A] time = 0.527349, size = 285, normalized size = 1.22

$$\frac{8b^3\text{PolyLog}\left(2,-e^{-2\tanh^{-1}\left(c\sqrt{x}\right)}\right)+2b\tanh^{-1}\left(c\sqrt{x}\right)\left(3a^2c^4x^2+2abc\sqrt{x}\left(c^2x+3\right)+b^2\left(c^2x-1\right)-8b^2\log\left(e^{-2\tanh^{-1}(\cdot}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(a + b*ArcTanh[c*Sqrt[x]])^3,x]
```

```
[Out] (-2*a*b^2 + 6*a^2*b*c*Sqrt[x] + 2*b^3*c*Sqrt[x] + 2*a*b^2*c^2*x + 2*a^2*b*c
^3*x^(3/2) + 2*a^3*c^4*x^2 + 2*b^2*(b*(-4 + 3*c*Sqrt[x] + c^3*x^(3/2)) + 3*
a*(-1 + c^4*x^2))*ArcTanh[c*Sqrt[x]]^2 + 2*b^3*(-1 + c^4*x^2)*ArcTanh[c*Sqr
t[x]]^3 + 2*b*ArcTanh[c*Sqrt[x]]*(3*a^2*c^4*x^2 + b^2*(-1 + c^2*x) + 2*a*b*
c*Sqrt[x]*(3 + c^2*x) - 8*b^2*Log[1 + E^(-2*ArcTanh[c*Sqrt[x]])]) + 3*a^2*b
*Log[1 - c*Sqrt[x]] - 3*a^2*b*Log[1 + c*Sqrt[x]] + 8*a*b^2*Log[1 - c^2*x] +
 8*b^3*PolyLog[2, -E^(-2*ArcTanh[c*Sqrt[x]])])/(4*c^4)
```

Maple [C] time = 0.286, size = 1339, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c*x^(1/2)))^3,x)
```

```
[Out] 1/2*b^3*x^2*arctanh(c*x^(1/2))^3+2/c^4*b^3*arctanh(c*x^(1/2))^2-4/c^4*b^3*d
ilog(1-I*(1+c*x^(1/2))/(-c^2*x+1)^(1/2))-4/c^4*b^3*dilog(1+I*(1+c*x^(1/2))/
(-c^2*x+1)^(1/2))-1/2/c^4*b^3*arctanh(c*x^(1/2))^3-3/8*I/c^4*b^3*Pi*csgn(I/
((1+c*x^(1/2))^2/(-c^2*x+1)+1))*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1))*csgn(I*(1
+c*x^(1/2))^2/(c^2*x-1)/((1+c*x^(1/2))^2/(-c^2*x+1)+1))*arctanh(c*x^(1/2))^
2-1/2*b^3*arctanh(c*x^(1/2))/c^4+1/2*b^3*x^(1/2)/c^3-3/4*I/c^4*b^3*Pi*arcta
nh(c*x^(1/2))^2+1/c*a*b^2*arctanh(c*x^(1/2))*x^(3/2)+3/c^3*a*b^2*x^(1/2)*ar
ctanh(c*x^(1/2))-3/4/c^4*a*b^2*ln(-1/2*c*x^(1/2)+1/2)*ln(1+c*x^(1/2))+3/4/c
^4*a*b^2*ln(-1/2*c*x^(1/2)+1/2)*ln(1/2+1/2*c*x^(1/2))+3/2/c^4*a*b^2*arctanh
(c*x^(1/2))*ln(c*x^(1/2)-1)-3/2/c^4*a*b^2*arctanh(c*x^(1/2))*ln(1+c*x^(1/2)
)-3/4/c^4*a*b^2*ln(c*x^(1/2)-1)*ln(1/2+1/2*c*x^(1/2))+1/2*a*b^2*x/c^2-3/4*I
/c^4*b^3*Pi*csgn(I/((1+c*x^(1/2))^2/(-c^2*x+1)+1))^3*arctanh(c*x^(1/2))^2+3
/4*I/c^4*b^3*Pi*csgn(I/((1+c*x^(1/2))^2/(-c^2*x+1)+1))^2*arctanh(c*x^(1/2))
^2+3/8*I/c^4*b^3*Pi*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1))^3*arctanh(c*x^(1/2))^
2+3/8*I/c^4*b^3*Pi*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1)/((1+c*x^(1/2))^2/(-c^2*
x+1)+1))^3*arctanh(c*x^(1/2))^2-4/c^4*b^3*arctanh(c*x^(1/2))*ln(1+I*(1+c*x^
(1/2))/(-c^2*x+1)^(1/2))-4/c^4*b^3*arctanh(c*x^(1/2))*ln(1-I*(1+c*x^(1/2))/
(-c^2*x+1)^(1/2))+3/4/c^4*b^3*arctanh(c*x^(1/2))^2*ln(c*x^(1/2)-1)-3/4/c^4*
b^3*arctanh(c*x^(1/2))^2*ln(1+c*x^(1/2))+3/2/c^4*b^3*arctanh(c*x^(1/2))^2*l
n((1+c*x^(1/2))/(-c^2*x+1)^(1/2))+3/2*a*b^2*x^2*arctanh(c*x^(1/2))^2+3/2*a^
2*b*x^2*arctanh(c*x^(1/2))+2/c^4*a*b^2*ln(1+c*x^(1/2))+3/4/c^4*a^2*b*ln(c*x
^(1/2)-1)-3/4/c^4*a^2*b*ln(1+c*x^(1/2))+3/8/c^4*a*b^2*ln(c*x^(1/2)-1)^2+3/8
/c^4*a*b^2*ln(1+c*x^(1/2))^2+2/c^4*a*b^2*ln(c*x^(1/2)-1)+1/2/c^2*b^3*arctan
h(c*x^(1/2))*x+3/2/c^3*b^3*arctanh(c*x^(1/2))^2*x^(1/2)+1/2/c*b^3*arctanh(c
*x^(1/2))^2*x^(3/2)+3/2/c^3*a^2*b*x^(1/2)+1/2/c*a^2*b*x^(3/2)-3/8*I/c^4*b^3
*Pi*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1))*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1)/((1+
c*x^(1/2))^2/(-c^2*x+1)+1))^2*arctanh(c*x^(1/2))^2+3/4*I/c^4*b^3*Pi*csgn(I*
(1+c*x^(1/2))/(-c^2*x+1)^(1/2))*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1))^2*arctanh
(c*x^(1/2))^2+3/8*I/c^4*b^3*Pi*csgn(I*(1+c*x^(1/2))/(-c^2*x+1)^(1/2))^2*csg
n(I*(1+c*x^(1/2))^2/(c^2*x-1))*arctanh(c*x^(1/2))^2+3/8*I/c^4*b^3*Pi*csgn(I
/((1+c*x^(1/2))^2/(-c^2*x+1)+1))*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1)/((1+c*x^(
1/2))^2/(-c^2*x+1)+1))^2*arctanh(c*x^(1/2))^2+1/2*x^2*a^3-1/2/c^4*b^3
```

Maxima [B] time = 3.39122, size = 1598, normalized size = 6.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^(1/2)))^3,x, algorithm="maxima")
```

```
[Out] 1/2*a^3*x^2 - 1/32*a*b^2*c*((3*c^3*x^2 + 10*c*x - 2*(3*c^3*x^2 + 4*c^2*x^(3
/2) + 6*c*x + 12*sqrt(x))*log(c*sqrt(x) + 1))/c^4 - 14*log(c*sqrt(x) + 1)/c
^5 - 14*log(c*sqrt(x) - 1)/c^5) - 1/16*(12*x^2*log(c*sqrt(x) + 1) - c*((3*c
^3*x^2 - 4*c^2*x^(3/2) + 6*c*x - 12*sqrt(x))/c^4 + 12*log(c*sqrt(x) + 1)/c^
5))*a*b^2*log(-c*sqrt(x) + 1) + 1/16*(12*x^2*log(c*sqrt(x) + 1) - c*((3*c^3
*x^2 - 4*c^2*x^(3/2) + 6*c*x - 12*sqrt(x))/c^4 + 12*log(c*sqrt(x) + 1)/c^5)
)*a^2*b - 1/16*(12*x^2*log(-c*sqrt(x) + 1) - c*((3*c^3*x^2 + 4*c^2*x^(3/2)
+ 6*c*x + 12*sqrt(x))/c^4 + 12*log(c*sqrt(x) - 1)/c^5))*a^2*b + 1/192*(9*(8
*log(-c*sqrt(x) + 1)^2 - 4*log(-c*sqrt(x) + 1) + 1)*(c*sqrt(x) - 1)^4 + 32*
(9*log(-c*sqrt(x) + 1)^2 - 6*log(-c*sqrt(x) + 1) + 2)*(c*sqrt(x) - 1)^3 + 2
16*(2*log(-c*sqrt(x) + 1)^2 - 2*log(-c*sqrt(x) + 1) + 1)*(c*sqrt(x) - 1)^2
+ 288*(log(-c*sqrt(x) + 1)^2 - 2*log(-c*sqrt(x) + 1) + 2)*(c*sqrt(x) - 1))*
```

```
a*b^2/c^4 - 1/4608*(9*(32*log(-c*sqrt(x) + 1)^3 - 24*log(-c*sqrt(x) + 1)^2
+ 12*log(-c*sqrt(x) + 1) - 3)*(c*sqrt(x) - 1)^4 + 128*(9*log(-c*sqrt(x) + 1
)^3 - 9*log(-c*sqrt(x) + 1)^2 + 6*log(-c*sqrt(x) + 1) - 2)*(c*sqrt(x) - 1)^
3 + 432*(4*log(-c*sqrt(x) + 1)^3 - 6*log(-c*sqrt(x) + 1)^2 + 6*log(-c*sqrt(
x) + 1) - 3)*(c*sqrt(x) - 1)^2 + 1152*(log(-c*sqrt(x) + 1)^3 - 3*log(-c*sqr
t(x) + 1)^2 + 6*log(-c*sqrt(x) + 1) - 6)*(c*sqrt(x) - 1))*b^3/c^4 + 2*(log(
c*sqrt(x) + 1)*log(-1/2*c*sqrt(x) + 1/2) + dilog(1/2*c*sqrt(x) + 1/2))*b^3/
c^4 - 319/384*b^3*log(c*sqrt(x) - 1)/c^4 + 1/16*(25*a*b^2 - 4*b^3)*log(c*sq
rt(x) + 1)/c^4 + 1/4608*(288*(b^3*c^4*x^2 - b^3)*log(c*sqrt(x) + 1)^3 + 27*
(8*a*b^2*c^4 - b^3*c^4)*x^2 + 576*(3*a*b^2*c^4*x^2 + b^3*c^3*x^(3/2) + 3*b^
3*c*sqrt(x) - 3*a*b^2 + 4*b^3)*log(c*sqrt(x) + 1)^2 - 72*(3*b^3*c^4*x^2 - 4
*b^3*c^3*x^(3/2) + 6*b^3*c^2*x - 12*b^3*c*sqrt(x) + 7*b^3 - 12*(b^3*c^4*x^2
 - b^3)*log(c*sqrt(x) + 1))*log(-c*sqrt(x) + 1)^2 - 4*(168*a*b^2*c^3 + 37*b
^3*c^3)*x^(3/2) + 6*(312*a*b^2*c^2 - 115*b^3*c^2)*x - 288*(3*a*b^2*c^4*x^2
- 4*a*b^2*c^3*x^(3/2) - 12*a*b^2*c*sqrt(x) + 2*(3*a*b^2*c^2 - 2*b^3*c^2)*x)
*log(c*sqrt(x) + 1) + 12*(9*b^3*c^4*x^2 + 28*b^3*c^3*x^(3/2) - 18*b^3*c^2*x
 + 300*b^3*c*sqrt(x) - 72*(b^3*c^4*x^2 - b^3)*log(c*sqrt(x) + 1)^2 - 96*(b^
3*c^3*x^(3/2) + 3*b^3*c*sqrt(x) + 4*b^3)*log(c*sqrt(x) + 1))*log(-c*sqrt(x)
 + 1) - 12*(600*a*b^2*c + 223*b^3*c)*sqrt(x))/c^4
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(b^3x\,\mathrm{artanh}\left(c\sqrt{x}\right)^3+3\,ab^2x\,\mathrm{artanh}\left(c\sqrt{x}\right)^2+3\,a^2bx\,\mathrm{artanh}\left(c\sqrt{x}\right)+a^3x,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^(1/2)))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x*arctanh(c*sqrt(x))^3 + 3*a*b^2*x*arctanh(c*sqrt(x))^2 + 3*a^
2*b*x*arctanh(c*sqrt(x)) + a^3*x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(a+b\,\mathrm{atanh}\left(c\sqrt{x}\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c*x**(1/2)))**3,x)
```

```
[Out] Integral(x*(a + b*atanh(c*sqrt(x)))**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\,\mathrm{artanh}\left(c\sqrt{x}\right)+a\right)^3x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^(1/2)))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*sqrt(x)) + a)^3*x, x)
```

3.205 $\int \left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3 \, dx$

Optimal. Leaf size=142

$$-\frac{3b^3\text{PolyLog}\left(2,1-\frac{2}{1-c\sqrt{x}}\right)}{c^2}-\frac{6b^2\log\left(\frac{2}{1-c\sqrt{x}}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{c^2}+\frac{3b\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2}{c^2}-\frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right.}{c^2}$$

```
[Out] (3*b*(a + b*ArcTanh[c*Sqrt[x]])^2)/c^2 + (3*b*Sqrt[x]*(a + b*ArcTanh[c*Sqrt
[x]])^2)/c - (a + b*ArcTanh[c*Sqrt[x]])^3/c^2 + x*(a + b*ArcTanh[c*Sqrt[x]]
)^3 - (6*b^2*(a + b*ArcTanh[c*Sqrt[x]])*Log[2/(1 - c*Sqrt[x])])/c^2 - (3*b^
3*PolyLog[2, 1 - 2/(1 - c*Sqrt[x])])/c^2
```

Rubi [F] time = 0.0063135, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcTanh[c*Sqrt[x]])^3,x]
```

```
[Out] Defer[Int][(a + b*ArcTanh[c*Sqrt[x]])^3, x]
```

Rubi steps

$$\int \left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3 \, dx=\int \left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3 \, dx$$

Mathematica [A] time = 0.276754, size = 201, normalized size = 1.42

$$\frac{6b^3\text{PolyLog}\left(2,-e^{-2\tanh^{-1}\left(c\sqrt{x}\right)}\right)+a\left(2a^2c^2x+6abc\sqrt{x}+3ab\log\left(1-c\sqrt{x}\right)-3ab\log\left(c\sqrt{x}+1\right)+6b^2\log\left(1-c^2x\right)\right)+6}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*Sqrt[x]])^3,x]
```

```
[Out] (6*b^2*(-1 + c*Sqrt[x])*(a + b + a*c*Sqrt[x])*ArcTanh[c*Sqrt[x]]^2 + 2*b^3*
(-1 + c^2*x)*ArcTanh[c*Sqrt[x]]^3 + 6*b*ArcTanh[c*Sqrt[x]]*(2*a*b*c*Sqrt[x]
 + a^2*c^2*x - 2*b^2*Log[1 + E^(-2*ArcTanh[c*Sqrt[x]])]) + a*(6*a*b*c*Sqrt[
x] + 2*a^2*c^2*x + 3*a*b*Log[1 - c*Sqrt[x]] - 3*a*b*Log[1 + c*Sqrt[x]] + 6*
b^2*Log[1 - c^2*x]) + 6*b^3*PolyLog[2, -E^(-2*ArcTanh[c*Sqrt[x]])])/(2*c^2)
```

Maple [C] time = 0.388, size = 6235, normalized size = 43.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^(1/2)))^3,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{3}{2}\left(c\left(\frac{2\sqrt{x}}{c^2}-\frac{\log\left(c\sqrt{x}+1\right)}{c^3}+\frac{\log\left(c\sqrt{x}-1\right)}{c^3}\right)+2\,x\,\mathrm{artanh}\left(c\sqrt{x}\right)\right)a^2b+\frac{3}{4}\left(4\,c\left(\frac{2\sqrt{x}}{c^2}-\frac{\log\left(c\sqrt{x}+1\right)}{c^3}+\frac{\log\left(c\sqrt{x}-1\right)}{c^3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^3,x, algorithm="maxima")
```

```
[Out] 3/2*(c*(2*sqrt(x)/c^2 - log(c*sqrt(x) + 1)/c^3 + log(c*sqrt(x) - 1)/c^3) +
2*x*arctanh(c*sqrt(x)))*a^2*b + 3/4*(4*c*(2*sqrt(x)/c^2 - log(c*sqrt(x) + 1
)/c^3 + log(c*sqrt(x) - 1)/c^3)*arctanh(c*sqrt(x)) + 4*x*arctanh(c*sqrt(x))
^2 - (2*(log(c*sqrt(x) - 1) - 2)*log(c*sqrt(x) + 1) - log(c*sqrt(x) + 1)^2
- log(c*sqrt(x) - 1)^2 - 4*log(c*sqrt(x) - 1))/c^2)*a*b^2 + a^3*x - 1/32*b^
3*(((4*log(-c*sqrt(x) + 1)^3 - 6*log(-c*sqrt(x) + 1)^2 + 6*log(-c*sqrt(x) +
 1) - 3)*(c*sqrt(x) - 1)^2 + 8*(log(-c*sqrt(x) + 1)^3 - 3*log(-c*sqrt(x) +
1)^2 + 6*log(-c*sqrt(x) + 1) - 6)*(c*sqrt(x) - 1))/c^2 - 4*integrate(log(c*
sqrt(x) + 1)^3 - 3*log(c*sqrt(x) + 1)^2*log(-c*sqrt(x) + 1) + 3*log(c*sqrt(
x) + 1)*log(-c*sqrt(x) + 1)^2, x))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(b^3\,\mathrm{artanh}\left(c\sqrt{x}\right)^3+3\,ab^2\,\mathrm{artanh}\left(c\sqrt{x}\right)^2+3\,a^2b\,\mathrm{artanh}\left(c\sqrt{x}\right)+a^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*arctanh(c*sqrt(x))^3 + 3*a*b^2*arctanh(c*sqrt(x))^2 + 3*a^2*b*
arctanh(c*sqrt(x)) + a^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\,\mathrm{atanh}\left(c\sqrt{x}\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**(1/2)))**3,x)
```

```
[Out] Integral((a + b*atanh(c*sqrt(x)))**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\,\mathrm{artanh}\left(c\sqrt{x}\right)+a\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*sqrt(x)) + a)^3, x)
```

3.206 $\int \frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3}{x}\,dx$

Optimal. Leaf size=224

$$3b^2\text{PolyLog}\left(3,1-\frac{2}{1-c\sqrt{x}}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)-3b^2\text{PolyLog}\left(3,\frac{2}{1-c\sqrt{x}}-1\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)-3b\text{PolyL}$$

```
[Out] 4*ArcTanh[1 - 2/(1 - c*Sqrt[x])]*(a + b*ArcTanh[c*Sqrt[x]])^3 - 3*b*(a + b*
ArcTanh[c*Sqrt[x]])^2*PolyLog[2, 1 - 2/(1 - c*Sqrt[x])] + 3*b*(a + b*ArcTan
h[c*Sqrt[x]])^2*PolyLog[2, -1 + 2/(1 - c*Sqrt[x])] + 3*b^2*(a + b*ArcTanh[c
*Sqrt[x]])*PolyLog[3, 1 - 2/(1 - c*Sqrt[x])] - 3*b^2*(a + b*ArcTanh[c*Sqrt[
x]])*PolyLog[3, -1 + 2/(1 - c*Sqrt[x])] - (3*b^3*PolyLog[4, 1 - 2/(1 - c*Sq
rt[x])])/2 + (3*b^3*PolyLog[4, -1 + 2/(1 - c*Sqrt[x])])/2
```

Rubi [A] time = 0.511527, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {6095, 5914, 6052, 5948, 6058, 6062, 6610}

$$3b^2\text{PolyLog}\left(3,1-\frac{2}{1-c\sqrt{x}}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)-3b^2\text{PolyLog}\left(3,\frac{2}{1-c\sqrt{x}}-1\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)-3b\text{PolyL}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*Sqrt[x]])^3/x,x]
```


```
[Out] 4*ArcTanh[1 - 2/(1 - c*Sqrt[x])]*(a + b*ArcTanh[c*Sqrt[x]])^3 - 3*b*(a + b*
ArcTanh[c*Sqrt[x]])^2*PolyLog[2, 1 - 2/(1 - c*Sqrt[x])] + 3*b*(a + b*ArcTan
h[c*Sqrt[x]])^2*PolyLog[2, -1 + 2/(1 - c*Sqrt[x])] + 3*b^2*(a + b*ArcTanh[c
*Sqrt[x]])*PolyLog[3, 1 - 2/(1 - c*Sqrt[x])] - 3*b^2*(a + b*ArcTanh[c*Sqrt[
x]])*PolyLog[3, -1 + 2/(1 - c*Sqrt[x])] - (3*b^3*PolyLog[4, 1 - 2/(1 - c*Sq
rt[x])])/2 + (3*b^3*PolyLog[4, -1 + 2/(1 - c*Sqrt[x])])/2
```


Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[
1/n, Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
 n}, x] && IGtQ[p, 0]
```

Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
 e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6062

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[k + 1, u])/(2
*c*d), x] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[k + 1,
u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && Eq
Q[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3}{x}\,dx = 2\,\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{x}\,dx, x, \sqrt{x}\right)$$

$$= 4\tanh^{-1}\left(1-\frac{2}{1-c\sqrt{x}}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3 - (12bc)\,\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2 \text{ta}}{1-c^2x}\right.$$

$$= 4\tanh^{-1}\left(1-\frac{2}{1-c\sqrt{x}}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3 + (6bc)\,\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2 \log}{1-c^2x^2}\right.$$

$$= 4\tanh^{-1}\left(1-\frac{2}{1-c\sqrt{x}}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3 - 3b\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2 \text{Li}_2\left(1-\frac{2}{1-c}\right.$$

$$= 4\tanh^{-1}\left(1-\frac{2}{1-c\sqrt{x}}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3 - 3b\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2 \text{Li}_2\left(1-\frac{2}{1-c}\right.$$

$$= 4\tanh^{-1}\left(1-\frac{2}{1-c\sqrt{x}}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3 - 3b\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2 \text{Li}_2\left(1-\frac{2}{1-c}\right.$$

Mathematica [A] time = 0.199004, size = 248, normalized size = 1.11

$$\frac{3}{2}b\left(2\text{PolyLog}\left(2,\frac{c\sqrt{x}+1}{1-c\sqrt{x}}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2 - 2\text{PolyLog}\left(2,\frac{c\sqrt{x}+1}{c\sqrt{x}-1}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2 + b\left(-2\text{PolyLog}\left(3\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*Sqrt[x]])^3/x,x]
```

```
[Out] 4*ArcTanh[1 + 2/(-1 + c*Sqrt[x])]*(a + b*ArcTanh[c*Sqrt[x]])^3 + (3*b*(2*(a
 + b*ArcTanh[c*Sqrt[x]])^2*PolyLog[2, (1 + c*Sqrt[x])/(1 - c*Sqrt[x])] - 2*
(a + b*ArcTanh[c*Sqrt[x]])^2*PolyLog[2, (1 + c*Sqrt[x])/(-1 + c*Sqrt[x])] +
 b*(-2*(a + b*ArcTanh[c*Sqrt[x]])*PolyLog[3, (1 + c*Sqrt[x])/(1 - c*Sqrt[x]
)] + 2*(a + b*ArcTanh[c*Sqrt[x]])*PolyLog[3, (1 + c*Sqrt[x])/(-1 + c*Sqrt[x
])] + b*(PolyLog[4, (1 + c*Sqrt[x])/(1 - c*Sqrt[x])] - PolyLog[4, (1 + c*Sq
rt[x])/(-1 + c*Sqrt[x])]))))/2
```

Maple [C] time = 0.173, size = 1542, normalized size = 6.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^(1/2)))^3/x,x)
```

```
[Out] 3*I*a*b^2*Pi*csgn(I*((1+c*x^(1/2))^2/(-c^2*x+1)-1))*csgn(I/((1+c*x^(1/2))^2
/(-c^2*x+1)+1))*csgn(I*((1+c*x^(1/2))^2/(-c^2*x+1)-1)/((1+c*x^(1/2))^2/(-c^
2*x+1)+1))*arctanh(c*x^(1/2))^2+2*b^3*arctanh(c*x^(1/2))^3*ln(1+(1+c*x^(1/2
))/(-c^2*x+1)^(1/2))+6*b^3*arctanh(c*x^(1/2))^2*polylog(2,-(1+c*x^(1/2))/(-
c^2*x+1)^(1/2))-12*b^3*arctanh(c*x^(1/2))*polylog(3,-(1+c*x^(1/2))/(-c^2*x+
1)^(1/2))+2*b^3*ln(c*x^(1/2))*arctanh(c*x^(1/2))^3-2*b^3*arctanh(c*x^(1/2))
^3*ln((1+c*x^(1/2))^2/(-c^2*x+1)-1)-3*b^3*arctanh(c*x^(1/2))^2*polylog(2,-(
1+c*x^(1/2))^2/(-c^2*x+1))+3*b^3*arctanh(c*x^(1/2))*polylog(3,-(1+c*x^(1/2)
)^2/(-c^2*x+1))-3*a^2*b*dilog(c*x^(1/2))-3*a^2*b*dilog(1+c*x^(1/2))+3*a*b^2
*polylog(3,-(1+c*x^(1/2))^2/(-c^2*x+1))-12*a*b^2*polylog(3,(1+c*x^(1/2))/(-
c^2*x+1)^(1/2))-12*a*b^2*polylog(3,-(1+c*x^(1/2))/(-c^2*x+1)^(1/2))+2*b^3*a
rctanh(c*x^(1/2))^3*ln(1-(1+c*x^(1/2))/(-c^2*x+1)^(1/2))+6*b^3*arctanh(c*x^
(1/2))^2*polylog(2,(1+c*x^(1/2))/(-c^2*x+1)^(1/2))-12*b^3*arctanh(c*x^(1/2)
)*polylog(3,(1+c*x^(1/2))/(-c^2*x+1)^(1/2))-6*a*b^2*arctanh(c*x^(1/2))^2*ln
((1+c*x^(1/2))^2/(-c^2*x+1)-1)+6*a*b^2*arctanh(c*x^(1/2))^2*ln(1-(1+c*x^(1/
2))/(-c^2*x+1)^(1/2))+12*a*b^2*arctanh(c*x^(1/2))*polylog(2,(1+c*x^(1/2))/(
-c^2*x+1)^(1/2))+6*a*b^2*arctanh(c*x^(1/2))^2*ln(1+(1+c*x^(1/2))/(-c^2*x+1)
^(1/2))+12*a*b^2*arctanh(c*x^(1/2))*polylog(2,-(1+c*x^(1/2))/(-c^2*x+1)^(1/
2))+6*a*b^2*ln(c*x^(1/2))*arctanh(c*x^(1/2))^2-6*a*b^2*arctanh(c*x^(1/2))*p
olylog(2,-(1+c*x^(1/2))^2/(-c^2*x+1))-I*b^3*Pi*csgn(I*((1+c*x^(1/2))^2/(-c^
2*x+1)-1))*csgn(I*((1+c*x^(1/2))^2/(-c^2*x+1)-1)/((1+c*x^(1/2))^2/(-c^2*x+1
)+1))^2*arctanh(c*x^(1/2))^3-I*b^3*Pi*csgn(I/((1+c*x^(1/2))^2/(-c^2*x+1)+1)
)*csgn(I*((1+c*x^(1/2))^2/(-c^2*x+1)-1)/((1+c*x^(1/2))^2/(-c^2*x+1)+1))^2*a
rctanh(c*x^(1/2))^3+3*I*a*b^2*Pi*csgn(I*((1+c*x^(1/2))^2/(-c^2*x+1)-1)/((1+
c*x^(1/2))^2/(-c^2*x+1)+1))^3*arctanh(c*x^(1/2))^2+6*a^2*b*ln(c*x^(1/2))*ar
ctanh(c*x^(1/2))-3*a^2*b*ln(c*x^(1/2))*ln(1+c*x^(1/2))+I*b^3*Pi*csgn(I*((1+
c*x^(1/2))^2/(-c^2*x+1)-1))*csgn(I/((1+c*x^(1/2))^2/(-c^2*x+1)+1))*csgn(I*(
(1+c*x^(1/2))^2/(-c^2*x+1)-1)/((1+c*x^(1/2))^2/(-c^2*x+1)+1))*arctanh(c*x^(
1/2))^3-3*I*a*b^2*Pi*csgn(I*((1+c*x^(1/2))^2/(-c^2*x+1)-1))*csgn(I*((1+c*x^
(1/2))^2/(-c^2*x+1)-1)/((1+c*x^(1/2))^2/(-c^2*x+1)+1))^2*arctanh(c*x^(1/2))
^2-3*I*a*b^2*Pi*csgn(I/((1+c*x^(1/2))^2/(-c^2*x+1)+1))*csgn(I*((1+c*x^(1/2)
)^2/(-c^2*x+1)-1)/((1+c*x^(1/2))^2/(-c^2*x+1)+1))^2*arctanh(c*x^(1/2))^2+2*
a^3*ln(c*x^(1/2))-3/2*b^3*polylog(4,-(1+c*x^(1/2))^2/(-c^2*x+1))+12*b^3*pol
ylog(4,(1+c*x^(1/2))/(-c^2*x+1)^(1/2))+12*b^3*polylog(4,-(1+c*x^(1/2))/(-c^
2*x+1)^(1/2))+I*b^3*Pi*csgn(I*((1+c*x^(1/2))^2/(-c^2*x+1)-1)/((1+c*x^(1/2))
^2/(-c^2*x+1)+1))^3*arctanh(c*x^(1/2))^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{8}b^3\int \frac{\log\left(c\sqrt{x}+1\right)^3}{x}\,dx - \frac{3}{8}b^3\int \frac{\log\left(c\sqrt{x}+1\right)^2\log\left(-c\sqrt{x}+1\right)}{x}\,dx + \frac{3}{8}b^3\int \frac{\log\left(c\sqrt{x}+1\right)\log\left(-c\sqrt{x}+1\right)^2}{x}\,dx - \frac{1}{8}b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^3/x,x, algorithm="maxima")
```

```
[Out] 1/8*b^3*integrate(log(c*sqrt(x) + 1)^3/x, x) - 3/8*b^3*integrate(log(c*sqrt
(x) + 1)^2*log(-c*sqrt(x) + 1)/x, x) + 3/8*b^3*integrate(log(c*sqrt(x) + 1)
*log(-c*sqrt(x) + 1)^2/x, x) - 1/8*b^3*integrate(log(-c*sqrt(x) + 1)^3/x, x
) + 3/4*a*b^2*integrate(log(c*sqrt(x) + 1)^2/x, x) - 3/2*a*b^2*integrate(lo
g(c*sqrt(x) + 1)*log(-c*sqrt(x) + 1)/x, x) + 3/4*a*b^2*integrate(log(-c*sqr
t(x) + 1)^2/x, x) + 3/2*a^2*b*integrate(log(c*sqrt(x) + 1)/x, x) - 3/2*a^2*
b*integrate(log(-c*sqrt(x) + 1)/x, x) + a^3*log(x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3\,\text{artanh}\left(c\sqrt{x}\right)^3 + 3\,ab^2\,\text{artanh}\left(c\sqrt{x}\right)^2 + 3\,a^2b\,\text{artanh}\left(c\sqrt{x}\right) + a^3}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^3/x,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctanh(c*sqrt(x))^3 + 3*a*b^2*arctanh(c*sqrt(x))^2 + 3*a^2*b
*arctanh(c*sqrt(x)) + a^3)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + b\,\text{atanh}\left(c\sqrt{x}\right)\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**(1/2)))**3/x,x)
```

```
[Out] Integral((a + b*atanh(c*sqrt(x)))**3/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\,\text{artanh}\left(c\sqrt{x}\right) + a\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^3/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*sqrt(x)) + a)^3/x, x)
```

3.207 $\int \frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3}{x^2}\,dx$

Optimal. Leaf size=142

$$-3b^3c^2\text{PolyLog}\left(2,\frac{2}{c\sqrt{x}+1}-1\right)+6b^2c^2\log\left(2-\frac{2}{c\sqrt{x}+1}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)+3bc^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^2+c^2$$

```
[Out] 3*b*c^2*(a + b*ArcTanh[c*Sqrt[x]])^2 - (3*b*c*(a + b*ArcTanh[c*Sqrt[x]])^2)
/Sqrt[x] + c^2*(a + b*ArcTanh[c*Sqrt[x]])^3 - (a + b*ArcTanh[c*Sqrt[x]])^3/
x + 6*b^2*c^2*(a + b*ArcTanh[c*Sqrt[x]])*Log[2 - 2/(1 + c*Sqrt[x])] - 3*b^3
*c^2*PolyLog[2, -1 + 2/(1 + c*Sqrt[x])]
```

Rubi [F] time = 0.0222653, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcTanh[c*Sqrt[x]])^3/x^2,x]
```

```
[Out] Defer[Int][(a + b*ArcTanh[c*Sqrt[x]])^3/x^2, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3}{x^2}\,dx=\int \frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3}{x^2}\,dx$$

Mathematica [A] time = 0.307345, size = 230, normalized size = 1.62

$$-6b^3c^2x\text{PolyLog}\left(2,e^{-2\tanh^{-1}\left(c\sqrt{x}\right)}\right)+a\left(-2a^2-3abc^2x\log\left(1-c\sqrt{x}\right)+3abc^2x\log\left(c\sqrt{x}+1\right)-6abc\sqrt{x}+12b^2c^2x\log\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*Sqrt[x]])^3/x^2,x]
```

```
[Out] (6*b^2*(-1 + c*Sqrt[x])*(a + a*c*Sqrt[x] + b*c*Sqrt[x])*ArcTanh[c*Sqrt[x]]^
2 + 2*b^3*(-1 + c^2*x)*ArcTanh[c*Sqrt[x]]^3 - 6*b*ArcTanh[c*Sqrt[x]]*(a^2 +
 2*a*b*c*Sqrt[x] - 2*b^2*c^2*x*Log[1 - E^(-2*ArcTanh[c*Sqrt[x]])]) + a*(-2*
a^2 - 6*a*b*c*Sqrt[x] - 3*a*b*c^2*x*Log[1 - c*Sqrt[x]] + 3*a*b*c^2*x*Log[1
+ c*Sqrt[x]] + 12*b^2*c^2*x*Log[(c*Sqrt[x])/Sqrt[1 - c^2*x]]) - 6*b^3*c^2*x
*PolyLog[2, E^(-2*ArcTanh[c*Sqrt[x]])])/(2*x)
```

Maple [C] time = 0.412, size = 5199, normalized size = 36.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^(1/2)))^3/x^2,x)
```

```
[Out] result too large to display
```

Maxima [B] time = 5.86528, size = 713, normalized size = 5.02

$$-3\left(\log\left(c\sqrt{x}+1\right)\log\left(-\frac{1}{2}c\sqrt{x}+\frac{1}{2}\right)+\mathrm{Li}_2\left(\frac{1}{2}c\sqrt{x}+\frac{1}{2}\right)\right)b^3c^2-3\left(\log\left(c\sqrt{x}\right)\log\left(-c\sqrt{x}+1\right)+\mathrm{Li}_2\left(-c\sqrt{x}+1\right)\right)b^3c^2+3($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^3/x^2,x, algorithm="maxima")
```

```
[Out] -3*(log(c*sqrt(x) + 1)*log(-1/2*c*sqrt(x) + 1/2) + dilog(1/2*c*sqrt(x) + 1/
2))*b^3*c^2 - 3*(log(c*sqrt(x))*log(-c*sqrt(x) + 1) + dilog(-c*sqrt(x) + 1)
)*b^3*c^2 + 3*(log(c*sqrt(x) + 1)*log(-c*sqrt(x)) + dilog(c*sqrt(x) + 1))*b
^3*c^2 - 3*a*b^2*c^2*log(c*sqrt(x) - 1) - 3/4*((2*c*log(c*sqrt(x) - 1) - c*
log(x) + 2/sqrt(x))*c - 2*log(-c*sqrt(x) + 1)/x)*a^2*b - a^3/x + 3/2*(a^2*b
*c^2 - 2*a*b^2*c^2)*log(c*sqrt(x) + 1) - 3/4*(a^2*b*c^2 - 4*a*b^2*c^2)*log(
x) - 1/8*(12*a^2*b*c*sqrt(x) - (b^3*c^2*x - b^3)*log(c*sqrt(x) + 1)^3 + (b^
3*c^2*x - b^3)*log(-c*sqrt(x) + 1)^3 + 6*(b^3*c*sqrt(x) + a*b^2 - (a*b^2*c^
2 - b^3*c^2)*x)*log(c*sqrt(x) + 1)^2 + 3*(2*b^3*c*sqrt(x) + 2*a*b^2 - 2*(a*
b^2*c^2 + b^3*c^2)*x - (b^3*c^2*x - b^3)*log(c*sqrt(x) + 1))*log(-c*sqrt(x)
 + 1)^2 + 12*(2*a*b^2*c*sqrt(x) + a^2*b)*log(c*sqrt(x) + 1) - 3*(8*a*b^2*c*
sqrt(x) - (b^3*c^2*x - b^3)*log(c*sqrt(x) + 1)^2 + 4*(b^3*c*sqrt(x) + a*b^2
 - (a*b^2*c^2 - b^3*c^2)*x)*log(c*sqrt(x) + 1))*log(-c*sqrt(x) + 1))/x
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^3\,\mathrm{artanh}\left(c\sqrt{x}\right)^3+3\,ab^2\,\mathrm{artanh}\left(c\sqrt{x}\right)^2+3\,a^2b\,\mathrm{artanh}\left(c\sqrt{x}\right)+a^3}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^3/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctanh(c*sqrt(x))^3 + 3*a*b^2*arctanh(c*sqrt(x))^2 + 3*a^2*b
*arctanh(c*sqrt(x)) + a^3)/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+b\,\mathrm{atanh}\left(c\sqrt{x}\right)\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**(1/2)))**3/x**2,x)
```

```
[Out] Integral((a + b*atanh(c*sqrt(x)))**3/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \operatorname{artanh}\left(c\sqrt{x}\right) + a\right)^3}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^3/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*sqrt(x)) + a)^3/x^2, x)
```

3.208 $\int \frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3}{x^3}\,dx$

Optimal. Leaf size=234

$$-2b^3c^4\text{PolyLog}\left(2,\frac{2}{c\sqrt{x}+1}-1\right)-\frac{b^2c^2\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)}{2x}+4b^2c^4\log\left(2-\frac{2}{c\sqrt{x}+1}\right)\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)+\frac{1}{2}c^4\left(a\right.$$

```
[Out] -(b^3*c^3)/(2*Sqrt[x]) + (b^3*c^4*ArcTanh[c*Sqrt[x]])/2 - (b^2*c^2*(a + b*A
rcTanh[c*Sqrt[x]]))/(2*x) + 2*b*c^4*(a + b*ArcTanh[c*Sqrt[x]])^2 - (b*c*(a
+ b*ArcTanh[c*Sqrt[x]])^2)/(2*x^(3/2)) - (3*b*c^3*(a + b*ArcTanh[c*Sqrt[x]]
)^2)/(2*Sqrt[x]) + (c^4*(a + b*ArcTanh[c*Sqrt[x]])^3)/2 - (a + b*ArcTanh[c*
Sqrt[x]])^3/(2*x^2) + 4*b^2*c^4*(a + b*ArcTanh[c*Sqrt[x]])*Log[2 - 2/(1 + c
*Sqrt[x])] - 2*b^3*c^4*PolyLog[2, -1 + 2/(1 + c*Sqrt[x])]
```

Rubi [F] time = 0.0232819, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcTanh[c*Sqrt[x]])^3/x^3,x]
```

```
[Out] Defer[Int][(a + b*ArcTanh[c*Sqrt[x]])^3/x^3, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3}{x^3}\,dx=\int \frac{\left(a+b\tanh^{-1}\left(c\sqrt{x}\right)\right)^3}{x^3}\,dx$$

Mathematica [A] time = 0.692146, size = 333, normalized size = 1.42

$$-\frac{8b^3c^4x^2\text{PolyLog}\left(2,e^{-2\tanh^{-1}\left(c\sqrt{x}\right)}\right)+2b\tanh^{-1}\left(c\sqrt{x}\right)\left(3a^2+2abc\sqrt{x}\left(3c^2x+1\right)-8b^2c^4x^2\log\left(1-e^{-2\tanh^{-1}\left(c\sqrt{x}\right)}\right)+b\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*Sqrt[x]])^3/x^3,x]
```

```
[Out] -(2*a^3 + 2*a^2*b*c*Sqrt[x] + 2*a*b^2*c^2*x + 6*a^2*b*c^3*x^(3/2) + 2*b^3*c
^3*x^(3/2) - 2*a*b^2*c^4*x^2 - 2*b^2*(b*c*Sqrt[x]*(-1 - 3*c^2*x + 4*c^3*x^(
3/2)) + 3*a*(-1 + c^4*x^2))*ArcTanh[c*Sqrt[x]]^2 - 2*b^3*(-1 + c^4*x^2)*Arc
Tanh[c*Sqrt[x]]^3 + 2*b*ArcTanh[c*Sqrt[x]]*(3*a^2 + b^2*c^2*x*(1 - c^2*x) +
 2*a*b*c*Sqrt[x]*(1 + 3*c^2*x) - 8*b^2*c^4*x^2*Log[1 - E^(-2*ArcTanh[c*Sqrt
[x]])]) + 3*a^2*b*c^4*x^2*Log[1 - c*Sqrt[x]] - 3*a^2*b*c^4*x^2*Log[1 + c*Sq
rt[x]] - 16*a*b^2*c^4*x^2*Log[(c*Sqrt[x])/Sqrt[1 - c^2*x]] + 8*b^3*c^4*x^2*
PolyLog[2, E^(-2*ArcTanh[c*Sqrt[x]])])/(4*x^2)
```

Maple [C] time = 0.31, size = 1365, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^(1/2)))^3/x^3,x)
```

```
[Out] -3/8*I*c^4*b^3*Pi*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1)/((1+c*x^(1/2))^2/(-c^2*x
+1)+1))^3*arctanh(c*x^(1/2))^2-3/4*I*c^4*b^3*Pi*csgn(I/((1+c*x^(1/2))^2/(-c
^2*x+1)+1))^2*arctanh(c*x^(1/2))^2-3/8*I*c^4*b^3*Pi*csgn(I*(1+c*x^(1/2))^2/
(c^2*x-1))^3*arctanh(c*x^(1/2))^2+3/4*I*c^4*b^3*Pi*csgn(I/((1+c*x^(1/2))^2/
(-c^2*x+1)+1))^3*arctanh(c*x^(1/2))^2-1/2*a^3/x^2+3/4*c^4*b^3*arctanh(c*x^(
1/2))^2*ln(1+c*x^(1/2))-3/2*c^4*b^3*arctanh(c*x^(1/2))^2*ln((1+c*x^(1/2))/(
-c^2*x+1)^(1/2))-3/2*a*b^2/x^2*arctanh(c*x^(1/2))^2-3/2*a^2*b/x^2*arctanh(c
*x^(1/2))-1/2*c^2*b^3*arctanh(c*x^(1/2))/x-1/2*c*b^3*arctanh(c*x^(1/2))^2/x
^(3/2)-3/2*c^3*b^3*arctanh(c*x^(1/2))^2/x^(1/2)-3/2*c^3*a^2*b/x^(1/2)-1/2*c
*a^2*b/x^(3/2)+4*c^4*a*b^2*ln(c*x^(1/2))-3/8*c^4*a*b^2*ln(c*x^(1/2)-1)^2-3/
8*c^4*a*b^2*ln(1+c*x^(1/2))^2-2*c^4*a*b^2*ln(c*x^(1/2)-1)-2*c^4*a*b^2*ln(1+
c*x^(1/2))-3/4*c^4*a^2*b*ln(c*x^(1/2)-1)+3/4*c^4*a^2*b*ln(1+c*x^(1/2))+4*c^
4*b^3*arctanh(c*x^(1/2))*ln(1+(1+c*x^(1/2))/(-c^2*x+1)^(1/2))-1/2*c^4*b^3/(
c*x^(1/2)+1-(-c^2*x+1)^(1/2))*(-c^2*x+1)^(1/2)+1/2*c^4*b^3/((-c^2*x+1)^(1/2
)+c*x^(1/2)+1)*(-c^2*x+1)^(1/2)-3/4*c^4*b^3*arctanh(c*x^(1/2))^2*ln(c*x^(1/
2)-1)+3/8*I*c^4*b^3*Pi*csgn(I/((1+c*x^(1/2))^2/(-c^2*x+1)+1))*csgn(I*(1+c*x
^(1/2))^2/(c^2*x-1))*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1)/((1+c*x^(1/2))^2/(-c^
2*x+1)+1))*arctanh(c*x^(1/2))^2-1/2*b^3/x^2*arctanh(c*x^(1/2))^3+4*c^4*b^3*
dilog(1+(1+c*x^(1/2))/(-c^2*x+1)^(1/2))-4*c^4*b^3*dilog((1+c*x^(1/2))/(-c^2
*x+1)^(1/2))-2*c^4*b^3*arctanh(c*x^(1/2))^2+1/2*c^4*b^3*arctanh(c*x^(1/2))^
3+1/2*b^3*c^4*arctanh(c*x^(1/2))-3/8*I*c^4*b^3*Pi*csgn(I/((1+c*x^(1/2))^2/(
-c^2*x+1)+1))*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1)/((1+c*x^(1/2))^2/(-c^2*x+1)+
1))^2*arctanh(c*x^(1/2))^2-3/8*I*c^4*b^3*Pi*csgn(I*(1+c*x^(1/2))/(-c^2*x+1)
^(1/2))^2*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1))*arctanh(c*x^(1/2))^2+3/8*I*c^4*
b^3*Pi*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1))*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1)/(
(1+c*x^(1/2))^2/(-c^2*x+1)+1))^2*arctanh(c*x^(1/2))^2-3/4*I*c^4*b^3*Pi*csgn
(I*(1+c*x^(1/2))/(-c^2*x+1)^(1/2))*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1))^2*arct
anh(c*x^(1/2))^2+3/4*c^4*a*b^2*ln(-1/2*c*x^(1/2)+1/2)*ln(1+c*x^(1/2))-3/4*c
^4*a*b^2*ln(-1/2*c*x^(1/2)+1/2)*ln(1/2+1/2*c*x^(1/2))-3/2*c^4*a*b^2*arctanh
(c*x^(1/2))*ln(c*x^(1/2)-1)+3/2*c^4*a*b^2*arctanh(c*x^(1/2))*ln(1+c*x^(1/2)
)+3/4*c^4*a*b^2*ln(c*x^(1/2)-1)*ln(1/2+1/2*c*x^(1/2))-c*a*b^2*arctanh(c*x^(
1/2))/x^(3/2)-3*c^3*a*b^2/x^(1/2)*arctanh(c*x^(1/2))+3/4*I*c^4*b^3*Pi*arcta
nh(c*x^(1/2))^2-1/2*c^2*a*b^2/x
```

Maxima [B] time = 6.64872, size = 949, normalized size = 4.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^3/x^3,x, algorithm="maxima")
```

```
[Out] -2*(log(c*sqrt(x) + 1)*log(-1/2*c*sqrt(x) + 1/2) + dilog(1/2*c*sqrt(x) + 1/
2))*b^3*c^4 - 2*(log(c*sqrt(x))*log(-c*sqrt(x) + 1) + dilog(-c*sqrt(x) + 1)
)*b^3*c^4 + 2*(log(c*sqrt(x) + 1)*log(-c*sqrt(x)) + dilog(c*sqrt(x) + 1))*b
^3*c^4 - 1/8*((6*c^3*log(c*sqrt(x) - 1) - 3*c^3*log(x) + (6*c^2*x + 3*c*sqr
t(x) + 2)/x^(3/2))*c - 6*log(-c*sqrt(x) + 1)/x^2)*a^2*b + 1/4*(3*a^2*b*c^4
```

```
- 8*a*b^2*c^4 + b^3*c^4)*log(c*sqrt(x) + 1) - 1/4*(8*a*b^2*c^4 + b^3*c^4)*l
og(c*sqrt(x) - 1) - 1/8*(3*a^2*b*c^4 - 16*a*b^2*c^4)*log(x) - 1/2*a^3/x^2 -
 1/16*(4*a^2*b*c*sqrt(x) - (b^3*c^4*x^2 - b^3)*log(c*sqrt(x) + 1)^3 + (b^3*
c^4*x^2 - b^3)*log(-c*sqrt(x) + 1)^3 + 2*(3*b^3*c^3*x^(3/2) + b^3*c*sqrt(x)
 + 3*a*b^2 - (3*a*b^2*c^4 - 4*b^3*c^4)*x^2)*log(c*sqrt(x) + 1)^2 + (6*b^3*c
^3*x^(3/2) + 2*b^3*c*sqrt(x) + 6*a*b^2 - 2*(3*a*b^2*c^4 + 4*b^3*c^4)*x^2 -
3*(b^3*c^4*x^2 - b^3)*log(c*sqrt(x) + 1))*log(-c*sqrt(x) + 1)^2 + 4*(3*a^2*
b*c^3 + 2*b^3*c^3)*x^(3/2) - 2*(3*a^2*b*c^2 - 4*a*b^2*c^2)*x + 4*(6*a*b^2*c
^3*x^(3/2) + b^3*c^2*x + 2*a*b^2*c*sqrt(x) + 3*a^2*b)*log(c*sqrt(x) + 1) -
(24*a*b^2*c^3*x^(3/2) + 4*b^3*c^2*x + 8*a*b^2*c*sqrt(x) - 3*(b^3*c^4*x^2 -
b^3)*log(c*sqrt(x) + 1)^2 + 4*(3*b^3*c^3*x^(3/2) + b^3*c*sqrt(x) + 3*a*b^2
- (3*a*b^2*c^4 - 4*b^3*c^4)*x^2)*log(c*sqrt(x) + 1))*log(-c*sqrt(x) + 1))/x
^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3 \operatorname{artanh}\left(c\sqrt{x}\right)^3 + 3\,ab^2 \operatorname{artanh}\left(c\sqrt{x}\right)^2 + 3\,a^2 b \operatorname{artanh}\left(c\sqrt{x}\right) + a^3}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^3/x^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctanh(c*sqrt(x))^3 + 3*a*b^2*arctanh(c*sqrt(x))^2 + 3*a^2*b
*arctanh(c*sqrt(x)) + a^3)/x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**(1/2)))**3/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \operatorname{artanh}\left(c\sqrt{x}\right) + a\right)^3}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^3/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*sqrt(x)) + a)^3/x^3, x)
```

3.209 $\int x^{3/2}\tanh^{-1}\left(\sqrt{x}\right)\,dx$

Optimal. Leaf size=38

$$\frac{x^2}{10}+\frac{2}{5}x^{5/2}\tanh^{-1}\left(\sqrt{x}\right)+\frac{x}{5}+\frac{1}{5}\log(1-x)$$

```
[Out] x/5 + x^2/10 + (2*x^(5/2)*ArcTanh[Sqrt[x]])/5 + Log[1 - x]/5
```

Rubi [A] time = 0.0162326, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6097, 43}

$$\frac{x^2}{10}+\frac{2}{5}x^{5/2}\tanh^{-1}\left(\sqrt{x}\right)+\frac{x}{5}+\frac{1}{5}\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*ArcTanh[Sqrt[x]],x]
```

```
[Out] x/5 + x^2/10 + (2*x^(5/2)*ArcTanh[Sqrt[x]])/5 + Log[1 - x]/5
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{3/2}\tanh^{-1}\left(\sqrt{x}\right)\,dx &= \frac{2}{5}x^{5/2}\tanh^{-1}\left(\sqrt{x}\right)-\frac{1}{5}\int\frac{x^2}{1-x}\,dx\\&=\frac{2}{5}x^{5/2}\tanh^{-1}\left(\sqrt{x}\right)-\frac{1}{5}\int\left(-1+\frac{1}{1-x}-x\right)dx\\&=\frac{x}{5}+\frac{x^2}{10}+\frac{2}{5}x^{5/2}\tanh^{-1}\left(\sqrt{x}\right)+\frac{1}{5}\log(1-x)\end{aligned}$$

Mathematica [A] time = 0.0159567, size = 31, normalized size = 0.82

$$\frac{1}{10}\left(4x^{5/2}\tanh^{-1}\left(\sqrt{x}\right)+(x+2)x+2\log(1-x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*ArcTanh[Sqrt[x]],x]
```

```
[Out] (x*(2 + x) + 4*x^(5/2)*ArcTanh[Sqrt[x]] + 2*Log[1 - x])/10
```

Maple [A] time = 0.024, size = 35, normalized size = 0.9

$$\frac{2}{5}x^{\frac{5}{2}}\operatorname{Artanh}\left(\sqrt{x}\right)+\frac{x^2}{10}+\frac{x}{5}+\frac{1}{5}\ln\left(-1+\sqrt{x}\right)+\frac{1}{5}\ln\left(1+\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*arctanh(x^(1/2)),x)
```

```
[Out] 2/5*x^(5/2)*arctanh(x^(1/2))+1/10*x^2+1/5*x+1/5*ln(-1+x^(1/2))+1/5*ln(1+x^(
1/2))
```

Maxima [A] time = 0.970501, size = 32, normalized size = 0.84

$$\frac{2}{5}\,x^{\frac{5}{2}}\operatorname{artanh}\left(\sqrt{x}\right)+\frac{1}{10}\,x^2+\frac{1}{5}\,x+\frac{1}{5}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctanh(x^(1/2)),x, algorithm="maxima")
```

```
[Out] 2/5*x^(5/2)*arctanh(sqrt(x)) + 1/10*x^2 + 1/5*x + 1/5*log(x - 1)
```

Fricas [A] time = 1.65311, size = 112, normalized size = 2.95

$$\frac{1}{5}\,x^{\frac{5}{2}}\log\left(-\frac{x+2\,\sqrt{x}+1}{x-1}\right)+\frac{1}{10}\,x^2+\frac{1}{5}\,x+\frac{1}{5}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctanh(x^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/5*x^(5/2)*log(-(x + 2*sqrt(x) + 1)/(x - 1)) + 1/10*x^2 + 1/5*x + 1/5*log(
x - 1)
```

Sympy [B] time = 18.1094, size = 128, normalized size = 3.37

$$\frac{4x^{\frac{7}{2}}\operatorname{atanh}\left(\sqrt{x}\right)}{10x-10}-\frac{4x^{\frac{5}{2}}\operatorname{atanh}\left(\sqrt{x}\right)}{10x-10}+\frac{x^3}{10x-10}+\frac{x^2}{10x-10}+\frac{4x\log\left(\sqrt{x}+1\right)}{10x-10}-\frac{4x\operatorname{atanh}\left(\sqrt{x}\right)}{10x-10}-\frac{x}{10x-10}-\frac{4\log\left(\sqrt{x}\right.}{10x-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*atanh(x**(1/2)),x)
```

```
[Out] 4*x**(7/2)*atanh(sqrt(x))/(10*x - 10) - 4*x**(5/2)*atanh(sqrt(x))/(10*x - 1
0) + x**3/(10*x - 10) + x**2/(10*x - 10) + 4*x*log(sqrt(x) + 1)/(10*x - 10)
 - 4*x*atanh(sqrt(x))/(10*x - 10) - x/(10*x - 10) - 4*log(sqrt(x) + 1)/(10*
```

```
x - 10) + 4*atanh(sqrt(x))/(10*x - 10) - 1/(10*x - 10)
```

<u>**Giac**</u> **[A]** time = 1.17898, size = 49, normalized size = 1.29

$$\frac{1}{5}\,x^{\frac{5}{2}}\log\left(-\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)+\frac{1}{10}\,x^{2}+\frac{1}{5}\,x+\frac{1}{5}\,\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctanh(x^(1/2)),x, algorithm="giac")
```

```
[Out] 1/5*x^(5/2)*log(-(sqrt(x) + 1)/(sqrt(x) - 1)) + 1/10*x^2 + 1/5*x + 1/5*log(
abs(x - 1))
```

3.210 $\int \sqrt{x} \tanh^{-1}\left(\sqrt{x}\right) dx$

Optimal. Leaf size=31

$$\frac{2}{3}x^{3/2}\tanh^{-1}\left(\sqrt{x}\right)+\frac{x}{3}+\frac{1}{3}\log(1-x)$$

```
[Out] x/3 + (2*x^(3/2)*ArcTanh[Sqrt[x]])/3 + Log[1 - x]/3
```

Rubi [A] time = 0.0129167, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6097, 43}

$$\frac{2}{3}x^{3/2}\tanh^{-1}\left(\sqrt{x}\right)+\frac{x}{3}+\frac{1}{3}\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*ArcTanh[Sqrt[x]],x]
```

```
[Out] x/3 + (2*x^(3/2)*ArcTanh[Sqrt[x]])/3 + Log[1 - x]/3
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\tanh^{-1}\left(\sqrt{x}\right) dx &= \frac{2}{3}x^{3/2}\tanh^{-1}\left(\sqrt{x}\right)-\frac{1}{3}\int \frac{x}{1-x}\,dx\\ &= \frac{2}{3}x^{3/2}\tanh^{-1}\left(\sqrt{x}\right)-\frac{1}{3}\int \left(-1+\frac{1}{1-x}\right) dx\\ &= \frac{x}{3}+\frac{2}{3}x^{3/2}\tanh^{-1}\left(\sqrt{x}\right)+\frac{1}{3}\log(1-x)\end{aligned}$$

Mathematica [A] time = 0.0111748, size = 25, normalized size = 0.81

$$\frac{1}{3}\left(2x^{3/2}\tanh^{-1}\left(\sqrt{x}\right)+x+\log(1-x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*ArcTanh[Sqrt[x]],x]
```

```
[Out] (x + 2*x^(3/2)*ArcTanh[Sqrt[x]] + Log[1 - x])/3
```

Maple [A] time = 0.024, size = 30, normalized size = 1.

$$\frac{2}{3}x^{\frac{3}{2}}\operatorname{Artanh}\left(\sqrt{x}\right)+\frac{x}{3}+\frac{1}{3}\ln\left(-1+\sqrt{x}\right)+\frac{1}{3}\ln\left(1+\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x^(1/2))*x^(1/2),x)
```

```
[Out] 2/3*x^(3/2)*arctanh(x^(1/2))+1/3*x+1/3*ln(-1+x^(1/2))+1/3*ln(1+x^(1/2))
```

Maxima [A] time = 0.97717, size = 26, normalized size = 0.84

$$\frac{2}{3}x^{\frac{3}{2}}\operatorname{artanh}\left(\sqrt{x}\right)+\frac{1}{3}x+\frac{1}{3}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x^(1/2))*x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*x^(3/2)*arctanh(sqrt(x)) + 1/3*x + 1/3*log(x - 1)
```

Fricas [A] time = 1.70567, size = 97, normalized size = 3.13

$$\frac{1}{3}x^{\frac{3}{2}}\log\left(-\frac{x+2\sqrt{x}+1}{x-1}\right)+\frac{1}{3}x+\frac{1}{3}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x^(1/2))*x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*x^(3/2)*log(-(x + 2*sqrt(x) + 1)/(x - 1)) + 1/3*x + 1/3*log(x - 1)
```

Sympy [A] time = 1.71664, size = 39, normalized size = 1.26

$$\frac{2x^{\frac{3}{2}}\operatorname{atanh}\left(\sqrt{x}\right)}{3}+\frac{x}{3}+\frac{2\log\left(\sqrt{x}+1\right)}{3}-\frac{2\operatorname{atanh}\left(\sqrt{x}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x**(1/2))*x**(1/2),x)
```

```
[Out] 2*x**(3/2)*atanh(sqrt(x))/3 + x/3 + 2*log(sqrt(x) + 1)/3 - 2*atanh(sqrt(x))
/3
```

Giac [A] time = 1.16795, size = 42, normalized size = 1.35

$$\frac{1}{3}\,x^{\frac{3}{2}}\log\left(-\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)+\frac{1}{3}\,x+\frac{1}{3}\,\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x^(1/2))*x^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*x^(3/2)*log(-(sqrt(x) + 1)/(sqrt(x) - 1)) + 1/3*x + 1/3*log(abs(x - 1))
```

3.211 $\int \frac{\tanh^{-1}(\sqrt{x})}{\sqrt{x}}\,dx$

Optimal. Leaf size=20

$$\log(1-x)+2\sqrt{x}\tanh^{-1}\left(\sqrt{x}\right)$$

```
[Out] 2*Sqrt[x]*ArcTanh[Sqrt[x]] + Log[1 - x]
```

Rubi [A] time = 0.0091329, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6097, 31}

$$\log(1-x)+2\sqrt{x}\tanh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Sqrt[x]]/Sqrt[x],x]
```

```
[Out] 2*Sqrt[x]*ArcTanh[Sqrt[x]] + Log[1 - x]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}\,dx &= 2\sqrt{x}\tanh^{-1}\left(\sqrt{x}\right)-\int \frac{1}{1-x}\,dx\\ &= 2\sqrt{x}\tanh^{-1}\left(\sqrt{x}\right)+\log(1-x)\end{aligned}$$

Mathematica [A] time = 0.0084416, size = 20, normalized size = 1.

$$\log(1-x)+2\sqrt{x}\tanh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Sqrt[x]]/Sqrt[x],x]
```

```
[Out] 2*Sqrt[x]*ArcTanh[Sqrt[x]] + Log[1 - x]
```

Maple [A] time = 0.025, size = 17, normalized size = 0.9

$$\ln(1-x)+2\,\mathrm{Artanh}\left(\sqrt{x}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x^(1/2))/x^(1/2),x)
```

```
[Out] ln(1-x)+2*arctanh(x^(1/2))*x^(1/2)
```

Maxima [A] time = 0.954426, size = 22, normalized size = 1.1

$$2\sqrt{x}\,\mathrm{artanh}\left(\sqrt{x}\right)+\log(-x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x^(1/2))/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x)*arctanh(sqrt(x)) + log(-x + 1)
```

Fricas [A] time = 1.70806, size = 76, normalized size = 3.8

$$\sqrt{x}\log\left(-\frac{x+2\sqrt{x}+1}{x-1}\right)+\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x^(1/2))/x^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(x)*log(-(x + 2*sqrt(x) + 1)/(x - 1)) + log(x - 1)
```

Sympy [B] time = 0.892376, size = 87, normalized size = 4.35

$$\frac{2x^{\frac{3}{2}}\,\mathrm{atanh}\left(\sqrt{x}\right)}{x-1}-\frac{2\sqrt{x}\,\mathrm{atanh}\left(\sqrt{x}\right)}{x-1}+\frac{2x\log\left(\sqrt{x}+1\right)}{x-1}-\frac{2x\,\mathrm{atanh}\left(\sqrt{x}\right)}{x-1}-\frac{2\log\left(\sqrt{x}+1\right)}{x-1}+\frac{2\,\mathrm{atanh}\left(\sqrt{x}\right)}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x**(1/2))/x**(1/2),x)
```

```
[Out] 2*x**(3/2)*atanh(sqrt(x))/(x - 1) - 2*sqrt(x)*atanh(sqrt(x))/(x - 1) + 2*x*
log(sqrt(x) + 1)/(x - 1) - 2*x*atanh(sqrt(x))/(x - 1) - 2*log(sqrt(x) + 1)/
(x - 1) + 2*atanh(sqrt(x))/(x - 1)
```

Giac [A] time = 1.18187, size = 34, normalized size = 1.7

$$\sqrt{x}\log\left(-\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)+\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x^(1/2))/x^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(x)*log(-(sqrt(x) + 1)/(sqrt(x) - 1)) + log(abs(x - 1))
```

3.212 $\int \frac{\tanh^{-1}(\sqrt{x})}{x^{3/2}} dx$

Optimal. Leaf size=24

$$-\log(1-x)+\log(x)-\frac{2\tanh^{-1}(\sqrt{x})}{\sqrt{x}}$$

```
[Out] (-2*ArcTanh[Sqrt[x]])/Sqrt[x] - Log[1 - x] + Log[x]
```

Rubi [A] time = 0.0102569, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6097, 36, 31, 29}

$$-\log(1-x)+\log(x)-\frac{2\tanh^{-1}(\sqrt{x})}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Sqrt[x]]/x^(3/2),x]
```

```
[Out] (-2*ArcTanh[Sqrt[x]])/Sqrt[x] - Log[1 - x] + Log[x]
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^{-1}(\sqrt{x})}{x^{3/2}} dx &= -\frac{2\tanh^{-1}(\sqrt{x})}{\sqrt{x}}+\int \frac{1}{(1-x)x} dx \\
&= -\frac{2\tanh^{-1}(\sqrt{x})}{\sqrt{x}}+\int \frac{1}{1-x} dx+\int \frac{1}{x} dx \\
&= -\frac{2\tanh^{-1}(\sqrt{x})}{\sqrt{x}}-\log(1-x)+\log(x)
\end{aligned}$$

Mathematica [A] time = 0.0176779, size = 24, normalized size = 1.

$$-\log(1-x)+\log(x)-\frac{2\tanh^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Sqrt[x]]/x^(3/2),x]
```

```
[Out] (-2*ArcTanh[Sqrt[x]])/Sqrt[x] - Log[1 - x] + Log[x]
```

Maple [A] time = 0.03, size = 29, normalized size = 1.2

$$-2\frac{\operatorname{Artanh}\left(\sqrt{x}\right)}{\sqrt{x}}-\ln\left(-1+\sqrt{x}\right)+\ln(x)-\ln\left(1+\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x^(1/2))/x^(3/2),x)
```

```
[Out] -2*arctanh(x^(1/2))/x^(1/2)-ln(-1+x^(1/2))+ln(x)-ln(1+x^(1/2))
```

Maxima [A] time = 0.958226, size = 24, normalized size = 1.

$$-\frac{2\operatorname{artanh}\left(\sqrt{x}\right)}{\sqrt{x}}-\log(x-1)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x^(1/2))/x^(3/2),x, algorithm="maxima")
```

```
[Out] -2*arctanh(sqrt(x))/sqrt(x) - log(x - 1) + log(x)
```

Fricas [A] time = 1.80816, size = 100, normalized size = 4.17

$$-\frac{x\log(x-1)-x\log(x)+\sqrt{x}\log\left(-\frac{x+2\sqrt{x}+1}{x-1}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x^(1/2))/x^(3/2),x, algorithm="fricas")
```

```
[Out] -(x*log(x - 1) - x*log(x) + sqrt(x)*log(-(x + 2*sqrt(x) + 1)/(x - 1)))/x
```

Sympy [B] time = 2.41666, size = 126, normalized size = 5.25

$$-\frac{2x^{\frac{3}{2}}\operatorname{atanh}\left(\sqrt{x}\right)}{x^2-x}+\frac{2\sqrt{x}\operatorname{atanh}\left(\sqrt{x}\right)}{x^2-x}+\frac{x^2\log(x)}{x^2-x}-\frac{2x^2\log\left(\sqrt{x}+1\right)}{x^2-x}+\frac{2x^2\operatorname{atanh}\left(\sqrt{x}\right)}{x^2-x}-\frac{x\log(x)}{x^2-x}+\frac{2x\log\left(\sqrt{x}+\right.}{x^2-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x**(1/2))/x**(3/2),x)
```

```
[Out] -2*x**(3/2)*atanh(sqrt(x))/(x**2 - x) + 2*sqrt(x)*atanh(sqrt(x))/(x**2 - x)
 + x**2*log(x)/(x**2 - x) - 2*x**2*log(sqrt(x) + 1)/(x**2 - x) + 2*x**2*ata
nh(sqrt(x))/(x**2 - x) - x*log(x)/(x**2 - x) + 2*x*log(sqrt(x) + 1)/(x**2 -
 x) - 2*x*atanh(sqrt(x))/(x**2 - x)
```

Giac [A] time = 1.16425, size = 41, normalized size = 1.71

$$-\frac{\log\left(-\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)}{\sqrt{x}} + \log\left(x\right) - \log\left(|x - 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x^(1/2))/x^(3/2),x, algorithm="giac")
```

```
[Out] -log(-(sqrt(x) + 1)/(sqrt(x) - 1))/sqrt(x) + log(x) - log(abs(x - 1))
```

3.213 $\int x^3 \left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right) dx$

Optimal. Leaf size=190

$$\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)+\frac{b\log\left(c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1\right)}{16c^{8/3}}-\frac{b\log\left(c^{2/3}x+\sqrt[3]{c}\sqrt{x}+1\right)}{16c^{8/3}}-\frac{\sqrt{3}b\tan^{-1}\left(\frac{1-2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)}{8c^{8/3}}+\frac{\sqrt{3}b\tan$$

```
[Out] (3*b*x^(5/2))/(20*c) - (Sqrt[3]*b*ArcTan[(1 - 2*c^(1/3)*Sqrt[x])/Sqrt[3]])/
(8*c^(8/3)) + (Sqrt[3]*b*ArcTan[(1 + 2*c^(1/3)*Sqrt[x])/Sqrt[3]])/(8*c^(8/3
)) - (b*ArcTanh[c^(1/3)*Sqrt[x]])/(4*c^(8/3)) + (x^4*(a + b*ArcTanh[c*x^(3/
2)]))/4 + (b*Log[1 - c^(1/3)*Sqrt[x] + c^(2/3)*x])/(16*c^(8/3)) - (b*Log[1
+ c^(1/3)*Sqrt[x] + c^(2/3)*x])/(16*c^(8/3))
```

Rubi [A] time = 0.300586, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {6097, 321, 329, 296, 634, 618, 204, 628, 206}

$$\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)+\frac{b\log\left(c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1\right)}{16c^{8/3}}-\frac{b\log\left(c^{2/3}x+\sqrt[3]{c}\sqrt{x}+1\right)}{16c^{8/3}}-\frac{\sqrt{3}b\tan^{-1}\left(\frac{1-2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)}{8c^{8/3}}+\frac{\sqrt{3}b\tan$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcTanh[c*x^(3/2)]),x]
```


```
[Out] (3*b*x^(5/2))/(20*c) - (Sqrt[3]*b*ArcTan[(1 - 2*c^(1/3)*Sqrt[x])/Sqrt[3]])/
(8*c^(8/3)) + (Sqrt[3]*b*ArcTan[(1 + 2*c^(1/3)*Sqrt[x])/Sqrt[3]])/(8*c^(8/3
)) - (b*ArcTanh[c^(1/3)*Sqrt[x]])/(4*c^(8/3)) + (x^4*(a + b*ArcTanh[c*x^(3/
2)]))/4 + (b*Log[1 - c^(1/3)*Sqrt[x] + c^(2/3)*x])/(16*c^(8/3)) - (b*Log[1
+ c^(1/3)*Sqrt[x] + c^(2/3)*x])/(16*c^(8/3))
```


Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
```

```
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^3\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)dx=\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)-\frac{1}{8}(3bc)\int\frac{x^{9/2}}{1-c^2x^3}dx$$
$$=\frac{3bx^{5/2}}{20c}+\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)-\frac{(3b)\int\frac{x^{3/2}}{1-c^2x^3}dx}{8c}$$
$$=\frac{3bx^{5/2}}{20c}+\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)-\frac{(3b)\,\mathrm{Subst}\left(\int\frac{x^4}{1-c^2x^6}dx,x,\sqrt{x}\right)}{4c}$$
$$=\frac{3bx^{5/2}}{20c}+\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)-\frac{b\,\mathrm{Subst}\left(\int\frac{1}{1-c^{2/3}x^2}dx,x,\sqrt{x}\right)}{4c^{7/3}}-\frac{b\,\mathrm{Subst}\left(\int\right.}{}$$
$$=\frac{3bx^{5/2}}{20c}-\frac{b\tanh^{-1}\left(\sqrt[3]{c}\sqrt{x}\right)}{4c^{8/3}}+\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)+\frac{b\,\mathrm{Subst}\left(\int\frac{-\sqrt[3]{c}+2c^{2/3}x}{1-\sqrt[3]{c}x+c^{2/3}x^2}\right.}{16c^{8/3}}$$
$$=\frac{3bx^{5/2}}{20c}-\frac{b\tanh^{-1}\left(\sqrt[3]{c}\sqrt{x}\right)}{4c^{8/3}}+\frac{1}{4}x^4\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)+\frac{b\log\left(1-\sqrt[3]{c}\sqrt{x}+c^{2/3}x\right)}{16c^{8/3}}$$
$$=\frac{3bx^{5/2}}{20c}-\frac{\sqrt{3}b\tan^{-1}\left(\frac{1-2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)}{8c^{8/3}}+\frac{\sqrt{3}b\tan^{-1}\left(\frac{1+2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)}{8c^{8/3}}-\frac{b\tanh^{-1}\left(\sqrt[3]{c}\sqrt{x}\right)}{4c^{8/3}}+\frac{1}{4}x^4$$

Mathematica [A] time = 0.0534598, size = 222, normalized size = 1.17

$$\frac{ax^4}{4}+\frac{b\log\left(1-\sqrt[3]{c}\sqrt{x}\right)}{8c^{8/3}}-\frac{b\log\left(\sqrt[3]{c}\sqrt{x}+1\right)}{8c^{8/3}}+\frac{b\log\left(c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1\right)}{16c^{8/3}}-\frac{b\log\left(c^{2/3}x+\sqrt[3]{c}\sqrt{x}+1\right)}{16c^{8/3}}+\frac{\sqrt{3}b\tan^{-1}\left(2\right.}{8c^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcTanh[c*x^(3/2)]),x]
```

```
[Out] (3*b*x^(5/2))/(20*c) + (a*x^4)/4 + (Sqrt[3]*b*ArcTan[(-1 + 2*c^(1/3)*Sqrt[x
])/Sqrt[3]]])/(8*c^(8/3)) + (Sqrt[3]*b*ArcTan[(1 + 2*c^(1/3)*Sqrt[x]])/Sqrt[3
]])/(8*c^(8/3)) + (b*x^4*ArcTanh[c*x^(3/2)])/4 + (b*Log[1 - c^(1/3)*Sqrt[x]
])/(8*c^(8/3)) - (b*Log[1 + c^(1/3)*Sqrt[x]])/(8*c^(8/3)) + (b*Log[1 - c^(1
/3)*Sqrt[x] + c^(2/3)*x])/(16*c^(8/3)) - (b*Log[1 + c^(1/3)*Sqrt[x] + c^(2/
3)*x])/(16*c^(8/3))
```

Maple [A] time = 0.036, size = 194, normalized size = 1.

$$\frac{x^4a}{4}+\frac{x^4b}{4}\mathrm{Artanh}\left(cx^{\frac{3}{2}}\right)+\frac{3b}{20c}x^{\frac{5}{2}}+\frac{b}{8c^3}\ln\left(\sqrt{x}-\sqrt[3]{c^{-1}}\right)\frac{1}{\sqrt[3]{c^{-1}}}-\frac{b}{16c^3}\ln\left(x+\sqrt[3]{c^{-1}}\sqrt{x}+\left(c^{-1}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{c^{-1}}}+\frac{b\sqrt{3}}{8c^3}\mathrm{arcta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c*x^(3/2))),x)
```

```
[Out] 1/4*x^4*a+1/4*x^4*b*arctanh(c*x^(3/2))+3/20*b*x^(5/2)/c+1/8*b/c^3/(1/c)^(1/
3)*ln(x^(1/2)-(1/c)^(1/3))-1/16*b/c^3/(1/c)^(1/3)*ln(x+(1/c)^(1/3)*x^(1/2)+
(1/c)^(2/3))+1/8*b/c^3*3^(1/2)/(1/c)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3
)*x^(1/2)+1))-1/8*b/c^3/(1/c)^(1/3)*ln(x^(1/2)+(1/c)^(1/3))+1/16*b/c^3/(1/c
)^(1/3)*ln(x-(1/c)^(1/3)*x^(1/2)+(1/c)^(2/3))+1/8*b/c^3*3^(1/2)/(1/c)^(1/3)
*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x^(1/2)-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^(3/2))),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 10.9902, size = 4629, normalized size = 24.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^(3/2))),x, algorithm="fricas")
```

```
[Out] 1/160*(40*a*c*x^4 + 24*b*x^(5/2) - 20*sqrt(3)*sqrt(((1/2)^(1/3)*(b^3 - (c^8
 - 1)*b^3/c^8 + b^3/c^8)^(1/3)*(I*sqrt(3) + 1) + 2*b)^2 - 4*((1/2)^(1/3)*(b
^3 - (c^8 - 1)*b^3/c^8 + b^3/c^8)^(1/3)*(I*sqrt(3) + 1) + 2*b)*b + 4*b^2)*c
*arctan(1/24*(4*sqrt(3)*sqrt(((1/2)^(1/3)*(b^3 - (c^8 - 1)*b^3/c^8 + b^3/c^
8)^(1/3)*(I*sqrt(3) + 1) + 2*b)^2*b^2*c^5*sqrt(x) + 4*b^4*c^5*sqrt(x) + 4*b
^4*c^2 + 4*b^4*x - 2*(2*b^3*c^5*sqrt(x) + b^3*c^2)*((1/2)^(1/3)*(b^3 - (c^8
 - 1)*b^3/c^8 + b^3/c^8)^(1/3)*(I*sqrt(3) + 1) + 2*b))*sqrt(((1/2)^(1/3)*(b
^3 - (c^8 - 1)*b^3/c^8 + b^3/c^8)^(1/3)*(I*sqrt(3) + 1) + 2*b)^2 - 4*((1/2)
^(1/3)*(b^3 - (c^8 - 1)*b^3/c^8 + b^3/c^8)^(1/3)*(I*sqrt(3) + 1) + 2*b)*b +
 4*b^2)*c^3 - sqrt(3)*(((1/2)^(1/3)*(b^3 - (c^8 - 1)*b^3/c^8 + b^3/c^8)^(1/
3)*(I*sqrt(3) + 1) + 2*b)^2*c^8 - 4*((1/2)^(1/3)*(b^3 - (c^8 - 1)*b^3/c^8 +
 b^3/c^8)^(1/3)*(I*sqrt(3) + 1) + 2*b)*b*c^8 + 4*b^2*c^8 + 8*b^2*c^3*sqrt(x
))*sqrt(((1/2)^(1/3)*(b^3 - (c^8 - 1)*b^3/c^8 + b^3/c^8)^(1/3)*(I*sqrt(3) +
 1) + 2*b)^2 - 4*((1/2)^(1/3)*(b^3 - (c^8 - 1)*b^3/c^8 + b^3/c^8)^(1/3)*(I*
sqrt(3) + 1) + 2*b)*b + 4*b^2))/b^3) - 10*((1/2)^(1/3)*(b^3 - (c^8 - 1)*b^3
/c^8 + b^3/c^8)^(1/3)*(I*sqrt(3) + 1) + 2*b)*c*log(-1/4*((1/2)^(1/3)*(b^3 -
 (c^8 - 1)*b^3/c^8 + b^3/c^8)^(1/3)*(I*sqrt(3) + 1) + 2*b)^2*c^5 + ((1/2)^(
1/3)*(b^3 - (c^8 - 1)*b^3/c^8 + b^3/c^8)^(1/3)*(I*sqrt(3) + 1) + 2*b)*b*c^5
 - b^2*c^5 + b^2*sqrt(x)) - 20*(4*(-1/1024*b^3 + 1/1024*(c^8 - 1)*b^3/c^8 +
 1/1024*b^3/c^8)^(1/3)*(I*sqrt(3) + 1) - b)*c*log((4*(-1/1024*b^3 + 1/1024*
(c^8 - 1)*b^3/c^8 + 1/1024*b^3/c^8)^(1/3)*(I*sqrt(3) + 1) - b)^2*c^5 + 2*(4
*(-1/1024*b^3 + 1/1024*(c^8 - 1)*b^3/c^8 + 1/1024*b^3/c^8)^(1/3)*(I*sqrt(3)
 + 1) - b)*b*c^5 + b^2*c^5 + b^2*sqrt(x)) - 40*sqrt(3*(4*(-1/1024*b^3 + 1/1
024*(c^8 - 1)*b^3/c^8 + 1/1024*b^3/c^8)^(1/3)*(I*sqrt(3) + 1) - b)^2 + 6*(4
*(-1/1024*b^3 + 1/1024*(c^8 - 1)*b^3/c^8 + 1/1024*b^3/c^8)^(1/3)*(I*sqrt(3)
 + 1) - b)*b + 3*b^2)*c*arctan(1/3*((4*(-1/1024*b^3 + 1/1024*(c^8 - 1)*b^3/
c^8 + 1/1024*b^3/c^8)^(1/3)*(I*sqrt(3) + 1) - b)^2*c^8 + 2*(4*(-1/1024*b^3
+ 1/1024*(c^8 - 1)*b^3/c^8 + 1/1024*b^3/c^8)^(1/3)*(I*sqrt(3) + 1) - b)*b*c
^8 + b^2*c^8 - 2*b^2*c^3*sqrt(x) + sqrt(-4*(4*(-1/1024*b^3 + 1/1024*(c^8 -
1)*b^3/c^8 + 1/1024*b^3/c^8)^(1/3)*(I*sqrt(3) + 1) - b)^2*b^2*c^5*sqrt(x) -
 4*b^4*c^5*sqrt(x) + 4*b^4*c^2 + 4*b^4*x - 4*(2*b^3*c^5*sqrt(x) - b^3*c^2)*
(4*(-1/1024*b^3 + 1/1024*(c^8 - 1)*b^3/c^8 + 1/1024*b^3/c^8)^(1/3)*(I*sqrt(
3) + 1) - b))*c^3)*sqrt(3*(4*(-1/1024*b^3 + 1/1024*(c^8 - 1)*b^3/c^8 + 1/10
24*b^3/c^8)^(1/3)*(I*sqrt(3) + 1) - b)^2 + 6*(4*(-1/1024*b^3 + 1/1024*(c^8
- 1)*b^3/c^8 + 1/1024*b^3/c^8)^(1/3)*(I*sqrt(3) + 1) - b)*b + 3*b^2)/b^3) +
 5*(((1/2)^(1/3)*(b^3 - (c^8 - 1)*b^3/c^8 + b^3/c^8)^(1/3)*(I*sqrt(3) + 1)
+ 2*b)*c - 6*b*c)*log(((1/2)^(1/3)*(b^3 - (c^8 - 1)*b^3/c^8 + b^3/c^8)^(1/3
```

```
)*(I*sqrt(3) + 1) + 2*b)^2*b^2*c^5*sqrt(x) + 4*b^4*c^5*sqrt(x) + 4*b^4*c^2
+ 4*b^4*x - 2*(2*b^3*c^5*sqrt(x) + b^3*c^2)*((1/2)^(1/3)*(b^3 - (c^8 - 1)*b
^3/c^8 + b^3/c^8)^(1/3)*(I*sqrt(3) + 1) + 2*b)) + 10*((4*(-1/1024*b^3 + 1/1
024*(c^8 - 1)*b^3/c^8 + 1/1024*b^3/c^8)^(1/3)*(I*sqrt(3) + 1) - b)*c + 3*b*
c)*log(-4*(4*(-1/1024*b^3 + 1/1024*(c^8 - 1)*b^3/c^8 + 1/1024*b^3/c^8)^(1/3
)*(I*sqrt(3) + 1) - b)^2*b^2*c^5*sqrt(x) - 4*b^4*c^5*sqrt(x) + 4*b^4*c^2 +
4*b^4*x - 4*(2*b^3*c^5*sqrt(x) - b^3*c^2)*(4*(-1/1024*b^3 + 1/1024*(c^8 - 1
)*b^3/c^8 + 1/1024*b^3/c^8)^(1/3)*(I*sqrt(3) + 1) - b)) + 20*(b*c*x^4 - b*c
)*log(-(c^2*x^3 + 2*c*x^(3/2) + 1)/(c^2*x^3 - 1)))/c
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c*x**(3/2))),x)
```

```
[Out] Timed out
```

Giac [C] time = 1.49211, size = 296, normalized size = 1.56

$$\frac{1}{4}ax^4 + \frac{1}{320}\left(40\,x^4\log\left(-\frac{cx^{\frac{3}{2}}+1}{cx^{\frac{3}{2}}-1}\right) + c\left(\frac{48\,x^{\frac{5}{2}}}{c^2} - \frac{10\,\sqrt{3}\left(-i\,\sqrt{3}-1\right)^2\arctan\left(\frac{\sqrt{3}\left(2\,\sqrt{x}+\left(-\frac{1}{c}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{1}{c}\right)^{\frac{1}{3}}}\right)}{c^{\frac{11}{3}}} + \frac{5\left(-i\,\sqrt{3}-1\right)^2\log\left(x+\right.}{c^{\frac{1}{3}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^(3/2))),x, algorithm="giac")
```

```
[Out] 1/4*a*x^4 + 1/320*(40*x^4*log(-(c*x^(3/2) + 1)/(c*x^(3/2) - 1)) + c*(48*x^(
5/2)/c^2 - 10*sqrt(3)*(-I*sqrt(3) - 1)^2*arctan(1/3*sqrt(3)*(2*sqrt(x) + (-
1/c)^(1/3))/(-1/c)^(1/3))/c^(11/3) + 5*(-I*sqrt(3) - 1)^2*log(x + sqrt(x)*(
-1/c)^(1/3) + (-1/c)^(2/3))/c^(11/3) - 40*(-1/c)^(2/3)*log(abs(sqrt(x) - (-
1/c)^(1/3)))/c^3 + 40*sqrt(3)*abs(c)^(4/3)*arctan(1/3*sqrt(3)*c^(1/3)*(2*sq
rt(x) + 1/c^(1/3)))/c^5 - 20*abs(c)^(4/3)*log(x + sqrt(x)/c^(1/3) + 1/c^(2/
3))/c^5 + 40*log(abs(sqrt(x) - 1/c^(1/3)))/c^(11/3)))*b
```

3.214 $\int x^2 \left(a + b \tanh^{-1}\left(cx^{3/2}\right)\right) dx$

Optimal. Leaf size=49

$$\frac{1}{3}x^3\left(a + b\tanh^{-1}\left(cx^{3/2}\right)\right) - \frac{b\tanh^{-1}\left(cx^{3/2}\right)}{3c^2} + \frac{bx^{3/2}}{3c}$$

```
[Out] (b*x^(3/2))/(3*c) - (b*ArcTanh[c*x^(3/2)])/(3*c^2) + (x^3*(a + b*ArcTanh[c*
x^(3/2)]))/3
```

Rubi [A] time = 0.0337865, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {6097, 321, 329, 275, 206}

$$\frac{1}{3}x^3\left(a + b\tanh^{-1}\left(cx^{3/2}\right)\right) - \frac{b\tanh^{-1}\left(cx^{3/2}\right)}{3c^2} + \frac{bx^{3/2}}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcTanh[c*x^(3/2)]),x]
```

```
[Out] (b*x^(3/2))/(3*c) - (b*ArcTanh[c*x^(3/2)])/(3*c^2) + (x^3*(a + b*ArcTanh[c*
x^(3/2)]))/3
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int x^2\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)dx &= \frac{1}{3}x^3\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)-\frac{1}{2}(bc)\int\frac{x^{7/2}}{1-c^2x^3}\,dx\\
&= \frac{bx^{3/2}}{3c}+\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)-\frac{b\int\frac{\sqrt{x}}{1-c^2x^3}\,dx}{2c}\\
&= \frac{bx^{3/2}}{3c}+\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)-\frac{b\,\text{Subst}\left(\int\frac{x^2}{1-c^2x^6}\,dx,x,\sqrt{x}\right)}{c}\\
&= \frac{bx^{3/2}}{3c}+\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)-\frac{b\,\text{Subst}\left(\int\frac{1}{1-c^2x^2}\,dx,x,x^{3/2}\right)}{3c}\\
&= \frac{bx^{3/2}}{3c}-\frac{b\tanh^{-1}\left(cx^{3/2}\right)}{3c^2}+\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)
\end{aligned}$$

Mathematica [A] time = 0.0263497, size = 75, normalized size = 1.53

$$\frac{ax^3}{3}+\frac{b\log\left(1-cx^{3/2}\right)}{6c^2}-\frac{b\log\left(cx^{3/2}+1\right)}{6c^2}+\frac{bx^{3/2}}{3c}+\frac{1}{3}bx^3\tanh^{-1}\left(cx^{3/2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcTanh[c*x^(3/2)]),x]
```

```
[Out] (b*x^(3/2))/(3*c) + (a*x^3)/3 + (b*x^3*ArcTanh[c*x^(3/2)])/3 + (b*Log[1 - c
*x^(3/2)])/(6*c^2) - (b*Log[1 + c*x^(3/2)])/(6*c^2)
```

Maple [A] time = 0.029, size = 57, normalized size = 1.2

$$\frac{x^3a}{3}+\frac{bx^3}{3}\text{Artanh}\left(cx^{\frac{3}{2}}\right)+\frac{b}{3c}x^{\frac{3}{2}}+\frac{b}{6c^2}\ln\left(cx^{\frac{3}{2}}-1\right)-\frac{b}{6c^2}\ln\left(cx^{\frac{3}{2}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c*x^(3/2))),x)
```

```
[Out] 1/3*x^3*a+1/3*b*x^3*arctanh(c*x^(3/2))+1/3*b*x^(3/2)/c+1/6/c^2*b*ln(c*x^(3/
2)-1)-1/6/c^2*b*ln(c*x^(3/2)+1)
```

Maxima [A] time = 0.995113, size = 78, normalized size = 1.59

$$\frac{1}{3}ax^3+\frac{1}{6}\left(2x^3\,\text{artanh}\left(cx^{\frac{3}{2}}\right)+c\left(\frac{2x^{\frac{3}{2}}}{c^2}-\frac{\log\left(cx^{\frac{3}{2}}+1\right)}{c^3}+\frac{\log\left(cx^{\frac{3}{2}}-1\right)}{c^3}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^(3/2))),x, algorithm="maxima")
```

```
[Out] 1/3*a*x^3 + 1/6*(2*x^3*arctanh(c*x^(3/2)) + c*(2*x^(3/2)/c^2 - log(c*x^(3/2
) + 1)/c^3 + log(c*x^(3/2) - 1)/c^3))*b
```

Fricas [A] time = 1.85607, size = 142, normalized size = 2.9

$$\frac{2\,ac^2x^3 + 2\,bcx^{\frac{3}{2}} + \left(bc^2x^3 - b\right)\log\left(-\frac{c^2x^3+2\,cx^{\frac{3}{2}}+1}{c^2x^3-1}\right)}{6\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^(3/2))),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*c^2*x^3 + 2*b*c*x^(3/2) + (b*c^2*x^3 - b)*log(-(c^2*x^3 + 2*c*x^(3
/2) + 1)/(c^2*x^3 - 1)))/c^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c*x**(3/2))),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22804, size = 97, normalized size = 1.98

$$\frac{1}{3}\,ax^3 + \frac{1}{6}\left(x^3\log\left(-\frac{cx^{\frac{3}{2}}+1}{cx^{\frac{3}{2}}-1}\right) + c\left(\frac{2\,x^{\frac{3}{2}}}{c^2} - \frac{\log\left(\left|cx^{\frac{3}{2}}+1\right|\right)}{c^3} + \frac{\log\left(\left|cx^{\frac{3}{2}}-1\right|\right)}{c^3}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^(3/2))),x, algorithm="giac")
```

```
[Out] 1/3*a*x^3 + 1/6*(x^3*log(-(c*x^(3/2) + 1)/(c*x^(3/2) - 1)) + c*(2*x^(3/2)/c
^2 - log(abs(c*x^(3/2) + 1))/c^3 + log(abs(c*x^(3/2) - 1))/c^3))*b
```

3.215 $\int x\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)dx$

Optimal. Leaf size=190

$$\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)+\frac{b\log\left(c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1\right)}{8c^{4/3}}-\frac{b\log\left(c^{2/3}x+\sqrt[3]{c}\sqrt{x}+1\right)}{8c^{4/3}}+\frac{\sqrt{3}b\tan^{-1}\left(\frac{1-2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)}{4c^{4/3}}-\frac{\sqrt{3}b\tan}{}$$

```
[Out] (3*b*Sqrt[x])/(2*c) + (Sqrt[3]*b*ArcTan[(1 - 2*c^(1/3)*Sqrt[x])/Sqrt[3]])/(
4*c^(4/3)) - (Sqrt[3]*b*ArcTan[(1 + 2*c^(1/3)*Sqrt[x])/Sqrt[3]])/(4*c^(4/3)
) - (b*ArcTanh[c^(1/3)*Sqrt[x]])/(2*c^(4/3)) + (x^2*(a + b*ArcTanh[c*x^(3/2
)]))/2 + (b*Log[1 - c^(1/3)*Sqrt[x] + c^(2/3)*x])/(8*c^(4/3)) - (b*Log[1 +
c^(1/3)*Sqrt[x] + c^(2/3)*x])/(8*c^(4/3))
```

Rubi [A] time = 0.238218, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {6097, 321, 329, 210, 634, 618, 204, 628, 206}

$$\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)+\frac{b\log\left(c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1\right)}{8c^{4/3}}-\frac{b\log\left(c^{2/3}x+\sqrt[3]{c}\sqrt{x}+1\right)}{8c^{4/3}}+\frac{\sqrt{3}b\tan^{-1}\left(\frac{1-2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)}{4c^{4/3}}-\frac{\sqrt{3}b\tan}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcTanh[c*x^(3/2)]),x]
```


```
[Out] (3*b*Sqrt[x])/(2*c) + (Sqrt[3]*b*ArcTan[(1 - 2*c^(1/3)*Sqrt[x])/Sqrt[3]])/(
4*c^(4/3)) - (Sqrt[3]*b*ArcTan[(1 + 2*c^(1/3)*Sqrt[x])/Sqrt[3]])/(4*c^(4/3)
) - (b*ArcTanh[c^(1/3)*Sqrt[x]])/(2*c^(4/3)) + (x^2*(a + b*ArcTanh[c*x^(3/2
)]))/2 + (b*Log[1 - c^(1/3)*Sqrt[x] + c^(2/3)*x])/(8*c^(4/3)) - (b*Log[1 +
c^(1/3)*Sqrt[x] + c^(2/3)*x])/(8*c^(4/3))
```


Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
```

```
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)dx=\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)-\frac{1}{4}(3bc)\int\frac{x^{5/2}}{1-c^2x^3}\,dx$$

$$=\frac{3b\sqrt{x}}{2c}+\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)-\frac{(3b)\int\frac{1}{\sqrt{x}\left(1-c^2x^3\right)}\,dx}{4c}$$

$$=\frac{3b\sqrt{x}}{2c}+\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)-\frac{(3b)\,\mathrm{Subst}\left(\int\frac{1}{1-c^2x^6}\,dx,x,\sqrt{x}\right)}{2c}$$

$$=\frac{3b\sqrt{x}}{2c}+\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)-\frac{b\,\mathrm{Subst}\left(\int\frac{1}{1-c^{2/3}x^2}\,dx,x,\sqrt{x}\right)}{2c}-\frac{b\,\mathrm{Subst}\left(\int\frac{1-}{1-\sqrt[3]{c}x}\right.}{2c}$$

$$=\frac{3b\sqrt{x}}{2c}-\frac{b\tanh^{-1}\left(\sqrt[3]{c}\sqrt{x}\right)}{2c^{4/3}}+\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)+\frac{b\,\mathrm{Subst}\left(\int\frac{-\sqrt[3]{c}+2c^{2/3}x}{1-\sqrt[3]{c}x+c^{2/3}x^2}\,dx,x,\right.}{8c^{4/3}}$$

$$=\frac{3b\sqrt{x}}{2c}-\frac{b\tanh^{-1}\left(\sqrt[3]{c}\sqrt{x}\right)}{2c^{4/3}}+\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)+\frac{b\log\left(1-\sqrt[3]{c}\sqrt{x}+c^{2/3}x\right)}{8c^{4/3}}-\frac{b}{}$$

$$=\frac{3b\sqrt{x}}{2c}+\frac{\sqrt{3}b\tan^{-1}\left(\frac{1-2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)}{4c^{4/3}}-\frac{\sqrt{3}b\tan^{-1}\left(\frac{1+2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)}{4c^{4/3}}-\frac{b\tanh^{-1}\left(\sqrt[3]{c}\sqrt{x}\right)}{2c^{4/3}}+\frac{1}{2}x^2\left(a+\right.$$

Mathematica [A] time = 0.0337743, size = 222, normalized size = 1.17

$$\frac{ax^2}{2}+\frac{b\log\left(1-\sqrt[3]{c}\sqrt{x}\right)}{4c^{4/3}}-\frac{b\log\left(\sqrt[3]{c}\sqrt{x}+1\right)}{4c^{4/3}}+\frac{b\log\left(c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1\right)}{8c^{4/3}}-\frac{b\log\left(c^{2/3}x+\sqrt[3]{c}\sqrt{x}+1\right)}{8c^{4/3}}-\frac{\sqrt{3}b\tan^{-1}\left(\frac{2}{}\right.}{4c^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcTanh[c*x^(3/2)]),x]
```

```
[Out] (3*b*Sqrt[x])/(2*c) + (a*x^2)/2 - (Sqrt[3]*b*ArcTan[(-1 + 2*c^(1/3)*Sqrt[x]
)/Sqrt[3]])/(4*c^(4/3)) - (Sqrt[3]*b*ArcTan[(1 + 2*c^(1/3)*Sqrt[x])/Sqrt[3]
])/(4*c^(4/3)) + (b*x^2*ArcTanh[c*x^(3/2)])/2 + (b*Log[1 - c^(1/3)*Sqrt[x]]
)/(4*c^(4/3)) - (b*Log[1 + c^(1/3)*Sqrt[x]])/(4*c^(4/3)) + (b*Log[1 - c^(1/
3)*Sqrt[x] + c^(2/3)*x])/(8*c^(4/3)) - (b*Log[1 + c^(1/3)*Sqrt[x] + c^(2/3)
*x])/(8*c^(4/3))
```

Maple [A] time = 0.036, size = 194, normalized size = 1.

$$\frac{ax^2}{2}+\frac{bx^2}{2}\mathrm{Artanh}\left(cx^{\frac{3}{2}}\right)+\frac{3b}{2c}\sqrt{x}+\frac{b}{4c^2}\ln\left(\sqrt{x}-\sqrt[3]{c^{-1}}\right)\left(c^{-1}\right)^{-\frac{2}{3}}-\frac{b}{8c^2}\ln\left(x+\sqrt[3]{c^{-1}}\sqrt{x}+\left(c^{-1}\right)^{\frac{2}{3}}\right)\left(c^{-1}\right)^{-\frac{2}{3}}-\frac{b\sqrt{3}}{4c^2}\mathrm{arc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c*x^(3/2))),x)
```

```
[Out] 1/2*a*x^2+1/2*b*x^2*arctanh(c*x^(3/2))+3/2*b*x^(1/2)/c+1/4*b/c^2/(1/c)^(2/3
)*ln(x^(1/2)-(1/c)^(1/3))-1/8*b/c^2/(1/c)^(2/3)*ln(x+(1/c)^(1/3)*x^(1/2)+(1
/c)^(2/3))-1/4*b/c^2/(1/c)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*
x^(1/2)+1))-1/4*b/c^2/(1/c)^(2/3)*ln(x^(1/2)+(1/c)^(1/3))+1/8*b/c^2/(1/c)^(
2/3)*ln(x-(1/c)^(1/3)*x^(1/2)+(1/c)^(2/3))-1/4*b/c^2/(1/c)^(2/3)*3^(1/2)*ar
ctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x^(1/2)-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^(3/2))),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 10.5287, size = 4730, normalized size = 24.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^(3/2))),x, algorithm="fricas")
```

```
[Out] 1/16*(8*a*c*x^2 + 4*sqrt(3)*sqrt(((1/2)^(1/3)*(b^3 - (c^4 - 1)*b^3/c^4 + b^
3/c^4)^(1/3)*(I*sqrt(3) + 1) + 2*b)^2 - 4*((1/2)^(1/3)*(b^3 - (c^4 - 1)*b^3
/c^4 + b^3/c^4)^(1/3)*(I*sqrt(3) + 1) + 2*b)*b + 4*b^2)*c*arctan(-1/24*(2*s
qrt(3)*sqrt(((1/2)^(1/3)*(b^3 - (c^4 - 1)*b^3/c^4 + b^3/c^4)^(1/3)*(I*sqrt(
3) + 1) + 2*b)^2*c^2 + 4*b^2*c^2 + 4*b^2*c*sqrt(x) + 4*b^2*x - 2*(2*b*c^2 +
 b*c*sqrt(x))*((1/2)^(1/3)*(b^3 - (c^4 - 1)*b^3/c^4 + b^3/c^4)^(1/3)*(I*sqr
t(3) + 1) + 2*b))*(((1/2)^(1/3)*(b^3 - (c^4 - 1)*b^3/c^4 + b^3/c^4)^(1/3)*(
I*sqrt(3) + 1) + 2*b)*c^3 - 2*b*c^3)*sqrt(((1/2)^(1/3)*(b^3 - (c^4 - 1)*b^3
/c^4 + b^3/c^4)^(1/3)*(I*sqrt(3) + 1) + 2*b)^2 - 4*((1/2)^(1/3)*(b^3 - (c^4
 - 1)*b^3/c^4 + b^3/c^4)^(1/3)*(I*sqrt(3) + 1) + 2*b)*b + 4*b^2) + sqrt(3)*
(((1/2)^(1/3)*(b^3 - (c^4 - 1)*b^3/c^4 + b^3/c^4)^(1/3)*(I*sqrt(3) + 1) + 2
*b)^2*c^4 + 4*b^2*c^4 + 8*b^2*c^3*sqrt(x) - 4*(b*c^4 + b*c^3*sqrt(x))*((1/2
)^(1/3)*(b^3 - (c^4 - 1)*b^3/c^4 + b^3/c^4)^(1/3)*(I*sqrt(3) + 1) + 2*b))*s
qrt(((1/2)^(1/3)*(b^3 - (c^4 - 1)*b^3/c^4 + b^3/c^4)^(1/3)*(I*sqrt(3) + 1)
+ 2*b)^2 - 4*((1/2)^(1/3)*(b^3 - (c^4 - 1)*b^3/c^4 + b^3/c^4)^(1/3)*(I*sqrt
(3) + 1) + 2*b)*b + 4*b^2))/b^3) - 2*((1/2)^(1/3)*(b^3 - (c^4 - 1)*b^3/c^4
+ b^3/c^4)^(1/3)*(I*sqrt(3) + 1) + 2*b)*c*log(1/2*((1/2)^(1/3)*(b^3 - (c^4
- 1)*b^3/c^4 + b^3/c^4)^(1/3)*(I*sqrt(3) + 1) + 2*b)*c - b*c + b*sqrt(x)) -
 4*(2*(-1/128*b^3 + 1/128*(c^4 - 1)*b^3/c^4 + 1/128*b^3/c^4)^(1/3)*(I*sqrt(
3) + 1) - b)*c*log((2*(-1/128*b^3 + 1/128*(c^4 - 1)*b^3/c^4 + 1/128*b^3/c^4
)^(1/3)*(I*sqrt(3) + 1) - b)*c + b*c + b*sqrt(x)) - 8*sqrt(3*(2*(-1/128*b^3
 + 1/128*(c^4 - 1)*b^3/c^4 + 1/128*b^3/c^4)^(1/3)*(I*sqrt(3) + 1) - b)^2 +
6*(2*(-1/128*b^3 + 1/128*(c^4 - 1)*b^3/c^4 + 1/128*b^3/c^4)^(1/3)*(I*sqrt(3
) + 1) - b)*b + 3*b^2)*c*arctan(-1/3*(2*sqrt((2*(-1/128*b^3 + 1/128*(c^4 -
1)*b^3/c^4 + 1/128*b^3/c^4)^(1/3)*(I*sqrt(3) + 1) - b)^2*c^2 + b^2*c^2 - b^
2*c*sqrt(x) + b^2*x + (2*b*c^2 - b*c*sqrt(x))*(2*(-1/128*b^3 + 1/128*(c^4 -
 1)*b^3/c^4 + 1/128*b^3/c^4)^(1/3)*(I*sqrt(3) + 1) - b))*((2*(-1/128*b^3 +
1/128*(c^4 - 1)*b^3/c^4 + 1/128*b^3/c^4)^(1/3)*(I*sqrt(3) + 1) - b)*c^3 + b
*c^3)*sqrt(3*(2*(-1/128*b^3 + 1/128*(c^4 - 1)*b^3/c^4 + 1/128*b^3/c^4)^(1/3
)*(I*sqrt(3) + 1) - b)^2 + 6*(2*(-1/128*b^3 + 1/128*(c^4 - 1)*b^3/c^4 + 1/1
28*b^3/c^4)^(1/3)*(I*sqrt(3) + 1) - b)*b + 3*b^2) + ((2*(-1/128*b^3 + 1/128
*(c^4 - 1)*b^3/c^4 + 1/128*b^3/c^4)^(1/3)*(I*sqrt(3) + 1) - b)^2*c^4 + b^2*
c^4 - 2*b^2*c^3*sqrt(x) + 2*(b*c^4 - b*c^3*sqrt(x))*(2*(-1/128*b^3 + 1/128*
(c^4 - 1)*b^3/c^4 + 1/128*b^3/c^4)^(1/3)*(I*sqrt(3) + 1) - b))*sqrt(3*(2*(-
1/128*b^3 + 1/128*(c^4 - 1)*b^3/c^4 + 1/128*b^3/c^4)^(1/3)*(I*sqrt(3) + 1)
- b)^2 + 6*(2*(-1/128*b^3 + 1/128*(c^4 - 1)*b^3/c^4 + 1/128*b^3/c^4)^(1/3)*
(I*sqrt(3) + 1) - b)*b + 3*b^2))/b^3) + (((1/2)^(1/3)*(b^3 - (c^4 - 1)*b^3/
c^4 + b^3/c^4)^(1/3)*(I*sqrt(3) + 1) + 2*b)*c - 6*b*c)*log(((1/2)^(1/3)*(b^
3 - (c^4 - 1)*b^3/c^4 + b^3/c^4)^(1/3)*(I*sqrt(3) + 1) + 2*b)^2*c^2 + 4*b^2
*c^2 + 4*b^2*c*sqrt(x) + 4*b^2*x - 2*(2*b*c^2 + b*c*sqrt(x))*((1/2)^(1/3)*(
b^3 - (c^4 - 1)*b^3/c^4 + b^3/c^4)^(1/3)*(I*sqrt(3) + 1) + 2*b)) + 2*((2*(-
1/128*b^3 + 1/128*(c^4 - 1)*b^3/c^4 + 1/128*b^3/c^4)^(1/3)*(I*sqrt(3) + 1)
- b)*c + 3*b*c)*log(4*(2*(-1/128*b^3 + 1/128*(c^4 - 1)*b^3/c^4 + 1/128*b^3/
c^4)^(1/3)*(I*sqrt(3) + 1) - b)^2*c^2 + 4*b^2*c^2 - 4*b^2*c*sqrt(x) + 4*b^2
*x + 4*(2*b*c^2 - b*c*sqrt(x))*(2*(-1/128*b^3 + 1/128*(c^4 - 1)*b^3/c^4 + 1
/128*b^3/c^4)^(1/3)*(I*sqrt(3) + 1) - b)) + 4*(b*c*x^2 - b*c)*log(-(c^2*x^3
 + 2*c*x^(3/2) + 1)/(c^2*x^3 - 1)) + 24*b*sqrt(x))/c
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c*x**(3/2))),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\,\mathrm{artanh}\left(cx^{\frac{3}{2}}\right)+a\right)x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^(3/2))),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^(3/2)) + a)*x, x)
```

3.216 $\int \left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)\,dx$

Optimal. Leaf size=170

$$ax+\frac{b\log\left(c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1\right)}{4c^{2/3}}-\frac{b\log\left(c^{2/3}x+\sqrt[3]{c}\sqrt{x}+1\right)}{4c^{2/3}}-\frac{\sqrt{3}b\tan^{-1}\left(\frac{1-2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)}{2c^{2/3}}+\frac{\sqrt{3}b\tan^{-1}\left(\frac{2\sqrt[3]{c}\sqrt{x}+1}{\sqrt{3}}\right)}{2c^{2/3}}-\frac{b\tanh^{-1}\left(\right.}{c^{2/3}}$$

```
[Out] a*x - (Sqrt[3]*b*ArcTan[(1 - 2*c^(1/3)*Sqrt[x])/Sqrt[3]])/(2*c^(2/3)) + (Sq
rt[3]*b*ArcTan[(1 + 2*c^(1/3)*Sqrt[x])/Sqrt[3]])/(2*c^(2/3)) - (b*ArcTanh[c
^(1/3)*Sqrt[x]])/c^(2/3) + b*x*ArcTanh[c*x^(3/2)] + (b*Log[1 - c^(1/3)*Sqrt
[x] + c^(2/3)*x])/(4*c^(2/3)) - (b*Log[1 + c^(1/3)*Sqrt[x] + c^(2/3)*x])/(4
*c^(2/3))
```

Rubi [A] time = 0.287018, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {6091, 329, 296, 634, 618, 204, 628, 206}

$$ax+\frac{b\log\left(c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1\right)}{4c^{2/3}}-\frac{b\log\left(c^{2/3}x+\sqrt[3]{c}\sqrt{x}+1\right)}{4c^{2/3}}-\frac{\sqrt{3}b\tan^{-1}\left(\frac{1-2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)}{2c^{2/3}}+\frac{\sqrt{3}b\tan^{-1}\left(\frac{2\sqrt[3]{c}\sqrt{x}+1}{\sqrt{3}}\right)}{2c^{2/3}}-\frac{b\tanh^{-1}\left(\right.}{c^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcTanh[c*x^(3/2)],x]
```

```
[Out] a*x - (Sqrt[3]*b*ArcTan[(1 - 2*c^(1/3)*Sqrt[x])/Sqrt[3]])/(2*c^(2/3)) + (Sq
rt[3]*b*ArcTan[(1 + 2*c^(1/3)*Sqrt[x])/Sqrt[3]])/(2*c^(2/3)) - (b*ArcTanh[c
^(1/3)*Sqrt[x]])/c^(2/3) + b*x*ArcTanh[c*x^(3/2)] + (b*Log[1 - c^(1/3)*Sqrt
[x] + c^(2/3)*x])/(4*c^(2/3)) - (b*Log[1 + c^(1/3)*Sqrt[x] + c^(2/3)*x])/(4
*c^(2/3))
```

Rule 6091

```
Int[ArcTanh[(c_.)*(x_)^(n_)], x_Symbol] :> Simp[x*ArcTanh[c*x^n], x] - Dist
[c*n, Int[x^n/(1 - c^2*x^(2*n)), x], x] /; FreeQ[{c, n}, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)dx &= ax+b\int \tanh^{-1}\left(cx^{3/2}\right)dx \\
&= ax+bx\tanh^{-1}\left(cx^{3/2}\right)-\frac{1}{2}(3bc)\int \frac{x^{3/2}}{1-c^2x^3}\,dx \\
&= ax+bx\tanh^{-1}\left(cx^{3/2}\right)-(3bc)\operatorname{Subst}\left(\int \frac{x^4}{1-c^2x^6}\,dx,x,\sqrt{x}\right) \\
&= ax+bx\tanh^{-1}\left(cx^{3/2}\right)-\frac{b\operatorname{Subst}\left(\int \frac{1}{1-c^{2/3}x^2}\,dx,x,\sqrt{x}\right)}{\sqrt[3]{c}}-\frac{b\operatorname{Subst}\left(\int \frac{-\frac{1}{2}-\frac{\sqrt[3]{c}x}{2}}{1-\sqrt[3]{c}x+c^{2/3}x^2}\,dx,x,\right.}{\sqrt[3]{c}} \\
&= ax-\frac{b\tanh^{-1}\left(\sqrt[3]{c}\sqrt{x}\right)}{c^{2/3}}+bx\tanh^{-1}\left(cx^{3/2}\right)+\frac{b\operatorname{Subst}\left(\int \frac{-\sqrt[3]{c}+2c^{2/3}x}{1-\sqrt[3]{c}x+c^{2/3}x^2}\,dx,x,\sqrt{x}\right)}{4c^{2/3}}-\frac{b\operatorname{Su}}{} \\
&= ax-\frac{b\tanh^{-1}\left(\sqrt[3]{c}\sqrt{x}\right)}{c^{2/3}}+bx\tanh^{-1}\left(cx^{3/2}\right)+\frac{b\log\left(1-\sqrt[3]{c}\sqrt{x}+c^{2/3}x\right)}{4c^{2/3}}-\frac{b\log\left(1+\sqrt[3]{\right.}}{4c} \\
&= ax-\frac{\sqrt{3}b\tan^{-1}\left(\frac{1-2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)}{2c^{2/3}}+\frac{\sqrt{3}b\tan^{-1}\left(\frac{1+2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)}{2c^{2/3}}-\frac{b\tanh^{-1}\left(\sqrt[3]{c}\sqrt{x}\right)}{c^{2/3}}+bx\tanh^{-1}\left(\right.
\end{aligned}
$$

Mathematica [A] time = 0.100893, size = 141, normalized size = 0.83

$$
ax-\frac{b\left(-\log\left(c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1\right)+\log\left(c^{2/3}x+\sqrt[3]{c}\sqrt{x}+1\right)+2\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)-2\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{c}\sqrt{x}+1}{\sqrt{3}}\right)+4\tanh^{-}\right.}{4c^{2/3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcTanh[c*x^(3/2)],x]
```

```
[Out] a*x + b*x*ArcTanh[c*x^(3/2)] - (b*(2*Sqrt[3]*ArcTan[(1 - 2*c^(1/3)*Sqrt[x])
/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*c^(1/3)*Sqrt[x])/Sqrt[3]] + 4*ArcTanh[c
^(1/3)*Sqrt[x]] - Log[1 - c^(1/3)*Sqrt[x] + c^(2/3)*x] + Log[1 + c^(1/3)*Sq
rt[x] + c^(2/3)*x]))/(4*c^(2/3))
```

Maple [A] time = 0.032, size = 179, normalized size = 1.1

$$ax + bx\operatorname{Artanh}\left(cx^{\frac{3}{2}}\right) + \frac{b}{2c}\ln\left(\sqrt{x} - \sqrt[3]{c^{-1}}\right)\frac{1}{\sqrt[3]{c^{-1}}} - \frac{b}{4c}\ln\left(x + \sqrt[3]{c^{-1}}\sqrt{x} + \left(c^{-1}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{c^{-1}}} + \frac{b\sqrt{3}}{2c}\arctan\left(\frac{\sqrt{3}}{3}\left(2\frac{\sqrt{x}}{\sqrt[3]{c^{-1}}} + \right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arctanh(c*x^(3/2)),x)
```

```
[Out] a*x+b*x*arctanh(c*x^(3/2))+1/2*b/c/(1/c)^(1/3)*ln(x^(1/2)-(1/c)^(1/3))-1/4*
b/c/(1/c)^(1/3)*ln(x+(1/c)^(1/3)*x^(1/2)+(1/c)^(2/3))+1/2*b*3^(1/2)/c/(1/c)
^(1/3)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x^(1/2)+1))-1/2*b/c/(1/c)^(1/3)*ln
(x^(1/2)+(1/c)^(1/3))+1/4*b/c/(1/c)^(1/3)*ln(x-(1/c)^(1/3)*x^(1/2)+(1/c)^(2
/3))+1/2*b*3^(1/2)/c/(1/c)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x^(1/2)-
1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c*x^(3/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 10.7687, size = 4289, normalized size = 25.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c*x^(3/2)),x, algorithm="fricas")
```

```
[Out] a*x - 1/2*sqrt(3)*sqrt(((1/2)^(1/3)*(b^3 - (c^2 - 1)*b^3/c^2 + b^3/c^2)^(1/
3)*(I*sqrt(3) + 1) + 2*b)^2 - 4*((1/2)^(1/3)*(b^3 - (c^2 - 1)*b^3/c^2 + b^3
/c^2)^(1/3)*(I*sqrt(3) + 1) + 2*b)*b + 4*b^2)*arctan(1/24*(4*sqrt(3)*sqrt((
(1/2)^(1/3)*(b^3 - (c^2 - 1)*b^3/c^2 + b^3/c^2)^(1/3)*(I*sqrt(3) + 1) + 2*b
)^2*b^2*c*sqrt(x) + 4*b^4*c*sqrt(x) + 4*b^4*x + 4*b^4 - 2*(2*b^3*c*sqrt(x)
+ b^3)*((1/2)^(1/3)*(b^3 - (c^2 - 1)*b^3/c^2 + b^3/c^2)^(1/3)*(I*sqrt(3) +
1) + 2*b))*sqrt(((1/2)^(1/3)*(b^3 - (c^2 - 1)*b^3/c^2 + b^3/c^2)^(1/3)*(I*s
qrt(3) + 1) + 2*b)^2 - 4*((1/2)^(1/3)*(b^3 - (c^2 - 1)*b^3/c^2 + b^3/c^2)^(
1/3)*(I*sqrt(3) + 1) + 2*b)*b + 4*b^2)*c - sqrt(3)*(((1/2)^(1/3)*(b^3 - (c^
2 - 1)*b^3/c^2 + b^3/c^2)^(1/3)*(I*sqrt(3) + 1) + 2*b)^2*c^2 - 4*((1/2)^(1/
```

```
3)*(b^3 - (c^2 - 1)*b^3/c^2 + b^3/c^2)^(1/3)*(I*sqrt(3) + 1) + 2*b)*b*c^2 +
 4*b^2*c^2 + 8*b^2*c*sqrt(x))*sqrt(((1/2)^(1/3)*(b^3 - (c^2 - 1)*b^3/c^2 +
b^3/c^2)^(1/3)*(I*sqrt(3) + 1) + 2*b)^2 - 4*((1/2)^(1/3)*(b^3 - (c^2 - 1)*b
^3/c^2 + b^3/c^2)^(1/3)*(I*sqrt(3) + 1) + 2*b)*b + 4*b^2))/b^3) + 1/8*((1/2
)^(1/3)*(b^3 - (c^2 - 1)*b^3/c^2 + b^3/c^2)^(1/3)*(I*sqrt(3) + 1) - 4*b)*lo
g(((1/2)^(1/3)*(b^3 - (c^2 - 1)*b^3/c^2 + b^3/c^2)^(1/3)*(I*sqrt(3) + 1) +
2*b)^2*b^2*c*sqrt(x) + 4*b^4*c*sqrt(x) + 4*b^4*x + 4*b^4 - 2*(2*b^3*c*sqrt(
x) + b^3)*((1/2)^(1/3)*(b^3 - (c^2 - 1)*b^3/c^2 + b^3/c^2)^(1/3)*(I*sqrt(3)
 + 1) + 2*b)) + 1/4*((-1/16*b^3 + 1/16*(c^2 - 1)*b^3/c^2 + 1/16*b^3/c^2)^(1
/3)*(I*sqrt(3) + 1) + 2*b)*log(-4*((-1/16*b^3 + 1/16*(c^2 - 1)*b^3/c^2 + 1/
16*b^3/c^2)^(1/3)*(I*sqrt(3) + 1) - b)^2*b^2*c*sqrt(x) - 4*b^4*c*sqrt(x) +
4*b^4*x + 4*b^4 - 4*(2*b^3*c*sqrt(x) - b^3)*((-1/16*b^3 + 1/16*(c^2 - 1)*b^
3/c^2 + 1/16*b^3/c^2)^(1/3)*(I*sqrt(3) + 1) - b)) - 1/4*((1/2)^(1/3)*(b^3 -
 (c^2 - 1)*b^3/c^2 + b^3/c^2)^(1/3)*(I*sqrt(3) + 1) + 2*b)*log(-1/4*((1/2)^
(1/3)*(b^3 - (c^2 - 1)*b^3/c^2 + b^3/c^2)^(1/3)*(I*sqrt(3) + 1) + 2*b)^2*c
+ ((1/2)^(1/3)*(b^3 - (c^2 - 1)*b^3/c^2 + b^3/c^2)^(1/3)*(I*sqrt(3) + 1) +
2*b)*b*c - b^2*c + b^2*sqrt(x)) - 1/2*((-1/16*b^3 + 1/16*(c^2 - 1)*b^3/c^2
+ 1/16*b^3/c^2)^(1/3)*(I*sqrt(3) + 1) - b)*log(((-1/16*b^3 + 1/16*(c^2 - 1)
*b^3/c^2 + 1/16*b^3/c^2)^(1/3)*(I*sqrt(3) + 1) - b)^2*c + 2*((-1/16*b^3 + 1
/16*(c^2 - 1)*b^3/c^2 + 1/16*b^3/c^2)^(1/3)*(I*sqrt(3) + 1) - b)*b*c + b^2*
c + b^2*sqrt(x)) + 1/2*(b*x - b)*log(-(c^2*x^3 + 2*c*x^(3/2) + 1)/(c^2*x^3
- 1)) - sqrt(3*((-1/16*b^3 + 1/16*(c^2 - 1)*b^3/c^2 + 1/16*b^3/c^2)^(1/3)*(
I*sqrt(3) + 1) - b)^2 + 6*((-1/16*b^3 + 1/16*(c^2 - 1)*b^3/c^2 + 1/16*b^3/c
^2)^(1/3)*(I*sqrt(3) + 1) - b)*b + 3*b^2)*arctan(1/3*(((-1/16*b^3 + 1/16*(c
^2 - 1)*b^3/c^2 + 1/16*b^3/c^2)^(1/3)*(I*sqrt(3) + 1) - b)^2*c^2 + 2*((-1/1
6*b^3 + 1/16*(c^2 - 1)*b^3/c^2 + 1/16*b^3/c^2)^(1/3)*(I*sqrt(3) + 1) - b)*b
*c^2 + b^2*c^2 - 2*b^2*c*sqrt(x) + sqrt(-4*((-1/16*b^3 + 1/16*(c^2 - 1)*b^3
/c^2 + 1/16*b^3/c^2)^(1/3)*(I*sqrt(3) + 1) - b)^2*b^2*c*sqrt(x) - 4*b^4*c*s
qrt(x) + 4*b^4*x + 4*b^4 - 4*(2*b^3*c*sqrt(x) - b^3)*((-1/16*b^3 + 1/16*(c^
2 - 1)*b^3/c^2 + 1/16*b^3/c^2)^(1/3)*(I*sqrt(3) + 1) - b))*c)*sqrt(3*((-1/1
6*b^3 + 1/16*(c^2 - 1)*b^3/c^2 + 1/16*b^3/c^2)^(1/3)*(I*sqrt(3) + 1) - b)^2
 + 6*((-1/16*b^3 + 1/16*(c^2 - 1)*b^3/c^2 + 1/16*b^3/c^2)^(1/3)*(I*sqrt(3)
+ 1) - b)*b + 3*b^2)/b^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*atanh(c*x**(3/2)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19899, size = 251, normalized size = 1.48

$$\frac{1}{4}\left(\left(c\left(\frac{2\sqrt{3}|c|^{\frac{1}{3}}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\sqrt{x}+\frac{1}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{c^2}+\frac{2\sqrt{3}|c|^{\frac{1}{3}}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\sqrt{x}-\frac{1}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{c^2}-\frac{|c|^{\frac{1}{3}}\log\left(x+\frac{\sqrt{x}}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{c^2}+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c*x^(3/2)),x, algorithm="giac")
```

```
[Out] 1/4*(c*(2*sqrt(3)*abs(c)^(1/3)*arctan(1/3*sqrt(3)*(2*sqrt(x) + 1/abs(c)^(1/
3))*abs(c)^(1/3))/c^2 + 2*sqrt(3)*abs(c)^(1/3)*arctan(1/3*sqrt(3)*(2*sqrt(x
) - 1/abs(c)^(1/3))*abs(c)^(1/3))/c^2 - abs(c)^(1/3)*log(x + sqrt(x)/abs(c)
^(1/3) + 1/abs(c)^(2/3))/c^2 + abs(c)^(1/3)*log(x - sqrt(x)/abs(c)^(1/3) +
1/abs(c)^(2/3))/c^2 - 2*abs(c)^(1/3)*log(sqrt(x) + 1/abs(c)^(1/3))/c^2 + 2*
abs(c)^(1/3)*log(abs(sqrt(x) - 1/abs(c)^(1/3)))/c^2) + 2*x*log(-(c*x^(3/2)
+ 1)/(c*x^(3/2) - 1)))*b + a*x
```

3.217 $\int \frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{x}\,dx$

Optimal. Leaf size=34

$$-\frac{1}{3}b\text{PolyLog}\left(2,-cx^{3/2}\right)+\frac{1}{3}b\text{PolyLog}\left(2,cx^{3/2}\right)+a\log(x)$$

```
[Out] a*Log[x] - (b*PolyLog[2, -(c*x^(3/2))])/3 + (b*PolyLog[2, c*x^(3/2)])/3
```

Rubi [A] time = 0.033444, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6095, 5912}

$$-\frac{1}{3}b\text{PolyLog}\left(2,-cx^{3/2}\right)+\frac{1}{3}b\text{PolyLog}\left(2,cx^{3/2}\right)+a\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^(3/2)])/x,x]
```

```
[Out] a*Log[x] - (b*PolyLog[2, -(c*x^(3/2))])/3 + (b*PolyLog[2, c*x^(3/2)])/3
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[
1/n, Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
 n}, x] && IGtQ[p, 0]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{x}\,dx &= \frac{2}{3}\text{Subst}\left(\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx,x,x^{3/2}\right)\\ &= a\log(x)-\frac{1}{3}b\text{Li}_2\left(-cx^{3/2}\right)+\frac{1}{3}b\text{Li}_2\left(cx^{3/2}\right)\end{aligned}$$

Mathematica [A] time = 0.0196923, size = 32, normalized size = 0.94

$$\frac{1}{3}b\left(\text{PolyLog}\left(2,cx^{3/2}\right)-\text{PolyLog}\left(2,-cx^{3/2}\right)\right)+a\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^(3/2)])/x,x]
```

```
[Out] a*Log[x] + (b*(-PolyLog[2, -(c*x^(3/2))] + PolyLog[2, c*x^(3/2)]))/3
```

Maple [B] time = 0.036, size = 63, normalized size = 1.9

$$\frac{2a}{3}\ln\left(cx^{\frac{3}{2}}\right)+\frac{2b}{3}\ln\left(cx^{\frac{3}{2}}\right)\operatorname{Artanh}\left(cx^{\frac{3}{2}}\right)-\frac{b}{3}\operatorname{dilog}\left(cx^{\frac{3}{2}}\right)-\frac{b}{3}\operatorname{dilog}\left(cx^{\frac{3}{2}}+1\right)-\frac{b}{3}\ln\left(cx^{\frac{3}{2}}\right)\ln\left(cx^{\frac{3}{2}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^(3/2)))/x,x)
```

```
[Out] 2/3*a*ln(c*x^(3/2))+2/3*b*ln(c*x^(3/2))*arctanh(c*x^(3/2))-1/3*b*dilog(c*x^
(3/2))-1/3*b*dilog(c*x^(3/2)+1)-1/3*b*ln(c*x^(3/2))*ln(c*x^(3/2)+1)
```

Maxima [B] time = 1.59449, size = 84, normalized size = 2.47

$$-\frac{1}{3}\left(\log\left(cx^{\frac{3}{2}}\right)\log\left(-cx^{\frac{3}{2}}+1\right)+\operatorname{Li}_2\left(-cx^{\frac{3}{2}}+1\right)\right)b+\frac{1}{3}\left(\log\left(cx^{\frac{3}{2}}+1\right)\log\left(-cx^{\frac{3}{2}}\right)+\operatorname{Li}_2\left(cx^{\frac{3}{2}}+1\right)\right)b+a\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(3/2)))/x,x, algorithm="maxima")
```

```
[Out] -1/3*(log(c*x^(3/2))*log(-c*x^(3/2) + 1) + dilog(-c*x^(3/2) + 1))*b + 1/3*(
log(c*x^(3/2) + 1)*log(-c*x^(3/2)) + dilog(c*x^(3/2) + 1))*b + a*log(x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{b\operatorname{artanh}\left(cx^{\frac{3}{2}}\right)+a}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(3/2)))/x,x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*x^(3/2)) + a)/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**(3/2)))/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{b\operatorname{artanh}\left(cx^{\frac{3}{2}}\right)+a}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(3/2)))/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^(3/2)) + a)/x, x)
```

3.218 $\int \frac{a+b\tanh^{-1}(cx^{3/2})}{x^2}\,dx$

Optimal. Leaf size=172

$$-\frac{a+b\tanh^{-1}(cx^{3/2})}{x}-\frac{1}{4}bc^{2/3}\log\left(c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1\right)+\frac{1}{4}bc^{2/3}\log\left(c^{2/3}x+\sqrt[3]{c}\sqrt{x}+1\right)-\frac{1}{2}\sqrt{3}bc^{2/3}\tan^{-1}\left(\frac{1-2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)$$

```
[Out] -(Sqrt[3]*b*c^(2/3)*ArcTan[(1 - 2*c^(1/3)*Sqrt[x])/Sqrt[3]])/2 + (Sqrt[3]*b
*c^(2/3)*ArcTan[(1 + 2*c^(1/3)*Sqrt[x])/Sqrt[3]])/2 + b*c^(2/3)*ArcTanh[c^(
1/3)*Sqrt[x]] - (a + b*ArcTanh[c*x^(3/2)])/x - (b*c^(2/3)*Log[1 - c^(1/3)*S
qrt[x] + c^(2/3)*x])/4 + (b*c^(2/3)*Log[1 + c^(1/3)*Sqrt[x] + c^(2/3)*x])/4
```

Rubi [A] time = 0.220878, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6097, 329, 210, 634, 618, 204, 628, 206}

$$-\frac{a+b\tanh^{-1}(cx^{3/2})}{x}-\frac{1}{4}bc^{2/3}\log\left(c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1\right)+\frac{1}{4}bc^{2/3}\log\left(c^{2/3}x+\sqrt[3]{c}\sqrt{x}+1\right)-\frac{1}{2}\sqrt{3}bc^{2/3}\tan^{-1}\left(\frac{1-2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^(3/2)])/x^2,x]
```

```
[Out] -(Sqrt[3]*b*c^(2/3)*ArcTan[(1 - 2*c^(1/3)*Sqrt[x])/Sqrt[3]])/2 + (Sqrt[3]*b
*c^(2/3)*ArcTan[(1 + 2*c^(1/3)*Sqrt[x])/Sqrt[3]])/2 + b*c^(2/3)*ArcTanh[c^(
1/3)*Sqrt[x]] - (a + b*ArcTanh[c*x^(3/2)])/x - (b*c^(2/3)*Log[1 - c^(1/3)*S
qrt[x] + c^(2/3)*x])/4 + (b*c^(2/3)*Log[1 + c^(1/3)*Sqrt[x] + c^(2/3)*x])/4
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{x^2}\,dx &= -\frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{x}+\frac{1}{2}(3bc)\int \frac{1}{\sqrt{x}\left(1-c^2x^3\right)}\,dx\\
&= -\frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{x}+(3bc)\,\text{Subst}\left(\int \frac{1}{1-c^2x^6}\,dx,x,\sqrt{x}\right)\\
&= -\frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{x}+(bc)\,\text{Subst}\left(\int \frac{1}{1-c^{2/3}x^2}\,dx,x,\sqrt{x}\right)+(bc)\,\text{Subst}\left(\int \frac{1-\frac{\sqrt[3]{}}{}}{1-\sqrt[3]{c}x+}\right.\\
&= bc^{2/3}\tanh^{-1}\left(\sqrt[3]{c}\sqrt{x}\right)-\frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{x}-\frac{1}{4}\left(bc^{2/3}\right)\text{Subst}\left(\int \frac{-\sqrt[3]{c}+2c^{2/3}x}{1-\sqrt[3]{c}x+c^{2/3}x^2}\,dx,x\right.\\
&= bc^{2/3}\tanh^{-1}\left(\sqrt[3]{c}\sqrt{x}\right)-\frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{x}-\frac{1}{4}bc^{2/3}\log\left(1-\sqrt[3]{c}\sqrt{x}+c^{2/3}x\right)+\frac{1}{4}bc^{2/3}\text{lo}\\
&= -\frac{1}{2}\sqrt{3}bc^{2/3}\tan^{-1}\left(\frac{1-2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)+\frac{1}{2}\sqrt{3}bc^{2/3}\tan^{-1}\left(\frac{1+2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)+bc^{2/3}\tanh^{-1}\left(\sqrt[3]{c}\sqrt{x}\right)\end{aligned}$$

Mathematica [A] time = 0.0338964, size = 205, normalized size = 1.19

$$-\frac{a}{x}-\frac{1}{2}bc^{2/3}\log\left(1-\sqrt[3]{c}\sqrt{x}\right)+\frac{1}{2}bc^{2/3}\log\left(\sqrt[3]{c}\sqrt{x}+1\right)-\frac{1}{4}bc^{2/3}\log\left(c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1\right)+\frac{1}{4}bc^{2/3}\log\left(c^{2/3}x+\sqrt[3]{c}\sqrt{x}+1\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^(3/2)])/x^2,x]
```

```
[Out] -(a/x) + (Sqrt[3]*b*c^(2/3)*ArcTan[(-1 + 2*c^(1/3)*Sqrt[x])/Sqrt[3]])/2 + (
Sqrt[3]*b*c^(2/3)*ArcTan[(1 + 2*c^(1/3)*Sqrt[x])/Sqrt[3]])/2 - (b*ArcTanh[c
*x^(3/2)])/x - (b*c^(2/3)*Log[1 - c^(1/3)*Sqrt[x]])/2 + (b*c^(2/3)*Log[1 +
c^(1/3)*Sqrt[x]])/2 - (b*c^(2/3)*Log[1 - c^(1/3)*Sqrt[x] + c^(2/3)*x])/4 +
(b*c^(2/3)*Log[1 + c^(1/3)*Sqrt[x] + c^(2/3)*x])/4
```

Maple [A] time = 0.034, size = 167, normalized size = 1.

$$-\frac{a}{x}-\frac{b}{x}\mathrm{Artanh}\left(cx^{\frac{3}{2}}\right)-\frac{b}{2}\ln\left(\sqrt{x}-\sqrt[3]{c^{-1}}\right)\left(c^{-1}\right)^{-\frac{2}{3}}+\frac{b}{4}\ln\left(x+\sqrt[3]{c^{-1}}\sqrt{x}+\left(c^{-1}\right)^{\frac{2}{3}}\right)\left(c^{-1}\right)^{-\frac{2}{3}}+\frac{b\sqrt{3}}{2}\arctan\left(\frac{\sqrt{3}}{3}\left(2\frac{\sqrt{x}}{\sqrt[3]{c^{-1}}}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^(3/2)))/x^2,x)
```

```
[Out] -a/x-b/x*arctanh(c*x^(3/2))-1/2*b/(1/c)^(2/3)*ln(x^(1/2)-(1/c)^(1/3))+1/4*b
/(1/c)^(2/3)*ln(x+(1/c)^(1/3)*x^(1/2)+(1/c)^(2/3))+1/2*b/(1/c)^(2/3)*3^(1/2
)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x^(1/2)+1))+1/2*b/(1/c)^(2/3)*ln(x^(1/2
)+(1/c)^(1/3))-1/4*b/(1/c)^(2/3)*ln(x-(1/c)^(1/3)*x^(1/2)+(1/c)^(2/3))+1/2*
b/(1/c)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x^(1/2)-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(3/2)))/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.93343, size = 647, normalized size = 3.76

$$-\frac{2\sqrt{3}\left(-c^{2}\right)^{\frac{1}{3}}bx\arctan\left(\frac{2\sqrt{3}\left(-c^{2}\right)^{\frac{2}{3}}\sqrt{x}+\sqrt{3}c}{3c}\right)-2\sqrt{3}b\left(c^{2}\right)^{\frac{1}{3}}x\arctan\left(\frac{2\sqrt{3}\left(c^{2}\right)^{\frac{2}{3}}\sqrt{x}-\sqrt{3}c}{3c}\right)+\left(-c^{2}\right)^{\frac{1}{3}}bx\log\left(c^{2}x-\left(-c^{2}\right)^{\frac{1}{3}}c\sqrt{x}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(3/2)))/x^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(3)*(-c^2)^(1/3)*b*x*arctan(1/3*(2*sqrt(3)*(-c^2)^(2/3)*sqrt(x)
 + sqrt(3)*c)/c) - 2*sqrt(3)*b*(c^2)^(1/3)*x*arctan(1/3*(2*sqrt(3)*(c^2)^(2
/3)*sqrt(x) - sqrt(3)*c)/c) + (-c^2)^(1/3)*b*x*log(c^2*x - (-c^2)^(1/3)*c*s
qrt(x) + (-c^2)^(2/3)) + b*(c^2)^(1/3)*x*log(c^2*x - (c^2)^(1/3)*c*sqrt(x)
+ (c^2)^(2/3)) - 2*(-c^2)^(1/3)*b*x*log(c*sqrt(x) + (-c^2)^(1/3)) - 2*b*(c^
2)^(1/3)*x*log(c*sqrt(x) + (c^2)^(1/3)) + 2*b*log(-(c^2*x^3 + 2*c*x^(3/2) +
 1)/(c^2*x^3 - 1)) + 4*a)/x
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**(3/2)))/x**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.46984, size = 232, normalized size = 1.35

$$\frac{1}{4}\left(\frac{2\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\sqrt{x}+\frac{1}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{|c|^{\frac{1}{3}}}+\frac{2\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\sqrt{x}-\frac{1}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{|c|^{\frac{1}{3}}}+\frac{\log\left(x+\frac{\sqrt{x}}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{|c|^{\frac{1}{3}}}-\frac{\log\left(x-\frac{\sqrt{x}}{|c|^{\frac{1}{3}}}\right.}{|c|^{\frac{1}{3}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(3/2)))/x^2,x, algorithm="giac")
```

```
[Out] 1/4*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*sqrt(x) + 1/abs(c)^(1/3))*abs(c)^(1/3)
)/abs(c)^(1/3) + 2*sqrt(3)*arctan(1/3*sqrt(3)*(2*sqrt(x) - 1/abs(c)^(1/3))*
abs(c)^(1/3))/abs(c)^(1/3) + log(x + sqrt(x)/abs(c)^(1/3) + 1/abs(c)^(2/3))
/abs(c)^(1/3) - log(x - sqrt(x)/abs(c)^(1/3) + 1/abs(c)^(2/3))/abs(c)^(1/3)
 + 2*log(sqrt(x) + 1/abs(c)^(1/3))/abs(c)^(1/3) - 2*log(abs(sqrt(x) - 1/abs
(c)^(1/3)))/abs(c)^(1/3))*b*c - 1/2*b*log(-(c*x^(3/2) + 1)/(c*x^(3/2) - 1))
/x - a/x
```

3.219 $\int \frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{x^3}\,dx$

Optimal. Leaf size=188

$$-\frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{2x^2}-\frac{1}{8}bc^{4/3}\log\left(c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1\right)+\frac{1}{8}bc^{4/3}\log\left(c^{2/3}x+\sqrt[3]{c}\sqrt{x}+1\right)+\frac{1}{4}\sqrt{3}bc^{4/3}\tan^{-1}\left(\frac{1-2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)$$

```
[Out] (-3*b*c)/(2*Sqrt[x]) + (Sqrt[3]*b*c^(4/3)*ArcTan[(1 - 2*c^(1/3)*Sqrt[x])/Sq
rt[3]])/4 - (Sqrt[3]*b*c^(4/3)*ArcTan[(1 + 2*c^(1/3)*Sqrt[x])/Sqrt[3]])/4 +
 (b*c^(4/3)*ArcTanh[c^(1/3)*Sqrt[x]])/2 - (a + b*ArcTanh[c*x^(3/2)])/(2*x^2
) - (b*c^(4/3)*Log[1 - c^(1/3)*Sqrt[x] + c^(2/3)*x])/8 + (b*c^(4/3)*Log[1 +
 c^(1/3)*Sqrt[x] + c^(2/3)*x])/8
```

Rubi [A] time = 0.286581, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {6097, 325, 329, 296, 634, 618, 204, 628, 206}

$$-\frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{2x^2}-\frac{1}{8}bc^{4/3}\log\left(c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1\right)+\frac{1}{8}bc^{4/3}\log\left(c^{2/3}x+\sqrt[3]{c}\sqrt{x}+1\right)+\frac{1}{4}\sqrt{3}bc^{4/3}\tan^{-1}\left(\frac{1-2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^(3/2)])/x^3,x]
```

```
[Out] (-3*b*c)/(2*Sqrt[x]) + (Sqrt[3]*b*c^(4/3)*ArcTan[(1 - 2*c^(1/3)*Sqrt[x])/Sq
rt[3]])/4 - (Sqrt[3]*b*c^(4/3)*ArcTan[(1 + 2*c^(1/3)*Sqrt[x])/Sqrt[3]])/4 +
 (b*c^(4/3)*ArcTanh[c^(1/3)*Sqrt[x]])/2 - (a + b*ArcTanh[c*x^(3/2)])/(2*x^2
) - (b*c^(4/3)*Log[1 - c^(1/3)*Sqrt[x] + c^(2/3)*x])/8 + (b*c^(4/3)*Log[1 +
 c^(1/3)*Sqrt[x] + c^(2/3)*x])/8
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
```

```
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{x^3}\,dx = -\frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{2x^2}+\frac{1}{4}(3bc)\int \frac{1}{x^{3/2}\left(1-c^2x^3\right)}\,dx$$
$$= -\frac{3bc}{2\sqrt{x}}-\frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{2x^2}+\frac{1}{4}\left(3bc^3\right)\int \frac{x^{3/2}}{1-c^2x^3}\,dx$$
$$= -\frac{3bc}{2\sqrt{x}}-\frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{2x^2}+\frac{1}{2}\left(3bc^3\right)\text{Subst}\left(\int \frac{x^4}{1-c^2x^6}\,dx,x,\sqrt{x}\right)$$
$$= -\frac{3bc}{2\sqrt{x}}-\frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{2x^2}+\frac{1}{2}\left(bc^{5/3}\right)\text{Subst}\left(\int \frac{1}{1-c^{2/3}x^2}\,dx,x,\sqrt{x}\right)+\frac{1}{2}\left(bc^{5/3}\right)\text{Subst}$$
$$= -\frac{3bc}{2\sqrt{x}}+\frac{1}{2}bc^{4/3}\tanh^{-1}\left(\sqrt[3]{c}\sqrt{x}\right)-\frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{2x^2}-\frac{1}{8}\left(bc^{4/3}\right)\text{Subst}\left(\int \frac{-\sqrt[3]{c}+2c^{2/3}}{1-\sqrt[3]{c}x+c^{2/3}}\right.$$
$$= -\frac{3bc}{2\sqrt{x}}+\frac{1}{2}bc^{4/3}\tanh^{-1}\left(\sqrt[3]{c}\sqrt{x}\right)-\frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{2x^2}-\frac{1}{8}bc^{4/3}\log\left(1-\sqrt[3]{c}\sqrt{x}+c^{2/3}x\right)+\frac{1}{8}$$
$$= -\frac{3bc}{2\sqrt{x}}+\frac{1}{4}\sqrt{3}bc^{4/3}\tan^{-1}\left(\frac{1-2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)-\frac{1}{4}\sqrt{3}bc^{4/3}\tan^{-1}\left(\frac{1+2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)+\frac{1}{2}bc^{4/3}\tanh^{-1}($$

Mathematica [A] time = 0.058352, size = 220, normalized size = 1.17

$$-\frac{a}{2x^2}-\frac{1}{4}bc^{4/3}\log\left(1-\sqrt[3]{c}\sqrt{x}\right)+\frac{1}{4}bc^{4/3}\log\left(\sqrt[3]{c}\sqrt{x}+1\right)-\frac{1}{8}bc^{4/3}\log\left(c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1\right)+\frac{1}{8}bc^{4/3}\log\left(c^{2/3}x+\sqrt[3]{c}\sqrt{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^(3/2)])/x^3,x]
```

```
[Out] -a/(2*x^2) - (3*b*c)/(2*Sqrt[x]) - (Sqrt[3]*b*c^(4/3)*ArcTan[(-1 + 2*c^(1/3
)*Sqrt[x])/Sqrt[3]])/4 - (Sqrt[3]*b*c^(4/3)*ArcTan[(1 + 2*c^(1/3)*Sqrt[x])/
Sqrt[3]])/4 - (b*ArcTanh[c*x^(3/2)])/(2*x^2) - (b*c^(4/3)*Log[1 - c^(1/3)*S
qrt[x]])/4 + (b*c^(4/3)*Log[1 + c^(1/3)*Sqrt[x]])/4 - (b*c^(4/3)*Log[1 - c^
(1/3)*Sqrt[x] + c^(2/3)*x])/8 + (b*c^(4/3)*Log[1 + c^(1/3)*Sqrt[x] + c^(2/3
)*x])/8
```

Maple [A] time = 0.037, size = 180, normalized size = 1.

$$-\frac{a}{2\,x^2}-\frac{b}{2\,x^2}\text{Artanh}\left(cx^{\frac{3}{2}}\right)-\frac{3\,bc}{2}\frac{1}{\sqrt{x}}-\frac{bc}{4}\ln\left(\sqrt{x}-\sqrt[3]{c^{-1}}\right)\frac{1}{\sqrt[3]{c^{-1}}}+\frac{bc}{8}\ln\left(x+\sqrt[3]{c^{-1}}\sqrt{x}+\left(c^{-1}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{c^{-1}}}-\frac{bc\sqrt{3}}{4}\arctan\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^(3/2)))/x^3,x)
```

```
[Out] -1/2*a/x^2-1/2*b/x^2*arctanh(c*x^(3/2))-3/2*b*c/x^(1/2)-1/4*b*c/(1/c)^(1/3)
*ln(x^(1/2)-(1/c)^(1/3))+1/8*b*c/(1/c)^(1/3)*ln(x+(1/c)^(1/3)*x^(1/2)+(1/c)
^(2/3))-1/4*b*c*3^(1/2)/(1/c)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x^(1/
2)+1))+1/4*b*c/(1/c)^(1/3)*ln(x^(1/2)+(1/c)^(1/3))-1/8*b*c/(1/c)^(1/3)*ln(x
-(1/c)^(1/3)*x^(1/2)+(1/c)^(2/3))-1/4*b*c*3^(1/2)/(1/c)^(1/3)*arctan(1/3*3^
(1/2)*(2/(1/c)^(1/3)*x^(1/2)-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(3/2)))/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.99585, size = 618, normalized size = 3.29

$$-\frac{2\sqrt{3}b(-c)^{\frac{1}{3}}cx^2\arctan\left(\frac{2}{3}\sqrt{3}(-c)^{\frac{1}{3}}\sqrt{x}-\frac{1}{3}\sqrt{3}\right)+2\sqrt{3}bc^{\frac{4}{3}}x^2\arctan\left(\frac{2}{3}\sqrt{3}c^{\frac{1}{3}}\sqrt{x}-\frac{1}{3}\sqrt{3}\right)+b(-c)^{\frac{1}{3}}cx^2\log\left(cx+(-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(3/2)))/x^3,x, algorithm="fricas")
```

```
[Out] -1/8*(2*sqrt(3)*b*(-c)^(1/3)*c*x^2*arctan(2/3*sqrt(3)*(-c)^(1/3)*sqrt(x) -
1/3*sqrt(3)) + 2*sqrt(3)*b*c^(4/3)*x^2*arctan(2/3*sqrt(3)*c^(1/3)*sqrt(x) -
 1/3*sqrt(3)) + b*(-c)^(1/3)*c*x^2*log(c*x + (-c)^(2/3)*sqrt(x) - (-c)^(1/3
)) + b*c^(4/3)*x^2*log(c*x - c^(2/3)*sqrt(x) + c^(1/3)) - 2*b*(-c)^(1/3)*c*
x^2*log(c*sqrt(x) - (-c)^(2/3)) - 2*b*c^(4/3)*x^2*log(c*sqrt(x) + c^(2/3))
+ 12*b*c*x^(3/2) + 2*b*log(-(c^2*x^3 + 2*c*x^(3/2) + 1)/(c^2*x^3 - 1)) + 4*
a)/x^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**(3/2)))/x**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.51374, size = 270, normalized size = 1.44

$$-\frac{1}{8}bc^3\left(\frac{2\sqrt{3}|c|^{\frac{1}{3}}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\sqrt{x}+\frac{1}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{c^2}+\frac{2\sqrt{3}|c|^{\frac{1}{3}}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\sqrt{x}-\frac{1}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{c^2}-\frac{|c|^{\frac{1}{3}}\log\left(x+\frac{\sqrt{x}}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(3/2)))/x^3,x, algorithm="giac")
```

```
[Out] -1/8*b*c^3*(2*sqrt(3)*abs(c)^(1/3)*arctan(1/3*sqrt(3)*(2*sqrt(x) + 1/abs(c)
^(1/3))*abs(c)^(1/3))/c^2 + 2*sqrt(3)*abs(c)^(1/3)*arctan(1/3*sqrt(3)*(2*sq
```

```
rt(x) - 1/abs(c)^(1/3))*abs(c)^(1/3))/c^2 - abs(c)^(1/3)*log(x + sqrt(x)/ab
s(c)^(1/3) + 1/abs(c)^(2/3))/c^2 + abs(c)^(1/3)*log(x - sqrt(x)/abs(c)^(1/3
) + 1/abs(c)^(2/3))/c^2 - 2*abs(c)^(1/3)*log(sqrt(x) + 1/abs(c)^(1/3))/c^2
+ 2*abs(c)^(1/3)*log(abs(sqrt(x) - 1/abs(c)^(1/3)))/c^2) - 1/4*b*log(-(c*x^
(3/2) + 1)/(c*x^(3/2) - 1))/x^2 - 1/2*(3*b*c*x^(3/2) + a)/x^2
```

3.220 $\int \frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{x^4}\,dx$

Optimal. Leaf size=47

$$-\frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{3x^3}+\frac{1}{3}bc^2\tanh^{-1}\left(cx^{3/2}\right)-\frac{bc}{3x^{3/2}}$$

```
[Out] -(b*c)/(3*x^(3/2)) + (b*c^2*ArcTanh[c*x^(3/2)])/3 - (a + b*ArcTanh[c*x^(3/2
)])/(3*x^3)
```

Rubi [A] time = 0.0322121, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {6097, 325, 329, 275, 206}

$$-\frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{3x^3}+\frac{1}{3}bc^2\tanh^{-1}\left(cx^{3/2}\right)-\frac{bc}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^(3/2)])/x^4,x]
```

```
[Out] -(b*c)/(3*x^(3/2)) + (b*c^2*ArcTanh[c*x^(3/2)])/3 - (a + b*ArcTanh[c*x^(3/2
)])/(3*x^3)
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{x^4}\,dx &= -\frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{3x^3}+\frac{1}{2}(bc)\int \frac{1}{x^{5/2}\left(1-c^2x^3\right)}\,dx\\
&= -\frac{bc}{3x^{3/2}}-\frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{3x^3}+\frac{1}{2}\left(bc^3\right)\int \frac{\sqrt{x}}{1-c^2x^3}\,dx\\
&= -\frac{bc}{3x^{3/2}}-\frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{3x^3}+\left(bc^3\right)\mathrm{Subst}\left(\int \frac{x^2}{1-c^2x^6}\,dx,x,\sqrt{x}\right)\\
&= -\frac{bc}{3x^{3/2}}-\frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{3x^3}+\frac{1}{3}\left(bc^3\right)\mathrm{Subst}\left(\int \frac{1}{1-c^2x^2}\,dx,x,x^{3/2}\right)\\
&= -\frac{bc}{3x^{3/2}}+\frac{1}{3}bc^2\tanh^{-1}\left(cx^{3/2}\right)-\frac{a+b\tanh^{-1}\left(cx^{3/2}\right)}{3x^3}
\end{aligned}$$

Mathematica [A] time = 0.0289697, size = 73, normalized size = 1.55

$$-\frac{a}{3x^3}-\frac{1}{6}bc^2\log\left(1-cx^{3/2}\right)+\frac{1}{6}bc^2\log\left(cx^{3/2}+1\right)-\frac{bc}{3x^{3/2}}-\frac{b\tanh^{-1}\left(cx^{3/2}\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^(3/2)])/x^4,x]
```

```
[Out] -a/(3*x^3) - (b*c)/(3*x^(3/2)) - (b*ArcTanh[c*x^(3/2)])/(3*x^3) - (b*c^2*Lo
g[1 - c*x^(3/2)])/6 + (b*c^2*Log[1 + c*x^(3/2)])/6
```

Maple [A] time = 0.036, size = 55, normalized size = 1.2

$$-\frac{a}{3x^3}-\frac{b}{3x^3}\mathrm{Artanh}\left(cx^{\frac{3}{2}}\right)-\frac{bc^2}{6}\ln\left(cx^{\frac{3}{2}}-1\right)+\frac{bc^2}{6}\ln\left(cx^{\frac{3}{2}}+1\right)-\frac{bc}{3}x^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^(3/2)))/x^4,x)
```

```
[Out] -1/3*a/x^3-1/3*b/x^3*arctanh(c*x^(3/2))-1/6*b*c^2*ln(c*x^(3/2)-1)+1/6*b*c^2
*ln(c*x^(3/2)+1)-1/3*b*c/x^(3/2)
```

Maxima [A] time = 0.984514, size = 69, normalized size = 1.47

$$\frac{1}{6}\left(\left(c\log\left(cx^{\frac{3}{2}}+1\right)-c\log\left(cx^{\frac{3}{2}}-1\right)-\frac{2}{x^{\frac{3}{2}}}\right)c-\frac{2\,\mathrm{artanh}\left(cx^{\frac{3}{2}}\right)}{x^3}\right)b-\frac{a}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(3/2)))/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*((c*log(c*x^(3/2) + 1) - c*log(c*x^(3/2) - 1) - 2/x^(3/2))*c - 2*arctan
h(c*x^(3/2))/x^3)*b - 1/3*a/x^3
```

Fricas [A] time = 1.87849, size = 132, normalized size = 2.81

$$-\frac{2\,bcx^{\frac{3}{2}} - \left(bc^2x^3 - b\right)\log\left(-\frac{c^2x^3+2\,cx^{\frac{3}{2}}+1}{c^2x^3-1}\right) + 2\,a}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(3/2)))/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(2*b*c*x^(3/2) - (b*c^2*x^3 - b)*log(-(c^2*x^3 + 2*c*x^(3/2) + 1)/(c^2
*x^3 - 1)) + 2*a)/x^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**(3/2)))/x**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2122, size = 90, normalized size = 1.91

$$\frac{1}{6}\,bc^2\log\left(cx^{\frac{3}{2}}+1\right) - \frac{1}{6}\,bc^2\log\left(cx^{\frac{3}{2}}-1\right) - \frac{b\log\left(-\frac{cx^{\frac{3}{2}}+1}{cx^{\frac{3}{2}}-1}\right)}{6\,x^3} - \frac{bcx^{\frac{3}{2}}+a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(3/2)))/x^4,x, algorithm="giac")
```

```
[Out] 1/6*b*c^2*log(c*x^(3/2) + 1) - 1/6*b*c^2*log(c*x^(3/2) - 1) - 1/6*b*log(-(c
*x^(3/2) + 1)/(c*x^(3/2) - 1))/x^3 - 1/3*(b*c*x^(3/2) + a)/x^3
```

3.221 $\int x^2 \left(a+b \tanh^{-1}\left(cx^{3/2}\right)\right)^2 dx$

Optimal. Leaf size=101

$$-\frac{\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)^2}{3c^2}+\frac{2abx^{3/2}}{3c}+\frac{1}{3}x^3\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)^2+\frac{b^2\log\left(1-c^2x^3\right)}{3c^2}+\frac{2b^2x^{3/2}\tanh^{-1}\left(cx^{3/2}\right)}{3c}$$

```
[Out] (2*a*b*x^(3/2))/(3*c) + (2*b^2*x^(3/2)*ArcTanh[c*x^(3/2)])/(3*c) - (a + b*A
rcTanh[c*x^(3/2)])^2/(3*c^2) + (x^3*(a + b*ArcTanh[c*x^(3/2)])^2)/3 + (b^2*
Log[1 - c^2*x^3])/(3*c^2)
```

Rubi [F] time = 0.0244631, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^2\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)^2 dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2*(a + b*ArcTanh[c*x^(3/2)])^2,x]
```

```
[Out] Defer[Int][x^2*(a + b*ArcTanh[c*x^(3/2)])^2, x]
```

Rubi steps

$$\int x^2\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)^2 dx=\int x^2\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)^2 dx$$

Mathematica [A] time = 0.0928518, size = 122, normalized size = 1.21

$$\frac{a^2c^2x^3+2abcx^{3/2}+b(a+b)\log\left(1-cx^{3/2}\right)-ab\log\left(cx^{3/2}+1\right)+2bcx^{3/2}\tanh^{-1}\left(cx^{3/2}\right)\left(acx^{3/2}+b\right)+b^2\left(c^2x^3-1\right)\tanh}{3c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcTanh[c*x^(3/2)])^2,x]
```

```
[Out] (2*a*b*c*x^(3/2) + a^2*c^2*x^3 + 2*b*c*x^(3/2)*(b + a*c*x^(3/2))*ArcTanh[c*
x^(3/2)] + b^2*(-1 + c^2*x^3)*ArcTanh[c*x^(3/2)]^2 + b*(a + b)*Log[1 - c*x^
(3/2)] - a*b*Log[1 + c*x^(3/2)] + b^2*Log[1 + c*x^(3/2)])/(3*c^2)
```

Maple [B] time = 0.049, size = 284, normalized size = 2.8

$$\frac{x^3a^2}{3}+\frac{x^3b^2}{3}\left(\mathrm{Artanh}\left(cx^{\frac{3}{2}}\right)\right)^2+\frac{2b^2}{3c}x^{\frac{3}{2}}\mathrm{Artanh}\left(cx^{\frac{3}{2}}\right)+\frac{b^2}{3c^2}\mathrm{Artanh}\left(cx^{\frac{3}{2}}\right)\ln\left(cx^{\frac{3}{2}}-1\right)-\frac{b^2}{3c^2}\mathrm{Artanh}\left(cx^{\frac{3}{2}}\right)\ln\left(cx^{\frac{3}{2}}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c*x^(3/2)))^2,x)
```

```
[Out] 1/3*x^3*a^2+1/3*b^2*x^3*arctanh(c*x^(3/2))^2+2/3*b^2*x^(3/2)*arctanh(c*x^(3
/2))/c+1/3/c^2*b^2*arctanh(c*x^(3/2))*ln(c*x^(3/2)-1)-1/3/c^2*b^2*arctanh(c
*x^(3/2))*ln(c*x^(3/2)+1)-1/6/c^2*b^2*ln(c*x^(3/2)-1)*ln(1/2+1/2*c*x^(3/2))
+1/12/c^2*b^2*ln(c*x^(3/2)-1)^2+1/3/c^2*b^2*ln(c*x^(3/2)-1)+1/3/c^2*b^2*ln(
c*x^(3/2)+1)-1/6/c^2*b^2*ln(-1/2*c*x^(3/2)+1/2)*ln(c*x^(3/2)+1)+1/6/c^2*b^2
*ln(-1/2*c*x^(3/2)+1/2)*ln(1/2+1/2*c*x^(3/2))+1/12/c^2*b^2*ln(c*x^(3/2)+1)^
2+2/3*a*b*x^3*arctanh(c*x^(3/2))+2/3*a*b*x^(3/2)/c+1/3/c^2*a*b*ln(c*x^(3/2)
-1)-1/3/c^2*a*b*ln(c*x^(3/2)+1)
```

Maxima [B] time = 1.00881, size = 251, normalized size = 2.49

$$\frac{1}{3}b^2x^3\operatorname{artanh}\left(cx^{\frac{3}{2}}\right)^2+\frac{1}{3}a^2x^3+\frac{1}{3}\left(2x^3\operatorname{artanh}\left(cx^{\frac{3}{2}}\right)+c\left(\frac{2x^{\frac{3}{2}}}{c^2}-\frac{\log\left(cx^{\frac{3}{2}}+1\right)}{c^3}+\frac{\log\left(cx^{\frac{3}{2}}-1\right)}{c^3}\right)\right)ab+\frac{1}{12}\left(4c\left(\frac{2x^{\frac{3}{2}}}{c^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^(3/2)))^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*x^3*arctanh(c*x^(3/2))^2 + 1/3*a^2*x^3 + 1/3*(2*x^3*arctanh(c*x^(3/
2)) + c*(2*x^(3/2)/c^2 - log(c*x^(3/2) + 1)/c^3 + log(c*x^(3/2) - 1)/c^3))*
a*b + 1/12*(4*c*(2*x^(3/2)/c^2 - log(c*x^(3/2) + 1)/c^3 + log(c*x^(3/2) - 1
)/c^3)*arctanh(c*x^(3/2)) - (2*(log(c*x^(3/2) - 1) - 2)*log(c*x^(3/2) + 1)
- log(c*x^(3/2) + 1)^2 - log(c*x^(3/2) - 1)^2 - 4*log(c*x^(3/2) - 1))/c^2)*
b^2
```

Fricas [B] time = 1.99773, size = 402, normalized size = 3.98

$$\frac{4a^2c^2x^3+8abcx^{\frac{3}{2}}+\left(b^2c^2x^3-b^2\right)\log\left(-\frac{c^2x^3+2cx^{\frac{3}{2}}+1}{c^2x^3-1}\right)^2+4\left(abc^2-ab+b^2\right)\log\left(cx^{\frac{3}{2}}+1\right)-4\left(abc^2-ab-b^2\right)\log\left(cx\right.}{12c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^(3/2)))^2,x, algorithm="fricas")
```

```
[Out] 1/12*(4*a^2*c^2*x^3 + 8*a*b*c*x^(3/2) + (b^2*c^2*x^3 - b^2)*log(-(c^2*x^3 +
 2*c*x^(3/2) + 1)/(c^2*x^3 - 1))^2 + 4*(a*b*c^2 - a*b + b^2)*log(c*x^(3/2)
+ 1) - 4*(a*b*c^2 - a*b - b^2)*log(c*x^(3/2) - 1) + 4*(a*b*c^2*x^3 + b^2*c*
x^(3/2) - a*b*c^2)*log(-(c^2*x^3 + 2*c*x^(3/2) + 1)/(c^2*x^3 - 1)))/c^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c*x**(3/2)))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\,\text{artanh}\left(cx^{\frac{3}{2}}\right)+a\right)^{2} x^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^(3/2)))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^(3/2)) + a)^2*x^2, x)
```

3.222 $\int \frac{\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)^2}{x}\,dx$

Optimal. Leaf size=156

$$-\frac{2}{3}b\text{PolyLog}\left(2,1-\frac{2}{1-cx^{3/2}}\right)\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)+\frac{2}{3}b\text{PolyLog}\left(2,\frac{2}{1-cx^{3/2}}-1\right)\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)+\frac{1}{3}b^2\text{Po}$$

```
[Out] (4*(a + b*ArcTanh[c*x^(3/2)])^2*ArcTanh[1 - 2/(1 - c*x^(3/2))])/3 - (2*b*(a
 + b*ArcTanh[c*x^(3/2)])*PolyLog[2, 1 - 2/(1 - c*x^(3/2))])/3 + (2*b*(a + b
*ArcTanh[c*x^(3/2)])*PolyLog[2, -1 + 2/(1 - c*x^(3/2))])/3 + (b^2*PolyLog[3
, 1 - 2/(1 - c*x^(3/2))])/3 - (b^2*PolyLog[3, -1 + 2/(1 - c*x^(3/2))])/3
```

Rubi [A] time = 0.319783, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6095, 5914, 6052, 5948, 6058, 6610}

$$-\frac{2}{3}b\text{PolyLog}\left(2,1-\frac{2}{1-cx^{3/2}}\right)\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)+\frac{2}{3}b\text{PolyLog}\left(2,\frac{2}{1-cx^{3/2}}-1\right)\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)+\frac{1}{3}b^2\text{Po}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^(3/2)])^2/x,x]
```

```
[Out] (4*(a + b*ArcTanh[c*x^(3/2)])^2*ArcTanh[1 - 2/(1 - c*x^(3/2))])/3 - (2*b*(a
 + b*ArcTanh[c*x^(3/2)])*PolyLog[2, 1 - 2/(1 - c*x^(3/2))])/3 + (2*b*(a + b
*ArcTanh[c*x^(3/2)])*PolyLog[2, -1 + 2/(1 - c*x^(3/2))])/3 + (b^2*PolyLog[3
, 1 - 2/(1 - c*x^(3/2))])/3 - (b^2*PolyLog[3, -1 + 2/(1 - c*x^(3/2))])/3
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[
1/n, Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
 n}, x] && IGtQ[p, 0]
```

Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
 e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)^2}{x}dx=\frac{2}{3}\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x}dx,x,x^{3/2}\right)$$

$$=\frac{4}{3}\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx^{3/2}}\right)-\frac{1}{3}(8bc)\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(cx)\right)}{1-c}\right.$$

$$=\frac{4}{3}\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx^{3/2}}\right)+\frac{1}{3}(4bc)\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(cx)\right)}{1-c^2x^2}\right.$$

$$=\frac{4}{3}\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx^{3/2}}\right)-\frac{2}{3}b\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)\text{Li}_2\left(1-\frac{}{1-}\right.$$

$$=\frac{4}{3}\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx^{3/2}}\right)-\frac{2}{3}b\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)\text{Li}_2\left(1-\frac{}{1-}\right.$$

Mathematica [A] time = 0.132115, size = 167, normalized size = 1.07

$$\frac{1}{3}\left(b\left(2\text{PolyLog}\left(2,\frac{cx^{3/2}+1}{1-cx^{3/2}}\right)\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)-2\text{PolyLog}\left(2,\frac{cx^{3/2}+1}{cx^{3/2}-1}\right)\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)+b\left(\text{PolyLog}\left(3,\right.\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^(3/2)])^2/x,x]
```

```
[Out] (4*(a + b*ArcTanh[c*x^(3/2)])^2*ArcTanh[1 + 2/(-1 + c*x^(3/2))] + b*(2*(a +
 b*ArcTanh[c*x^(3/2)])*PolyLog[2, (1 + c*x^(3/2))/(1 - c*x^(3/2))] - 2*(a +
 b*ArcTanh[c*x^(3/2)])*PolyLog[2, (1 + c*x^(3/2))/(-1 + c*x^(3/2))] + b*(-P
olyLog[3, (1 + c*x^(3/2))/(1 - c*x^(3/2))] + PolyLog[3, (1 + c*x^(3/2))/(-1
 + c*x^(3/2))])))/3
```

Maple [C] time = 0.334, size = 785, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^(3/2)))^2/x,x)
```

```
[Out] 2/3*a^2*ln(c*x^(3/2))+2/3*b^2*ln(c*x^(3/2))*arctanh(c*x^(3/2))^2-2/3*b^2*ar
ctanh(c*x^(3/2))*polylog(2,-(c*x^(3/2)+1)^2/(-c^2*x^3+1))+1/3*b^2*polylog(3
,-(c*x^(3/2)+1)^2/(-c^2*x^3+1))-2/3*b^2*arctanh(c*x^(3/2))^2*ln((c*x^(3/2)+
1)^2/(-c^2*x^3+1)-1)+2/3*b^2*arctanh(c*x^(3/2))^2*ln(1-(c*x^(3/2)+1)/(-c^2*
x^3+1)^(1/2))+4/3*b^2*arctanh(c*x^(3/2))*polylog(2,(c*x^(3/2)+1)/(-c^2*x^3+
1)^(1/2))-4/3*b^2*polylog(3,(c*x^(3/2)+1)/(-c^2*x^3+1)^(1/2))+2/3*b^2*arcta
nh(c*x^(3/2))^2*ln(1+(c*x^(3/2)+1)/(-c^2*x^3+1)^(1/2))+4/3*b^2*arctanh(c*x^
(3/2))*polylog(2,-(c*x^(3/2)+1)/(-c^2*x^3+1)^(1/2))-4/3*b^2*polylog(3,-(c*x
^(3/2)+1)/(-c^2*x^3+1)^(1/2))-1/3*I*b^2*Pi*csgn(I*((c*x^(3/2)+1)^2/(-c^2*x^
3+1)-1))*csgn(I*((c*x^(3/2)+1)^2/(-c^2*x^3+1)-1)/((c*x^(3/2)+1)^2/(-c^2*x^3
+1)+1))^2*arctanh(c*x^(3/2))^2+1/3*I*b^2*Pi*csgn(I*((c*x^(3/2)+1)^2/(-c^2*x
^3+1)-1))*csgn(I/((c*x^(3/2)+1)^2/(-c^2*x^3+1)+1))*csgn(I*((c*x^(3/2)+1)^2/
(-c^2*x^3+1)-1)/((c*x^(3/2)+1)^2/(-c^2*x^3+1)+1))*arctanh(c*x^(3/2))^2+1/3*
I*b^2*Pi*csgn(I*((c*x^(3/2)+1)^2/(-c^2*x^3+1)-1)/((c*x^(3/2)+1)^2/(-c^2*x^3
+1)+1))^3*arctanh(c*x^(3/2))^2-1/3*I*b^2*Pi*csgn(I/((c*x^(3/2)+1)^2/(-c^2*x
^3+1)+1))*csgn(I*((c*x^(3/2)+1)^2/(-c^2*x^3+1)-1)/((c*x^(3/2)+1)^2/(-c^2*x^
3+1)+1))^2*arctanh(c*x^(3/2))^2-2/3*a*b*ln(c*x^(3/2))*ln(c*x^(3/2)+1)+4/3*a
*b*ln(c*x^(3/2))*arctanh(c*x^(3/2))-2/3*a*b*dilog(c*x^(3/2))-2/3*a*b*dilog(
c*x^(3/2)+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{4}b^2\int\frac{\log\left(cx^{\frac{3}{2}}+1\right)^2}{x}\,dx-\frac{1}{2}b^2\int\frac{\log\left(cx^{\frac{3}{2}}+1\right)\log\left(-cx^{\frac{3}{2}}+1\right)}{x}\,dx+\frac{1}{4}b^2\int\frac{\log\left(-cx^{\frac{3}{2}}+1\right)^2}{x}\,dx+ab\int\frac{\log\left(cx^{\frac{3}{2}}+\right.}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(3/2)))^2/x,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*integrate(log(c*x^(3/2) + 1)^2/x, x) - 1/2*b^2*integrate(log(c*x^(3
/2) + 1)*log(-c*x^(3/2) + 1)/x, x) + 1/4*b^2*integrate(log(-c*x^(3/2) + 1)^
2/x, x) + a*b*integrate(log(c*x^(3/2) + 1)/x, x) - a*b*integrate(log(-c*x^(
3/2) + 1)/x, x) + a^2*log(x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{b^2\operatorname{artanh}\left(cx^{\frac{3}{2}}\right)^2+2\,ab\operatorname{artanh}\left(cx^{\frac{3}{2}}\right)+a^2}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(3/2)))^2/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x^(3/2))^2 + 2*a*b*arctanh(c*x^(3/2)) + a^2)/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**(3/2)))**2/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \operatorname{artanh}\left(cx^{\frac{3}{2}}\right) + a\right)^2}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(3/2)))^2/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^(3/2)) + a)^2/x, x)
```

3.223 $\int \frac{\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)^2}{x^4}\,dx$

Optimal. Leaf size=96

$$\frac{1}{3}c^2\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)^2-\frac{2bc\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)}{3x^{3/2}}-\frac{\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)^2}{3x^3}-\frac{1}{3}b^2c^2\log\left(1-c^2x^3\right)+b^2c^2\log(x$$

```
[Out] (-2*b*c*(a + b*ArcTanh[c*x^(3/2)]))/(3*x^(3/2)) + (c^2*(a + b*ArcTanh[c*x^(
3/2)])^2)/3 - (a + b*ArcTanh[c*x^(3/2)])^2/(3*x^3) + b^2*c^2*Log[x] - (b^2*
c^2*Log[1 - c^2*x^3])/3
```

Rubi [F] time = 0.0237288, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)^2}{x^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcTanh[c*x^(3/2)])^2/x^4,x]
```

```
[Out] Defer[Int][(a + b*ArcTanh[c*x^(3/2)])^2/x^4, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)^2}{x^4}\,dx=\int \frac{\left(a+b\tanh^{-1}\left(cx^{3/2}\right)\right)^2}{x^4}\,dx$$

Mathematica [A] time = 0.139447, size = 123, normalized size = 1.28

$$\frac{1}{3}\left(-\frac{a^2}{x^3}-bc^2(a+b)\log\left(1-cx^{3/2}\right)+bc^2(a-b)\log\left(cx^{3/2}+1\right)-\frac{2abc}{x^{3/2}}-\frac{2b\tanh^{-1}\left(cx^{3/2}\right)\left(a+bcx^{3/2}\right)}{x^3}+\frac{b^2\left(c^2x^3-1\right)}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^(3/2)])^2/x^4,x]
```

```
[Out] (-(a^2/x^3) - (2*a*b*c)/x^(3/2) - (2*b*(a + b*c*x^(3/2))*ArcTanh[c*x^(3/2)]
)/x^3 + (b^2*(-1 + c^2*x^3)*ArcTanh[c*x^(3/2)]^2)/x^3 + 3*b^2*c^2*Log[x] -
b*(a + b)*c^2*Log[1 - c*x^(3/2)] + (a - b)*b*c^2*Log[1 + c*x^(3/2)])/3
```

Maple [F] time = 0.28, size = 0, normalized size = 0.

$$\int \frac{1}{x^4}\left(a+b\mathrm{Artanh}\left(cx^{\frac{3}{2}}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^(3/2)))^2/x^4,x)

[Out] int((a+b*arctanh(c*x^(3/2)))^2/x^4,x)
```

Maxima [B] time = 0.998003, size = 236, normalized size = 2.46

$$\frac{1}{3}\left(\left(c\log\left(cx^{\frac{3}{2}}+1\right)-c\log\left(cx^{\frac{3}{2}}-1\right)-\frac{2}{x^{\frac{3}{2}}}\right)c-\frac{2\,\operatorname{artanh}\left(cx^{\frac{3}{2}}\right)}{x^3}\right)ab+\frac{1}{12}\left(\left(2\left(\log\left(cx^{\frac{3}{2}}-1\right)-2\right)\log\left(cx^{\frac{3}{2}}+1\right)-\log\left(cx^{\frac{3}{2}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(3/2)))^2/x^4,x, algorithm="maxima")

[Out] 1/3*((c*log(c*x^(3/2) + 1) - c*log(c*x^(3/2) - 1) - 2/x^(3/2))*c - 2*arctan
h(c*x^(3/2))/x^3)*a*b + 1/12*((2*(log(c*x^(3/2) - 1) - 2)*log(c*x^(3/2) + 1
) - log(c*x^(3/2) + 1)^2 - log(c*x^(3/2) - 1)^2 - 4*log(c*x^(3/2) - 1) + 12
*log(x))*c^2 + 4*(c*log(c*x^(3/2) + 1) - c*log(c*x^(3/2) - 1) - 2/x^(3/2))*
c*arctanh(c*x^(3/2)))*b^2 - 1/3*b^2*arctanh(c*x^(3/2))^2/x^3 - 1/3*a^2/x^3
```

Fricas [B] time = 1.89078, size = 402, normalized size = 4.19

$$\frac{24\,b^2c^2x^3\log\left(\sqrt{x}\right)+4\left(ab-b^2\right)c^2x^3\log\left(cx^{\frac{3}{2}}+1\right)-4\left(ab+b^2\right)c^2x^3\log\left(cx^{\frac{3}{2}}-1\right)-8\,abcx^{\frac{3}{2}}+\left(b^2c^2x^3-b^2\right)\log\left(-\frac{c^2x^3+2}{c^2x}\right.}{12\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(3/2)))^2/x^4,x, algorithm="fricas")

[Out] 1/12*(24*b^2*c^2*x^3*log(sqrt(x)) + 4*(a*b - b^2)*c^2*x^3*log(c*x^(3/2) + 1
) - 4*(a*b + b^2)*c^2*x^3*log(c*x^(3/2) - 1) - 8*a*b*c*x^(3/2) + (b^2*c^2*x
^3 - b^2)*log(-(c^2*x^3 + 2*c*x^(3/2) + 1)/(c^2*x^3 - 1))^2 - 4*a^2 - 4*(b^
2*c*x^(3/2) + a*b)*log(-(c^2*x^3 + 2*c*x^(3/2) + 1)/(c^2*x^3 - 1)))/x^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**(3/2)))**2/x**4,x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(b\,\operatorname{artanh}\left(cx^{\frac{3}{2}}\right)+a\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(3/2)))^2/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^(3/2)) + a)^2/x^4, x)
```

3.224 $\int x^2 \left(a + b \tanh^{-1}(cx^n)\right) dx$

Optimal. Leaf size=66

$$\frac{1}{3}x^3\left(a+b\tanh^{-1}(cx^n)\right)-\frac{bcnx^{n+3}\text{Hypergeometric2F1}\left(1,\frac{n+3}{2n},\frac{3(n+1)}{2n},c^2x^{2n}\right)}{3(n+3)}$$

```
[Out] (x^3*(a + b*ArcTanh[c*x^n]))/3 - (b*c*n*x^(3 + n)*Hypergeometric2F1[1, (3 +
 n)/(2*n), (3*(1 + n))/(2*n), c^2*x^(2*n)])/(3*(3 + n))
```

Rubi [A] time = 0.0305945, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6097, 364}

$$\frac{1}{3}x^3\left(a+b\tanh^{-1}(cx^n)\right)-\frac{bcnx^{n+3}\,{}_2F_1\left(1,\frac{n+3}{2n};\frac{3(n+1)}{2n};c^2x^{2n}\right)}{3(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcTanh[c*x^n]),x]
```

```
[Out] (x^3*(a + b*ArcTanh[c*x^n]))/3 - (b*c*n*x^(3 + n)*Hypergeometric2F1[1, (3 +
 n)/(2*n), (3*(1 + n))/(2*n), c^2*x^(2*n)])/(3*(3 + n))
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+b\tanh^{-1}(cx^n)\right)dx&=\frac{1}{3}x^3\left(a+b\tanh^{-1}(cx^n)\right)-\frac{1}{3}(bcn)\int\frac{x^{2+n}}{1-c^2x^{2n}}\,dx\\&=\frac{1}{3}x^3\left(a+b\tanh^{-1}(cx^n)\right)-\frac{bcnx^{3+n}\,{}_2F_1\left(1,\frac{3+n}{2n};\frac{3(1+n)}{2n};c^2x^{2n}\right)}{3(3+n)}\end{aligned}$$

Mathematica [A] time = 0.0519168, size = 73, normalized size = 1.11

$$-\frac{bcnx^{n+3}\text{Hypergeometric2F1}\left(1,\frac{n+3}{2n},\frac{n+3}{2n}+1,c^2x^{2n}\right)}{3(n+3)}+\frac{ax^3}{3}+\frac{1}{3}bx^3\tanh^{-1}(cx^n)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcTanh[c*x^n]),x]
```

```
[Out] (a*x^3)/3 + (b*x^3*ArcTanh[c*x^n])/3 - (b*c*n*x^(3 + n)*Hypergeometric2F1[1
, (3 + n)/(2*n), 1 + (3 + n)/(2*n), c^2*x^(2*n)])/(3*(3 + n))
```

Maple [F] time = 0.12, size = 0, normalized size = 0.

$$\int x^2 (a + b\text{Artanh}\,(cx^n))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c*x^n)),x)
```

```
[Out] int(x^2*(a+b*arctanh(c*x^n)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{3}\,ax^3 + \frac{1}{6}\left(x^3\log\left(cx^n+1\right) - x^3\log\left(-cx^n+1\right) + 3\,n\int\frac{x^2}{3\left(cx^n+1\right)}\,dx + 3\,n\int\frac{x^2}{3\left(cx^n-1\right)}\,dx\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^n)),x, algorithm="maxima")
```

```
[Out] 1/3*a*x^3 + 1/6*(x^3*log(c*x^n + 1) - x^3*log(-c*x^n + 1) + 3*n*integrate(1
/3*x^2/(c*x^n + 1), x) + 3*n*integrate(1/3*x^2/(c*x^n - 1), x))*b
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(bx^2\,\text{artanh}\,(cx^n) + ax^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^n)),x, algorithm="fricas")
```

```
[Out] integral(b*x^2*arctanh(c*x^n) + a*x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c*x**n)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\operatorname{artanh}(cx^n) + a)x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^n)),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^n) + a)*x^2, x)
```

3.225 $\int x\left(a+b\tanh^{-1}\left(cx^n\right)\right)\,dx$

Optimal. Leaf size=67

$$\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(cx^n\right)\right)-\frac{bcnx^{n+2}\text{Hypergeometric2F1}\left(1,\frac{n+2}{2n},\frac{1}{2}\left(\frac{2}{n}+3\right),c^2x^{2n}\right)}{2(n+2)}$$

```
[Out] (x^2*(a + b*ArcTanh[c*x^n]))/2 - (b*c*n*x^(2 + n)*Hypergeometric2F1[1, (2 + n)/(2*n), (3 + 2/n)/2, c^2*x^(2*n)])/(2*(2 + n))
```

Rubi [A] time = 0.0243278, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6097, 364}

$$\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(cx^n\right)\right)-\frac{bcnx^{n+2}\,{}_2F_1\left(1,\frac{n+2}{2n};\frac{1}{2}\left(3+\frac{2}{n}\right);c^2x^{2n}\right)}{2(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcTanh[c*x^n]),x]
```

```
[Out] (x^2*(a + b*ArcTanh[c*x^n]))/2 - (b*c*n*x^(2 + n)*Hypergeometric2F1[1, (2 + n)/(2*n), (3 + 2/n)/2, c^2*x^(2*n)])/(2*(2 + n))
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x\left(a+b\tanh^{-1}\left(cx^n\right)\right)\,dx&=\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(cx^n\right)\right)-\frac{1}{2}(bcn)\int\frac{x^{1+n}}{1-c^2x^{2n}}\,dx\\&=\frac{1}{2}x^2\left(a+b\tanh^{-1}\left(cx^n\right)\right)-\frac{bcnx^{2+n}\,{}_2F_1\left(1,\frac{2+n}{2n};\frac{1}{2}\left(3+\frac{2}{n}\right);c^2x^{2n}\right)}{2(2+n)}\end{aligned}$$

Mathematica [A] time = 0.0445054, size = 73, normalized size = 1.09

$$-\frac{bcnx^{n+2}\text{Hypergeometric2F1}\left(1,\frac{n+2}{2n},\frac{n+2}{2n}+1,c^2x^{2n}\right)}{2(n+2)}+\frac{ax^2}{2}+\frac{1}{2}bx^2\tanh^{-1}\left(cx^n\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcTanh[c*x^n]),x]
```

```
[Out] (a*x^2)/2 + (b*x^2*ArcTanh[c*x^n])/2 - (b*c*n*x^(2 + n)*Hypergeometric2F1[1
, (2 + n)/(2*n), 1 + (2 + n)/(2*n), c^2*x^(2*n)])/(2*(2 + n))
```

Maple [F] time = 0.21, size = 0, normalized size = 0.

$$\int x\,(a+b\mathrm{Artanh}\,(cx^n))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c*x^n)),x)
```

```
[Out] int(x*(a+b*arctanh(c*x^n)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\,ax^2+\frac{1}{4}\left(x^2\log\left(cx^n+1\right)-x^2\log\left(-cx^n+1\right)+2\,n\int\frac{x}{2\left(cx^n+1\right)}\,dx+2\,n\int\frac{x}{2\left(cx^n-1\right)}\,dx\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^n)),x, algorithm="maxima")
```

```
[Out] 1/2*a*x^2 + 1/4*(x^2*log(c*x^n + 1) - x^2*log(-c*x^n + 1) + 2*n*integrate(1
/2*x/(c*x^n + 1), x) + 2*n*integrate(1/2*x/(c*x^n - 1), x))*b
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(bx\,\mathrm{artanh}\left(cx^n\right)+ax,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^n)),x, algorithm="fricas")
```

```
[Out] integral(b*x*arctanh(c*x^n) + a*x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(a+b\,\mathrm{atanh}\left(cx^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c*x**n)),x)
```

```
[Out] Integral(x*(a + b*atanh(c*x**n)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{artanh}(cx^n) + a)x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^n)),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^n) + a)*x, x)
```

3.226 $\int \left(a+b\tanh^{-1}\left(cx^n\right)\right)dx$

Optimal. Leaf size=58

$$-\frac{bcnx^{n+1}\text{Hypergeometric2F1}\left(1,\frac{n+1}{2n},\frac{1}{2}\left(\frac{1}{n}+3\right),c^2x^{2n}\right)}{n+1}+ax+bx\tanh^{-1}\left(cx^n\right)$$

```
[Out] a*x + b*x*ArcTanh[c*x^n] - (b*c*n*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2
*n), (3 + n^(-1))/2, c^2*x^(2*n)])/(1 + n)
```

Rubi [A] time = 0.0279346, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6091, 364}

$$ax-\frac{bcnx^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);c^2x^{2n}\right)}{n+1}+bx\tanh^{-1}\left(cx^n\right)$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcTanh[c*x^n],x]
```

```
[Out] a*x + b*x*ArcTanh[c*x^n] - (b*c*n*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2
*n), (3 + n^(-1))/2, c^2*x^(2*n)])/(1 + n)
```

Rule 6091

```
Int[ArcTanh[(c_.)*(x_)^(n_)], x_Symbol] :> Simp[x*ArcTanh[c*x^n], x] - Dist
[c*n, Int[x^n/(1 - c^2*x^(2*n)), x], x] /; FreeQ[{c, n}, x]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \left(a+b\tanh^{-1}\left(cx^n\right)\right)dx &= ax+b\int \tanh^{-1}\left(cx^n\right)dx\\
&= ax+bx\tanh^{-1}\left(cx^n\right)-(bcn)\int \frac{x^n}{1-c^2x^{2n}}\,dx\\
&= ax+bx\tanh^{-1}\left(cx^n\right)-\frac{bcnx^{1+n}\,{}_2F_1\left(1,\frac{1+n}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);c^2x^{2n}\right)}{1+n}
\end{aligned}$$

Mathematica [A] time = 0.0316411, size = 58, normalized size = 1.

$$-\frac{bcnx^{n+1}\text{Hypergeometric2F1}\left(1,\frac{n+1}{2n},\frac{1}{2}\left(\frac{1}{n}+3\right),c^2x^{2n}\right)}{n+1}+ax+bx\tanh^{-1}\left(cx^n\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcTanh[c*x^n],x]
```

```
[Out] a*x + b*x*ArcTanh[c*x^n] - (b*c*n*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/(2
*n), (3 + n^(-1))/2, c^2*x^(2*n)])/(1 + n)
```

Maple [F] time = 0.191, size = 0, normalized size = 0.

$$\int a + b\text{Artanh}\,(cx^n)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arctanh(c*x^n),x)
```

```
[Out] int(a+b*arctanh(c*x^n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\left(n\int\frac{1}{cx^n+1}\,dx+n\int\frac{1}{cx^n-1}\,dx+x\log\left(cx^n+1\right)-x\log\left(-cx^n+1\right)\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c*x^n),x, algorithm="maxima")
```

```
[Out] 1/2*(n*integrate(1/(c*x^n + 1), x) + n*integrate(1/(c*x^n - 1), x) + x*log(
c*x^n + 1) - x*log(-c*x^n + 1))*b + a*x
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(b\,\text{artanh}\,(cx^n)+a,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c*x^n),x, algorithm="fricas")
```

```
[Out] integral(b*arctanh(c*x^n) + a, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\,\text{atanh}\,(cx^n)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*atanh(c*x**n),x)
```

```
[Out] Integral(a + b*atanh(c*x**n), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int b \operatorname{artanh}(cx^n) + a\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c*x^n),x, algorithm="giac")
```

```
[Out] integrate(b*arctanh(c*x^n) + a, x)
```

3.227 $\int \frac{a+b\tanh^{-1}(cx^n)}{x}\,dx$

Optimal. Leaf size=36

$$-\frac{b\text{PolyLog}(2,-cx^n)}{2n}+\frac{b\text{PolyLog}(2,cx^n)}{2n}+a\log(x)$$

```
[Out] a*Log[x] - (b*PolyLog[2, -(c*x^n)])/(2*n) + (b*PolyLog[2, c*x^n])/(2*n)
```

Rubi [A] time = 0.0350596, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6095, 5912}

$$-\frac{b\text{PolyLog}(2,-cx^n)}{2n}+\frac{b\text{PolyLog}(2,cx^n)}{2n}+a\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^n])/x,x]
```

```
[Out] a*Log[x] - (b*PolyLog[2, -(c*x^n)])/(2*n) + (b*PolyLog[2, c*x^n])/(2*n)
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[
1/n, Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
 n}, x] && IGtQ[p, 0]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}(cx^n)}{x}\,dx &= \frac{\text{Subst}\left(\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx,x,x^n\right)}{n}\\ &= a\log(x)-\frac{b\text{Li}_2(-cx^n)}{2n}+\frac{b\text{Li}_2(cx^n)}{2n}\end{aligned}$$

Mathematica [C] time = 0.0750161, size = 39, normalized size = 1.08

$$\frac{bcx^n\text{HypergeometricPFQ}\left(\left\{\frac{1}{2},\frac{1}{2},1\right\},\left\{\frac{3}{2},\frac{3}{2}\right\},c^2x^{2n}\right)}{n}+a\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^n])/x,x]
```

```
[Out] (b*c*x^n*HypergeometricPFQ[{1/2, 1/2, 1}, {3/2, 3/2}, c^2*x^(2*n)])/n + a*L
og[x]
```

Maple [B] time = 0.036, size = 76, normalized size = 2.1

$$\frac{a\ln\left(cx^{n}\right)}{n}+\frac{b\ln\left(cx^{n}\right)\mathrm{Artanh}\left(cx^{n}\right)}{n}-\frac{b\mathrm{dilog}\left(cx^{n}\right)}{2n}-\frac{b\mathrm{dilog}\left(cx^{n}+1\right)}{2n}-\frac{b\ln\left(cx^{n}\right)\ln\left(cx^{n}+1\right)}{2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^n))/x,x)
```

```
[Out] 1/n*a*ln(c*x^n)+1/n*b*ln(c*x^n)*arctanh(c*x^n)-1/2/n*b*dilog(c*x^n)-1/2/n*b
*dilog(c*x^n+1)-1/2/n*b*ln(c*x^n)*ln(c*x^n+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\left(n\int\frac{\log\left(x\right)}{cxx^{n}+x}\,dx+n\int\frac{\log\left(x\right)}{cxx^{n}-x}\,dx+\log\left(cx^{n}+1\right)\log\left(x\right)-\log\left(-cx^{n}+1\right)\log\left(x\right)\right)b+a\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))/x,x, algorithm="maxima")
```

```
[Out] 1/2*(n*integrate(log(x)/(c*x*x^n + x), x) + n*integrate(log(x)/(c*x*x^n - x
), x) + log(c*x^n + 1)*log(x) - log(-c*x^n + 1)*log(x))*b + a*log(x)
```

Fricas [B] time = 1.81852, size = 456, normalized size = 12.67

$$-\frac{bn\log\left(c\cosh\left(n\log\left(x\right)\right)+c\sinh\left(n\log\left(x\right)\right)+1\right)\log\left(x\right)-bn\log\left(-c\cosh\left(n\log\left(x\right)\right)-c\sinh\left(n\log\left(x\right)\right)+1\right)\log\left(x\right)-b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))/x,x, algorithm="fricas")
```

```
[Out] -1/2*(b*n*log(c*cosh(n*log(x)) + c*sinh(n*log(x)) + 1)*log(x) - b*n*log(-c*
cosh(n*log(x)) - c*sinh(n*log(x)) + 1)*log(x) - b*n*log(x)*log(-(c*cosh(n*l
og(x)) + c*sinh(n*log(x)) + 1)/(c*cosh(n*log(x)) + c*sinh(n*log(x)) - 1)) -
 2*a*n*log(x) - b*dilog(c*cosh(n*log(x)) + c*sinh(n*log(x))) + b*dilog(-c*c
osh(n*log(x)) - c*sinh(n*log(x))))/n
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{a+b\,\mathrm{atanh}\left(cx^{n}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**n))/x,x)
```

```
[Out] Integral((a + b*atanh(c*x**n))/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{artanh}(cx^n) + a}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^n) + a)/x, x)
```

3.228 $\int \frac{a+b\tanh^{-1}(cx^n)}{x^2}\,dx$

Optimal. Leaf size=67

$$-\frac{bcnx^{n-1}\text{Hypergeometric2F1}\left(1,-\frac{1-n}{2n},\frac{1}{2}\left(3-\frac{1}{n}\right),c^2x^{2n}\right)}{1-n}-\frac{a+b\tanh^{-1}(cx^n)}{x}$$

```
[Out] -((a + b*ArcTanh[c*x^n])/x) - (b*c*n*x^(-1 + n)*Hypergeometric2F1[1, -(1 -
n)/(2*n), (3 - n^(-1))/2, c^2*x^(2*n)])/(1 - n)
```

Rubi [A] time = 0.032358, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6097, 364}

$$-\frac{a+b\tanh^{-1}(cx^n)}{x}-\frac{bcnx^{n-1}\,{}_2F_1\left(1,-\frac{1-n}{2n};\frac{1}{2}\left(3-\frac{1}{n}\right);c^2x^{2n}\right)}{1-n}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^n])/x^2,x]
```

```
[Out] -((a + b*ArcTanh[c*x^n])/x) - (b*c*n*x^(-1 + n)*Hypergeometric2F1[1, -(1 -
n)/(2*n), (3 - n^(-1))/2, c^2*x^(2*n)])/(1 - n)
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}(cx^n)}{x^2}\,dx &= -\frac{a+b\tanh^{-1}(cx^n)}{x}+(bcn)\int \frac{x^{-2+n}}{1-c^2x^{2n}}\,dx\\ &= -\frac{a+b\tanh^{-1}(cx^n)}{x}-\frac{bcnx^{-1+n}\,{}_2F_1\left(1,-\frac{1-n}{2n};\frac{1}{2}\left(3-\frac{1}{n}\right);c^2x^{2n}\right)}{1-n}\end{aligned}$$

Mathematica [A] time = 0.0609311, size = 66, normalized size = 0.99

$$\frac{bcnx^{n-1}\text{Hypergeometric2F1}\left(1,\frac{n-1}{2n},\frac{n-1}{2n}+1,c^2x^{2n}\right)}{n-1}-\frac{a}{x}-\frac{b\tanh^{-1}(cx^n)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^n])/x^2,x]
```

```
[Out] -(a/x) - (b*ArcTanh[c*x^n])/x + (b*c*n*x^(-1 + n)*Hypergeometric2F1[1, (-1
+ n)/(2*n), 1 + (-1 + n)/(2*n), c^2*x^(2*n)])/(-1 + n)
```

Maple [F] time = 0.102, size = 0, normalized size = 0.

$$\int \frac{a + b\mathrm{Artanh}\left(cx^{n}\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^n))/x^2,x)
```

```
[Out] int((a+b*arctanh(c*x^n))/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}\left(n\int \frac{1}{cx^{2}x^{n}+x^{2}}\,dx + n\int \frac{1}{cx^{2}x^{n}-x^{2}}\,dx + \frac{\log\left(cx^{n}+1\right)-\log\left(-cx^{n}+1\right)}{x}\right)b - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*(n*integrate(1/(c*x^2*x^n + x^2), x) + n*integrate(1/(c*x^2*x^n - x^2)
, x) + (log(c*x^n + 1) - log(-c*x^n + 1))/x)*b - a/x
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b\,\mathrm{artanh}\left(cx^{n}\right)+a}{x^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*x^n) + a)/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b\,\mathrm{atanh}\left(cx^{n}\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**n))/x**2,x)
```

```
[Out] Integral((a + b*atanh(c*x**n))/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\,\text{artanh}\,(cx^n) + a}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^n) + a)/x^2, x)
```

3.229 $\int \frac{a+b\tanh^{-1}(cx^n)}{x^3}\,dx$

Optimal. Leaf size=70

$$-\frac{bcnx^{n-2}\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(1-\frac{2}{n}\right),\frac{1}{2}\left(3-\frac{2}{n}\right),c^2x^{2n}\right)}{2(2-n)}-\frac{a+b\tanh^{-1}(cx^n)}{2x^2}$$

```
[Out] -(a + b*ArcTanh[c*x^n])/(2*x^2) - (b*c*n*x^(-2 + n)*Hypergeometric2F1[1, (1
 - 2/n)/2, (3 - 2/n)/2, c^2*x^(2*n)])/(2*(2 - n))
```

Rubi [A] time = 0.0311192, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6097, 364}

$$-\frac{a+b\tanh^{-1}(cx^n)}{2x^2}-\frac{bcnx^{n-2}\,{}_2F_1\left(1,\frac{1}{2}\left(1-\frac{2}{n}\right);\frac{1}{2}\left(3-\frac{2}{n}\right);c^2x^{2n}\right)}{2(2-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^n])/x^3,x]
```

```
[Out] -(a + b*ArcTanh[c*x^n])/(2*x^2) - (b*c*n*x^(-2 + n)*Hypergeometric2F1[1, (1
 - 2/n)/2, (3 - 2/n)/2, c^2*x^(2*n)])/(2*(2 - n))
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}(cx^n)}{x^3}\,dx&=-\frac{a+b\tanh^{-1}(cx^n)}{2x^2}+\frac{1}{2}(bcn)\int \frac{x^{-3+n}}{1-c^2x^{2n}}\,dx\\&=-\frac{a+b\tanh^{-1}(cx^n)}{2x^2}-\frac{bcnx^{-2+n}\,{}_2F_1\left(1,\frac{1}{2}\left(1-\frac{2}{n}\right);\frac{1}{2}\left(3-\frac{2}{n}\right);c^2x^{2n}\right)}{2(2-n)}\end{aligned}$$

Mathematica [A] time = 0.0411021, size = 73, normalized size = 1.04

$$\frac{bcnx^{n-2}\text{Hypergeometric2F1}\left(1,\frac{n-2}{2n},\frac{n-2}{2n}+1,c^2x^{2n}\right)}{2(n-2)}-\frac{a}{2x^2}-\frac{b\tanh^{-1}(cx^n)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^n])/x^3,x]
```

```
[Out] -a/(2*x^2) - (b*ArcTanh[c*x^n])/(2*x^2) + (b*c*n*x^(-2 + n)*Hypergeometric2
F1[1, (-2 + n)/(2*n), 1 + (-2 + n)/(2*n), c^2*x^(2*n)])/(2*(-2 + n))
```

Maple [F] time = 0.109, size = 0, normalized size = 0.

$$\int \frac{a + b\operatorname{Artanh}(cx^n)}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^n))/x^3,x)
```

```
[Out] int((a+b*arctanh(c*x^n))/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{4}\left(2\,n\int \frac{1}{2\left(cx^3x^n + x^3\right)}\, dx + 2\,n\int \frac{1}{2\left(cx^3x^n - x^3\right)}\, dx + \frac{\log\left(cx^n + 1\right) - \log\left(-cx^n + 1\right)}{x^2}\right)b - \frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))/x^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*n*integrate(1/2/(c*x^3*x^n + x^3), x) + 2*n*integrate(1/2/(c*x^3*x^
n - x^3), x) + (log(c*x^n + 1) - log(-c*x^n + 1))/x^2)*b - 1/2*a/x^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{b\operatorname{artanh}\left(cx^n\right) + a}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))/x^3,x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*x^n) + a)/x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**n))/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\,\mathrm{artanh}\left(cx^n\right)+a}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^n) + a)/x^3, x)
```

3.230 $\int \frac{a+b\tanh^{-1}(cx^n)}{x^4}\,dx$

Optimal. Leaf size=72

$$-\frac{bcnx^{n-3}\text{Hypergeometric2F1}\left(1,-\frac{3-n}{2n},-\frac{3(1-n)}{2n},c^2x^{2n}\right)}{3(3-n)}-\frac{a+b\tanh^{-1}(cx^n)}{3x^3}$$

```
[Out] -(a + b*ArcTanh[c*x^n])/(3*x^3) - (b*c*n*x^(-3 + n)*Hypergeometric2F1[1, -(
3 - n)/(2*n), (-3*(1 - n))/(2*n), c^2*x^(2*n)])/(3*(3 - n))
```

Rubi [A] time = 0.0342286, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6097, 364}

$$-\frac{a+b\tanh^{-1}(cx^n)}{3x^3}-\frac{bcnx^{n-3}\,{}_2F_1\left(1,-\frac{3-n}{2n};-\frac{3(1-n)}{2n};c^2x^{2n}\right)}{3(3-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^n])/x^4,x]
```


```
[Out] -(a + b*ArcTanh[c*x^n])/(3*x^3) - (b*c*n*x^(-3 + n)*Hypergeometric2F1[1, -(
3 - n)/(2*n), (-3*(1 - n))/(2*n), c^2*x^(2*n)])/(3*(3 - n))
```


Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}(cx^n)}{x^4}\,dx &= -\frac{a+b\tanh^{-1}(cx^n)}{3x^3}+\frac{1}{3}(bcn)\int \frac{x^{-4+n}}{1-c^2x^{2n}}\,dx\\ &= -\frac{a+b\tanh^{-1}(cx^n)}{3x^3}-\frac{bcnx^{-3+n}\,{}_2F_1\left(1,-\frac{3-n}{2n};-\frac{3(1-n)}{2n};c^2x^{2n}\right)}{3(3-n)}\end{aligned}$$

Mathematica [A] time = 0.0416435, size = 73, normalized size = 1.01

$$\frac{bcnx^{n-3}\text{Hypergeometric2F1}\left(1,\frac{n-3}{2n},\frac{n-3}{2n}+1,c^2x^{2n}\right)}{3(n-3)}-\frac{a}{3x^3}-\frac{b\tanh^{-1}(cx^n)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^n])/x^4,x]
```

```
[Out] -a/(3*x^3) - (b*ArcTanh[c*x^n])/(3*x^3) + (b*c*n*x^(-3 + n)*Hypergeometric2
F1[1, (-3 + n)/(2*n), 1 + (-3 + n)/(2*n), c^2*x^(2*n)])/(3*(-3 + n))
```

Maple [F] time = 0.112, size = 0, normalized size = 0.

$$\int \frac{a + b\text{Artanh}\left(cx^{n}\right)}{x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^n))/x^4,x)
```

```
[Out] int((a+b*arctanh(c*x^n))/x^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{6}\left(3\,n\int \frac{1}{3\left(cx^{4}x^{n}+x^{4}\right)}\, dx + 3\,n\int \frac{1}{3\left(cx^{4}x^{n}-x^{4}\right)}\, dx + \frac{\log\left(cx^{n}+1\right)-\log\left(-cx^{n}+1\right)}{x^{3}}\right)b - \frac{a}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))/x^4,x, algorithm="maxima")
```

```
[Out] -1/6*(3*n*integrate(1/3/(c*x^4*x^n + x^4), x) + 3*n*integrate(1/3/(c*x^4*x^
n - x^4), x) + (log(c*x^n + 1) - log(-c*x^n + 1))/x^3)*b - 1/3*a/x^3
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\,\text{artanh}\left(cx^{n}\right)+a}{x^{4}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))/x^4,x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*x^n) + a)/x^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**n))/x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{artanh}(cx^n) + a}{x^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^n) + a)/x^4, x)
```

3.231 $\int x \left(a+b\tanh^{-1}\left(cx^n\right)\right)^2 dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(x \left(a+b\tanh^{-1}\left(cx^n\right)\right)^2,x\right)$$

```
[Out] Unintegrable[x*(a + b*ArcTanh[c*x^n])^2, x]
```

Rubi [A] time = 0.0149523, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x \left(a+b\tanh^{-1}\left(cx^n\right)\right)^2 dx$$

Verification is Not applicable to the result.

```
[In] Int[x*(a + b*ArcTanh[c*x^n])^2,x]
```

```
[Out] Defer[Int][x*(a + b*ArcTanh[c*x^n])^2, x]
```

Rubi steps

$$\int x \left(a+b\tanh^{-1}\left(cx^n\right)\right)^2 dx=\int x \left(a+b\tanh^{-1}\left(cx^n\right)\right)^2 dx$$

Mathematica [A] time = 8.61828, size = 0, normalized size = 0.

$$\int x \left(a+b\tanh^{-1}\left(cx^n\right)\right)^2 dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x*(a + b*ArcTanh[c*x^n])^2,x]
```

```
[Out] Integrate[x*(a + b*ArcTanh[c*x^n])^2, x]
```

Maple [A] time = 0.162, size = 0, normalized size = 0.

$$\int x \left(a+b\text{Artanh}\left(cx^n\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c*x^n))^2,x)
```

```
[Out] int(x*(a+b*arctanh(c*x^n))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{1}{8}\,b^2x^2\log\left(-cx^n+1\right)^2+\frac{1}{2}\,a^2x^2-\int-\frac{\left(b^2cxx^n-b^2x\right)\log\left(cx^n+1\right)^2+4\left(abcxx^n-abx\right)\log\left(cx^n+1\right)+\left(4\,abx-\left(b^2cn+4\right.\right.}{4\left(cx^n-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^n))^2,x, algorithm="maxima")
```

```
[Out] 1/8*b^2*x^2*log(-c*x^n + 1)^2 + 1/2*a^2*x^2 - integrate(-1/4*((b^2*c*x*x^n
- b^2*x)*log(c*x^n + 1)^2 + 4*(a*b*c*x*x^n - a*b*x)*log(c*x^n + 1) + (4*a*b
*x - (b^2*c*n + 4*a*b*c)*x*x^n - 2*(b^2*c*x*x^n - b^2*x)*log(c*x^n + 1))*lo
g(-c*x^n + 1))/(c*x^n - 1), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(b^2x\,\text{artanh}\left(cx^n\right)^2+2\,abx\,\text{artanh}\left(cx^n\right)+a^2x,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^n))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x*arctanh(c*x^n)^2 + 2*a*b*x*arctanh(c*x^n) + a^2*x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c*x**n))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\left(b\,\text{artanh}\left(cx^n\right)+a\right)^2x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^n))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^n) + a)^2*x, x)
```

3.232 $\int \left(a+b\tanh^{-1}\left(cx^n\right)\right)^2\,dx$

Optimal. Leaf size=14

$$\text{Unintegrable}\left(\left(a+b\tanh^{-1}\left(cx^n\right)\right)^2,x\right)$$

```
[Out] Unintegrable[(a + b*ArcTanh[c*x^n])^2, x]
```

Rubi [A] time = 0.0063323, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \left(a+b\tanh^{-1}\left(cx^n\right)\right)^2\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcTanh[c*x^n])^2,x]

[Out] Defer[Int][(a + b*ArcTanh[c*x^n])^2, x]
```

Rubi steps

$$\int \left(a+b\tanh^{-1}\left(cx^n\right)\right)^2\,dx=\int \left(a+b\tanh^{-1}\left(cx^n\right)\right)^2\,dx$$

Mathematica [A] time = 1.89704, size = 0, normalized size = 0.

$$\int \left(a+b\tanh^{-1}\left(cx^n\right)\right)^2\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcTanh[c*x^n])^2,x]

[Out] Integrate[(a + b*ArcTanh[c*x^n])^2, x]
```

Maple [A] time = 0.187, size = 0, normalized size = 0.

$$\int (a+b\text{Artanh}\,(cx^n))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^n))^2,x)

[Out] int((a+b*arctanh(c*x^n))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{1}{4}\,b^2x\log\left(-cx^n+1\right)^2+a^2x-\int-\frac{\left(b^2cx^n-b^2\right)\log\left(cx^n+1\right)^2+4\left(abcx^n-ab\right)\log\left(cx^n+1\right)+2\left(2\,ab-\left(b^2cn+2\,abc\right)x^n-\right.}{4\left(cx^n-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*x*log(-c*x^n + 1)^2 + a^2*x - integrate(-1/4*((b^2*c*x^n - b^2)*log
(c*x^n + 1)^2 + 4*(a*b*c*x^n - a*b)*log(c*x^n + 1) + 2*(2*a*b - (b^2*c*n +
2*a*b*c)*x^n - (b^2*c*x^n - b^2)*log(c*x^n + 1))*log(-c*x^n + 1))/(c*x^n -
1), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(b^2\,\mathrm{artanh}\left(cx^n\right)^2+2\,ab\,\mathrm{artanh}\left(cx^n\right)+a^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*arctanh(c*x^n)^2 + 2*a*b*arctanh(c*x^n) + a^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\,\mathrm{atanh}\left(cx^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**n))**2,x)
```

```
[Out] Integral((a + b*atanh(c*x**n))**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\left(b\,\mathrm{artanh}\left(cx^n\right)+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^n) + a)^2, x)
```

3.233 $\int \frac{\left(a+b\tanh^{-1}(cx^n)\right)^2}{x}\,dx$

Optimal. Leaf size=148

$$-\frac{b\text{PolyLog}\left(2,1-\frac{2}{1-cx^n}\right)\left(a+b\tanh^{-1}(cx^n)\right)}{n}+\frac{b\text{PolyLog}\left(2,\frac{2}{1-cx^n}-1\right)\left(a+b\tanh^{-1}(cx^n)\right)}{n}+\frac{b^2\text{PolyLog}\left(3,1-\right.}{2n}$$

```
[Out] (2*(a + b*ArcTanh[c*x^n])^2*ArcTanh[1 - 2/(1 - c*x^n)])/n - (b*(a + b*ArcTa
nh[c*x^n])*PolyLog[2, 1 - 2/(1 - c*x^n)])/n + (b*(a + b*ArcTanh[c*x^n])*Pol
yLog[2, -1 + 2/(1 - c*x^n)])/n + (b^2*PolyLog[3, 1 - 2/(1 - c*x^n)])/(2*n)
- (b^2*PolyLog[3, -1 + 2/(1 - c*x^n)])/(2*n)
```

Rubi [A] time = 0.313463, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6095, 5914, 6052, 5948, 6058, 6610}

$$-\frac{b\text{PolyLog}\left(2,1-\frac{2}{1-cx^n}\right)\left(a+b\tanh^{-1}(cx^n)\right)}{n}+\frac{b\text{PolyLog}\left(2,\frac{2}{1-cx^n}-1\right)\left(a+b\tanh^{-1}(cx^n)\right)}{n}+\frac{b^2\text{PolyLog}\left(3,1-\right.}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^n])^2/x,x]
```

```
[Out] (2*(a + b*ArcTanh[c*x^n])^2*ArcTanh[1 - 2/(1 - c*x^n)])/n - (b*(a + b*ArcTa
nh[c*x^n])*PolyLog[2, 1 - 2/(1 - c*x^n)])/n + (b*(a + b*ArcTanh[c*x^n])*Pol
yLog[2, -1 + 2/(1 - c*x^n)])/n + (b^2*PolyLog[3, 1 - 2/(1 - c*x^n)])/(2*n)
- (b^2*PolyLog[3, -1 + 2/(1 - c*x^n)])/(2*n)
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[
1/n, Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
 n}, x] && IGtQ[p, 0]
```

Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
 e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
```

```
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx^n)\right)^2}{x}dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x}dx, x, x^n\right)}{n}$$

$$= \frac{2\left(a+b\tanh^{-1}(cx^n)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx^n}\right)}{n} - \frac{(4bc)\,\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(cx)\right)\tanh^{-1}\left(1-\frac{2}{1-cx}\right)}{1-c^2x^2} d\right.}{n}$$

$$= \frac{2\left(a+b\tanh^{-1}(cx^n)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx^n}\right)}{n} + \frac{(2bc)\,\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1-cx}\right)}{1-c^2x^2}dx, x, x\right.}{n}$$

$$= \frac{2\left(a+b\tanh^{-1}(cx^n)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx^n}\right)}{n} - \frac{b\left(a+b\tanh^{-1}(cx^n)\right)\text{Li}_2\left(1-\frac{2}{1-cx^n}\right)}{n} + \frac{b\,(a}{}$$

$$= \frac{2\left(a+b\tanh^{-1}(cx^n)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx^n}\right)}{n} - \frac{b\left(a+b\tanh^{-1}(cx^n)\right)\text{Li}_2\left(1-\frac{2}{1-cx^n}\right)}{n} + \frac{b\,(a}{}$$

Mathematica [A] time = 0.128001, size = 146, normalized size = 0.99

$$\frac{\frac{1}{2}b\left(2\text{PolyLog}\left(2,\frac{cx^n+1}{1-cx^n}\right)\left(a+b\tanh^{-1}(cx^n)\right)-2\text{PolyLog}\left(2,\frac{cx^n+1}{cx^n-1}\right)\left(a+b\tanh^{-1}(cx^n)\right)+b\left(\text{PolyLog}\left(3,\frac{cx^n+1}{cx^n-1}\right)-\text{Pol}\right.\right.}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^n])^2/x,x]
```

```
[Out] (2*(a + b*ArcTanh[c*x^n])^2*ArcTanh[1 + 2/(-1 + c*x^n)] + (b*(2*(a + b*ArcT
anh[c*x^n])*PolyLog[2, (1 + c*x^n)/(1 - c*x^n)] - 2*(a + b*ArcTanh[c*x^n])*
PolyLog[2, (1 + c*x^n)/(-1 + c*x^n)] + b*(-PolyLog[3, (1 + c*x^n)/(1 - c*x^
n)] + PolyLog[3, (1 + c*x^n)/(-1 + c*x^n)])))/2)/n
```

Maple [C] time = 0.33, size = 880, normalized size = 6.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^n))^2/x,x)
```

```
[Out] 1/n*a^2*ln(c*x^n)+1/n*b^2*ln(c*x^n)*arctanh(c*x^n)^2-1/n*b^2*arctanh(c*x^n)
*polylog(2,-(c*x^n+1)^2/(-c^2*(x^n)^2+1))+1/2/n*b^2*polylog(3,-(c*x^n+1)^2/
(-c^2*(x^n)^2+1))-1/n*b^2*arctanh(c*x^n)^2*ln((c*x^n+1)^2/(-c^2*(x^n)^2+1)-
1)+1/n*b^2*arctanh(c*x^n)^2*ln(1-(c*x^n+1)/(-c^2*(x^n)^2+1)^(1/2))+2/n*b^2*
arctanh(c*x^n)*polylog(2,(c*x^n+1)/(-c^2*(x^n)^2+1)^(1/2))-2/n*b^2*polylog(
3,(c*x^n+1)/(-c^2*(x^n)^2+1)^(1/2))+1/n*b^2*arctanh(c*x^n)^2*ln(1+(c*x^n+1)
/(-c^2*(x^n)^2+1)^(1/2))+2/n*b^2*arctanh(c*x^n)*polylog(2,-(c*x^n+1)/(-c^2*
(x^n)^2+1)^(1/2))-2/n*b^2*polylog(3,-(c*x^n+1)/(-c^2*(x^n)^2+1)^(1/2))+1/2*
I/n*b^2*Pi*csgn(I*((c*x^n+1)^2/(-c^2*(x^n)^2+1)-1)/((c*x^n+1)^2/(-c^2*(x^n)
^2+1)+1))^3*arctanh(c*x^n)^2+1/2*I/n*b^2*Pi*csgn(I*((c*x^n+1)^2/(-c^2*(x^n)
^2+1)-1))*csgn(I/((c*x^n+1)^2/(-c^2*(x^n)^2+1)+1))*csgn(I*((c*x^n+1)^2/(-c^
2*(x^n)^2+1)-1)/((c*x^n+1)^2/(-c^2*(x^n)^2+1)+1))*arctanh(c*x^n)^2-1/2*I/n*
b^2*Pi*csgn(I/((c*x^n+1)^2/(-c^2*(x^n)^2+1)+1))*csgn(I*((c*x^n+1)^2/(-c^2*(
x^n)^2+1)-1)/((c*x^n+1)^2/(-c^2*(x^n)^2+1)+1))^2*arctanh(c*x^n)^2-1/2*I/n*b
^2*Pi*csgn(I*((c*x^n+1)^2/(-c^2*(x^n)^2+1)-1))*csgn(I*((c*x^n+1)^2/(-c^2*(x
^n)^2+1)-1)/((c*x^n+1)^2/(-c^2*(x^n)^2+1)+1))^2*arctanh(c*x^n)^2-1/n*a*b*ln
(c*x^n)*ln(c*x^n+1)+2/n*a*b*ln(c*x^n)*arctanh(c*x^n)-1/n*a*b*dilog(c*x^n)-1
/n*a*b*dilog(c*x^n+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{4}b^2\log\left(-cx^n+1\right)^2\log\left(x\right)+a^2\log\left(x\right)-\int-\frac{\left(b^2cx^n-b^2\right)\log\left(cx^n+1\right)^2+4\left(abcx^n-ab\right)\log\left(cx^n+1\right)+2\left(2\,ab-\left(b^2\right.\right.}{4\left(cxx^n-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))^2/x,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*log(-c*x^n + 1)^2*log(x) + a^2*log(x) - integrate(-1/4*((b^2*c*x^n
- b^2)*log(c*x^n + 1)^2 + 4*(a*b*c*x^n - a*b)*log(c*x^n + 1) + 2*(2*a*b - (
b^2*c*n*log(x) + 2*a*b*c)*x^n - (b^2*c*x^n - b^2)*log(c*x^n + 1))*log(-c*x^
n + 1))/(c*x*x^n - x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^2\operatorname{artanh}\left(cx^n\right)^2+2\,ab\operatorname{artanh}\left(cx^n\right)+a^2}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))^2/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x^n)^2 + 2*a*b*arctanh(c*x^n) + a^2)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+b\operatorname{atanh}\left(cx^n\right)\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**n))**2/x,x)
```

```
[Out] Integral((a + b*atanh(c*x**n))**2/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{artanh}(cx^n) + a)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))^2/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^n) + a)^2/x, x)
```

3.234 $\int \frac{\left(a+b\tanh^{-1}(cx^n)\right)^2}{x^2}\,dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{\left(a+b\tanh^{-1}\left(cx^n\right)\right)^2}{x^2},x\right)$$

```
[Out] Unintegrable[(a + b*ArcTanh[c*x^n])^2/x^2, x]
```

Rubi [A] time = 0.02298, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^n\right)\right)^2}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcTanh[c*x^n])^2/x^2,x]
```

```
[Out] Defer[Int][(a + b*ArcTanh[c*x^n])^2/x^2, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^n\right)\right)^2}{x^2}\,dx=\int \frac{\left(a+b\tanh^{-1}\left(cx^n\right)\right)^2}{x^2}\,dx$$

Mathematica [A] time = 12.3515, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\tanh^{-1}\left(cx^n\right)\right)^2}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcTanh[c*x^n])^2/x^2,x]
```

```
[Out] Integrate[(a + b*ArcTanh[c*x^n])^2/x^2, x]
```

Maple [A] time = 0.109, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\,\mathrm{Artanh}\left(cx^n\right)\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^n))^2/x^2,x)
```

```
[Out] int((a+b*arctanh(c*x^n))^2/x^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{b^2\log\left(-cx^n+1\right)^2}{4\,x}-\frac{a^2}{x}-\int-\frac{\left(b^2cx^n-b^2\right)\log\left(cx^n+1\right)^2+4\left(abcx^n-ab\right)\log\left(cx^n+1\right)+2\left(2\,ab+\left(b^2cn-2\,abc\right)x^n-\left(\right.\right.}{4\left(cx^2x^n-x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))^2/x^2,x, algorithm="maxima")
```

```
[Out] -1/4*b^2*log(-c*x^n + 1)^2/x - a^2/x - integrate(-1/4*((b^2*c*x^n - b^2)*lo
g(c*x^n + 1)^2 + 4*(a*b*c*x^n - a*b)*log(c*x^n + 1) + 2*(2*a*b + (b^2*c*n -
 2*a*b*c)*x^n - (b^2*c*x^n - b^2)*log(c*x^n + 1))*log(-c*x^n + 1))/(c*x^2*x
^n - x^2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^2\,\mathrm{artanh}\left(cx^n\right)^2+2\,ab\,\mathrm{artanh}\left(cx^n\right)+a^2}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))^2/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x^n)^2 + 2*a*b*arctanh(c*x^n) + a^2)/x^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+b\,\mathrm{atanh}\left(cx^n\right)\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**n))**2/x**2,x)
```

```
[Out] Integral((a + b*atanh(c*x**n))**2/x**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(b\,\mathrm{artanh}\left(cx^n\right)+a\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^n) + a)^2/x^2, x)
```

3.235 $\int \frac{\left(a+b\tanh^{-1}(cx^n)\right)^2}{x^3}\,dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{\left(a+b\tanh^{-1}(cx^n)\right)^2}{x^3},x\right)$$

```
[Out] Unintegrable[(a + b*ArcTanh[c*x^n])^2/x^3, x]
```

Rubi [A] time = 0.0233473, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\tanh^{-1}(cx^n)\right)^2}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcTanh[c*x^n])^2/x^3,x]
```

```
[Out] Defer[Int][(a + b*ArcTanh[c*x^n])^2/x^3, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx^n)\right)^2}{x^3}\,dx=\int \frac{\left(a+b\tanh^{-1}(cx^n)\right)^2}{x^3}\,dx$$

Mathematica [A] time = 12.3165, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\tanh^{-1}(cx^n)\right)^2}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcTanh[c*x^n])^2/x^3,x]
```

```
[Out] Integrate[(a + b*ArcTanh[c*x^n])^2/x^3, x]
```

Maple [A] time = 0.109, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\,\mathrm{Artanh}\,(cx^n)\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^n))^2/x^3,x)
```

```
[Out] int((a+b*arctanh(c*x^n))^2/x^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{b^2\log\left(-cx^n+1\right)^2}{8\,x^2}-\frac{a^2}{2\,x^2}-\int-\frac{\left(b^2cx^n-b^2\right)\log\left(cx^n+1\right)^2+4\left(abcx^n-ab\right)\log\left(cx^n+1\right)+\left(4\,ab+\left(b^2cn-4\,abc\right)x^n-2\right.}{4\left(cx^3x^n-x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))^2/x^3,x, algorithm="maxima")
```

```
[Out] -1/8*b^2*log(-c*x^n + 1)^2/x^2 - 1/2*a^2/x^2 - integrate(-1/4*((b^2*c*x^n -
 b^2)*log(c*x^n + 1)^2 + 4*(a*b*c*x^n - a*b)*log(c*x^n + 1) + (4*a*b + (b^2
*c*n - 4*a*b*c)*x^n - 2*(b^2*c*x^n - b^2)*log(c*x^n + 1))*log(-c*x^n + 1))/
(c*x^3*x^n - x^3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2\,\text{artanh}\left(cx^n\right)^2+2\,ab\,\text{artanh}\left(cx^n\right)+a^2}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))^2/x^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x^n)^2 + 2*a*b*arctanh(c*x^n) + a^2)/x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**n))**2/x**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(b\,\text{artanh}\left(cx^n\right)+a\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))^2/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^n) + a)^2/x^3, x)
```

3.236 $\int \frac{\tanh^{-1}(ax^n)}{x} \, dx$

Optimal. Leaf size=30

$$\frac{\text{PolyLog}(2, ax^n)}{2n} - \frac{\text{PolyLog}(2, -ax^n)}{2n}$$

```
[Out] -PolyLog[2, -(a*x^n)]/(2*n) + PolyLog[2, a*x^n]/(2*n)
```

Rubi [A] time = 0.0213019, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6095, 5912}

$$\frac{\text{PolyLog}(2, ax^n)}{2n} - \frac{\text{PolyLog}(2, -ax^n)}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x^n]/x,x]
```

```
[Out] -PolyLog[2, -(a*x^n)]/(2*n) + PolyLog[2, a*x^n]/(2*n)
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[
1/n, Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
 n}, x] && IGtQ[p, 0]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{\tanh^{-1}(ax^n)}{x} \, dx &= \frac{\text{Subst}\left(\int \frac{\tanh^{-1}(ax)}{x} \, dx, x, x^n\right)}{n} \\ &= -\frac{\text{Li}_2(-ax^n)}{2n} + \frac{\text{Li}_2(ax^n)}{2n} \end{aligned}$$

Mathematica [C] time = 0.0386013, size = 33, normalized size = 1.1

$$\frac{ax^n \text{HypergeometricPFQ}\left(\left\{\frac{1}{2}, \frac{1}{2}, 1\right\}, \left\{\frac{3}{2}, \frac{3}{2}\right\}, a^2 x^{2n}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x^n]/x,x]
```

```
[Out] (a*x^n*HypergeometricPFQ[{1/2, 1/2, 1}, {3/2, 3/2}, a^2*x^(2*n)])/n
```

Maple [B] time = 0.034, size = 61, normalized size = 2.

$$\frac{\ln\left(ax^{n}\right)\operatorname{Artanh}\left(ax^{n}\right)}{n}-\frac{\operatorname{dilog}\left(ax^{n}\right)}{2n}-\frac{\operatorname{dilog}\left(ax^{n}+1\right)}{2n}-\frac{\ln\left(ax^{n}\right)\ln\left(ax^{n}+1\right)}{2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x^n)/x,x)
```

```
[Out] 1/n*ln(a*x^n)*arctanh(a*x^n)-1/2/n*dilog(a*x^n)-1/2/n*dilog(a*x^n+1)-1/2/n*
ln(a*x^n)*ln(a*x^n+1)
```

Maxima [B] time = 1.26638, size = 198, normalized size = 6.6

$$-\frac{1}{2}an\left(\frac{\log\left(\frac{ax^{n}+1}{a}\right)}{an}-\frac{\log\left(\frac{ax^{n}-1}{a}\right)}{an}\right)\log\left(x\right)+\frac{1}{2}an\left(\frac{\log\left(ax^{n}+1\right)\log\left(x\right)-\log\left(ax^{n}-1\right)\log\left(x\right)}{an}-\frac{n\log\left(ax^{n}+1\right)\log\left(x\right)+}{an^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x^n)/x,x, algorithm="maxima")
```

```
[Out] -1/2*a*n*(log((a*x^n + 1)/a)/(a*n) - log((a*x^n - 1)/a)/(a*n))*log(x) + 1/2
*a*n*((log(a*x^n + 1)*log(x) - log(a*x^n - 1)*log(x))/(a*n) - (n*log(a*x^n
+ 1)*log(x) + dilog(-a*x^n))/(a*n^2) + (n*log(-a*x^n + 1)*log(x) + dilog(a*
x^n))/(a*n^2)) + arctanh(a*x^n)*log(x)
```

Fricas [B] time = 1.73074, size = 423, normalized size = 14.1

$$-\frac{n\log\left(a\cosh\left(n\log\left(x\right)\right)+a\sinh\left(n\log\left(x\right)\right)+1\right)\log\left(x\right)-n\log\left(-a\cosh\left(n\log\left(x\right)\right)-a\sinh\left(n\log\left(x\right)\right)+1\right)\log\left(x\right)-n\,\mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x^n)/x,x, algorithm="fricas")
```

```
[Out] -1/2*(n*log(a*cosh(n*log(x)) + a*sinh(n*log(x)) + 1)*log(x) - n*log(-a*cosh
(n*log(x)) - a*sinh(n*log(x)) + 1)*log(x) - n*log(x)*log(-(a*cosh(n*log(x))
 + a*sinh(n*log(x)) + 1)/(a*cosh(n*log(x)) + a*sinh(n*log(x)) - 1)) - dilog
(a*cosh(n*log(x)) + a*sinh(n*log(x))) + dilog(-a*cosh(n*log(x)) - a*sinh(n*
log(x))))/n
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{atanh}\left(ax^{n}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x**n)/x,x)
```

```
[Out] Integral(atanh(a*x**n)/x, x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{artanh}(ax^n)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x^n)/x,x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x^n)/x, x)
```

3.237 $\int \frac{\tanh^{-1}(ax^5)}{x}\,dx$

Optimal. Leaf size=24

$$\frac{1}{10}\text{PolyLog}\left(2,ax^5\right)-\frac{1}{10}\text{PolyLog}\left(2,-ax^5\right)$$

```
[Out] -PolyLog[2, -(a*x^5)]/10 + PolyLog[2, a*x^5]/10
```

Rubi [A] time = 0.0212865, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6095, 5912}

$$\frac{1}{10}\text{PolyLog}\left(2,ax^5\right)-\frac{1}{10}\text{PolyLog}\left(2,-ax^5\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x^5]/x,x]
```

```
[Out] -PolyLog[2, -(a*x^5)]/10 + PolyLog[2, a*x^5]/10
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[
1/n, Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
 n}, x] && IGtQ[p, 0]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}\left(ax^5\right)}{x}\,dx&=\frac{1}{5}\text{Subst}\left(\int \frac{\tanh^{-1}(ax)}{x}\,dx,x,x^5\right)\\&=-\frac{1}{10}\text{Li}_2\left(-ax^5\right)+\frac{\text{Li}_2\left(ax^5\right)}{10}\end{aligned}$$

Mathematica [A] time = 0.0112493, size = 22, normalized size = 0.92

$$\frac{1}{10}\left(\text{PolyLog}\left(2,ax^5\right)-\text{PolyLog}\left(2,-ax^5\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x^5]/x,x]
```

```
[Out] (-PolyLog[2, -(a*x^5)] + PolyLog[2, a*x^5])/10
```

Maple [C] time = 0.095, size = 85, normalized size = 3.5

$$\ln(x)\,\mathrm{Artanh}\left(a x^5\right)+\frac{1}{2}\sum_{\_R1=\mathrm{RootOf}\left(a\_Z^5-1\right)}\ln(x)\ln\left(\frac{\_R1-x}{\_R1}\right)+\mathrm{dilog}\left(\frac{\_R1-x}{\_R1}\right)-\frac{1}{2}\sum_{\_R1=\mathrm{RootOf}\left(a\_Z^5+1\right)}\ln(x)\ln\left(\frac{}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x^5)/x,x)
```

```
[Out] ln(x)*arctanh(a*x^5)+1/2*sum(ln(x)*ln((_R1-x)/_R1)+dilog((_R1-x)/_R1),_R1=R
ootOf(_Z^5*a-1))-1/2*sum(ln(x)*ln((_R1-x)/_R1)+dilog((_R1-x)/_R1),_R1=RootO
f(_Z^5*a+1))
```

Maxima [B] time = 0.989419, size = 140, normalized size = 5.83

$$-\frac{1}{2}a\left(\frac{\log\left(ax^5+1\right)}{a}-\frac{\log\left(ax^5-1\right)}{a}\right)\log(x)-\frac{1}{10}a\left(\frac{\log\left(ax^5-1\right)\log\left(ax^5\right)+\mathrm{Li}_2\left(-ax^5+1\right)}{a}-\frac{\log\left(ax^5+1\right)\log\left(-ax\right.}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x^5)/x,x, algorithm="maxima")
```

```
[Out] -1/2*a*(log(a*x^5 + 1)/a - log(a*x^5 - 1)/a)*log(x) - 1/10*a*((log(a*x^5 -
1)*log(a*x^5) + dilog(-a*x^5 + 1))/a - (log(a*x^5 + 1)*log(-a*x^5) + dilog(
a*x^5 + 1))/a) + arctanh(a*x^5)*log(x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\mathrm{artanh}\left(ax^5\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x^5)/x,x, algorithm="fricas")
```

```
[Out] integral(arctanh(a*x^5)/x, x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: KeyError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x**5)/x,x)
```

```
[Out] Exception raised: KeyError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}\left(ax^5\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x^5)/x,x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x^5)/x, x)
```

3.238 $\int \tanh^{-1}\left(\frac{1}{x}\right)\,dx$

Optimal. Leaf size=19

$$\frac{1}{2}\log\left(1-x^2\right)+x\tanh^{-1}\left(\frac{1}{x}\right)$$

```
[Out] x*ArcTanh[x^(-1)] + Log[1 - x^2]/2
```

Rubi [A] time = 0.006243, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {6091, 263, 260}

$$\frac{1}{2}\log\left(1-x^2\right)+x\tanh^{-1}\left(\frac{1}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[x^(-1)],x]
```

```
[Out] x*ArcTanh[x^(-1)] + Log[1 - x^2]/2
```

Rule 6091

```
Int[ArcTanh[(c_.)*(x_)^(n_)], x_Symbol] :> Simp[x*ArcTanh[c*x^n], x] - Dist
[c*n, Int[x^n/(1 - c^2*x^(2*n)), x], x] /; FreeQ[{c, n}, x]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \tanh^{-1}\left(\frac{1}{x}\right)dx &= x\tanh^{-1}\left(\frac{1}{x}\right)+\int \frac{1}{\left(1-\frac{1}{x^2}\right)x}\,dx\\ &= x\tanh^{-1}\left(\frac{1}{x}\right)+\int \frac{x}{-1+x^2}\,dx\\ &= x\tanh^{-1}\left(\frac{1}{x}\right)+\frac{1}{2}\log\left(1-x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0020759, size = 17, normalized size = 0.89

$$\frac{1}{2}\log\left(x^2-1\right)+x\tanh^{-1}\left(\frac{1}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[x^(-1)],x]
```

```
[Out] x*ArcTanh[x^(-1)] + Log[-1 + x^2]/2
```

Maple [A] time = 0.043, size = 30, normalized size = 1.6

$$x\text{Artanh}\left(x^{-1}\right)+\frac{\ln\left(x^{-1}-1\right)}{2}-\ln\left(x^{-1}\right)+\frac{\ln\left(x^{-1}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(1/x),x)
```

```
[Out] x*arctanh(1/x)+1/2*ln(1/x-1)-ln(1/x)+1/2*ln(1/x+1)
```

Maxima [A] time = 0.954859, size = 20, normalized size = 1.05

$$x\,\text{artanh}\left(\frac{1}{x}\right)+\frac{1}{2}\,\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1/x),x, algorithm="maxima")
```

```
[Out] x*arctanh(1/x) + 1/2*log(x^2 - 1)
```

Fricas [A] time = 1.60687, size = 63, normalized size = 3.32

$$\frac{1}{2}\,x\log\left(\frac{x+1}{x-1}\right)+\frac{1}{2}\,\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1/x),x, algorithm="fricas")
```

```
[Out] 1/2*x*log((x + 1)/(x - 1)) + 1/2*log(x^2 - 1)
```

Sympy [A] time = 0.258037, size = 15, normalized size = 0.79

$$x\,\text{atanh}\left(\frac{1}{x}\right)+\log\left(x+1\right)-\text{atanh}\left(\frac{1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(1/x),x)
```

```
[Out] x*atanh(1/x) + log(x + 1) - atanh(1/x)
```

Giac [A] time = 1.17986, size = 38, normalized size = 2.

$$\frac{1}{2}\,x\log\left(-\frac{\frac{1}{x}+1}{\frac{1}{x}-1}\right)+\frac{1}{2}\,\log\left(\left|x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1/x),x, algorithm="giac")
```

```
[Out] 1/2*x*log(-(1/x + 1)/(1/x - 1)) + 1/2*log(abs(x^2 - 1))
```

3.239 $\int (dx)^m \left(a+b\tanh^{-1}\left(cx^n\right)\right)^3 dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left((dx)^m \left(a+b\tanh^{-1}\left(cx^n\right)\right)^3,x\right)$$

```
[Out] Unintegrable[(d*x)^m*(a + b*ArcTanh[c*x^n])^3, x]
```

Rubi [A] time = 0.0235968, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (dx)^m \left(a+b\tanh^{-1}\left(cx^n\right)\right)^3 dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m*(a + b*ArcTanh[c*x^n])^3,x]
```

```
[Out] Defer[Int][(d*x)^m*(a + b*ArcTanh[c*x^n])^3, x]
```

Rubi steps

$$\int (dx)^m \left(a+b\tanh^{-1}\left(cx^n\right)\right)^3 dx=\int (dx)^m \left(a+b\tanh^{-1}\left(cx^n\right)\right)^3 dx$$

Mathematica [A] time = 7.97124, size = 0, normalized size = 0.

$$\int (dx)^m \left(a+b\tanh^{-1}\left(cx^n\right)\right)^3 dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m*(a + b*ArcTanh[c*x^n])^3,x]
```

```
[Out] Integrate[(d*x)^m*(a + b*ArcTanh[c*x^n])^3, x]
```

Maple [A] time = 0.232, size = 0, normalized size = 0.

$$\int (dx)^m \left(a+b\text{Artanh}\left(cx^n\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctanh(c*x^n))^3,x)
```

```
[Out] int((d*x)^m*(a+b*arctanh(c*x^n))^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^n))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((dx)^m\, b^3\, \mathrm{artanh}\left(cx^n\right)^3 + 3\, (dx)^m\, ab^2\, \mathrm{artanh}\left(cx^n\right)^2 + 3\, (dx)^m\, a^2 b\, \mathrm{artanh}\left(cx^n\right) + (dx)^m\, a^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^n))^3,x, algorithm="fricas")
```

```
[Out] integral((d*x)^m*b^3*arctanh(c*x^n)^3 + 3*(d*x)^m*a*b^2*arctanh(c*x^n)^2 +
3*(d*x)^m*a^2*b*arctanh(c*x^n) + (d*x)^m*a^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atanh(c*x**n))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (b\, \mathrm{artanh}\left(cx^n\right) + a)^3\, (dx)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^n))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^n) + a)^3*(d*x)^m, x)
```

3.240 $\int (dx)^m \left(a+b\tanh^{-1}(cx^n)\right)^2 dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left((dx)^m \left(a+b\tanh^{-1}(cx^n)\right)^2,x\right)$$

```
[Out] Unintegrable[(d*x)^m*(a + b*ArcTanh[c*x^n])^2, x]
```

Rubi [A] time = 0.0241501, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (dx)^m \left(a+b\tanh^{-1}(cx^n)\right)^2 dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m*(a + b*ArcTanh[c*x^n])^2,x]
```

```
[Out] Defer[Int][(d*x)^m*(a + b*ArcTanh[c*x^n])^2, x]
```

Rubi steps

$$\int (dx)^m \left(a+b\tanh^{-1}(cx^n)\right)^2 dx = \int (dx)^m \left(a+b\tanh^{-1}(cx^n)\right)^2 dx$$

Mathematica [A] time = 5.79071, size = 0, normalized size = 0.

$$\int (dx)^m \left(a+b\tanh^{-1}(cx^n)\right)^2 dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m*(a + b*ArcTanh[c*x^n])^2,x]
```

```
[Out] Integrate[(d*x)^m*(a + b*ArcTanh[c*x^n])^2, x]
```

Maple [A] time = 0.214, size = 0, normalized size = 0.

$$\int (dx)^m (a+b\text{Artanh}(cx^n))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctanh(c*x^n))^2,x)
```

```
[Out] int((d*x)^m*(a+b*arctanh(c*x^n))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^n))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((dx)^m b^2 \operatorname{artanh}(cx^n)^2 + 2\,(dx)^m ab \operatorname{artanh}(cx^n) + (dx)^m a^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^n))^2,x, algorithm="fricas")
```

```
[Out] integral((d*x)^m*b^2*arctanh(c*x^n)^2 + 2*(d*x)^m*a*b*arctanh(c*x^n) + (d*x
)^m*a^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atanh(c*x**n))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{artanh}(cx^n) + a)^2 (dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^n))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^n) + a)^2*(d*x)^m, x)
```

3.241 $\int (dx)^m \left(a+b\tanh^{-1}(cx^n)\right)\,dx$

Optimal. Leaf size=84

$$\frac{x(dx)^m\left(a+b\tanh^{-1}(cx^n)\right)}{m+1}-\frac{bcnx^{n+1}(dx)^m\text{Hypergeometric2F1}\left(1,\frac{m+n+1}{2n},\frac{m+3n+1}{2n},c^2x^{2n}\right)}{(m+1)(m+n+1)}$$

```
[Out] (x*(d*x)^m*(a + b*ArcTanh[c*x^n]))/(1 + m) - (b*c*n*x^(1 + n)*(d*x)^m*Hyper
geometric2F1[1, (1 + m + n)/(2*n), (1 + m + 3*n)/(2*n), c^2*x^(2*n)])/((1 +
 m)*(1 + m + n))
```

Rubi [A] time = 0.0441134, antiderivative size = 88, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {6097, 20, 364}

$$\frac{(dx)^{m+1}\left(a+b\tanh^{-1}(cx^n)\right)}{d(m+1)}-\frac{bcnx^{n+1}(dx)^m\,{}_2F_1\left(1,\frac{m+n+1}{2n};\frac{m+3n+1}{2n};c^2x^{2n}\right)}{(m+1)(m+n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*ArcTanh[c*x^n]),x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*ArcTanh[c*x^n]))/(d*(1 + m)) - (b*c*n*x^(1 + n)*(d*x)
^m*Hypergeometric2F1[1, (1 + m + n)/(2*n), (1 + m + 3*n)/(2*n), c^2*x^(2*n)
])/((1 + m)*(1 + m + n))
```

Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int (dx)^m \left(a+b\tanh^{-1}(cx^n)\right) dx = \frac{(dx)^{1+m}\left(a+b\tanh^{-1}(cx^n)\right)}{d(1+m)} - \frac{(bcn)\int \frac{x^{-1+n}(dx)^{1+m}}{1-c^2x^{2n}}\,dx}{d(1+m)}$$
$$= \frac{(dx)^{1+m}\left(a+b\tanh^{-1}(cx^n)\right)}{d(1+m)} - \frac{(bcnx^{-m}(dx)^m)\int \frac{x^{m+n}}{1-c^2x^{2n}}\,dx}{1+m}$$
$$= \frac{(dx)^{1+m}\left(a+b\tanh^{-1}(cx^n)\right)}{d(1+m)} - \frac{bcnx^{1+n}(dx)^m\,{}_2F_1\left(1,\frac{1+m+n}{2n};\frac{1+m+3n}{2n};c^2x^{2n}\right)}{(1+m)(1+m+n)}$$

Mathematica [A] time = 0.0870418, size = 77, normalized size = 0.92

$$\frac{x(dx)^m\left((m+n+1)\left(a+b\tanh^{-1}(cx^n)\right)-bcnx^n\text{Hypergeometric2F1}\left(1,\frac{m+n+1}{2n},\frac{m+3n+1}{2n},c^2x^{2n}\right)\right)}{(m+1)(m+n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a + b*ArcTanh[c*x^n]),x]
```

```
[Out] (x*(d*x)^m*((1 + m + n)*(a + b*ArcTanh[c*x^n]) - b*c*n*x^n*Hypergeometric2F
1[1, (1 + m + n)/(2*n), (1 + m + 3*n)/(2*n), c^2*x^(2*n)]))/((1 + m)*(1 + m
 + n))
```

Maple [F] time = 0.279, size = 0, normalized size = 0.

$$\int (dx)^m \left(a+b\text{Artanh}(cx^n)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctanh(c*x^n)),x)
```

```
[Out] int((d*x)^m*(a+b*arctanh(c*x^n)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^n)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((dx)^m b\,\text{artanh}(cx^n)+(dx)^m a, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^n)),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m*b*arctanh(c*x^n) + (d*x)^m*a, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atanh(c*x**n)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{artanh}(cx^n) + a)(dx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^n)),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^n) + a)*(d*x)^m, x)
```

3.242 $\int \frac{(dx)^m}{a+b\tanh^{-1}(cx^n)}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{(dx)^m}{a+b\tanh^{-1}(cx^n)},x\right)$$

```
[Out] Unintegrable[(d*x)^m/(a + b*ArcTanh[c*x^n]), x]
```

Rubi [A] time = 0.0269594, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(dx)^m}{a+b\tanh^{-1}(cx^n)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m/(a + b*ArcTanh[c*x^n]),x]
```

```
[Out] Defer[Int][(d*x)^m/(a + b*ArcTanh[c*x^n]), x]
```

Rubi steps

$$\int \frac{(dx)^m}{a+b\tanh^{-1}(cx^n)}\,dx = \int \frac{(dx)^m}{a+b\tanh^{-1}(cx^n)}\,dx$$

Mathematica [A] time = 0.372964, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{a+b\tanh^{-1}(cx^n)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m/(a + b*ArcTanh[c*x^n]),x]
```

```
[Out] Integrate[(d*x)^m/(a + b*ArcTanh[c*x^n]), x]
```

Maple [A] time = 0.058, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{a+b\text{Artanh}(cx^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a+b*arctanh(c*x^n)),x)
```

```
[Out] int((d*x)^m/(a+b*arctanh(c*x^n)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{b\,\text{artanh}\,(cx^n) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c*x^n)),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/(b*arctanh(c*x^n) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx)^m}{b\,\text{artanh}\,(cx^n) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c*x^n)),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(b*arctanh(c*x^n) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*atanh(c*x**n)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{b\,\text{artanh}\,(cx^n) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c*x^n)),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b*arctanh(c*x^n) + a), x)
```

3.243 $\int \frac{(dx)^m}{\left(a+b\tanh^{-1}(cx^n)\right)^2}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{(dx)^m}{\left(a+b\tanh^{-1}(cx^n)\right)^2},x\right)$$

```
[Out] Unintegrable[(d*x)^m/(a + b*ArcTanh[c*x^n])^2, x]
```

Rubi [A] time = 0.0264206, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(dx)^m}{\left(a+b\tanh^{-1}(cx^n)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m/(a + b*ArcTanh[c*x^n])^2,x]

[Out] Defer[Int][(d*x)^m/(a + b*ArcTanh[c*x^n])^2, x]
```

Rubi steps

$$\int \frac{(dx)^m}{\left(a+b\tanh^{-1}(cx^n)\right)^2}\,dx=\int \frac{(dx)^m}{\left(a+b\tanh^{-1}(cx^n)\right)^2}\,dx$$

Mathematica [A] time = 1.7533, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left(a+b\tanh^{-1}(cx^n)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m/(a + b*ArcTanh[c*x^n])^2,x]

[Out] Integrate[(d*x)^m/(a + b*ArcTanh[c*x^n])^2, x]
```

Maple [A] time = 0.275, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{(a+b\operatorname{Artanh}(cx^n))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a+b*arctanh(c*x^n))^2,x)
```

```
[Out] int((d*x)^m/(a+b*arctanh(c*x^n))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{2\left(c^2d^mxe^{(m\log(x)+2n\log(x))}-d^mxx^m\right)}{b^2cnx^n\log\left(cx^n+1\right)-b^2cnx^n\log\left(-cx^n+1\right)+2\,abcnx^n}+\int-\frac{2\left(c^2d^m(m+n+1)e^{(m\log(x)+2n\log(x))}-d^m(m-n+1)x^m\right)}{b^2cnx^n\log\left(cx^n+1\right)-b^2cnx^n\log\left(-cx^n+1\right)+2\,abcnx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c*x^n))^2,x, algorithm="maxima")
```

```
[Out] 2*(c^2*d^m*x*e^(m*log(x) + 2*n*log(x)) - d^m*x*x^m)/(b^2*c*n*x^n*log(c*x^n
+ 1) - b^2*c*n*x^n*log(-c*x^n + 1) + 2*a*b*c*n*x^n) + integrate(-2*(c^2*d^m
*(m + n + 1)*e^(m*log(x) + 2*n*log(x)) - d^m*(m - n + 1)*x^m)/(b^2*c*n*x^n*
log(c*x^n + 1) - b^2*c*n*x^n*log(-c*x^n + 1) + 2*a*b*c*n*x^n), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(dx)^m}{b^2\,\mathrm{artanh}\left(cx^n\right)^2+2\,ab\,\mathrm{artanh}\left(cx^n\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c*x^n))^2,x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(b^2*arctanh(c*x^n)^2 + 2*a*b*arctanh(c*x^n) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*atanh(c*x**n))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{(dx)^m}{\left(b\,\mathrm{artanh}\left(cx^n\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c*x^n))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b*arctanh(c*x^n) + a)^2, x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
```

```
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
```

```
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
```

```
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
```

```
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
```

```
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
```

```
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len
   (flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                         #since this estimate of leaf count is bit lower than
```

```
                     #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

```

```
def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
```

```
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): # is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, " leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=", expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```